Flexible Supply Contracts via Options

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Abstract

We develop an option model to quantify and price a flexible supply contract, by which the buyer (a manufacturer), in addition to a committed order quantity, can purchase option contracts and decide whether or not to exercise them after demand is realized. We consider both call and put options, which generalize several widely practiced contracting schemes such as capacity reservation and buy-back/return policies. We focus on deriving (a) the optimal order decision of the buyer, in terms of both the committed order quantity and the number of option contracts; and (b) the optimal pricing decision of the seller (which supplies raw materials or components to the manufacturer), in terms of both the option price and the exercise price. We show that the option contracts shift part of the buyer's risk due to demand uncertainty to the supplier; and the supplier, in turn, is compensated by the additional revenue obtained from the options. We also show that a better alternative to the two parties' individual optimization is for them to negotiate a mechanism to share the profit improvement over the no-flexibility contract. Indeed, this profit sharing may achieve channel coordination.

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1 Introduction

Consider a supply chain consisting of a supplier and a buyer with the supplier selling raw materials or components to the buyer, a manufacturing firm, which in turn sells finished products to end customers with random demands. In a decentralized setting, each party will attempt to maximize its own profit objective and often based on its private information. It has been widely recognized that the supplier and the buyer can benefit from coordination and thereby improve the overall performance of the supply chain as a whole, as well as, though not necessarily always, the performance of each party individually. Coordination between the two parties can be achieved by various means, for example, information sharing. Marketing and negotiation strategies can also be designed to provide incentives that induce coordination.

Flexible supply contracts, the subject of our study here, constitute yet another effective means to facilitate coordination, thanks to the capacity of these contracts in accommodating different and often conflicting objectives through associating them with the right incentives. For example, quantity flexibility can be specified in a supply contract that allows the buyer to adjust its order quantities after the initial order is placed. Such flexibility enables the buyer to reduce its risk in overstock or understock, and naturally comes at extra cost to the buyer, which also gives the supplier incentive to offer it while undertaking more risk. Other forms of flexibility in supply contracts include capacity reservation and buy-back or return policies. Examples of flexible supply contracts have been reported as industrial practices at companies such as IBM Printer division (Bassok *et al.*, 1997), Sun Microsystems (Farlow *et al.*, 1995), Hewlett Packard (Tsay and Lovejoy, 1999), and Solectron, among many others.

With the success of using derivative instruments for risk management in the financial service industry, there has been much recent interest in exploring and extending the usage of options as a way to manage risks in other industries, including those closely associated with supply chain management. Indeed, as our results below will show, several existing forms of flexible supply contracts can be unified, and modeled with payoff functions that resemble call and put option contracts in the financial market.

In terms of pricing these options, however, there is a crucial distinction. Financial options are priced based on notions such as no-arbitrage and complete market, which support a certain martingale measure in computing the expected payoff function as the option price (refer to Hull 2002). These notions and ideas do not apply to the pricing of flexible supply contracts, which is often the result of a private negotiation process between two firms. The two parties bargain over prices and quantities of the orders, as well as costs and incentives associated with any flexibility in question. Indeed, the context is so different that it is not clear *a priori* whether in flexible supply contracts certain standard relations for financial options such as put-call parity will continue to hold or in what form.

The main contribution of this paper is to provide a formal approach to pricing flexible supply contracts. Specifically, we model the negotiation process between the supplier (seller) and the manufacturer (buyer) as a Stackelberg game, with the supplier being the leader. The equilibrium of the game takes the form of (a) the optimal order decision of the buyer, in terms of both the committed order quantity and the number of option contracts; and (b) the optimal pricing decision of the supplier, in terms of both the option price and the exercise price. In other words, the pricing of the option is, naturally, tied together with the buyer's ordering decisions, in the form of a game-theoretic equilibrium. This notion of equilibrium is reminiscent of the market equilibrium model of pricing financial options based on martingale measures, but it is also clearly quite different in that the equilibrium is associated with a two-player Stackelberg game. This difference notwithstanding, our model does lead to a parity relationship between the put and call options of flexible supply contracts.

Our model also generates considerable qualitative insights. First, it demonstrates that the options redistribute the risk among the two parties in shifting part of the buyer's risk due to demand uncertainty to the supplier; and the supplier, in turn, is compensated by the additional revenue obtained from the options. Second, it shows that a better alternative to the two parties' individual optimization is for them to negotiate a mechanism to share the profit improvement over the no-flexibility contract; and, furthermore, any sharing of this profit improvement can be represented by an option contract through a suitable choice of the contract parameters. Under mild conditions, this profit sharing mechanism will achieve channel coordination.

The rest of the paper is organized as follows. In the remaining part of this introduction, we briefly review the related literature. In §2, we start with a base model, a newsvendor formulation, which does not allow any flexibility. As preliminaries for later discussions, we also present the integrated supply chain model, followed by introducing the option model. In the next two sections, §3 and §4, we focus on the call option model, deriving the optimal decisions of the manufacturer (buyer) and the supplier (seller). These are followed by numerical studies in §5, where some of the results point to the profit sharing model detailed in §6. In §7, we examine the put option model, and derive the optimal solutions through a parity relation between the put and the call option models. Several possible extensions and follow-up issues are highlighted in the concluding section §8.

1.1 Literature Review

The majority of the literature relating to supply flexibility deals with the buyer's inventory decision making problem and/or the supplier's production problem under a given supply contract. The buyer's problem is usually formulated as a two-stage newsvendor problem with the initial order quantity being the decision variable in the first stage and an additional decision to update the initial order quantity within the range allowed by the quantity flexibility agreement in the second stage. The supplier's problem is to determine the production quantity in each of the two stages usually with different costs. Typically, the performance of a centralized supply chain is used as a benchmark for a decentralized supply chain where the supplier and the buyer make decisions individually based on their own interests. There are also different types of flexible contracts. For example, quantity flexibility, buy-back or returns, minimum commitment, and options are the types of flexible supply contracts that have appeared frequently in the recent literature. For a more general review of the supplier contract literature, we refer readers to Tsay, Nahmias & Agrawal (1999) for an excellent survey. Modeling and solution techniques for multi-period problems can also be found in Anupindi and Bassok (1999).

Brown and Lee (2003) study two-stage flexible supply contracts for advance reservation of capacity or advance procurement of supply. The objective is *not* to study the terms and conditions of new contracts or to coordinate benefits for the supply chain; rather, the paper emphasizes the impact of the so-called "demand signal quality" — essentially the correlation between demand signal and the actual demand — on order decisions.

One popular topic in the research of supply contracts with flexibility is to investigate various mechanisms that allow the supplier and the buyer to achieve channel coordination, i.e., to achieve the maximum joint profit that is equal to the total profit of the centralized supply chain in a decentralized setting. Eppen and Iyer (1997) analyze "backup agreements" in which the buyer is allowed a certain backup quantity in excess of its initial forecast at no premium, but pays a penalty for any of these units not purchased. They show that for certain parameter combinations, the use of backup agreements can lead to profit improvement for both parties.

Barnes-Schuster, Bassok and Anupindi (2002) provide an analysis to a two-period problem with options offered to provide flexibility to deal with demand uncertainty. Their paper focuses on deriving the sufficient conditions on the cost parameters that are required for channel coordination. It shows that in general channel coordination can be achieved only if the exercise price is piecewise linear. Araman, Kleinknecht and Akella (2001) consider the optimal procurement strategy using a mix of the long-term contracts and the spot market supply. They provide a necessary and sufficient condition for the contracts to achieve channel coordination. A new type of contract with a linear risk sharing agreement is introduced and shown to be able to achieve system efficiency and enable a range of profit split between the retailer and its long-term supplier. Ertogral and Wu (2001) analyze a bargaining game for supply chain contracting, where the buyer negotiates the order quantity and wholesale price with a supplier. They show that the channel coordinated solution is also optimal for both parties in *subgame perfect equilibrium*.

As illustrated by Barnes-Schuster *et al.*, individual rationality may be violated when channel coordination is achieved. Particularly, they conclude that the supplier makes zero profits if linear prices are used to achieve channel coordination in an option model. In such a case, the supplier is most likely unwilling to participate to achieve coordination. On the other hand, one can still maximize the joint profits of the two parties in a decentralized setting without necessarily achieving channel coordination, particularly when individual rationality is to be observed.

Existing studies in the literature focus mostly on deriving the conditions on prices for channel coordination. The issue of pricing the supply flexibility in a general setting and its role in supply contract negotiation has yet to be addressed in detail in the literature. A related model that addresses the option pricing issue in a slightly different setting is provided by Wu, Kleindorfer and Zhang (2002), where they consider a long-term supply contract between a seller and a buyer with a capacity limit specified in the contract. There is a reservation cost per unit of capacity that the buyer needs to pay in advance, as well as an execution cost per unit of output when the capacity is actually used. The paper by Wu *et al.* derives the seller's optimal bidding and buyer's optimal contracting strategies. An important difference between their model and ours is that there is no committed purchase quantity in the model of Wu *et al.*, while in our model the buyer is allowed to order a fixed quantity (charged at a base price) which both the supplier and buyer are committed to. The buyer can buy options to have the right to get an additional quantity of supply which can be exercised later if necessary.

A recent paper by Albeniz and Simchi-Levi (2003) studies a purchasing process between a buyer and many suppliers for option contracts in a single period supply environment. While the paper and ours do have some overlapping in topics studied, neither appears to be a subset or superset of the other; and in terms of both models and results, the distinction is much more pronounced than the commonality. For instance, their model includes multiple suppliers whereas ours focuses on a single supplier; their focus is on the equilibrium analysis in a Stackelberg game setting, while ours goes beyond the Stackelberg game by introducing a profit-sharing mechanism that allows the two parties to negotiate out the terms of the contract that are mutually beneficial. In addition, our analysis points out the limitation of the equilibrium solution in that it may not lead to channel coordination in general, and even when channel coordination is achieved the solution may still be unacceptable to the individual parties.

In our setting for the supply contract with options, we assume the base price is given and not part of the option/flexibility negotiation. (For instance, the base price follows from an earlier negotiation on a no-flexibility contract, or it was set in a broader framework involving other parties.) The supplier decides the price of options as well as the exercise price based on the manufacturer's initial order quantities for base purchases and options, while the manufacturer revises these quantities based on the prices that the supplier offers. Then the supplier is allowed to adjust the prices given the manufacturer's revised order quantities. The two parties exchange their offers back and forth until they reach an agreement. Furthermore, our model captures the impact of the competition from the spot market, which can be an alternative source for supply flexibility.

2 Overview of the Models

2.1 Notation and Given Data

Throughout the paper, we will use the following notation:

D	customer demand, supplied by the manufacturer
μ	expectation of D
σ	standard deviation of D
$F(\cdot)$	the distribution function of D
$\bar{F}(\cdot)$	$= 1 - F(\cdot)$
Z	the standard normal variate
$\Phi(\cdot)$	distribution function of the standard normal
$\phi(\cdot)$	density function of the standard normal
r	manufacturer's unit selling price
m	supplier's unit cost
w_0	unit base price charged by the supplier to the manufacturer
p_M	manufacturer's unit penalty for shortage
v_M	manufacturer's unit salvage value
v_S	supplier's unit salvage value

Consider a single-period, single-product model involving a manufacturer (buyer) and a supplier. At the beginning of the period, the manufacturer places an order to the supplier, based on its forecast of the demand. The supplier produces the order and delivers it to the manufacturer, before the end of the period, at which point demand is realized and supplied.

Let $D \geq 0$ denote the demand, a random variable with the distribution function, $F(\cdot)$, known at the beginning of the period. Let μ and σ denote the mean and the standard deviation of D. Each unit of the order costs m to the supplier, which sells it at a (wholesale) price of w_0

to the manufacturer, which turns it into a product that supplies demand at a (retail) price of r. At the end of the period, when supply and demand are balanced, any shortage incurs a penalty cost; and any surplus, a salvage value (or, inventory cost). These are denoted p_M (penalty) and v_M (salvage) for the manufacturer, and v_S for the supplier.

Throughout, we assume the following relations hold among the given data:

$$v_S \le m \le w_0, \qquad v_M \le w_0, \qquad w_0 \le r + p_M. \tag{1}$$

These inequalities simply rule out the trivial case in which the supplier or the manufacturer (or both) will have no incentive to supply any demand. Note, in particular, that p_M could be negative. For instance, if the manufacturer can buy additional units, after demand is realized, from the spot market, at a unit price of w_s. Then, $p_M = w_s - r$ can be negative if $w_s < r$. In this case, the third inequality in (1) simply stipulates that $w_0 \le w_s$.

To allow for sufficient model generality, we do not make any assumptions about the location of leftover inventory in terms of salvage values; specifically, we allow $v_S < v_M$, $v_S = v_M$, and $v_S > v_M$. (In the supply chain contracting literature, it is usually assumed that $v_M = v_S$; see Cachon (2004). Also refer to Lariviere (1999), where it is argued that any leftover inventory should always be salvaged at the same price as it can be salvaged in an integrated supply chain.)

2.2 The Newsvendor Model: No Flexibility

To start with, consider the base model, where there is no supply flexibility: the manufacturer can only order at the beginning of the period, and every unit is supplied to the manufacturer at the base price of w_0. This is the so-called newsvendor model. The manufacturer chooses its order quantity Q such that its expected profit is maximized:

$$\max_Q \; G_M^{NV}(Q) := r\mathsf{E}[D \wedge Q] + v_M \mathsf{E}[Q - D]^+ - p_M \mathsf{E}[D - Q]^+ - w_0 Q. \tag{2}$$

Making use of

$$D \wedge Q = Q - (Q - D)^+, \quad \text{and} \quad (D - Q)^+ = (Q - D)^+ - (Q - D),$$

we can rewrite the objective function in (2) as follows:

$$\max_Q \; G_M^{NV}(Q) = (r + p_M - w_0)Q - (r + p_M - v_M)\mathsf{E}[Q - D]^+ - p_M \mu. \tag{3}$$

Note that

$$\mathsf{E}[Q - D]^+ = \int_0^Q (Q - x)dF(x) = QF(Q) - \int_0^Q x dF(x) = \int_0^Q F(x)dx, \tag{4}$$

where the last equality follows from integration by parts. Hence, taking derivative w.r.t. Q on the objective function in (3) and letting it be zero, we have

$$(r + p_M - w_0) - (r + p_M - v_M)F(Q) = 0.$$

Since the objective functions in (3) is concave in Q (in particular, $[x]^+$ is a convex function), the solution to the above equation yields the optimal Q value:

$$Q_0 := F^{-1}\left(\frac{r + p_M - w_0}{r + p_M - v_M}\right). \tag{5}$$

Note that if $r + p_M = v_M$, which implies $w_0 = v_M$ in view of (1), the fraction on the right hand side above is defined as unity, resulting in an infinite Q_0 (or equal to the largest point in the support of the demand distribution), which is consistent with intuition.

The profit of the supplier in this case is simply

$$G_S^{NV}(Q) = (w_0 - m)Q, \tag{6}$$

as the supplier will produce and deliver the exact quantity ordered by the manufacturer, and undertake no risk at all.

When demand follows a normal distribution, we write $D = \mu + \sigma Z$, where Z is the standard normal variate, and Φ and ϕ below denote the distribution and density functions associated with Z. We have $F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$. Denote $\theta := \frac{r+p_M-w_0}{r+p_M-v_M}$. Then, we write

$$Q_0 = \mu + \sigma\Phi^{-1}(\theta) := \mu + k\sigma, \tag{7}$$

where k is often referred to as the "safety factor." In this case, (4) takes the following form:

$$\mathsf{E}[Q_0 - D]^+ = \sigma\mathsf{E}[k - Z]^+ = \sigma\int_{-\infty}^{k} \Phi(z)dz = \sigma[k\Phi(k) + \phi(k)], \tag{8}$$

where the second equality makes use of (4), and the third equality follows from integration by parts:

$$\begin{aligned}\int_a^b \Phi(x)dx &= b\Phi(b) - a\Phi(a) - \int_a^b x\phi(x)dx \\ &= b\Phi(b) - a\Phi(a) + \phi(b) - \phi(a).\end{aligned}$$

2.3 Integrated Supply Chain

Suppose both the supplier and the manufacturer constitute two consecutive stages of an integrated supply chain, which takes as input the raw materials (from exogenous sources), at cost

m, and supplies the finished product to external demand at a return of r. The penalty for not satisfying demand is p_M.

The unit salvage values are v_S and v_M, for the supplier and the manufacturer, respectively. Note that here we do not assume that $v_S \leq v_M$. Indeed, in certain applications, it can very well be that $v_M = 0$, i.e., manufactured goods, if unsold, will have no salvage value; whereas it will be relatively easy for the supplier to resell any surplus raw materials to other buyers.

Since v_S and v_M, are different, in the integrated supply chain, it is necessary to keep part of the order (or raw materials) at the first stage (the supplier) so as to get a better salvage value, if $v_S > v_M$. Let $Q + q$ be the total order quantity, of which q units are kept at the first stage (and the remaining Q units go to the second stage, the manufacturer). Those q units will only be used to supply demand when $D > Q$; otherwise, those units will be salvaged at v_S per unit.

The objective function for this integrated supply chain is:

$$\begin{aligned} G_I(Q,q) \quad := \quad & r\mathsf{E}[(Q+q)\wedge D] - p_M\mathsf{E}(D-Q-q)^+ - m(Q+q) \\ & + v_M\mathsf{E}(Q-D)^+ + v_s\mathsf{E}[(Q+q-D)^+ - (Q-D)^+]. \end{aligned}$$

Similar to §2.2, the above can be simplified to:

$$\begin{aligned} G_I(Q,q) \quad = \quad & (r+p_M-m)(Q+q) - (r+p_M)\int_0^{Q+q} F(x)dx \\ & + v_M \int_0^Q F(x)dx + v_S \int_Q^{Q+q} F(x)dx - p_M\mu. \end{aligned} \tag{9}$$

Clearly, when $v_S \leq v_M$, to maximize the above objective, we must have $q = 0$. For if $q > 0$, we can always reduce it to zero while increase Q to $Q + q$, and thereby increase the objective value. Similarly, when $v_S > v_M$, we must have $Q = 0$ in the optimal solution.

Hence, combining the two cases, we have the following objective, for the integrated supply chain:

$$\max_Q \; G_I(Q) := (r+p_M-m)Q - (r+p_M-\max(v_M,v_S))\mathsf{E}[Q-D]^+ - p_M\mu, \tag{10}$$

from which the optimal solution, denoted Q_I, is immediate:

$$Q_I = F^{-1}\left(\frac{r+p_M-m}{r+p_M-\max(v_M,v_S)}\right). \tag{11}$$

Note that in the integrated supply chain, the first two equations in (1) reduce to one:

$$m \geq \max(v_M, v_S). \tag{12}$$

That is, as there is no w_0 in the integrated supply chain, we assume $w_0 = m$. Also note that, in general, we have $Q_I \geq Q_0$.

Observe that the objective function in (10) relates to the objective functions in (3) and (6) as follows:

$$G_I(Q) \geq G_M^{NV}(Q) + G_S^{NV}(Q).$$

Since Q_I maximizes the left hand side, we have

$$G_I(Q_I) \geq G_M^{NV}(Q_0) + G_S^{NV}(Q_0). \tag{13}$$

That is, the profit of the integrated system dominates the sum of the manufacturer's profit and the supplier's profit. Channel coordination is achieved when (13) holds as an equality; i.e., when the supplier and the manufacturer make decisions individually (i.e., in a decentralized manner), but the sum of their individually maximized profits are equal to that of the integrated supply chain.

2.4 The Option Model

This will be the main model studied in this paper. There are two variations, the call option and the put option. We shall focus here and the next four sections on the call option model, as the put option can be related to the call option through a *parity* relationship established in §7.

The call option works as follows. At the beginning of the period, the manufacturer places an order of quantity Q, paying a price of w_0 for each unit. In addition, the manufacturer can also purchase from the supplier q (call) option contracts, at a cost of c per contract. Each option contract gives the manufacturer the right (but not the obligation) to receive an additional unit, at a cost w (exercise price of the option), from the supplier at the end of the period after demand is realized. Under this arrangement, the supplier is committed to producing the quantity $Q + q$. The supplier can salvage any unexercised options at the end of the period at a unit value of v_S. Clearly, this call option includes as a special case the existing practice of adding quantity flexibility to supply contracts, which will allow the buyer to order additional units, at a premium (corresponding to the option exercise price), up to a certain limit (corresponding to the number of option contracts), after the initial order is placed. (In the case of the put option, the manufacturer will have the right to sell, i.e., return, to the supplier any surplus units, up to q, at the exercise price, after demand is realized. The put option generalizes the existing practice of buy-back contracts. Refer to §7.)

We shall assume that the following relations,

$$c + w \geq w_0, \qquad c + v_M \leq w_0, \qquad r + p_M \geq c + w, \tag{14}$$

always hold. If the first inequality is violated, it would cost less to buy a unit via option than to place a regular order, which would make the regular order useless. If the second inequality is violated, i.e., if $w_0 - v_M < c$, then the option plan is never worthwhile, since buying a unit up front and (in the worst case) salvaging it later costs less costly than buying an option contract. As to the third inequality, consider the case of $p_M = w_s - r$ (recall w_s is the unit price from the spot market). Then, $r + p_M \geq c + w$ reduces to $w_s \geq c + w$; otherwise, the spot market will make the option plan superfluous.

The determination of c, w, Q and q is the result of the supply contract negotiation or bargaining process between the supplier and the manufacturer. This bargaining process can be modeled as a Stackelberg game, in which the supplier is the Stackelberg leader, meaning that the supplier will optimize its own profit when it decides on c and w while the manufacturer is the follower and has to accept the prices offered and thereby optimize its decision on Q and q. We further assume that both parties are rational, self-interested, and risk neutral (expected value maximizers).

In the next two sections, we study the optimal decisions of the manufacturer and the supplier, respectively.

3 Manufacturer's Order Decisions

The manufacturer's decision variables are (Q, q), so as to maximize the total expected profit:

$$\begin{aligned} G_M(Q,q) &:= r\mathsf{E}[D \wedge (Q+q)] + v_M \mathsf{E}[Q-D]^+ - w\mathsf{E}[(D-Q)^+ \wedge q] \\ &\quad - p_M \mathsf{E}[D-Q-q]^+ - w_0 Q - cq \\ &= -p_M \mu + (r + p_M - w_0)Q + (r + p_M - w - c)q \\ &\quad -(r + p_M - w)\mathsf{E}[Q+q-D]^+ - (w - v_M)\mathsf{E}[Q-D]^+. \end{aligned} \tag{15}$$

Note in this case, the total supply is up to $Q + q$ (hence the terms weighted by r and p_M), and the q option contracts cost cq up front, plus w for each one exercised after demand is realized (hence the term weighted by w).

Making use of (4), we can write the above objective as:

$$\begin{aligned} G_M(Q,q) &= -p_M \mu + (r + p_M - w_0)Q + (r + p_M - w - c)q \\ &\quad -(r + p_M - w) \int_0^{Q+q} F(x)dx - (w - v_M) \int_0^{Q} F(x)dx. \end{aligned} \tag{16}$$

Note that if we let $q = 0$, then the above reduces to the base model in (3).

Taking partial derivatives on the objective function in (16) w.r.t. Q and q, and setting them to zero, we get the following optimality equations:

$$\begin{aligned}(r + p_M - w_0) - (r + p_M - w)F(Q + q) - (w - v_M)F(Q) &= 0,\\ (r + p_M - w - c) - (r + p_M - w)F(Q + q) &= 0.\end{aligned}$$

Solving the two equations, we obtain:

$$Q = F^{-1}\Big(\frac{c + w - w_0}{w - v_M}\Big), \tag{17}$$

$$q = F^{-1}\Big(\frac{r + p_M - w - c}{r + p_M - w}\Big) - Q =: \tilde{Q} - Q. \tag{18}$$

For the above to be well defined, we need to have, in addition to the relations assumed in (1) and (14),

$$\frac{c + w - w_0}{w - v_M} \leq \frac{r + p_M - w - c}{r + p_M - w},$$

which reduces to:

$$(r + p_M - v_M)c + (w_0 - v_M)w \leq (r + p_M)(w_0 - v_M). \tag{19}$$

Proposition 1 *The objective function in (16) is (jointly) concave in (Q, q). Consequently, the manufacturer's optimal decisions on (Q, q) are as follows:*

(i) if (19) holds as a strict inequality, then the optimal Q and q follow (17) and (18);

(ii) if (19) holds as an equality, then the optimal $Q = Q_0$ in (5) and $q = 0$.

(iii) if (19) is violated, then the optimal $Q = Q_0$ in (5) and $q = 0$.

Proof. The (joint) concavity in (Q, q) follows directly from the $G_M(Q, q)$ expression in (15), taking into account that $[x]^+$ is an increasing and convex function (and hence preserves the convexity embodied in the linearity of $Q + q$) and that $w \geq v_M$ and $r + p_M \geq w$ as assumed in (14).

In case (i), Q and q are well defined following (17) and (18). In case (ii), (17) and (18) yield $q = 0$ and (19) entails $Q = Q_0$. In case (iii), (18) will yield a negative q. Therefore, due to the concavity of the objective, the optimal value must be at the boundary $q = 0$, and hence $Q = Q_0$ follows. □

In cases (ii) and (iii) of Proposition 1 the manufacturer has no incentive to adopt the option model. The expected profit of the manufacturer and the supplier in these cases is the same as in the newsvendor model.

Remark 2 There are two special cases of Proposition 1 (ii) that warrant special attention:

- $(c, w) = (0, r + p_M)$, which makes $\tilde{Q}$ undefined in (18). However, substituting this into (16) reduces the latter to the newsvendor objective function. Hence, the optimal solution is $Q = Q_0$ and $q = 0$.

- $(c, w) = (w_0 - v_M, v_M)$, which makes Q undefined in (17). Again, substituting this into (16) makes the objective interchangeable in Q and q. Hence, the optimal solution in this case is either $Q = Q_0$ and $q = 0$ or $Q = 0$ and $q = Q_0$, or any point in between.

Proposition 3 *The manufacturer's optimal decisions, (Q, q), satisfy the following properties:*

(a) Q is increasing in (c, w), q is decreasing in (c, w), and $Q + q$ is also decreasing in (c, w).

(b) $Q \leq Q_0 \leq Q + q$, where Q_0 is the newsvendor solution in (5).

Furthermore, the expected profit of the manufacturer is decreasing in (c, w).

Proof. (a) We only need to consider Case (i) in Proposition 1, since in the other case (Q, q) are constants. The argument of F^{-1} in (17) can be written as

$$1 - \frac{w_0 - (c + v_M)}{w - v_M},$$

which is increasing in both c and w, taking into account the inequalities in (14). (In particular, the first two inequalities there imply $w \geq v_M$.) Hence, Q is increasing in (c, w).

Similarly, we can write the argument of F^{-1} in (18) as

$$1 - \frac{c}{r + p_M - w},$$

which is decreasing in both c and w (again, taking into account (14)). Hence, $Q+q$ is decreasing in (c, w). This, along with the increasing property of Q, establishes that q is decreasing in (c, w).

(b) Again, we only need to consider Case (i) in Proposition 1, where $Q \leq Q_0$ follows from

$$\frac{c + w - w_0}{w - v_M} \leq \frac{r + p_M - w_0}{r + p_M - v_M},$$

which reduces to the inequality in (19). Similarly, $Q_0 \leq \tilde{Q} = Q + q$ follows from

$$\frac{r + p_M - w - c}{r + p_M - w} \geq \frac{r + p_M - w_0}{r + p_M - v_M},$$

which also reduces to (19).

For the last statement consider w first. From (16), we know that G_M involves w directly, and through Q and q. But,

$$\frac{\partial G_M}{\partial Q} \cdot \frac{\partial Q}{\partial w} = \frac{\partial G_M}{\partial q} \cdot \frac{\partial q}{\partial w} = 0,$$

since the derivatives of G_M w.r.t. Q and q are zero at optimality, we have

$$\frac{\partial G_M}{\partial w} = -q + \int_Q^{Q+q} F(x)dx \leq 0,$$

since $F(x) \leq 1$. Similarly, we have

$$\frac{\partial G_M}{\partial c} = -q + \frac{\partial G_M}{\partial Q} \cdot \frac{\partial Q}{\partial c} + \frac{\partial G_M}{\partial q} \cdot \frac{\partial q}{\partial c} = -q + 0 \leq 0;$$

hence, G_M is decreasing in c as well. □

Proposition 4 *Compared with the base case (no flexibility), the manufacturer's expected net profit is no less in the flexibility model. It is strictly higher if the inequality in (19) holds as a strict inequality.*

Proof. The first part is trivial, since letting $q = 0$ in (16), we recover the objective function of the newsvendor model in (3). That is, $Q = Q_0$ and $q = 0$ is a feasible solution to (16).

For the second part, it suffices to show

$$G_M(Q_0, q) > G_M^{NV}(Q_0),$$

for some q, since the left hand side will be dominated by the optimal objective value of the flexibility model. Comparing (3) and (16), we have

$$G_M(Q_0, q) - G_M^{NV}(Q_0) = (r + p_M - w - c)q - (r + p_M - w) \int_{Q_0}^{Q_0+q} F(x)dx.$$

Since the right hand side above is zero when $q = 0$, all we need is to establish that its derivative at $q = 0$ is positive, i.e., $r + p_M - w - c > (r + p_M - w)F(Q_0)$, or

$$\frac{r + p_M - w - c}{r + p_M - w} > \frac{r + p_M - w_0}{r + p_M - v_M}.$$

But from the proof of Proposition 3(b), we know the above is exactly the inequality in (19) holding as a strict inequality. □

4 Supplier's Pricing Decisions

The supplier wants to maximize the following objective function:

$$G_S(c,w) := w_0 Q + cq - m(Q+q) + w\mathsf{E}[(D-Q)^+ \wedge q] + v_S \mathsf{E}[q-(D-Q)^+]^+. \tag{20}$$

Note that

$$\begin{aligned} (D-Q)^+ \wedge q &= (D-Q)^+ - (D-Q-q)^+, \\ [q-(D-Q)^+]^+ &= q-(D-Q)^+ + (D-Q-q)^+. \end{aligned}$$

Hence, the supplier's objective function simplifies to:

$$G_S(c,w) = (w_0-m)Q + (c+v_S-m)q + (w-v_S)[\mathsf{E}(D-Q)^+ - \mathsf{E}(D-Q-q)^+].$$

From (4), we have

$$\mathsf{E}(D-Q)^+ = \mu - Q + \mathsf{E}(Q-D)^+ = \mu - \int_0^Q \bar{F}(x)dx.$$

Hence, the supplier's decision problem is as follows:

$$\max\, G_S(c,w) = (w_0-m)Q + (c+v_S-m)q + (w-v_S)\int_Q^{Q+q} \bar{F}(x)dx. \tag{21}$$

Note that if and when $Q = Q_0$ and $q = 0$, i.e., the manufacturer takes the newsvendor solution, then the supplier's profit also becomes what's in the newsvendor model:

$$G_S = (w_0 - m)Q_0 = G_S^{NV}(Q_0).$$

In addition, the decision variables, (c,w), should satisfy the following constraints, in view of (14) and (19):

$$\begin{aligned} c &\le w_0 - v_M, \qquad (22) \\ c + w &\ge w_0, \qquad (23) \\ (r+p_M-v_M)c + (w_0-v_M)w &\le (r+p_M)(w_0-v_M). \qquad (24) \end{aligned}$$

Note that the last inequality in (14) is superseded by the stronger one in (24), since

$$c + w \le \frac{c(r+p_M-v_M)}{w_0-v_M} + w \le r + p_M,$$

where the first inequality follows from $r + p_M \ge w_0$ (refer to (14)), and the second inequality is (24).

The supplier treats (Q, q) as functions of (c, w). Specifically, (Q, q) will follow the optimal solutions from the manufacturer's model in (17) and (18). Note that if the supplier knows that the manufacturer uses a Gaussian model to forecast demand, then knowing the manufacturer's order decisions (Q, q) is equivalent to knowing the demand distribution – the two parameters of the Gaussian distribution, its mean and variance, are uniquely determined by Q and q via (17) and (18).

Rewrite the objective function in (21) as follows:

$$\begin{aligned} G_S(c,w) &= (w_0 - m)Q + (c + w - m)q - (w - v_S)\int_Q^{Q+q} F(x)dx \\ &= (w_0 - w - c)Q + (c + w - m)\tilde{Q} - (w - v_S)\int_Q^{\tilde{Q}} F(x)dx. \end{aligned} \tag{25}$$

Taking partial derivatives upon the objective function w.r.t. c and w, we have:

$$\frac{\partial G_S}{\partial c} = q + [w_0 - w - c + (w - v_S)F(Q)]Q'_c + [c + w - m - (w - v_S)F(\tilde{Q})]\tilde{Q}'_c, \tag{26}$$

$$\begin{aligned} \frac{\partial G_S}{\partial w} &= q + [w_0 - w - c + (w - v_S)F(Q)]Q'_w + [c + w - m - (w - v_S)F(\tilde{Q})]\tilde{Q}'_w \\ &\quad - \int_Q^{\tilde{Q}} F(x)dx; \end{aligned} \tag{27}$$

where Q'_c, $\tilde{Q}'_c$, Q'_w and $\tilde{Q}'_w$ denote the partial derivatives of Q and $\tilde{Q}$ w.r.t. c and w. Let $f(x) := \frac{d}{dx}F(x)$ denote the probability density function (whenever it exists). From (17) and (18), we have

$$\begin{aligned} Q'_c &= [(w - v_M)f(Q)]^{-1}, \\ \tilde{Q}'_c &= [-(r + p_M - w)f(\tilde{Q})]^{-1}; \\ Q'_w &= Q'_c \bar{F}(Q), \\ \tilde{Q}'_w &= \tilde{Q}'_c \bar{F}(\tilde{Q}). \end{aligned}$$

Substituting the last two equations into (27), we have

$$\begin{aligned} \frac{\partial G_S}{\partial w} &= q + [w_0 - w - c + (w - v_S)F(Q)]\bar{F}(Q)Q'_c \\ &\quad + [c + w - m - (w - v_S)F(\tilde{Q})]\bar{F}(\tilde{Q})\tilde{Q}'_c - \int_Q^{\tilde{Q}} F(x)dx. \end{aligned}$$

Since $\frac{\partial G_S}{\partial c} = 0$ implies

$$[c + w - m - (w - v_S)F(\tilde{Q})]\tilde{Q}'_c = -q - [w_0 - w - c + (w - v_S)F(Q)]Q'_c, \tag{28}$$

we have

$$\frac{\partial G_S}{\partial w} = [w_0 - w - c + (w - v_S)F(Q)][\bar{F}(Q) - \bar{F}(\tilde{Q})]Q_c' + qF(\tilde{Q}) - \int_Q^{\tilde{Q}} F(x)dx.$$

Furthermore, from (17), we have

$$w_0 - w - c + (w - v_S)F(Q) = (v_M - v_S)F(Q).$$

Hence, $\frac{\partial G_S}{\partial w} = 0$ takes the following form:

$$(v_M - v_S)F(Q)[F(\tilde{Q}) - F(Q)]Q_c' + [qF(\tilde{Q}) - \int_Q^{\tilde{Q}} F(x)dx] = 0. \tag{29}$$

Note that when $v_M \geq v_S$, the first term on the left hand side above is non-negative ($Q_c' \geq 0$ follows from Proposition 3), and so is the other term. Hence, when $v_M \geq v_S$, we have $\frac{\partial G_S}{\partial w} \geq 0$. Consequently, the supplier will prefer a w as large as possible, only to be constrained by the inequality in (24). However, if this inequality becomes an equality, then we know the manufacturer will forego the options, and consequently leaving the supplier with no additional profit beyond the newsvendor model. Hence, the supplier will set the c value close to zero, and the w value just slightly below the spot price $r + p_M$. This way, the left hand side of (24) is slightly below its right hand side.

Proposition 5 *Given the demand distribution, the supplier's optimal decision* (c, w) *follows the two equations in (28) and (29) when* $v_M < v_S$. *When* $v_M \geq v_S$, *the supplier will charge a* c *value that is close to zero, and a* w *value that is just slightly below the spot price* $r + p_M$, *so that the inequality in (24) holds, with the left hand side only slightly less than the right hand side.*

The second case in the above Proposition appears to explain why in existing supply contracts with quantity flexibility, there is no up-front charge, i.e., $c = 0$. On the other hand, it also points to what is perhaps a disadvantageous position for the buyer in such contracts, to the extent that the supplier will end up with reaping virtually all the profit improvement (over the non-flexible contract). More along this line will be illustrated through examples in the next section.

5 Numerical Studies

To start with, consider an example with the following data:

$$v_S = v_M = 0,\ m = 50,\ r = 100,\ p_M = 50,\ \mu = 100,\ \sigma = 30.$$

The decisions of both the supplier (c, w) and the manufacturer $(Q, Q+q)$ in the option model are summarized in Table 1. The table also shows the optimal order decisions, Q_0, of the newsvendor model (NV) described in §2.2, and of the integrated supply chain (ISC) model, Q_I, from §2.3. Table 2 summarizes the corresponding optimal profits realized by the supplier, manufacturer, and the total supply chain. Also reported are the relative profit improvements (Δ) of the option model over the newsvendor model. G_{MS} denotes the combined profit of the supplier and the manufacturer. Negative values are stated in brackets. To avoid possible local optima, we used an exhaustive search to find the supplier's optimal (c, w) values, with a step-size of 0.05.

	supplier's decision		manufacturer's decision		NV	ISC
w_0	c	w	Q	$Q+q$	Q_0	Q_I
60	0.05	149.85	107.6	112.9	108	113
70	0.05	149.85	102.5	112.9	103	113
80	0.05	149.85	97.5	112.9	97	113
90	0.05	149.85	92.4	112.9	92	113
100	0.05	149.85	87.1	113.0	87	113

Table 1: Comparisons of the optimal decisions ($v_S = 0$).

	manufacturer's profit			supplier's profit			supply chain profit		
w_0	G_M	G_M^{NV}	Δ	G_S	G_S^{NV}	Δ	G_{MS}	G_{MS}^{NV}	Δ
60	2262	2262	0.0 %	1102	1076	2.4 %	3364	3338	0.8 %
70	1212	1212	0.0 %	2153	2050	5.0 %	3364	3262	3.2 %
80	212	212	0.0 %	3153	2925	7.8 %	3364	3137	7.3 %
90	(738)	(738)	0.0 %	4102	3696	11.0 %	3364	2958	13.7 %
100	(1635)	(1636)	0.1 %	4999	4354	14.8 %	3364	2718	23.8 %

Table 2: Comparisons of the optimal profits ($v_S = 0$).

Observe from the results that the supplier's optimal decision is always such that $c + w$ is close to $r + p_M$. (Recall, the latter can be interpreted as the unit price available in the spot market.) Indeed, according to Proposition 5, we know in this case (c, w) will fall below the line specified in (24), which is slightly stronger than $c + w \leq r + p_M$ as argued before. More specifically, to maximize its profit, the supplier in the above example charges an exercise price w that is only slightly lower than the spot market price, and uses a very minimal option price c to entice the manufacturer to buy into the flexibility.

Furthermore, observe that the manufacturer's total order quantity $Q + q$ is close to Q_I. In other words, the supplier's pricing decision pushes the combined profit of the two parties close to the profit of the integrated supply chain (which is the best that can be achieved), while the manufacturer's profit remains barely above its newsvendor level. The net effect is that the supplier gets to keep virtually the entirety of the profit improvement.

If $v_M > v_S$ the optimal decisions of the supplier and the manufacturer as well as the characteristics of the solution remain essentially the same as above, with a lesser profit improvement (again, mainly for the supplier).

Next, suppose the supplier has a substantial salvage value, $v_S = 30$ (as opposed to $v_S = 0$ in the example above) and $v_M = 0$. All other data remains the same. The results are summarized in Tables 3 and 4. All the observations described above are still valid, however there is one new twist. For low selling prices, $w_0 = 60$, the supplier charges a high option price c along with a minimal exercise price w. The intuition here is this. Recall that the supplier must produce the optional part q of the manufacturer's order, as well as the committed part Q. When the salvage value is high, the supplier can afford to give a low exercise price, and at the same time increase the option price as much as possible, i.e., constrained only by $c \le w_0 - v_M$ in (22).

	supplier's decision		manufacturer's decision		NV	ISC
w_0	c	w	Q	$Q+q$	Q_0	Q_I
60	58.4	1.7	43.7	108.1	108	129
70	0.05	149.7	102.5	129	103	129
80	0.05	149.7	97.4	129	97	129
90	0.05	149.7	92.3	129	92	129
100	0.05	149.7	87	129	87	129

Table 3: Comparisons of the optimal decisions ($v_S = 30$).

	manufacturer's profit			supplier's profit			supply chain profit		
w_0	G_M	G_M^{NV}	Δ	G_S	G_S^{NV}	Δ	G_{MS}	G_{MS}^{NV}	Δ
60	2285	2262	1.0 %	1540	1076	43.1 %	3825	3338	14.6 %
70	1213	1212	0.1 %	2491	2050	21.5 %	3704	3262	13.5 %
80	213	212	0.5 %	3566	2925	21.9 %	3779	3137	20.5 %
90	(736)	(738)	0.3 %	4581	3696	23.9 %	3845	2958	30.0 %
100	(1633)	(1636)	0.2 %	5537	4354	27.2 %	3904	2718	43.6 %

Table 4: Comparisons of the optimal profits ($v_S = 30$).

In view of the above results, we next impose a constraint on w: $w \le a w_0$, where $a > 0$ is a given parameter. We repeat the above examples, for both $v_S = 0$ and $v_S = 30$, and with a values of 0.7 and 1.2. The results are summarized in Tables (5) through (8). Qualitatively, the results are similar to the earlier ones without the additional constraint on w. In particular, the supplier's optimal exercise price w is at aw_0, unless w_0 is substantially below the spot market price (i.e., when $w_0 = 60$ or 70) and $v_S > v_M$, in which case the supplier opts for a high c value to compensate for the low w_0. Further, we observe that for a given salvage value v_S, the supplier can achieve a higher profit G_S when a becomes larger (at the expense of the manufacturer), and similarly, the sum of the profits of the supplier and manufacturer G_{MS} increases with a.

	supplier's decision					manufacturer's decision					supply chain		
w_0	c	w	G_S	G_S^{NV}	Δ	Q	$Q+q$	G_M	G_M^{NV}	Δ	G_{MS}	G_{MS}^{NV}	Δ
60	41.1	42	1083	1076	0.7%	104	109	2292	2262	1.3%	3375	3338	1.1%
70	42.7	49	2083	2050	1.6%	96	106	1258	1212	3.8%	3341	3262	2.4%
80	43.2	56	3008	2925	2.8%	88	103	287	212	5.7%	3295	3137	5.0%
90	42.8	63	3862	3696	4.5%	80	101	(621)	(738)	15.9%	3241	2958	9.6%
100	41.9	70	4644	4354	6.7%	71	98	(1465)	(1636)	10.5%	3179	2718	17.0%

Table 5: Comparisons of the optimal decisions ($a = 0.7$ and $v_S = 0$).

	supplier's decision					manufacturer's decision					supply chain		
w_0	c	w	G_S	G_S^{NV}	Δ	Q	$Q+q$	G_M	G_M^{NV}	Δ	G_{MS}	G_{MS}^{NV}	Δ
60	58.4	1.6	1540	1076	43.1%	44	108	2314	2262	2.3%	3854	3338	15.5%
70	67.7	2.3	2435	2050	18.8%	39	103	1268	1212	4.6%	3703	3262	13.5%
80	35.8	56	3314	2925	13.3%	76	109	464	212	118.9%	3778	3137	20.4%
90	36.7	63	4170	3696	12.8%	70	106	(449)	(738)	39.2%	3721	2958	25.8%
100	37.1	70	4950	4354	13.7%	62	103	(1304)	(1636)	20.3%	3646	2718	34.1%

Table 6: Comparisons of the optimal decisions ($a = 0.7$ and $v_S = 30$).

6 Channel Coordination: the Profit Sharing Model

From the results in §4, in particular Propositions 5 and the numerical results, we know that if $v_M \geq v_S$, the manufacturer does not gain any improvement in expected profit from the newsvendor model. These hence constitute the Nash equilibrium when either party individually optimizes its own objective.

In contrast, below we show how the supplier and the manufacturer can optimize in a *coor-*

	supplier's decision					manufacturer's decision					supply chain		
w_0	c	w	G_S	G_S^{NV}	Δ	Q	$Q+q$	G_M	G_M^{NV}	Δ	G_{MS}	G_{MS}^{NV}	Δ
60	28.6	72	1089	1076	1.2%	105	110	2290	2262	1.2%	3379	3338	1.2%
70	25.8	84	2107	2050	2.8%	98	108	1257	1212	3.7%	3364	3262	3.1%
80	21.8	96	3070	2925	5.0%	92	107	283	212	33.5%	3353	3137	6.9%
90	17.1	108	3987	3696	7.9%	86	107	(638)	(738)	13.6%	3349	2958	13.2%
100	12.0	120	4868	4354	11.8%	81	107	(1515)	(1636)	7.4%	3353	2718	23.4%

Table 7: Comparisons of the optimal decisions ($a = 1.2$ and $v_S = 0$).

	supplier's decision					manufacturer's decision					supply chain		
w_0	c	w	G_S	G_S^{NV}	Δ	Q	$Q+q$	G_M	G_M^{NV}	Δ	G_{MS}	G_{MS}^{NV}	Δ
60	58.4	1.65	1540	1076	43.1%	44	108	2314	2262	2.3%	3854	3338	15.5%
70	67.7	2.30	2435	2050	18.8%	39	103	1268	1212	3.6%	3703	3262	13.5%
80	15.1	96	3416	2925	16.8%	86	117	437	212	106.1%	3853	3137	22.8%
90	11.7	108	4380	3696	18.5%	82	118	(486)	(738)	34.1%	3894	2958	31.6%
100	7.9	120	5316	4354	22.1%	78	119	(1377)	(1636)	15.8%	3939	2718	44.9%

Table 8: Comparisons of the optimal decisions ($a = 1.2$ and $v_S = 30$).

dinated manner, as opposed to individually, so that they both will do better, in all parametric cases, than their newsvendor solutions.

First, note that combining the objective functions of both the manufacturer and the supplier in (16) and (21), we have:

$$
\begin{aligned}
G_{MS} &:= G_M(Q,q) + G_S(c,w) \\
&= (r + p_M - m)(Q+q) - (r + p_M)\int_0^{Q+q} F(x)dx \\
&\quad + v_M \int_0^{Q} F(x)dx + v_S \int_Q^{Q+q} F(x)dx - p_M\mu, \qquad (30)
\end{aligned}
$$

where Q and q follow (17) and (18), and c and w are only implicitly involved via Q and q. On the other hand, also notice that G_{MS} in the above expression has exactly the same form as G_I in (9), the objective function of the integrated supply chain.

Therefore, instead of pursuing their individual optimal solutions, the two parties can try to achieve channel coordination, in the sense of $G_{MS} = G_I$, or to minimize the gap $G_I - G_{MS}$. Throughout this section we denote $\hat{v} := \max(v_S, v_M) \leq m$

Proposition 6 *Suppose the supplier's decision on* (c, w) *lies on the line segment,*

$$(r + p_M - \hat{v})c + (m - \hat{v})w = (r + p_M)(m - \hat{v}), \tag{31}$$

between the points $(c_1, w_1) = (0, r_M + p_M)$ *and* (c_2, w_2)*, with*

$$c_2 = (m - \hat{v})\frac{r + p_M - w_0}{r + p_M - m} = (m - \hat{v})F(Q_0), \qquad w_2 = w_0 - c_2.$$

(Recall, Q_0*, following (5), is the manufacturer's newsvendor solution.) Also, suppose the manufacturer, given the supplier's decision, follows its optimal solution in (17) and (18). Then,*

(i) Every point on the line segment in (31) satisfies the constraints in (22) $\sim$ *(24).*

(ii) For every point on the line segment in (31) $(Q + q)(c, w) = Q_I$*. The manufacturer's firm order quantity* Q *is decreasing in* c*.*

(iii) When $\hat{v} = v_S = v_M$*, channel coordination is achieved on the whole line segment, namely,* $G_{MS} = G_I$*.*

(iv) When $\hat{v} = v_S$ *and* $v_S > v_M$*, channel coordination is achieved at* (c_2, w_2)*.*

(v) When $\hat{v} = v_M$ *and* $v_S < v_M$*, we have*

$$G_I - G_{MS} = (v_M - v_S)\int_{Q_0}^{Q_I} F(x)dx.$$

Proof. (i) First, the feasibility of (c_1, w_1) is trivial. Second, the line in (31), which starts at (c_1, w_1), has a slope that is steeper than the slope of the line in (24), and hence falls below the latter:

$$\frac{r + p_M - \hat{v}}{m - \hat{v}} \geq \frac{r + p_M - \hat{v}}{w_0 - \hat{v}} \geq \frac{r + p_M - v_M}{w_0 - v_M}.$$

The line in (31) then ends at (c_2, w_2), where it crosses $c + w = w_0$, the constraint in (23). Hence, every point on the line is feasible.

(ii) From (31), we have

$$\frac{r + p_M - c - w}{r + p_M - w} = \frac{r + p_M - m}{r + p_M - \hat{v}}.$$

Given the supplier's decision (c, w), the manufacturer follows its optimal decision to order

$$\tilde{Q} = F^{-1}\left(\frac{r + p_M - c - w}{r + p_M - w}\right) = F^{-1}\left(\frac{r + p_M - m}{r + p_M - \hat{v}}\right) = Q_I,$$

the order quantity in an integrated supply chain. The manufacturer's firm order quantity for an option contract on the line segment (31) is for every $c \in (0, c_2)$ given by

$$
\begin{aligned}
Q(c) &= F^{-1}\left(\frac{r+p_M-w_0-\frac{r+p_M-m}{m-v}c}{r+p_M-v-\frac{r+p_M-v_M}{m-v}c}\right) \\
&= F^{-1}\left(\frac{(r+p_M-w_0)(m-v)-(r+p_M-m)c}{(r+p_M-v)(m-v)-(r+p_M-v)c}\right) \\
&= F^{-1}\left(\frac{r+p_M-w_0}{r+p_M-v}-\frac{(w_0-m)c}{(r+p_M-v)(m-c-v)}\right), \\
Q'(c) &= -\frac{1}{f(Q(c))}\frac{(w_0-m)(m-v)}{(r+p_M-v)(m-v-c)^2} < 0.
\end{aligned}
$$

Hence, we have

$$G_I - G_{MS} = \hat{v}\int_0^{Q_I} F(x)dx - v_M\int_0^{Q} F(x)dx - v_S\int_Q^{Q_I} F(x)dx.$$

In (iii), when $\hat{v} = v_S = v_M$, the above becomes $G_I - G_{MS} = 0$. Hence, channel coordination is achieved.

In (iv), when $\hat{v} = v_S$ and $v_S > v_M$, the above becomes

$$G_I - G_{MS} = (v_S - v_M)\int_0^{Q} F(x)dx.$$

With (c_2, w_2), the manufacturer's decision becomes, $Q = 0$ and $q = \tilde{Q} = Q_I$, following (17) and (18). (Note, in particular, that $c_2 + w_2 = w_0$, and it can be directly verified that $w_2 > v_M$.)

In (v), when $v_M > v_S$, we have

$$G_I - G_{MS} = (v_M - v_S)\int_Q^{Q_I} F(x)dx.$$

Since the line in (31) and the line in (24) (with the latter holding as an equality) only meet at (c_1, w_1), there will always be a gap between Q and Q_I. The gap is minimized at (c_1, w_1), where from Remark 2, we know $Q = Q_0$, the newsvendor solution. □

Note that when $v_S \neq v_M$ it is crucial where leftover inventory is salvaged. It will be impossible to achieve the integrated supply chain's profit if the option contract results in some stock being salvaged at unfavorable terms. In case $v_S > v_M$ channel coordination is achieved if the manufacturer does not place a firm order and buys Q_I option contracts. In case (v), where $v_S < v_M$, any leftover inventory is salvaged at a lower price than in the integrated supply chain whenever realized demand is between Q and $Q + q$. However, the supplier cannot give sufficient incentive for the manufacturer to place a firm order of size Q_I.

Since channel coordination is achieved on the whole line segment (31) if $v_S = v_M$, the coordinating contract is not unique; rather, a continuum of coordinating contracts exists, resulting in different profit improvements for the manufacturer and the supplier.

Proposition 7 *Suppose as in Proposition 6, the supplier's decision (c, w) falls on the line in (31). Then, the expected profit of the manufacturer is decreasing in w and increasing in c; whereas the supplier's expected profit is increasing in w and decreasing in c.*

Proof. Consider the manufacturer's expected profit first. Same as in the proof of Proposition 3(b), except substitute c in the objective function G_M by

$$c = \frac{(r + p_M - w)(m - \hat{v})}{r + p_M - \hat{v}},$$

following (31). Taking into account that the derivatives of G_M w.r.t. Q and q are zero at optimality, we can modify the derivative, $\frac{\partial G_M}{\partial w}$, in the proof of Proposition 3(b) as follows:

$$\begin{aligned} \frac{dG_M}{dw} &= -\frac{r + p_M - m}{r + p_M - \hat{v}} q + \int_Q^{Q+q} F(x) dx \\ &= -F(Q_I)(Q_I - Q) + \int_Q^{Q_I} F(x)\, dx \leq 0 \end{aligned}$$

That is, G_M is decreasing in w. From the negative slope of the line in (31) it follows that G_M is increasing in c. Next, consider the supplier's expected profit. When $\hat{v} = v_S$, we know, from Proposition 6(iv),

$$G_S(c, w) = G_I - G_M(c, w) - (v_S - v_M) \int_0^{Q(c,w)} F(x)\, dx.$$

Since G_I is independent of (c, w), and $G_M(c, w) + (v_S - v_M) \int_0^{Q(c,w)} F(x)\, dx$ is just the manufacturer's expected profit function with v_M replaced by v_S which is still decreasing in w, G_S must be increasing in w.
When $\hat{v} = v_M$, from Proposition 6(v), we have

$$G_S(c, w) = G_I - G_M(c, w) - (v_M - v_S) \int_{Q(c,w)}^{Q_I} F(x)\, dx.$$

Here G_I is again constant. G_M is decreasing in w from our results above. $(v_M - v_S) \int_{Q(c,w)}^{Q_I} F(x)\, dx$ is decreasing in w, since Q is increasing in w of the line in (31) as can be deduced from Proposition 6(ii). Hence, G_S must be increasing in w. From the negative slope of the line in (31) it follows that G_S is decreasing in c for any v_S and v_M. □

Proposition 7 corresponds to Theorem 7 of Lariviere (1999), where the results are established in the context of buy-back contracts. Two interesting observations can be made. First, option contracts on the line segment (31), do not depend on the demand distribution, just like coordinating buy-back contracts in Lariviere (1999). Second, the manufacturer prefers greater flexibility with larger values of q, whereas the supplier's interest is just the opposite. In other words, each party wishes, quite naturally, to avoid as much as possible the risk of keeping excess stock.

In the case of $v_S = v_M$, a continuum of contracts exists that achieves channel coordination. This result, along with Proposition 7, further explains the phenomenon alluded to in Proposition 5, i.e., why the supplier wants to push c as low as possible so as to capture all the additional profit, but cannot quite set c to zero since this would revert to the no-flexibility newsvendor solution.

From the last two propositions, we know that when the supplier's decision falls on the line in (31), the manufacturer's expected profit is guaranteed to be no worse than its newsvendor solution, since the worst for the manufacturer happens at the end point $(c_1, w_1) = (0, r_M + p_M)$, where it opts for the newsvendor solution. The same, however, cannot be guaranteed for the supplier. Its worst case happens at the end point (c_2, w_2), which corresponds to, letting $Q = 0$ and $q = \tilde{Q} = Q_I$ in (25),

$$
\begin{aligned}
G_S(c_2, w_2) &= (c + v_S - m)Q_I + (w - v_S)\int_0^{Q_I} \bar{F}(x)dx \\
&= -(m - v_S)\bar{F}(Q_0)Q_I + (w - v_S)\int_0^{Q_I} \bar{F}(x)dx \\
&\leq -(m - v_S)\int_0^{Q_I} \bar{F}(x)dx + (w - v_S)\int_0^{Q_I} \bar{F}(x)dx \\
&= (w - m)\int_0^{Q_I} \bar{F}(x)dx \\
&\leq (w - m)\mathsf{E}(D).
\end{aligned}
$$

When $Q_0 \geq \mathsf{E}(D) = \mu$, a very likely scenario, then the above is dominated by the supplier's newsvendor profit, $(w_0 - m)Q_0$.

Therefore, to synthesize the above discussion, we propose that the supplier and the manufacturer work out an agreement to share their total expected profit G_{MS} as follows: the supplier receives αG_{MS} and the manufacturer receives $(1 - \alpha)G_{MS}$, with $\alpha \in (0, 1)$ being a parameter agreed upon by both parties, which must be such that both parties do no worse than the newsvendor solution. That is,

$$\alpha G_{MS} \geq G_S^{NV} \qquad \text{and} \qquad (1-\alpha) G_{MS} \geq G_M^{NV}. \tag{32}$$

From the above, we have the following boundaries on α

$$\frac{G_{MS} - G_M^{NV}}{G_{MS}} := \alpha_u \geq \alpha \geq \alpha_l := \frac{G_S^{NV}}{G_{MS}}.$$

One reasonable way to determine α is to require $\alpha/(1-\alpha) = G_S^{NV}/G_M^{NV}$ or,

$$\alpha = \frac{G_S^{NV}}{G_S^{NV} + G_M^{NV}}.$$

That is, each party receives the profit improvement proportionate to its profit in the newsvendor model. This will guarantee that α satisfies the constraints in (32), since $G_{MS} \geq G_S^{NV} + G_M^{NV}$.

To summarize, under this profit-sharing scheme, given the choice of α, the four decision variables, (c, w) and (Q, q), are determined by the following four equations:

- the equation in (31) that relates c and w;
- the two equations in (17) and (18) relating Q and $Q + q = Q_I$ to (c, w);
- the equation that $G_S = \alpha G_{MS}$, where G_S follows (21) and G_{MS} follows (30).

Figure 1 confirms that channel coordination can be achieved when $v_S = v_M$. Here the sum of optimal profits of the supplier and the manufacturer equals the profit of the integrated supply chain, and the two parties only need to decide how to split the total profit. For any α value determined through negotiation, the two parties can always find the corresponding option price c and exercise price w such that the desired profit-sharing scheme will be realized, i.e., the supplier's expected profit equals αG_{MS} and the manufacturer's $(1-\alpha)G_{MS}$. Note the α value is plotted against the second Y-axis on the right-hand side of the chart.

When $v_S > v_M$, channel coordination can still be achieved if the feasibility condition (32) is relaxed. This means one can maximize the expected total supply chain profit G_{MS} such that $G_{MS} = G_I$, at the price of the supplier's expected profit falling below its newsvendor profit G_S^{NV}. Figure 2 illustrates such a case. However, given (32), channel coordination cannot be achieved in this case simply because the supplier has no incentive to do so. But the two parties can still decide an α such that both of them will be better off than using the newsvendor solution.

When $v_S < v_M$, Figure 3 shows that there is a gap between the expected profit of the integrated supply chain G_I and the expected total profits of the two parties G_{MS}. In this case, channel coordination simply cannot be achieved, even with the supplier's profit falling below its newsvendor value.

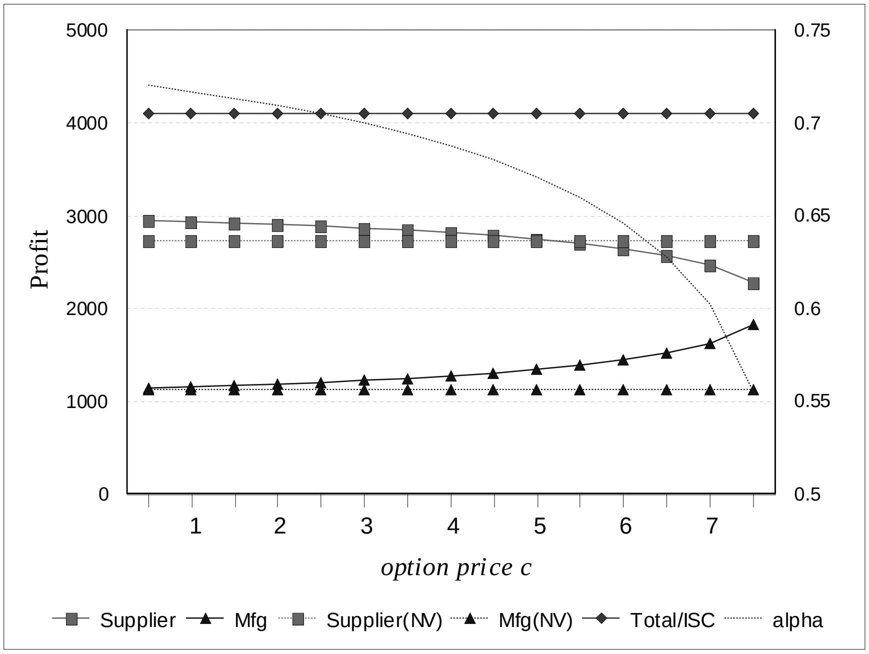

Figure 1: Profit sharing model with $v_S = v_M$

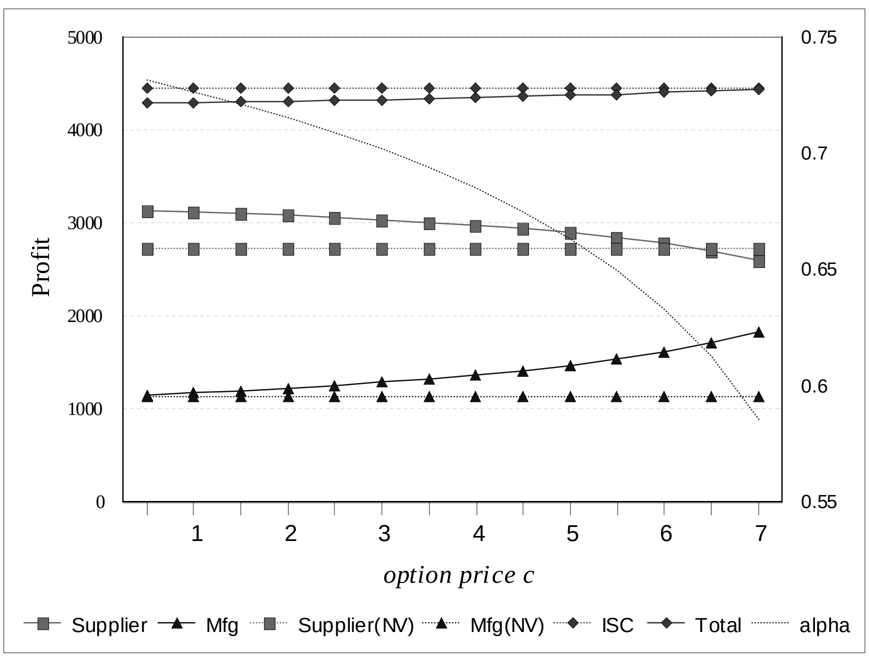

Figure 2: Profit sharing model with $v_S > v_M$

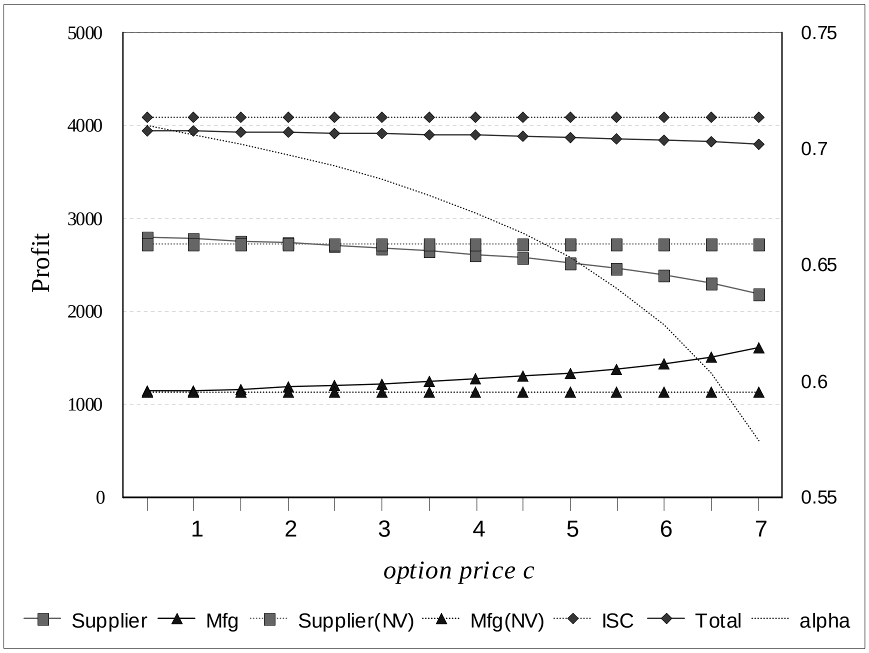

Figure 3: Profit sharing model with $v_S < v_M$

7 Put Option: the Put-Call Parity

In the put option model, the manufacturer, in addition to the up-front order quantity Q, at a unit price of w_0, purchases q put option contracts, at a unit price of p. Each such contract gives the manufacturer the right to return (i.e., sell back) to the supplier a surplus unit after demand is realized, at the exercise price of w. The supplier in this case is committed to producing the quantity Q and to taking back up to q units. As before, the supplier can salvage any returned units at a unit value of v_S.

Note that the put option contract is a generalization of the buy-back contract. With the buy-back contract, the supplier will buy back from the manufacturer, after demand is realized, any leftover units. Hence, this is equivalent to associating with every unit of the up-front order quantity Q a put option contract (at no additional charge), with the exercise price being the buy-back price.

To differentiate the put option from the call option, below we shall write the manufacturer's decision variables in the two models as (Q_p, q_p) and (Q_c, q_c).

The manufacturer's objective is to maximize the following expected profit:

$$\begin{aligned} G_M(Q_p, q_p) \quad &:= \quad r\mathsf{E}[D \wedge Q_p] + v_M \mathsf{E}[Q_p - q_p - D]^+ + w\mathsf{E}[(Q_p - D)^+ \wedge q_p] \\ &\qquad -p_M \mathsf{E}[D - Q_p]^+ - w_0 Q_p - p q_p \end{aligned}$$

$$
\begin{aligned}
&= -p_M\mu + (r + p_M - w_0)Q_p - pq_p \\
&\quad -(r + p_M - w)\mathsf{E}[Q_p - D]^+ - (w - v_M)\mathsf{E}[Q_p - q_p - D]^+. \qquad (33)
\end{aligned}
$$

The supplier wants to maximize the following objective function:

$$
\begin{aligned}
G_S(p, w) &:= w_0Q_p + pq_p - mQ_p - w\mathsf{E}[(Q_p - D)^+ \wedge q_p] + v_S\mathsf{E}[(Q_p - D)^+ \wedge q_p]^+ \\
&= (w_0 - m)Q_p + pq_p - (w - v_S)[\mathsf{E}(Q_p - D)^+ - \mathsf{E}(Q_p - q_p - D)^+]. \qquad (34)
\end{aligned}
$$

It turns out that the put option model relates directly to the call option model analyzed in the earlier sections through a parity relation as follows.

Proposition 8 *Suppose the following relations hold:*

$$
c - p = w_0 - w, \qquad (35)
$$

and

$$
Q_p = Q_c + q_c, \qquad q_p = q_c. \qquad (36)
$$

Then, the objective functions in (33,34) of the put option model are equal to the objective functions in (15,20) of the call option model:

$$
G_M(Q_p, q_p) = G_M(Q_c, q_c), \qquad G_S(p, w) = G_S(c, w). \qquad (37)
$$

Proof. Taking the difference between the two expressions in (15) and (33) and taking into account the relations in (36), we have

$$
\begin{aligned}
& G_M(Q_c, q_c) - G_M(Q_p, q_p) \\
&= -(r + p_M - w_0)q_c + (r + p_M - w - c + p)q_c \\
&= (p - c - w + w_0)q_c = 0,
\end{aligned}
$$

where the last equality follows from (35). Similarly, from (20) and (34), we have

$$
\begin{aligned}
& G_S(c, w) - G_S(p, w) \\
&= -(w_0 - m)q_c + (c - p + v_S - m)q_c + (w - v_S)q_c \\
&= (w - w_0 + c - p)q_c = 0. \qquad \square
\end{aligned}
$$

Making use of the above proposition, the solutions to the put option model can be summarized as follows:

Proposition 9 *In the put option model, the optimal decisions for the manufacturer are:*

$$Q_p = F^{-1}\Big(\frac{r+p_M-w_0-p}{r+p_M-w}\Big), \tag{38}$$

$$q_p = Q_p - F^{-1}\Big(\frac{p}{w-v_M}\Big). \tag{39}$$

Consequently, the relations in (35) and (36) do hold. And, the optimal decisions for the supplier follow those in the call option model, with the variables (c, w) changed to (p, w) following the parity relation in (35), and with (Q_c, q_c) replaced by (Q_p, q_p) via (36).

Proof. The manufacturer's optimal decisions, Q_p and q_p, are directly derived from taking derivatives on the objective function G_M in (33). Comparing Q_p and q_p with the decisions in the call option model confirms the relations in (35) and (36). Hence, the statement on the supplier's optimal decisions follows from Proposition 8. □

Note that the parity relation in (35) is in the same form as the put-call parity of financial options, specifically, European options on stocks paying no dividend, with w_0 being the stock price at time zero and w being the exercise price; refer to Hull (2002). (Here we have ignored the discounting of the exercise price, which is paid at the end of the period, to time zero.)

Also note that with the parity in (35), the inequalities in (14) that characterize the parameters in the call option model change to the following, which now govern the parametric relations for the put option:

$$p \geq 0, \qquad w-p \geq v_M, \qquad r+p_M \geq w_0+p,$$

The inequality in (24) takes the following form in the put option model:

$$(r+p_M-w_0)w - (r+p_M-v_M)p \quad \geq \quad (r+p_M-w_0)v_M.$$

Also, since $c > 0$ we deduce from (35) that

$$w - p \leq w_0.$$

Similarly, based on (35) and (36) and following Proposition 9, we can derive the supplier's decisions for the put option by modifying the solutions in the call option models.

The supplier's optimal decision (p, w) follows the following two equations, provided $v_M < v_S$:

$$[w_0 + p - m - (w-v_S)F(Q_p)]Q_p' = -q_p + [p - (w-v_S)F(Q_p-q_p)](Q_p' - q_p'),$$

and

$$(v_M - v_S)F(Q_p-q_p)[F(Q_p) - F(Q_p-q_p)](Q_p'-q_p') + [q_pF(Q_p) - \int_{Q_p-q_p}^{Q_p} F(x)dx] = 0;$$

where Q'_p and q'_p denote the partial derivatives of Q_p and q_p with respect to p:

$$Q'_p = -[(r + p_M - w)f(Q_p)]^{-1}, \qquad q'_p = Q'_p - [(w - v_M)f(Q_p - q_p)]^{-1}.$$

When $v_S \leq v_M$, the optimal (p, w) is a point inside the feasible region with w just below the spot price.

Next, we use the relations in (35) and (14) to derive numerical results for the put option model corresponding to the call option results presented in Tables 1 and 4. The results are summarized in Tables 9 and 10 for the cases $v_S = 0$ and $v_S = 30$, respectively. We have also verified these results by directly solving the optimization problems in (33) and (34) for the put option model. (The direct optimization returned the same results in most cases. In a couple of cases which it did not, the cause is numerical, basically due to discretization.)

	call option				put option			
w_0	c	w	Q_c	q_c	p	w	Q_p	q_p
60	0.01	149.97	107.60	5.32	89.98	149.97	112.92	5.32
70	0.01	149.97	102.51	10.41	79.98	149.97	112.92	10.41
80	0.01	149.97	97.49	15.43	69.98	149.97	112.92	15.43
90	0.01	149.97	92.40	20.53	59.98	149.97	112.93	20.53
100	0.01	149.97	87.07	25.85	49.98	149.97	112.92	25.85

Table 9: Numerical example for put options corresponding to Table 1

	call option				put option			
w_0	c	w	Q_c	q_c	p	w	Q_p	q_p
60	59.32	0.69	34.49	73.32	0.01	0.69	107.81	73.32
70	0.01	149.94	102.50	26.52	79.95	149.94	129.02	26.52
80	0.01	149.94	97.48	31.54	69.95	149.94	129.02	31.54
90	0.01	149.94	92.39	36.64	59.95	149.94	129.03	36.64
100	0.01	149.94	87.06	41.96	49.95	149.94	129.02	41.96

Table 10: Numerical example for put options corresponding to Table 3

Similar to the call option case, when put option and exercise prices are both determined by the supplier, it can be observed that the buyer (manufacturer) may not have a significant profit gain by using put options over its newsvendor profit. Hence, as in the call option model, we restrict w within a certain range to allow the buyer to share some profit gains. Tables 11 and 12 show the corresponding results for a put-option model with $a = 0.7$ and $a = 1.2$ respectively.

	call option				put option			
w_0	c	w	Q_c	q_c	p	w	Q_p	q_p
60	41.1	42	104	5	23.1	42	109	5
70	42.7	49	96	10	21.7	49	106	10
80	43.2	56	88	15	19.2	56	103	15
90	42.8	63	80	21	15.8	63	101	21
100	41.9	70	71	27	11.9	70	98	27

Table 11: Numerical example for put options corresponding to Table 5

	call option				put option			
w_0	c	w	Q_c	q_c	p	w	Q_p	q_p
60	28.6	72	105	5	40.6	72	105	110
70	25.8	84	98	10	39.8	84	98	108
80	21.8	96	92	15	37.8	96	92	107
90	17.1	108	86	21	35.1	108	86	107
100	12.0	120	81	26	32.0	120	81	107

Table 12: Numerical example for put options corresponding to Table 7

Furthermore, a profit sharing scheme similar to the one described in §6 for call options can be designed for put options as well.

8 Concluding Remarks

Our results suggest that for the option contract to be effective, the exercise price w should either be fixed (e.g., as a fraction of w_0) or constrained (e.g., $w \leq w_0$). Otherwise, the supplier will reap most of the profit improvement, leaving the manufacturer with little incentive to buy the options. Instead of having the supplier and the manufacturer pursue a Stackelberg game to come up with pricing and ordering decisions respectively, a better alternative is for the two parties to coordinate and maximize their combined profits, aiming at channel coordination if possible. The two parties can then negotiate a mechanism to share the profit improvement over the newsvendor model. We have shown that this mechanism can be translated equivalently into an option contract, in terms of the pricing and ordering decisions of the two parties.

Combining the call and put options, we can readily extend our models to construct a flexible contract that will allow the manufacturer (buyer) to purchase both call and put options, with quantities q_c and q_p, respectively, in addition to the up-front quantity Q. This way, the manufacturer can acquire up to q_c more units should the realized demand be higher than Q, or return to the seller up to q_p units if the demand turns out to be lower than Q. Thus, the manufacturer will have to decide on three variables, (Q, q_c, q_p). The supplier, in turn, will have four decision variables: call and put option prices, c and p; and the two exercise prices, w_c and w_p.

We have not addressed the issue of risk profiles associated with the two parties' decisions. For instance, although the supplier is the main beneficiary of the option model, the improvement is in terms of *expected* profit, whereas in its newsvendor solution, the profit is deterministic, i.e., there is no risk involved. Hence, it is important to characterize what is the risk associated with the profit improvement the supplier can expect from the option model. This can take several forms, such as the variance of the profit, or the probability that the profit will exceed that of its newsvendor solution. These will be the subject of our further studies.

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