

Outline

Twelve lectures on a select set of topics in financial engineering, in particular, derivative pricing and risk hedging. The focus is on stochastic modeling, analysis, and numerical solution techniques.

Prerequisite:

Basic probability models and elementary stochastic processes (at the level of IEOR4606).

Texts:

J.C. HULL, *Options, Futures, and other Derivatives*, Prentice Hall, Englewood Cliffs, NJ, newest edition.

S.E. SHREVE, *Stochastic Calculus for Finance II, Continuous-Time Models*, Springer, New York, 2004.

Supplemented by lecture notes.

References:

Investments, W.F. Sharpe, G.J. Alexander, and J.V. Bailey, Prentice Hall, Englewood Cliffs, NJ, newest edition.

Stochastic Calculus and Financial Applications, J. Michael Steele, Springer-Verlag, New York, 2000.

Homework:

11 sets, each with about 10 problems, assigned at every class, and due in two weeks, unless otherwise specified; full credit if returned on time.

Midterm Exam:

October 24 (class hours); a closed book exam, but an “aid sheet” is allowed.

Final Exam:

Tentative: December 19 (class hours); same format as the midterm.

Grading:

20% homework, 40% midterm, 40% final.

Contact:

Office Hours: Monday, 1:30-2:30p; Tuesday, 3-4p; or by appointment.

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Topics and Schedule (*subject to changes*):

- normal and lognormal distributions, stock price models, Brownian motion and geometric BM, relations to random walk; (lecture 1)
- Itô's calculus; option pricing, risk-neutral valuation, put-call parity; Black-Scholes partial differential equations and solutions, sensitivity: the Greek letters; (lectures 2,3)
- variations of the Black-Scholes model: index options, currency options, futures options; American options, inequalities (lecture 4);
- binomial trees; dividend models; finite difference methods; delta hedging; VaR; (lectures 5,6)
- risk hedging, self-financing; multi-dimensional Itô's calculus, the product rule; (lecture 7)
- conditional expectation w.r.t. sigma algebra, filtration, martingales, stochastic integral; change of measures, Girsanov theory; (lecture 8)
- equivalent martingale measures, market price of risk; martingale representation theorem; existence and uniqueness of the risk-neutral measure; (lectures 9,10)
- numeraire, Siegel's paradox, applications of change of measures, interest-rate derivatives. (lectures 11,12)