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Myopic loss aversion, reference point, and money illusion

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We use the portfolio selection model presented in He and Zhou [*Manage. Sci.*, 2011, **57**, 315–331] and the NYSE equity and US treasury bond returns for the period 1926–1990 to revisit Benartzi and Thaler's myopic loss aversion theory. Through an extensive empirical study, we find that in addition to the agent's loss aversion and evaluation period, his reference point also has a significant effect on optimal asset allocation. We demonstrate that the agent's optimal allocation to equities is consistent with market observation when he has reasonable values of degree of loss aversion, evaluation period and reference point. We also find that the optimal allocation to equities is sensitive to these parameters. We then examine the implications of money illusion for asset allocation. Finally, we extend the model to a dynamic setting.

Keywords: Myopic loss aversion; Cumulative prospect theory (CPT); Evaluation period; Reference point; Money illusion; Portfolio selection

JEL Classification: D03, G11

1. Introduction

In recent years, many researchers have been devoted to the area of behavioural finance in which various principles of behavioural psychology are employed to explain numerous puzzles in portfolio selection and asset pricing. One of the most notable works in this area is the myopic loss aversion theory presented by Benartzi and Thaler (1995). This theory was built upon the principles of two of the most important theories in behavioural finance: mental accounting and cumulative prospect theory (CPT) (CPT; Tversky and Kahneman 1992), and Benartzi and Thaler (1995) employed this theory to explain the equity premium puzzle.¶

The work of Benartzi and Thaler (1995) has been followed and generalized by many subsequent works, such as the studies of Barberis *et al.* (2001) and Barberis and Huang (2001). Although these works show that CPT and other behavioural theories are promising in explaining some of the puzzles in asset pricing, particularly the equity premium puzzle, until recently an extensive study of portfolio choice, which is a foundation for asset pricing in behavioural finance, had not been carried out. As a result, the studies of behavioural asset pricing either lack a comprehensive portfolio selection model or assume a representative agent in order to avoid having to solve portfolio selection problems. Benartzi and Thaler (1995), for example, performed portfolio selection analysis not based on analytical or numerical argument. Indeed, they computed 'the prospective utility of each portfolio mix between 100% bonds and 100% stocks, in 10 % increments' (Benartzi and Thaler 1995, p. 84) and presented the 'optimal' allocation graphically.‖

Recently, some progress has been made in the area of behavioural portfolio selection. De Giorgi *et al.* (2011), Levy and Levy (2004), Del Vigna (2013) studied single-period portfolio selection problems from the standpoint of CPT, assuming the returns of risky assets to follow normal distributions. Barberis and Huang (2008b) considered similar problems, but allowing for an additional risky asset that follows a Bernoulli distribution. Pirvu and Schulze (2012) assumed the risky asset

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¶The puzzle was first articulated by Mehra and Prescott (1985), who found the historical equity premium of the S&P 500 index for the period 1889–1979 to be 6.18%, much higher than could be accounted for by a standard consumption-based utility maximization model. Subsequent empirical studies have confirmed that the equity premium puzzle is robust across different time periods and different countries; see for instance Fama and French (2002), Lettau *et al.* (2008), Campbell (2003), and Mehra and Prescott (2003) for the recent developments on this topic.

‖It is well known in optimization that such a graphical solution may significantly deviate from the true optimum if the objective function is not well behaved.

returns to follow the family of elliptically symmetric distributions. Hens and Bachmann (2011) considered the case in which risky asset returns follow the Bernoulli distribution. Zakamouline and Koekebakker (2009) connected single-period portfolio selection problems of CPT agents with the mean-variance analysis. Bernard and Ghossoub (2010) and He and Zhou (2011) studied single period single risky asset portfolio selection problems with the risky asset return following a general distribution. On the other hand, dynamic multi-period CPT portfolio choice models have been investigated by Gomes (2005) and by Barberis and Xiong (2009), whereas their continuous-time counterparts have been considered by Berkelaar *et al.* (2004) and by Jin and Zhou (2008, 2010).

The main contribution of our work is to use a comprehensive CPT portfolio choice model to show both the power and limitation of the myopic loss aversion theory and explore beyond this theory. More precisely, we follow the single-period CPT portfolio selection model used in He and Zhou (2011) and use the data of NYSE equity and US treasury bond returns for the period 1926–1990 to empirically investigate the myopic loss aversion theory.† In the model, an agent invests once at the beginning of a period, choosing a combination of a risky stock and a risk-free account, in order to maximize the preference value of his wealth at the end of the period. The preference is represented by CPT. First, we find that given the historical equity premium, our model does not exclude the possibility of the agent taking infinite leverage on the stock when allowed. However, infinite leverage is rarely observed in reality, which in turn suggests a limitation of the single-period myopic loss aversion theory. We then revisit the myopic loss theory proposed by Benartzi and Thaler (1995) assuming a piecewise linear utility function and confirm that loss aversion and mental accounting contribute to an agent being unwilling to bear the risk associated with holding equities. In addition, we find that the reference point, which was *not* considered in Benartzi and Thaler (1995) as an important factor in deciding asset allocation, also has a significant effect.‡ The optimal allocation to equities in our model is consistent with the historical equity premium data when reasonable values of the loss aversion degree, evaluation period and reference point are chosen, confirming the power of the myopic loss aversion theory. However, the optimal allocation is sensitive to these parameters and this sensitivity may explain the high equity volatility in multi-period models because a small shift in the representative agent's loss aversion, evaluation period or reference point over time may cause a large movement in his equity holdings, thus leading to a dramatic change of equity prices in equilibrium.

Our numerical study is based on data-sets from Siegel (1992), which are available in both nominal dollar and real dollar terms. Because returns are usually reported by the media and discussed among investors in nominal dollars, it is likely that investors use nominal returns in their mental accounts. This phenomenon has been documented in the literature under the heading *money illusion* (see e.g. Shafir *et al.* 1997). While most of our experiments will be based on nominal figures, toward the end of this paper we will use real dollar figures to test the effect of money illusion on portfolio selection. We find that when investors set initial wealth to be their reference points, money illusion makes the investors who use nominal returns invest substantially more in equities than those who use real returns.

In the last part of the paper, we demonstrate that the single-period portfolio selection model can be embedded into a consumption-based dynamic model. The need for consumption prevents the agent from taking infinite leverage; so the dynamic model overcomes one of the limitations of the single-period myopic loss aversion theory. We examine the effect of mental accounting on asset allocation in the dynamic setting and draw the same conclusions as in the single-period model. Our consumption-based dynamic model is related to the studies of Barberis and Huang (2009, 2008a), Barberis *et al.* (2006), and De Giorgi and Legg (2012), in which portfolio choice problems featuring narrow framing, loss aversion and recursive utility were formulated and solved. However, compared to ours, their studies did not investigate the effect of the evaluation period and overlooked the role the reference point plays in asset allocation.

The remainder of this paper is organized as follows: section 2 briefly reviews the myopic loss aversion theory proposed by Benartzi and Thaler (1995). In section 3 we briefly recall the CPT of Tversky and Kahneman (1992) that underlies our model. Section 4 is devoted to model description, solutions, model parameters and data. In section 5 we test whether the agent will take infinite leverage on the stock before we investigate the optimal allocation to the stock in section 6. Section 7 is devoted to studying the effect of money illusion. In section 8, we present the dynamic model. Finally, section 9 concludes the paper.

2. Myopic loss aversion

Benartzi and Thaler (1995) proposed the *myopic loss aversion* theory based on two concepts in behavioural psychology: *loss aversion* and *mental accounting*. Loss aversion is the phenomenon by which the disutility of one unit loss is significantly larger than the utility of one unit gain (see e.g. Tversky and Kahneman 1991). Benartzi and Thaler (1995) modelled loss aversion within the framework of CPT, which is a central piece of behavioural finance. Mental accounting, on the other hand, is 'the set of cognitive operations used by individuals to organize, evaluate, and keep track of financial activities' (Thaler 1999, p. 183). For instance, individuals may have separate mental

†We choose this period for our numerical study so as to compare our results with Benartzi and Thaler (1995), where the same data-set was used.

‡The importance of the reference point in decision-making has been recognized in the literature and some studies have been done in the determination of the reference point; see for instance Kőszegi and Rabin (2006) and De Giorgi and Post (2011). However, the impact of reference point is still largely overlooked in the portfolio selection literature as many works in this field assume the reference point to be simply the risk-free payoff. On the other hand, the reference point in CPT is related to the habit formation theory (Abel 1990; Constantinides 1990; Campbell and Cochrane 1999; Ravina 2007). The former is different from the latter in that the reference point distinguishes gains and losses and individuals have different risk attitudes towards gains and losses and are loss averse. CPT, with the reference point involved, has become a popular model in decision-making and financial applications; see for instance the survey by Barberis and Thaler (2003).

accounts for different categories of expenses, such as living expenses, entertainment expenses, etc. Mental accounting is a result of limited cognitive resources that force individuals to break large tasks down into smaller pieces and to ignore the correlations between them. It has been observed in the literature that mental accounting is a robust and widespread phenomenon and has many implications in real life (see Thaler 1999, and the references therein). Benartzi and Thaler (1995) argued that, as a result of the mental accounting, investors tend to evaluate the performance of their investments in a short period known as *evaluation period*. Therefore, mental accounting renders investors *myopic*, in sharp contrast to the classical assumption that investors have the objective of maximizing long-term consumption.

Benartzi and Thaler (1995) presented the concept of myopic loss aversion as an explanation for the equity premium puzzle. In particular, they showed that when the degree of loss aversion was 2.25 and the evaluation period was one year, the optimal allocation to stocks was roughly 50% of the initial wealth given the historical equity premium value. Because it had been observed that the most frequent allocation between stocks and bonds was 50–50 for both institutions and individuals, the authors concluded that myopic loss aversion was consistent with the historical equity premium. They argued further that not only individual investors but also institutional investors exhibited myopic loss aversion because institutions were ultimately operated by human beings.

Since Benartzi and Thaler (1995), many researchers have confirmed the phenomenon of myopic loss aversion using both experimental and real data. Relevant studies in this regard include Thaler *et al.* (1997), Gneezy and Potters (1997), and Benartzi and Thaler (1999). All of their results reinforce the argument in Benartzi and Thaler (1995) that because of mental accounting investors tend to evaluate their investment performance in a short period and that mental accounting eventually makes stocks less favourable, resulting in a larger equity premium.

3. Cumulative prospect theory

The most prominent approach to modelling an individual's preference is that of expected utility theory (EUT). In this theory, individuals evaluate risky prospects by computing their expected utilities. Von Neumann and Morgenstern (1947) showed that EUT can be axiomatized from several normative principles to which individuals are supposed to conform. Therefore, EUT is considered to be a model of rational preference.

Laboratory evidence, however, shows that individuals frequently violate the principles underlying EUT. First, individuals evaluate wealth or consumption in comparison to certain benchmarks rather than in their absolute values (i.e. reference points define gains and losses). Secondly, individuals behave differently with respect to gains and losses. In particular, they are risk averse and risk seeking, respectively, regarding gains and losses that occur with moderate or high probabilities. Moreover, they are distinctly more sensitive to losses than to gains, a behaviour known as loss aversion. Thirdly, individuals tend to overweight the largest and smallest payoffs when they occur with small probabilities.

Based on these tendencies, Kahneman and Tversky (1979) proposed prospect theory (PT), which was later elaborated by Tversky and Kahneman (1992) into CPT. In CPT, a random prospect X is evaluated according to its *CPT value*, which is defined as

$$\begin{aligned} V(X) := &\int_B^{+\infty} u_+(x-B)d[-w_+(1-F(x))] \\ &- \int_{-\infty}^{B} u_-(B-x)d[w_-(F(x))], \end{aligned} \tag{1}$$

where $F(\cdot)$ is the cumulative distribution function (CDF) of X. In (1), B models the reference point that divides X into the gain part $\max(X-B, 0)$ and the loss part $\min(X-B, 0)$. The *utility function* $u_+(\cdot)$ and *disutility function* $u_-(\cdot)$ are concave, increasing and satisfy $u_\pm(0) = 0$.† The overall utility function $u(\cdot)$, defined by $u(x) = u_+(x), x \geq 0$ and $u(x) = -u_-(x), x \leq 0$, is therefore *S-shaped*. The *probability weighting functions* $w_\pm(\cdot)$ are mappings from [0, 1] to [0, 1] and are *reversed S-shaped*. The particular shapes of the utility function and probability weighting functions in CPT are consistent with the aforementioned evidence regarding individual behaviuor.

In this paper, we consider the following functional forms of the (dis)utility functions and probability weighting functions:

$$u_+(x) = x^\alpha, \quad u_-(x) = kx^\alpha, \quad w_\pm(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{1/\delta}}, \tag{2}$$

where $k > 1$ is the *loss aversion degree*, $\alpha \in (0, 1]$ is the curvature parameter of the (dis)utility functions for both gains and losses and $\delta \in (0, 1)$ is the shape parameter of the probability weighting functions for gains and losses.‡

4. Model and data

4.1. *Portfolio selection model*

We follow here the portfolio selection model employed in He and Zhou (2011). Consider a market consisting of one risky asset (stock) and one risk-free account and an agent with an investment planning horizon from date 0 to date T. The risk-free gross return over this period is a deterministic quantity, $r(T)$ (i.e. \$1 invested in the risk-free account returns \$ $r(T)$ at T). The stock excess return, $R(T) - r(T)$, is a random variable following a CDF, For simplicity, we assume that shorting is not allowed in this market.§ For now, we assume that there is no restriction on the levels of stock position and leverage. It follows from the no-arbitrage rule that

$$0 < F_T(0) \equiv P(R(T) \leq r(T)) < 1. \tag{3}$$

†These are called *value functions* in the Kahneman–Tversky terminology. In this paper, however, we use the term *utility function* to distinguish this function from the *CPT value function* defined below.

‡Tversky and Kahneman (1992) adopted these forms but with different curvature parameters and shape parameters for (dis)utility functions and for probability weighting functions for gains and losses. However, their calibration results from experimental data showed that there was no significant difference between the parameters for gains and those for losses. Similar evidence can be found in Abdellaoui *et al.* (2007) and in the references therein.

§Proposition 6 in He and Zhou (2011) provides two sufficient conditions under which shorting will not happen even if it is allowed.

An agent who is initially endowed with an amount, $\widetilde{x}_0$, invests once at $t = 0$ to maximize the CPT value of his terminal wealth at $t = T$. He has a reference point, denoted B, in his terminal wealth to distinguish gains from losses. Suppose an amount, θ, is invested in the stock and the remainder in the risk-free account and set $x_0 = r(T)\widetilde{x}_0 - B$. This quantity, x_0, is the deviation of the reference point from the risk-free payoff. Then, the terminal wealth is

$$X(x_0, \theta, T) = x_0 + B + [R(T) - r(T)]\theta. \tag{4}$$

Now we evaluate the CPT value of (4) by applying (1), leading to a function of θ, called the *CPT value function*, which is denoted by $U(\theta)$. When $\theta = 0$,

$$U(0) = \begin{cases} u_+(x_0), & \text{if } x_0 \geq 0, \\ -u_-(-x_0), & \text{if } x_0 < 0. \end{cases} \tag{5}$$

When $\theta > 0$, by changing variables, one obtains from (1) that

$$\begin{aligned} U(\theta) = &\int_{-x_0/\theta}^{+\infty} u_+(\theta t + x_0) d[-w_+(1 - F_T(t))] \\ &- \int_{-\infty}^{-x_0/\theta} u_-(-\theta t - x_0) d[w_-(F_T(t))]. \end{aligned} \tag{6}$$

To make the CPT value function well-defined, some minimal conditions on the probability weighting functions and on the CDF of the stock excess return must be satisfied, such as Assumption 3 in He and Zhou (2011) and Assumptions 1 and 5 in Barberis and Huang (2008b).† In the following numerical analysis, these conditions are indeed satisfied with the parameter values that we use.

The CPT portfolio choice model is:

$$\max_{\theta \geq 0} \; U(\theta). \tag{P}$$

This problem was analysed in He and Zhou (2011), and the following numerical analysis is based on the results obtained therein. For the reader's convenience, we reproduce the relevant results here.

Define

$$k_0 := \frac{\int_0^{+\infty} t^\alpha d[-w_+(1 - F_T(t))]}{\int_{-\infty}^0 (-t)^\alpha d[w_-(F_T(t))]}. \tag{7}$$

THEOREM 4.1 *If $k > k_0$, then there exists an optimal solution, θ^*, of (P) and $U(\theta^*) < \infty$. If $k < k_0$, then $\lim_{\theta \to \infty} U(\theta) = +\infty$, in which case the agent will take infinite leverage on the stock.*

Proof See Lemma 1, Theorem 2, and Corollary 1 in He and Zhou (2011). □

THEOREM 4.2 *Assume $\alpha = 1$ and*

$$k > \max \left\{ k_0, \sup_{p \in (0,1)} \frac{w'_+(1-p)}{w'_-(p)} \right\}.$$

We have the following conclusions:

(i) *$U(\cdot)$ is strictly concave on $[0, \infty)$ and satisfies $\lim_{\theta \to \infty} U(\theta) = -\infty$.*

(ii) *Suppose $x_0 \leq 0$. If*

$$\lambda^- := \int_{-\infty}^{+\infty} t d[w_-(F_T(t))] \leq 0, \tag{8}$$

then the unique optimal solution is $\theta^* = 0$. If $\lambda^- > 0$, then the function

$$\begin{aligned} h(v) := &\int_v^{+\infty} t d[-w_+(1 - F_T(t))] \\ &+ k \int_{-\infty}^{v} t d[w_-(F_T(t))], \quad v \in \mathbb{R} \end{aligned} \tag{9}$$

admits a unique root, $v_+^*(k, T)$, on $(0, +\infty)$, and the unique optimal solution is

$$\theta^* = \frac{-x_0}{v_+^*(k, T)}. \tag{10}$$

(iii) *Suppose $x_0 > 0$. If*

$$\lambda^+ := \int_{-\infty}^{+\infty} t d[-w_+(1 - F_T(t))] \leq 0, \tag{11}$$

then the unique optimal solution is $\theta^* = 0$. If $\lambda^+ > 0$, then $h(\cdot)$ admits a unique root, $v_-^*(k, T)$, on $(-\infty, 0)$, and the unique optimal solution is

$$\theta^* = \frac{-x_0}{v_-^*(k, T)}. \tag{12}$$

Proof By equation (26) in He and Zhou (2011), $U(\cdot)$ is strictly concave on $[0, +\infty)$. In particular, Assumption 4 in He and Zhou (2011) is satisfied. On the other hand, because $k > k_0$, by Theorem 2 in He and Zhou (2011), $\lim_{\theta \to \infty} U(\theta) = -\infty$.

Now consider the case in which $x_0 < 0$. By Proposition 3 in He and Zhou (2011), $U'(0+) = u'_-(-x_0)\lambda^-$. Thus, if $\lambda^- \leq 0$, $U'(0+) \leq 0$ and the unique optimal solution is $\theta^* = 0$ due to the concavity of $U(\cdot)$. If $\lambda^- > 0$, the optimal solution follows from Corollary 3 in He and Zhou (2011).

The case in which $x_0 < 0$ can be treated similarly. □

The critical value k_0 is determined by parts of both the agent preference set (the curvature parameter α and the probability weighting functions $w_\pm$) and the market. Theorem 4.1 shows that loss aversion degree relative to this critical level determines whether the agent will invest in the stock as much as possible or will strike a balance between the stock and the risk-free account. See He and Zhou (2011) on the importance of this critical value and see section 5 below, where we compute k_0 numerically and discuss its implications. Theorem 4.2 gives the optimal allocation to the stock when the (dis)utility functions are linear. This result will place the myopic loss aversion theory of Benartzi and Thaler (1995) on an analytically rigorous ground. Section 6 is devoted to a full discussion on it.

4.2. Data and model parameters

Next, we report the model parameters and the data that we use in the numerical analysis to follow. The curvature parameter of the (dis)utility functions, α and the shape parameter of

†Assumption 3 in He and Zhou (2011) requires that $F_T(\cdot)$ has a density function $f_T(\cdot)$ and there exists $\epsilon_0 > 0$ such that $w'_\pm(1 - F_T(x))f_T(x) = O(|x|^{-2-\epsilon_0})$ and $w'_\pm(F_T(x))f_T(x) = O(|x|^{-2-\epsilon_0})$ for $|x|$ sufficiently large and $0 < F_T(x) < 1$. Assumptions 1 and 5 in Barberis and Huang (2008b) assume that $F_T(\cdot)$ has finite second-order moment and $u(\cdot)$ and $w_\pm(\cdot)$ are defined as in (2) with $\alpha < 2\delta$.

the probability weighting functions, γ, have been estimated extensively from experimental data using a variety of methods and in a variety of contexts. In this regard, see, for instance, Tversky and Kahneman (1992), Camerer and Ho (1994), Wu and Gonzalez (1996), Abdellaoui (2000), Bleichrodt and Pinto (2000), Abdellaoui *et al.* (2007), Booij and van de Kuilen (2009), and Booij *et al.* (2010). The majority of these studies have estimated α at around 0.9, although others have obtained values as low as 0.5. On the other hand, the estimate of δ is relatively stable, varying from 0.6 to 0.7. Therefore, we set our benchmark value for δ at 0.65. For this δ, we have

$$\sup_{p\in(0,1)} \frac{w'_+(1-p)}{w'_-(p)} \leq 1.776$$

according to Proposition 5 in He and Zhou (2011). For α, we choose different values rather than fixing a benchmark value in our experiments because of its unstable estimation.†

The choice of the loss aversion degree k is tricky. Indeed, there are various definitions of loss aversion in the literature (see Abdellaoui *et al.* 2007, and the references therein). Most of the estimates fall within the range from 1 to 3 (though the definitions may differ), with exceptions exceeding 4 or even 8.‡ It is important to note that all of these estimates are based on *small* gains and losses.§ However, as argued in section 3.2 of He and Zhou (2011), the *large loss aversion degree* (LLAD), a measure of loss aversion for large gains and losses, is more relevant in the context of financial investment. In fact, the value of the large loss aversion degree probably differs substantially from that of the small one. Unfortunately, no estimation of the large loss aversion degree has been reported in the literature. If we were to use the available loss aversion degree estimates, we would be in a danger of misusing the small loss aversion degree in place of the large one. However, for the (dis)utility functions specified in (2), it happens that these two coincide theoretically. This is also one of the reasons we decided to take the (dis)utility functions in (2), and we will use the estimates of loss aversion in the literature for the value of k. As the available estimates do not agree on a single number, we will use different values for the loss aversion degree in our testing.¶

The investment horizon T is the evaluation period of the agent. As shown by Benartzi and Thaler (1995), the evaluation period is a consequence of mental accounting and plays an important role in influencing investment behaviour. Following Benartzi and Thaler (1995), we do not specify an evaluation period *a priori*. Instead, we infer the period from our numerical analysis by matching the historical equity premium.‖

We set the benchmark distribution of the stock total return, $R(T)$, to be *lognormal*. In other words, we assume that the log return, $\ln R(T)$, follows a normal distribution with mean μT and standard deviation $\sigma\sqrt{T}$, where μ and σ are the stock's annual expected log return and annual volatility, respectively. The continuously compounded expected return rate of the stock is then $\mu + \sigma^2/2$ per annum. The total return of the risk-free account, $r(T)$, is assumed to be e^{rT}, where r is the annual continuously compounded risk-free return rate. Thus, the continuously compounded *equity premium* is $\mu + \sigma^2/2 - r$ per annum.

The lognormal distribution might be criticized for its failure to produce a heavy tail, one of the stylized facts of equity returns. However, the heavy tail is typically observed for equity returns in short periods, usually within one month. For medium-term or long-term equity returns, there is no significant heavy tail.†† Because a typical evaluation period T is one year, as argued in Benartzi and Thaler (1995), lognormal distribution is suitable for our use. Even for those evaluation periods shorter than one month, we used a student t-distribution with degree of freedom 3 to test and found no significant difference from the lognormal distribution in our numerical results.‡‡ Thus we decided to use lognormal distribution to model the stock return.

The values of μ, σ, and r that we use are taken from Siegel (1992) in which the data-sets are a value-weighted index of all NYSE stocks from 1926 to 1990 and treasury bonds in the same period.§§ Here, we ignore the risk of treasury bonds and consider them to be good proxies for the risk-free account. These values, together with the corresponding stock return rates and equity premiums, are summarized in table 1 in both nominal dollar and real dollar terms. According to the money illusion theory, investors normally use nominal returns in their mental accounts. Thus, we decided to use nominal returns in most of our numerical experiments presented below.¶¶ The real returns are offered here as a comparison to the nominal returns and are used to test the effect of money illusion on portfolio selection in section 7.

†A concern in using these parameter values is that they were estimated in laboratories where the magnitude of payoffs was usually small. This raises a question as to whether these estimates, especially that of α, can be used directly in the context of financial investment, where gains and losses can be huge.

‡See tables 1 and 5 in Abdellaoui *et al.* (2007).

§Recently, assuming a single-period option trading model, Kliger and Levy (2009) and Gurevich *et al.* (2009) used option data to estimate the loss aversion agree. On the other hand, Abdellaoui *et al.* (2013) attempted to measure the loss aversion degree of a selected group of financial professionals *without* monetary payoffs. In addition, their experiment questions were formulated in terms of the company's money rather than the financial professionals' own money.

¶Benartzi and Thaler (1995) used $k = 2.25$ and $\alpha = 0.88$, estimates that they derived from Tversky and Kahneman (1992). Moreover, they assumed different shape parameters for the probability weighting functions for gains and losses, respectively, and took the parameter values to be 0.61 and 0.69, also as per the estimates in Tversky and Kahneman (1992).

‖In Benartzi and Thaler (1995), the period is one year.

††For instance, Cont (2001) noted that as the length of a period increased the distribution of log equity returns tended toward to a Gaussian law. Termed the *aggregational Gaussian*, this empirical finding is another stylized fact of equity returns.

‡‡Kon (1984) reported that the degree of freedom of the student-t distribution for *daily* returns ranged from 3.1177 to 5.5415. Thus, there is no reason to believe that the degree of freedom for monthly returns is lower than 3.

§§Siegel (1992) estimated the mean and standard deviation of the index return in one year to be 8.6 and 21.2%, respectively. To derive the values of μ and σ, we match the one-year expected return of the stock, $e^{\mu+\sigma^2/2}$, and the standard deviation, $e^{\mu+\sigma^2/2}\sqrt{e^{\sigma^2}-1}$, in our model with the estimates. A similar conversion is carried out for the risk-free return.

¶¶Benartzi and Thaler (1995) also used nominal returns for the same reason.

Table 1. Estimates of r, μ, and σ.

	Nominal dollars (%)	Real dollars (%)
r	4.78	1.78
μ	9.50	6.38
σ	18.69	19.34
Stock return rate	11.24	8.25
Equity premium	6.46	6.47

5. Test of model wellposedness

We first compute the critical value k_0 defined by (7). If $k > k_0$, i.e. the agent is sufficiently loss averse, then there is a trade-off between the stock desirability and avoidance of potential large losses, resulting in a *well-posed* model. Otherwise, if $k < k_0$, then the former outweighs the latter and the agent will take infinite leverage, leading to an *ill-posed* model. As argued in He and Zhou (2011), this possible ill-posedness suggests the importance of the interplay between investors and markets. The critical value k_0 depends not only on the agent preference set (utilities, probability weighting and evaluation period) but also on the investment opportunity set (asset return distributions).

We use the market parameters r, μ and σ presented in table 1 in nominal dollars (corresponding to an equity premium of 6.46%). We vary the evaluation period, T, from one month to two years. As explained in section 4.2, we choose different values for the curvature parameter α within the interval 0.4–1. Figure 1 shows the values of k_0 graphically.

We have two qualitative findings. First, with the curvature parameter α fixed, the critical value k_0 is increasing with respect to the evaluation period T. Recall that k_0 determines whether the agent will take infinite leverage on the stock. Thus, this finding reiterates the conclusion in Benartzi and Thaler (1995) that the evaluation period, which is derived from mental accounting, is important in portfolio selection. To see this, suppose that the agent has a loss aversion degree $k = 2.5$ and curvature parameter $\alpha = 1$. If his evaluation period is one year, then $k_0 < 2.5 = k$, indicating that he will hold a limited position in the stock. If his evaluation period is extended to one and one-half years, then $k_0 > 2.5 = k$, indicating that he will invest as much as possible in the stock.

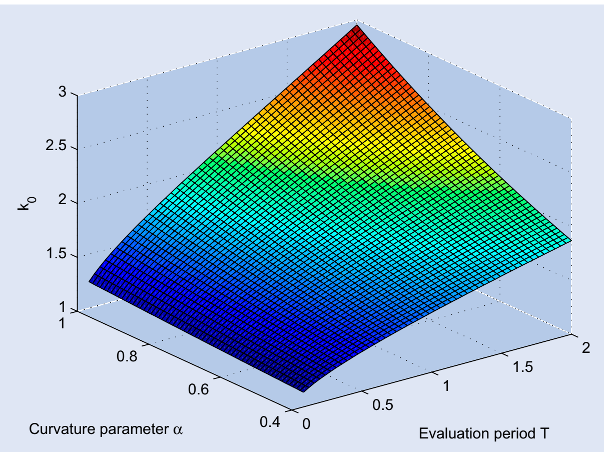

Figure 1. The critical value k_0 with T varying from one month to two years and α from 0.4 to 1.

Second, with the evaluation period T fixed, k_0 increases as the curvature parameter α increases. A larger α makes the agent less risk averse (i.e. makes the agent dislike mean-preserving spread) for gains of moderate probability and less risk seeking for losses of moderate probability. However, the overall effect of a larger α is that the agent increases his risk appetite and favours the stock more. This can be explained as follows. Since shorting is not allowed in our model, the possible gain of the stock is unbounded, while the possible loss is bounded. As a result, although a higher α makes the agent less risk averse for gains of moderate probability and less risk seeking for losses of moderate probability, its effect on evaluating the stock gain is larger than its effect on evaluating the stock loss, making the agent favour the stock more when α increases.† Moreover, figure 1 shows that k_0 is insensitive in α when the evaluation period is short. This suggests that for a short evaluation period the effect of the curvature of the (dis)utility functions is very limited. Thus, it may be reasonable to assume a piecewise linear utility function in this case.

Next, we check quantitatively whether k_0 is larger than a reasonable loss aversion degree k. In table 2 we provide the values of k_0 when the evaluation period varies from one month to two years. Three possible values of α are taken here: 1, 0.88 and 0.4. The first of these, $\alpha = 1$, is the same value used in Barberis and Huang (2001), Barberis *et al.* (2001), and Barberis and Xiong (2012). The second, $\alpha = 0.88$, was estimated by Tversky and Kahneman (1992) and was also used in Benartzi and Thaler (1995). Many subsequent estimates are close to this value. The third figure, $\alpha = 0.4$, is close to the lowest estimate found in the literature.

We can see from table 2 that in the case considered in Benartzi and Thaler (1995), where $\alpha = 0.88$ and $T = 1$, the value of k_0 is 2.00, i.e. less than the value of $k = 2.25$ assumed in that paper. Therefore, no infinite leverage takes place. However, in this case $k > k_0$ only by a small margin. If the evaluation period is extended to 18 months, $k_0 = 2.34 > k$, then the loss aversion is insufficient to prohibit the agent from taking infinite leverage on the stock.

The above analysis is based on a historical equity premium (in nominal dollars) of 6.46%. A slight change in the equity premium may switch the value of k_0 from $k_0 < k$ to $k_0 > k$, making the agent change from taking a finite exposure to the stock to an infinite one. Figure 2 depicts the value of k_0 with one-year evaluation period, curvature parameter $\alpha = 0.88$, and the equity premium varying from 5 to 9%. We can see that once the equity premium is larger than roughly 7.8% the stock becomes sufficiently attractive relative to the agent's risk appetite that a loss aversion degree of 2.25 can no longer prohibit the agent from taking infinite leverage on the stock.

We have observed that whether the agent would take infinite leverage on the stock is sensitive to the evaluation period, loss aversion degree and equity premium. The sensitivity to the first two variables shows their importance and thus reiterates the relevance of the myopic loss aversion theory presented in Benartzi and Thaler (1995); the sensitivity to the third variable shows how the market performance would influence investment decisions.

†We investigated an artificial stock return distribution in which the loss is unbounded and the gain is bounded and found that the critical value k_0 is decreasing in α, confirming our explanation.

Table 2. The critical value k_0 with T from one month to two years and $\alpha = 1$, 0.88, or 0.4.

	T								
α	1/12	3/12	6/12	9/12	12/12	15/12	18/12	21/12	24/12
1	1.24	1.46	1.71	1.93	2.14	2.35	2.55	2.75	2.95
0.88	1.22	1.41	1.63	1.82	2.00	2.17	2.34	2.51	2.68
0.4	1.14	1.25	1.36	1.46	1.55	1.64	1.72	1.79	1.87

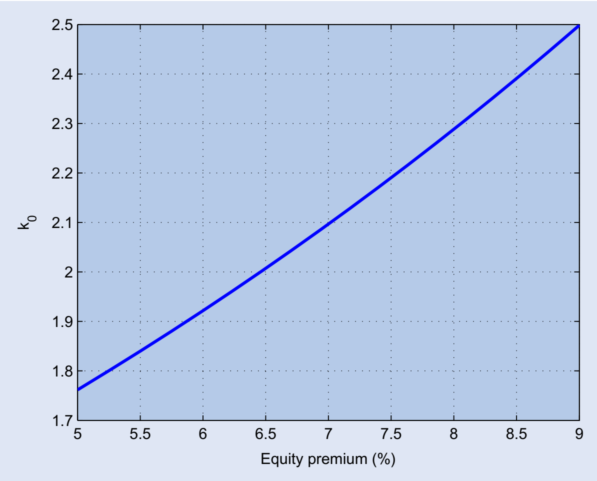

Figure 2. The critical value k_0 with one-year evaluation period and curvature parameter $\alpha = 0.88$. The equity premium rate varies from 5 to 9%.

The preceding analysis of the model's wellposedness (in particular, the aforementioned sensitivity) shows that, in theory, some investors will take infinite leverage. This, however, rarely occurs in practice. The following two aspects may have contributed to this inconsistency.

First, as discussed in section 4.2, we are actually equating the small loss aversion degree and the large one. While the latter is more relevant in financial investment, we have taken the estimates of the former from the literature. It is therefore possible that the large loss aversion degree is much larger, so much so that investors are still not willing to take infinite leverage even if the equity premium is considerably large.

Secondly, and more importantly, this inconsistency may indicate a limitation of the myopic loss aversion theory. Additional leverage constraints, explicit or implicit ones, can be useful to overcome this limitation. For instance, in the models examined in Barberis *et al.* (2001) and Barberis and Huang (2001), a leverage constraint is implicitly imposed because the agent must make sure that his wealth is nonnegative at the end of each period in order to fulfil the minimal consumption requirement.

6. Myopic loss aversion theory revisited

When the critical value k_0 is less than the loss aversion k, the optimal allocation to the stock is given in Theorem 4.2 when the (dis)utility functions are linear, i.e. $\alpha = 1$. Benartzi and Thaler (1995) demonstrated graphically that, given the historical equity premium in the period 1926–1990 and the one-year evaluation period, the optimal allocation to the stock, as a percentage of total wealth, was roughly 50%. Because real investment data show a norm of 50–50 allocation between stocks and bonds, Benartzi and Thaler (1995) concluded that myopic loss aversion was a resolution to the equity premium puzzle and that the reasonable evaluation period was one year. However, as we have discussed, an analytically or numerically rigorous portfolio selection analysis is lacking in Benartzi and Thaler (1995). Thanks to the model and solutions presented in section 4.1, we are able to perform a rigorous analysis here.†

It is worth mentioning that Benartzi and Thaler (1995) overlooked the effect of the reference point in CPT. Indeed, they set the reference point to be the initial wealth without emphasizing the role that it plays in asset allocation. Theorem 5 in He and Zhou (2011) shows that the reference point has a significant effect on optimal asset allocation. In particular, the optimal stock allocation is strictly increasing with respect to the deviation of the reference point from the risk-free payoff. This suggests that, along with mental accounting and loss aversion, the reference point is an important factor in portfolio selection and asset pricing.

The reference point in our setting can be articulated as a rate of return rather than as a dollar amount. For instance, an agent may want his investment to achieve a 5% annualized return rate. We call this 5% the *reference return rate* of the agent. Thus, if the agent has a reference return rate p and an evaluation period T, the corresponding reference point applied to the terminal wealth (recall that $\widetilde{x}_0$ is the initial endowment) is

$$B = \widetilde{x}_0 e^{pT}.$$

Therefore, the deviation of the reference point from the risk-free payoff is

$$x_0 = \widetilde{x}_0\left(e^{rT} - e^{pT}\right).$$

The optimal stock amount (10) and (12) can then be rewritten as

$$\theta^*(\widetilde{x}_0, k, T) = \begin{cases} c_+(k, T)\widetilde{x}_0, & \text{if } p \geq r, \\ c_-(k, T)\widetilde{x}_0, & \text{if } p < r, \end{cases}$$

where

$$c_+(k, T) = \frac{e^{pT} - e^{rT}}{v_+(k, T)}, \quad c_-(k, T) = \frac{e^{pT} - e^{rT}}{v_-(k, T)}.$$

†In Benartzi and Thaler (1995), the curvature parameter α was taken to be 0.88. Here, we consider the case in which $\alpha = 1$ instead because other cases remain unsolved. However, the impact of the loss aversion, evaluation period and reference point on the optimal stock allocation should not depend on the particular choice of α. Moreover, we observe from section 5 that changing α from 0.88 to 1 does not substantially alter the value of k_0, suggesting that the optimal stock allocation will not change markedly either.

Note that $c_+(k, T)$ and $c_-(k, T)$ are precisely the optimal weights of the stock allocation when the reference return rate is above and below the risk-free return rate, respectively.

The reference return rate varies from individual to individual. We consider two of the most plausible reference return rates: $p = 0\%$ and $p = 11.24\%$. The first of these rates, which was also used in Benartzi and Thaler (1995), refers to the case in which the agent identifies losses and gains by comparing his terminal wealth to his initial wealth. Because information about the initial wealth is clear and accessible in the cognitive system, it is natural for some investors to use this quantity as the benchmark for comparison. The second figure, given that 11.24% is the expected return rate of the stock (see table 1), is the case in which the agent wants to beat so-called market expectations. For instance, institutional investors such as pension funds commonly use this reference return rate.†

We compute the optimal allocation to the stock for these two reference return rates, respectively, when the evaluation period varies from one month to two years and the loss aversion degree varies from 1 to 4.‡ Figures 3 and 4 depict the results for three different values of the loss aversion degree: $k = 2.25$, 2.5 and 2.75, when the reference return rate is 0% and 11.24%, respectively. We first note that all the plots have vertical asymptotic lines corresponding to the evaluation periods for which the agent would take infinite exposure to the stock. We can also see that the optimal allocation to the stock is lower if the loss aversion degree k is higher, which was established analytically in Theorem 5 of He and Zhou (2011). On the other hand, the optimal stock allocation increases as the evaluation period becomes longer. Both of these observations lay a rigorous ground for the claim of Benartzi and Thaler (1995) that loss aversion and a short evaluation period contribute to an investor being unwilling to bear the risks associated with holding equities.

The optimal allocation to the stock is extremely sensitive to the evaluation period and to the loss aversion degree near the boundary beyond which the agent is willing to take infinite leverage. For instance, when the reference return rate is 0%, the evaluation period is one year, the loss aversion degree is 2.75 and the optimal weight is 20.75%. If the loss aversion degree decreases by 0.5–2.25 (18.18% decrease), the optimal weight more than doubles, to 43.07%. If the loss aversion degree decreases to 2.00, the agent is willing to invest in the stock as extensively as possible.

Another interesting observation is that the agent is not willing to invest in the stock when the evaluation period is very short (within two months) if his reference return rate is 0%. It is well known that an agent with the expected utility preference and a smooth utility function is *always* willing to invest in the stock so long as the equity premium is positive. Here, because the reference point is not the risk-free payoff, the investor is not at the kink of the utility function. Thus, the nonparticipation in the stock is due to the probability weighting

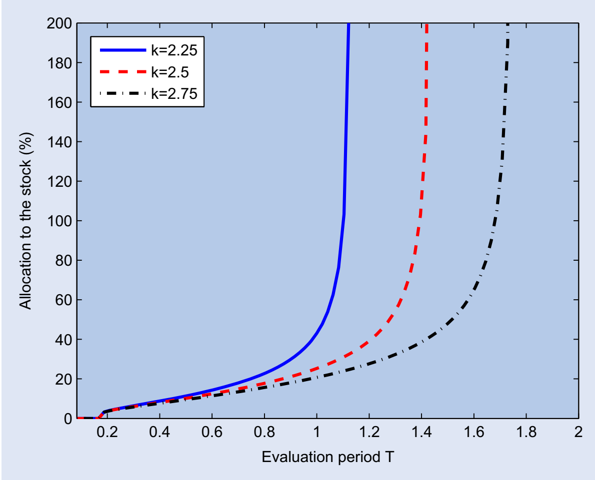

Figure 3. Optimal allocation to the stock, $c_-(p, T)$, when $p = 0\%$ and T varies from one month to two years. The loss aversion degree k is taken at three values: 2.25, 2.5 and 2.75.

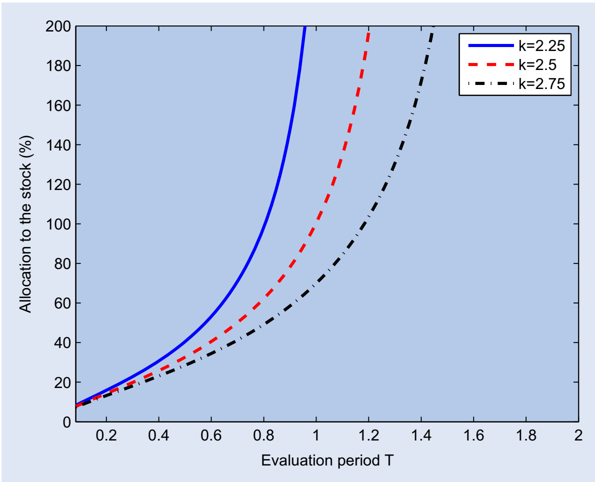

Figure 4. Optimal allocation to the stock, $c_+(p, T)$, when $p = 11.24\%$ and T varies from one month to two years. The loss aversion degree k is taken at three values: 2.25, 2.5 and 2.75.

functions that cause first-order risk aversion, as noted in Segal and Spivak (1990). The observation that the probability weighting function may lead to nonparticipation in equities was also made in Polkovnichenko (2005). Interestingly, if the agent's reference return rate is 11.24%, i.e. higher than the risk-free rate, he is willing to invest in the stock no matter how short the evaluation period is. This remarkably different attitudes toward investing in the stock when the evaluation period is short is due to the *asymmetry* between the probability weighting functions for gains and losses. Put more precisely, when the evaluation period is short, the agent's decision to invest in the stock is determined by its expected excess return *adjusted for probability weighting*. When the reference return rate is higher than the risk-free rate, the agent starts in the loss territory, so he applies the probability weighting for losses to the stock's expected excess return. Similarly, when the reference return rate is lower than the risk-free rate, the agent applies the probability weighting for gains. As a consequence, the

†The risk-free return rate is another likely reference return rate. However, in this case, it is optimal not to invest in the stock when $k > k_0$, as shown by Theorem 4.2(ii). Thus, we do not discuss this case here.

‡The assumption of Theorem 4.2 is not satisfied when $k < 1.776$. However, Assumption 4 in He and Zhou (2011) is still satisfied, so we can use the general results presented there to compute the optimal allocation.

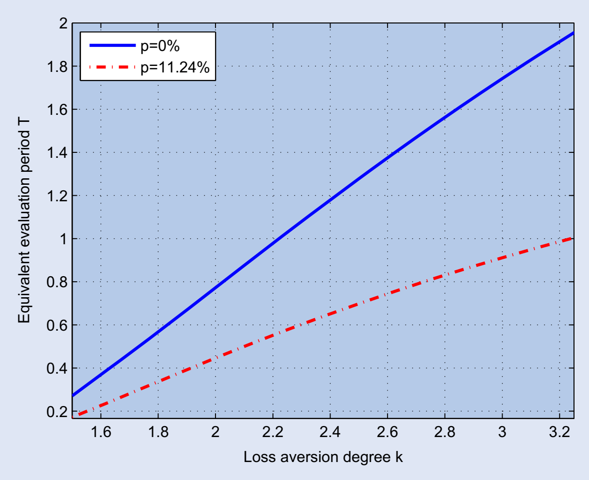

Figure 5. The equilibrium evaluation period of the agent that makes him allocate equally between the stock and the risk-free account. The loss aversion varies from 1.5 to 3.25.

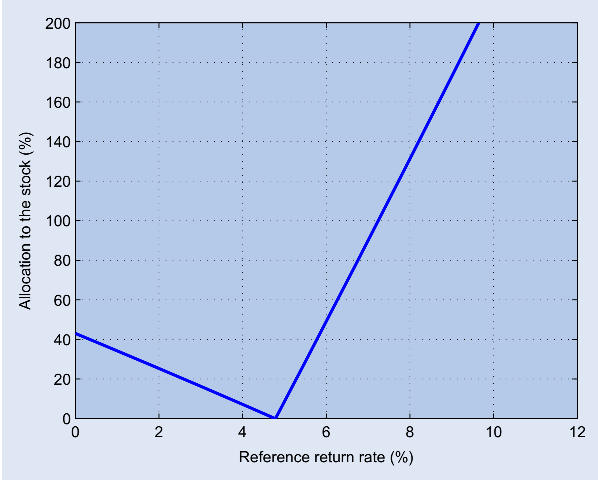

Figure 6. Allocation to the stock when the reference return rate varies from 0 to 12%. The evaluation period is fixed at one year and the loss aversion degree is fixed at 2.25.

agent evaluates the stock very differently when his reference return rate is higher or lower than the risk-free rate.†

Next, we use our model to find the evaluation period that would yield a 50–50 allocation between the stock and a risk-free asset. Because both institutions and individuals tend to allocate equally in equities and bonds (see Benartzi and Thaler 1995), the resulting evaluation period is the one that is consistent with the equity premium. Let us call this the *equilibrium evaluation period*. Figure 5 illustrates the length of this equilibrium evaluation period with the loss aversion degree varying from 1.5 to 3.25. We can see that if the reference return rate is 0% and the loss aversion degree is 2.25, the equilibrium evaluation period is roughly 1.05 years, which is very close to one year. Thus, our results confirm the observation in Benartzi and Thaler (1995) that one year is a plausible equilibrium evaluation period.

From figure 5 we can observe that the equilibrium evaluation period is strongly dependent on the reference return rate. For instance, if we fix the loss aversion degree at 2.25, the equilibrium evaluation period is roughly 0.4 years shorter when $p = 11.24\%$ than when $p = 0\%$. This is because the former reference return rate deviates from the risk-free rate much more substantially than the latter. Theorem 5 in He and Zhou (2011) indicates that the agent invests more in the stock in the former case because of the larger deviation from the risk-free payoff. As a result, the equilibrium evaluation period has to be shorter to force the agent to invest equally in the stock and risk-free asset. We further investigate the effect of the reference return rate by letting it vary from 0 to 14% and computing the optimal allocation to the stock. Figure 6 shows that the allocation to the stock is sensitive to the reference return rate. The V-shaped curve in figure 6 can be explained as follows. When the reference point is further away from the risk-free payoff, the agent is further away from the kink in the utility function. Because the kink models loss aversion, the agent becomes less risk averse overall when the reference point is away from the risk-free payoff, and therefore invests more in the stock. Quantitatively, an increase in the reference return rate from 8% to 12% will double the allocation to the stock. This suggests that, in addition to mental accounting and loss aversion, reference point is another important component in portfolio selection and asset pricing. For instance, from figure 6 we observe that when the reference return rate is 6%, the allocation to the stock is roughly 50%, showing that this reference return rate, together with a loss aversion degree of 2.25 and an evaluation period of one year, produces stock allocation that is consistent with market observations.

In summary, we confirm the observation by Benartzi and Thaler (1995) that loss aversion and mental accounting contribute to an agent being unwilling to bear the risks associated with holding equities, thus offering an explanation for the equity premium puzzle. Moreover, we find that the reference point also has a significant effect. On the other hand, optimal allocation to equities is highly sensitive to loss aversion degree, evaluation period, and reference point given the historical equity premium. This sensitivity highlights the importance of loss aversion and mental accounting (where the latter determines evaluation periods and reference points) in portfolio selection and asset pricing.

One might think that the sensitivity of the optimal allocation would weaken the argument for using the myopic loss aversion theory to explain the equity premium puzzle. However, this is not the case. This sensitivity may explain the high volatility of equity returns in multi-period models. Indeed, a small movement in the loss aversion and mental accounting of the representative agent may change his optimal equity allocation dramatically. As a consequence, equilibrium equity returns move likewise in dramatic ways.‡

†It follows from Theorem 4.2 that whether the agent invests in the stock depends on the sign of λ^- when the reference return rate is higher than the risk-free rate and that of λ^+ is otherwise. Because of the probability weighting functions, λ^- differs from λ^+ in general, unless $w_-(p) = 1 - w_+(1-p)$, $p \in [0, 1]$. The general inequality $w_-(p) \neq 1 - w_+(1-p)$ is what we here call 'asymmetry'.

‡Barberis *et al.* (2001) and Barberis and Huang (2001) assume that the loss aversion and reference point of the representative agent change dynamically according to prior gains or losses as a result of the so-called *house money* effect. The dynamic movement of the loss aversion and reference point produces a notably high equity volatility.

7. Money illusion

'Money illusion' is the name given to the tendency to think in terms of nominal dollars rather than real dollars. It owes its existence to the easier cognitive accessibility of nominal dollars than of real dollars. As (Shafir *et al.* 1997, p. 347) pointed out, 'people are generally aware that there is a difference between real and nominal values, but because at a single point in time, or over a short period, money is a salient and natural unit, people often think of transactions in predominantly nominal terms'. Therefore, nominal dollars and real dollars represent distinct mental account media.

Money illusion has numerous implications with respect to transactions, investments, social welfare and so on. Here, we have an opportunity to test money illusion in the context of CPT portfolio selection. We use the parameters in table 1 in *real* dollars, i.e. we assume that the agent uses real-dollar mental accounting, to reexamine the results in the previous sections. If the results differ significantly from the previous ones, then we can conclude that money illusion indeed has a significant effect.

We first compute the values of k_0 that determine whether the agent will take infinite leverage on the stock. It turns out that the results when the agent uses real-dollar mental accounting are almost the same as when he uses the nominal one. For instance, we compute the values of k_0 with T varying from one month to two years and $\alpha = 1$, 0.88 or 0.4 in real-dollar mental accounting. The results are shown in table 3. It is clear that the values of k_0 shown here are almost identical to those in table 2. Therefore, money illusion has little bearing on whether the agent will take infinite leverage. This result is not surprising because money illusion does not change the attractiveness of the stock *compared to* the risk-free asset. Only the equity premium, the excess stock return to the risk-free return, matters.†

Next, we investigate the effect of money illusion on the optimal stock allocation. If the agent sets his mental accounting so that the reference return rate is $p = r + q$ for some rate q, the money illusion will not be a factor in this case. Indeed, the deviation of the reference point from the risk-free point, x_0, which determines the optimal allocation, is approximately $\widetilde{x}_0 \cdot qT$, and this deviation is clearly invariant whichever mental accounting system (real-dollar or nominal-dollar) is used. Therefore, we can conclude that money illusion comes into effect only if the agent sets his reference point *not* relative to the risk-free payoff, i.e. not in a form of the risk-free payoff plus a surplus which is same in both the nominal-dollar mental accounting and real-dollar mental accounting. A commonly formed reference point, which is not relative to the risk-free payoff, is the initial wealth. In terms of reference return rates, this reference point corresponds to the case in which $p = 0\%$. Thus, in the following, we let $p = 0\%$ in order to investigate the effect of money illusion.

Fixing $k = 2.25$, we compute the optimal allocation to the stock and compare it to that in the nominal-dollar mental accounting. The results are shown in figure 7. We can see that when using nominal-dollar mental accounting, the allocation to the stock is much higher than when using real dollars. For instance, if the evaluation period is one year, the allocation to the stock is lower than 20% when using real-dollar mental accounting and is higher than 40% when using nominal dollars. Moreover, the difference increases as the evaluation period becomes longer. These results are consistent with the experimental results presented in Shafir *et al.* (1997), where the mean allocation to the risky fund was 42.3% in the no-inflation condition (corresponding to real-dollar mental accounting) and 71.5% in the inflation condition (corresponding to nominal-dollar mental accounting). Thus, when inflation is higher, money illusion makes the risky stock more attractive compared to the benchmark and thus induces the agent to invest more into it. As a result, in equilibrium, the equity premium is lower. This finding is consistent with the empirical fact that inflation and equity returns are negatively correlated (see e.g. Fama 1981).

To conclude, money illusion comes into play only if the agent has a reference point that is not relative to the risk-free payoff. In this case, the deviation of the reference point from the risk-free payoff is different in nominal dollars and in real dollars, leading to different optimal allocations. For instance, a zero reference return rate makes the deviation much larger in nominal dollars than in real dollars. Therefore, if some agents in the market form this reference return rate, they will invest more in equities and the equilibrium equity premium will be lower when using nominal dollars than when using real dollars. This finding also indicates that the equity premium would have been even higher had there been no money illusion.

8. A dynamic model

Some researchers criticized that the approach employed by Benartzi and Thaler (1995) was not very satisfying because it focused on a single-period setting. The same criticism could be applied to our study in the previous sections. In this section, we demonstrate that the single-period model can indeed be embedded into a consumption-based dynamic model, in the sense that the latter can be solved by solving the former. The study of the effect of mental accounting on asset allocation in this dynamic model is parallel to that in the single-period model. Another advantage of the consumption-based dynamic model is to prevent the agent from taking infinite leverage as is the case with the single-period model. The dynamic model we are going to present is similar to those employed by Barberis and Huang (2008a, 2009); Barberis *et al.* (2006); De Giorgi and Legg (2012) to study narrow framing. The new feature of our model is a general reference point that might be different from the risk-free payoff, the particular reference point that has been used in the aforementioned works.

Consider an agent who faces a multi-period problem to decide the optimal consumption and optimal allocation to a risk-free asset and a risky stock in each period. Suppose that the gross returns of the risk-free asset and the stock in the period from time t to $t+1$ are $R_{f,t+1}$ and R_{t+1}, respectively, that the agent's wealth at time t is W_t, and that the agent chooses to consume C_t and to invest θ_t dollar amount in the stock

†The critical value k_0 is determined by the distribution of $R(T) - r(T) \approx \ln R(T) - \ln r(T)$ (since $r(T)$ is close to 1), i.e. the difference between the stock return rate and the risk-free rate. This difference is almost the same when evaluated using nominal dollars and real dollars.

Table 3. The critical value k_0 with T from one month to two years and $\alpha = 1$, 0.88 or 0.4, using the parameters in table 1 in real dollars.

	T								
α	1/12	3/12	6/12	9/12	12/12	15/12	18/12	21/12	24/12
1	1.24	1.45	1.70	1.91	2.11	2.31	2.51	2.70	2.90
0.88	1.22	1.40	1.62	1.80	1.97	2.14	2.30	2.47	2.63
0.4	1.13	1.24	1.35	1.44	1.53	1.61	1.68	1.76	1.83

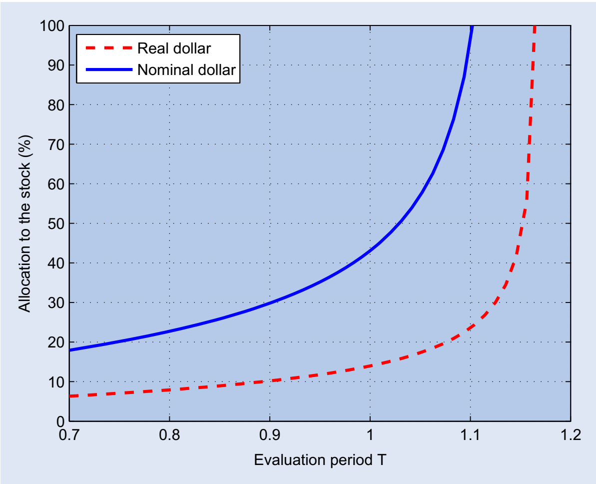

Figure 7. The allocation to the stock when $k = 2.25$. The evaluation period varies from 0.7 years to 1.2 years. The solid blue line represents the allocation to the stock when using nominal dollars. The dashed red line represents the allocation to the stock when using real dollars.

and the remainder in the risk-free asset at time t. Then, the agent's wealth at time $t+1$ is $W_{t+1} = (W_t - C_t)R_{f,t+1} + \theta_t(R_{t+1} - R_{f,t+1})$. The agent's (total) utility X_t is defined as an aggregation of the current consumption C_t, the certainty equivalent of the utility after this period X_{t+1} and the utility M_t derived from the gains or losses of the agent's investment in this period. Without M_t, the agent's utility becomes the *recursive utility* presented in Epstein and Zin (1989). The inclusion of M_t in the aggregation models that, in addition to the utility of consumption, investors also receive utility directly from the gain or loss that they experience in their investments.

The aggregation is modelled as follows:

$$X_t = H(C_t, m(X_{t+1}|\mathcal{F}_t) + b_0 M_t), \tag{13}$$

where $\mathcal{F}_t$ represents the information available at time t, $m(X_{t+1}|\mathcal{F}_t)$ is time-t certainty equivalent of time-$(t+1)$ (random) utility X_{t+1}, $b_0 \geq 0$ measures the relevance of capital gains and losses to the agent's utility and H is an aggregator. In the following, we use the specification of H and m proposed by Kreps and Porteus (1978):

$$H(c,z) = \left[(1-\beta)c^{1-\rho} + \beta z^{1-\rho}\right]^{\frac{1}{1-\rho}}, \quad \rho \geq 0,$$
$$m(X) = \left[\mathbb{E}(X^{1-\gamma})\right]^{\frac{1}{1-\gamma}}, \quad \gamma \geq 0.$$

The economic interpretations of the parameters γ and ρ are as follows: γ is the relative risk aversion and $1/\rho$ is the elasticity of intertemporal substitution.†

The utility induced by gains or losses, M_t, is defined to be $V(W_{t+1}) - K_t$, the CPT value of the random wealth at time $t+1$ adjusted by a constant K_t. The adjustment $K_t := V((W_t - C_t)R_{f,t+1})$ is made so that $M_t = 0$ if $W_{t+1} = (W_t - C_t)R_{f,t+1}$. As a result, the action of saving all the after-consumption wealth in the risk-free asset does not add to or reduce the total utility.‡

The agent's optimal consumption-investment problem at each time t can be formulated as follows:

$$\begin{aligned}&\underset{\{C_s\}_{s\geq t},\{\theta_s\}_{s\geq t}}{\text{Max}} \quad X_t,\\ &\text{Subject to} \quad W_{s+1} = (W_s - C_s)R_{f,s+1}\\ &\qquad\qquad + \theta_s\left(R_{s+1} - R_{f,s+1}\right), \quad s \geq t,\end{aligned} \tag{14}$$

where $\{C_s\}_{s\geq t}$ and $\{\theta_s\}_{s\geq t}$ represent the consumption stream and the dollar amount allocated to the stock after time t, respectively.

We use the methodology in Barberis and Huang (2009) to solve the agent's consumption-investment problem (14). Denote by J_t the optimal value of (14), i.e. the agent's optimal utility at time t. Thanks to the recursive nature of the utility (13), dynamic programming principle immediately yields the following Bellman equation§

$$\begin{aligned}J_t = \max_{C_t,\theta_t}\Big\{(1-\beta)C_t^{1-\rho} + \beta\Big[\left(\mathbb{E}(J_{t+1}^{1-\gamma}|\mathcal{F}_t)\right)^{\frac{1}{1-\gamma}}\\ + b_0(V(W_{t+1}) - V((W_t - C_t)R_{f,t+1}))\Big]^{1-\rho}\Big\}^{\frac{1}{1-\rho}}.\end{aligned} \tag{15}$$

†When $\rho = 1$, $H(c,z)$ is defined as $\exp[(1-\beta)\ln c + \beta \ln z]$. Similarly, when $\gamma = 1$, $m(X)$ is defined as $\exp[\mathbb{E}(\ln X)]$.

‡In the model presented in Barberis and Huang (2009) and others to study narrow framing, investors receive utility from the gain or loss of a risky stock rather than from the performance of the whole portfolio, and the authors focus their study on the case of risk-free reference point. As a result, if there is only one risky stock in the market, the CPT value of the portfolio wealth is same as the CPT value of the wealth in the stock, and hence is zero in the case of nonparticipation in the stock. Thus, in this case, the model presented here, which accommodates a general reference point in portfolio wealth, coincides with theirs.

§One might think that the nonlinear probability weighting function would cause time inconsistency, and hence invalidate the dynamic programming approach. However, in the model we present here, the issue of time inconsistency does not arise. Indeed, the probability weighting function is used only in the evaluation of gains and losses in each *single* period. The utilities across different periods are linked by the recursive utility, which by its very definition is a time-consistent model. Thus, the model here is time-consistent, and consequently we are able to apply dynamic programming to solve the problem.

To find tractable solutions, we assume that the CPT preference is specified by the utility function and probability weighting functions in (2). Moreover, we assume that $\alpha = 1$, i.e. the utility function is piecewise linear, and that the assumptions on the degree of loss aversion and probability weighting functions in Theorem 4.2 hold. In addition, we let the reference point B_{t+1} in the period from time t to $t+1$ take the following form: $B_{t+1} = (W_t - C_t)R_b$, where R_b is a benchmarked gross portfolio return that is same in every period. Thanks to the homogeneity of the aggregator H, certainty equivalent μ and CPT value function V, we are able to simplify the Bellman equation (15) by defining $c_t := C_t/W_t$, $\omega_t := \theta_t/(W_t - C_t)$ and $\psi_t := J_t/W_t$, which are the percentage consumption, the percentage allocation to the stock and the optimal utility per unit wealth, respectively. Plugging these variables into (15), we obtain

$$\psi_t = \max_{c_t,\omega_t}\Big\{(1-\beta)c_t^{1-\rho} + \beta(1-c_t)^{1-\rho} \times \Big[\Big(\mathbb{E}\big((\psi_{t+1}R_{p,t+1})^{1-\gamma}\,|\mathcal{F}_t\big)\Big)^{\frac{1}{1-\gamma}} + b_0(f(\omega_t) - v(R_{f,t+1}))\Big]^{1-\rho}\Big\}^{\frac{1}{1-\rho}}, \quad (16)$$

where $R_{p,t+1} := R_{f,t+1} + \omega_t(R_{t+1} - R_{f,t+1})$ is the gross return of the portfolio, $v(\cdot)$ is the CPT value function with the reference point to be R_b and $f(\omega_t) := v(R_{f,t+1} + \omega_t(R_{t+1} - R_{f,t+1}))$.

We further assume that $R_{f,t+1} \equiv R_f, t \geq 0$ and $R_{t+1}, t \geq 0$ are i.i.d. Under this assumption, the optimal utility per unit wealth at each time, which should depend only on the returns afterwards, is independent of the information at that time. As a result, we must have $\psi_t \equiv \psi, t \geq 0$. Therefore, equation (16) becomes

$$\psi = \max_{c_t,\omega_t}\Big\{(1-\beta)c_t^{1-\rho} + \beta(1-c_t)^{1-\rho} \times \Big[\psi\left(\mathbb{E}(R_{p,t+1}^{1-\gamma})\right)^{\frac{1}{1-\gamma}} + b_0(f(\omega_t) - v(R_f))\Big]^{1-\rho}\Big\}^{\frac{1}{1-\rho}}. \quad (17)$$

We can solve (17) by iteration: start from an initial guess of ψ, plug it in the right-hand side of (17), and obtain an updated value of ψ. In each iteration, one needs to solve the maximization problem in (17). Notice that the maximization over c_t and ω_t is separable. We can first solve

$$g(\psi) := \max_{\omega_t}\left[\psi\left(\mathbb{E}\left(R_{p,t+1}^{1-\gamma}\right)\right)^{\frac{1}{1-\gamma}} + b_0(f(\omega_t) - v(R_f))\right] \quad (18)$$

and then solve

$$\max_{c_t}\left[(1-\beta)c_t^{1-\rho} + \beta(1-c_t)^{1-\rho} g(\psi)^{1-\rho}\right]^{\frac{1}{1-\rho}}. \quad (19)$$

Because (19) can be solved analytically, in the following, we focus our discussion on how to solve (18). One can easily observe that $f(\omega_t) = U(\omega_t)$ with $\tilde{x}_0 = 1$, where $U(\cdot)$ is defined by (5) and (6) and $\tilde{x}_0$ is the initial wealth in the single-period model presented in section 4.1. Thus, by Theorem 4.2, $f(\cdot)$ is strictly concave. On the other hand, the first term in the bracket in (18), which is the certainty equivalent of a power utility function, is also concave in ω_t. As a result, the objective function in (18) is concave and the maximizer can be found easily. Note that the agent in the dynamic model cannot take infinite leverage on the stock, for otherwise his wealth may fall negative in which case he cannot consume in the future.

Next, we numerically investigate the effect of mental accounting on asset allocation in the dynamic model. We assume that the return of the stock is lognormal distributed and the market data are the same as presented in table 1 in nominal dollars. The length of each period in the dynamic model is set to be one year for the moment. The value of δ—the parameter for the probability weighting functions—is the same as used in section 4.2. We set the degree of loss aversion λ, the relative risk aversion γ, the reciprocal of elasticity of intertemporal substitution ρ, the parameter b_0 and the discount factor β to be 3, 3, 2, 0.1 and 0.9, respectively (the findings we obtain below, however, do not depend on these specific values). According to Barberis *et al.* (2006), this specification of model parameter values can explain nonparticipation in the stock market and realistic attitudes towards monetary gambles at the same time.

We choose a variety of reference return rates p ranging from 0 to 12%. The benchmark R_b in the dynamic model is defined by $R_b = e^p$ correspondingly. The optimal percentage allocation to the risky stock, ω_t^*, is depicted in figure 8. We can observe that the reference point has a significant effect on optimal asset allocation. The further the reference point deviates from the risk-free payoff, the more invested in the stock. This observation is consistent with the observation that we made in section 6 in a single-period setting. The optimal consumption depicted in figure 8 shows that the reference point has little effect on consumption. This is because the reference point is modelled as a gross return of the after-consumption wealth. Consequently, the reference point affects the allocation between the risk-free asset and the stock rather than affecting the consumption.

The effect of money illusion can be also tested in the dynamic model. As in section 7, we consider a zero reference return rate, which equates the referent point with the initial wealth. We compute that the percentage allocation to the stock is 26.33% if nominal returns are being used and is 8.89% if real returns are being used. Money illusion almost triples the allocation to

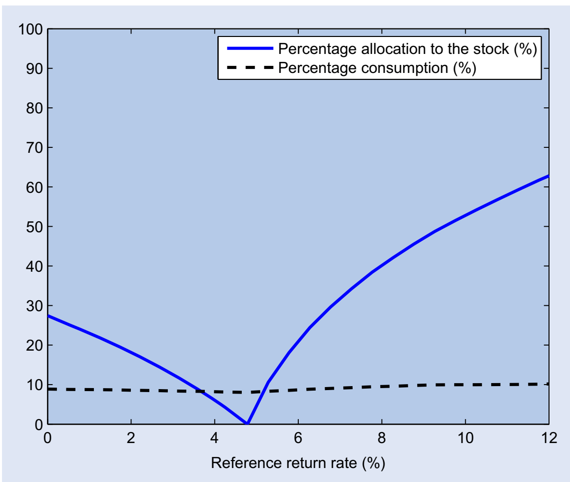

Figure 8. The percentage allocation to the stock and the percentage consumption with different reference return rates in the dynamic model. The solid line represents the allocation to the stock and the dashed line represents the consumption.

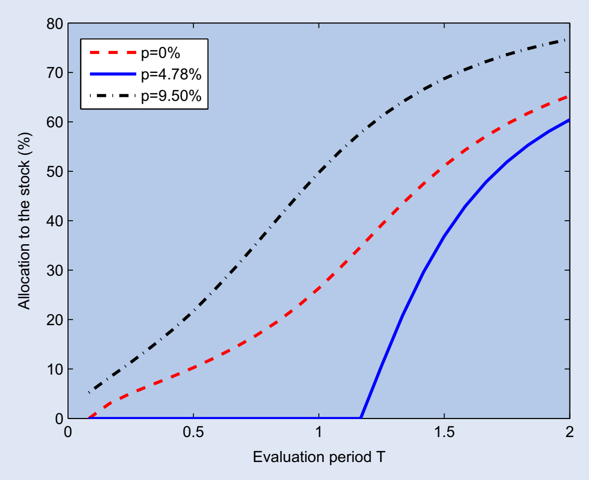

Figure 9. The percentage allocation to the stock with different evaluation periods in the dynamic model. The dashed line, solid line and dash-dotted line represent the allocation to the stock with a 0% reference return rate, a 4.78% reference return rate and a 9.50% reference return rate, respectively.

the stock, indicating a significant effect of money illusion on asset allocation.

Finally, we examine the effect of evaluation period on asset allocation in the dynamic model. We vary the length of each period T in the dynamic model from one month to two years. A smaller T indicates that the agent evaluates gains or losses more frequently. Therefore, T can be considered to be the agent's evaluation period. For each T, we let the discount factor β in the dynamic model be $e^{(\ln 0.9)T}$. The other parameter values are the same as before. The optimal percentage allocation to the stock is drawn in figure 9, with three different reference return rates being used. Whichever reference return rate is used, the allocation to the stock decreases as the evaluation period T becomes shorter. Therefore, we confirm in a dynamic setting that mental accounting contributes to an agent being unwilling to hold equities.

9. Conclusion

This paper uses the NYSE equity and US treasury bond returns for the period 1926–1990 to test the implications of the CPT portfolio choice model presented in He and Zhou (2011) for asset allocation. We have generalized the myopic loss aversion theory presented in Benartzi and Thaler (1995) by showing analytically and empirically that the agent's loss aversion, evaluation period and reference point have significant effects on asset allocation. We have demonstrated that although the optimal allocation, when the agent has reasonable loss aversion, evaluation period and reference point, is consistent with market observations, it is sensitive to these three factors. We have also shown the relevance of money illusion in portfolio selection. Finally, we have presented a dynamic version of the model.

In this paper, we used the NYSE equity and US treasury bond returns for the period 1926–1990 because the same data-set was used in Benartzi and Thaler (1995) with whose results we want to compare. We also looked at the US equity and bond market for the period 1991–2012 and found that the conclusions in this paper do not change.†

The dynamic model we presented in section 8 has a potential for studying the implication of reference points for asset pricing. The method proposed by Barberis and Huang (2009) can be used to extend the dynamic model to accommodate equilibrium analysis. One possible direction of future research is to study the implication of money illusion for asset pricing and to explain the historically observed negative correlation between inflation and equity returns.

In our dynamic model we assumed the reference point of the agent to be constant over time. A more realistic model is to allow the reference point to update over time. The update can either be exogenous, as proposed in Barberis *et al.* (2001) for the purpose of modelling the 'house money' effect, or be endogenous, as suggested by Kőszegi and Rabin (2006) and De Giorgi and Post (2011). We leave the study of how to incorporate the reference point update into our model and the implications of the resulting new model for future research.

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†We used monthly return data of the S&P 500 index from Yahoo and the 6-month US treasury bill rate data from http://www.federalreserve.gov/releases/h15/data.htm and obtained the following estimates (in nominal dollars) for the period 1991–2012: $r = 3.16\%$, $\mu = 6.96\%$, $\sigma = 14.97\%$ and equity premium is 5.21%.

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