Contractual Remedies to the Holdup Problem: A Dynamic Perspective

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Abstract

An important theme of modern contract theory is the role contracts play to protect parties from the risk of holdup and thereby encouraging their relationship specific investments. While this perspective has generated valuable insights about various contracts, the underlying models abstract from realistic investment dynamics. We reexamine the role of contracts in a dynamic model that endogenizes the timing of investments and trade. The resulting interaction between bargaining and investment significantly alters the insights learned from static models. We show that contracts that would exacerbate the parties' vulnerability to holdup — rather than those protecting them from the risk of holdup — can be desirable. Specifically, separate ownership of complementary assets can be optimal, an exclusivity agreement can protect the investments of its recipient, and trade contracts can be beneficial with a purely cooperative investment, much in contrast to the existing results (based on static models).

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1 Introduction

The holdup problem arises when a business partner makes relationship specific investments that are susceptible to ex post expropriation by his associates. Examples of such investments include acquisition of firm specific skills by workers, subcontractors' efforts to customize their parts to the special needs of manufacturers, and a firm's relocation of its plant adjacent to its partner. These investments create more surplus within a relationship than without. Hence, absent any special safeguard, the fear of holdup may lead the parties to underinvest relative to the efficient level. An important theme of the modern contract literature is how contracts can protect the investors from the risk of holdup.[1]

While this literature has generated valuable insights about various contracts, the underlying models abstract from the realistic investment dynamics present in many business relationships. Specifically, the extant models assume that the partners can invest only once, and only at a certain time. In practice, however, the timing of investment and bargaining is — at least to some extent — chosen endogenously by the parties, and the investment and bargaining stages are often intertwined. For instance, the Department of Defense may negotiate to order a weapons system from a contractor based on his current technical knowledge, or it may decide to wait until the latter invests more in R & D and develops a better technology. A similar dynamic interaction of investment and bargaining arises in the development of new building construction, advertising pilots or software projects.[2] We develop a model that allows for the timing of investment and trade to be endogenous. Specifically, in our model, an investing party accumulates specific investments in each period until either the parties agree to trade or their negotiation breaks down.

We study the effects of ex ante (incomplete) contracts in such an environment. In our

[1]A range of organizational and contract forms have been rationalized as safeguards against holdup: Examples include vertical integration (Klein, Crawford and Alchian, 1978; Williamson, 1979), a property rights allocation (Grossman and Hart, 1986; Hart and Moore, 1990), contracting on renegotiation rights (Chung, 1991; Aghion, Dewatripont and Rey, 1994), option contracts (Nöldeke and Schmidt, 1995), and trade contracts (Edlin and Reichelstein, 1996; Che and Hausch, 1999).

[2]Such dynamic interaction is also present in the publishing of academic articles. Consider the editorial procedure at the Berkeley Electronic Journals. Here submission is simultaneous to four vertically differentiated journals. Unless the initial submission is rejected, in principle the author is offered a choice of acceptance at a lower level or following a substantial revision — incremental investment after the negotiation has commenced — acceptance at a higher level.

model, two parties, 1 and 2, initially sign a contract, say m, that may specify various aspects of their relationship, such as asset ownership, future trade decisions or exclusivity of their trading relationship. This contract may be renegotiated over time, but it affects the outcome of renegotiation by determining its threat point. Specifically, party 1 (the investor) begins by making a sunk investment, say $k \in \mathbb{R}_+$, and then a randomly-chosen proposer offers to his partner a share of surplus that would obtain through trading. If that offer is accepted, trade occurs according to the agreement, and the game ends. If the offer is rejected, then negotiation breaks down irrevocably with probability $1-\delta \in (0,1)$. In that case, the contract in place takes effect, generating "contract payoffs," say $\psi_1(k,m)$ and $\psi_2(k,m)$, respectively, to the parties. With the remaining probability δ, the game moves to the next period without trade, and the same process is repeated; i.e., party 1 can make an incremental investment, which is followed by a new bargaining round with a random proposer, and the game goes on until either trade occurs or bargaining breaks down.

Absent investment, the unique subgame perfect equilibrium of this bargaining model replicates the Nash Bargaining Solution (NBS), just as in Binmore, Rubinstein and Wolinsky (1986).[3] Thus, our full-fledged model can be seen as making the stake of bargaining in their model endogenous through investment.

If the investment is allowed to occur only in the first period or equivalently if $\delta = 0$ (i.e., when breakdown is certain following disagreement), the model coincides with the majority of existing contract models which use the NBS with the contract payoffs serving as the threat point.[4] Our full-fledged model provides a natural dynamic extension of these that endogenizes the timing of investment and trade decisions. For this reason, the equilibrium results of our model are easily comparable to those in much of the contract literature.

We show that allowing for investment dynamics yields a new insight on the effects of contracts. The recent contract literature has focused on the inappropriability of returns *at the margin* as a source of inefficiency and a rationale for contract intervention. More precisely,

[3]Binmore, Rubinstein and Wolinsky (1986) present the alternating-offer version of this model. They need to take the limit as the breakdown probability tends to zero in order to eliminate the first-mover advantage and to show the full equivalence with the NBS. Instead, we assume that in each bargaining round the proposer is chosen randomly, which yields the same qualitative effect.

[4]Examples include Grossman and Hart (1986), Hart and Moore (1990), Edlin and Reichelstein (1996), Che and Hausch (1999), Hart and Moore (1999), Segal (1999), and Segal and Whinston (2000, 2002). The alternative approach treats contracts as affecting the outside option payoffs of bargaining (MacLeod and Malcomson, 1993; Chiu, 1998; de Meza and Lockwood, 1998).

the literature has focused on how parties' contract payoffs, $\psi_i(k,m)$, vary with investment, k. If an investor's contract payoff increases with her specific investment, it effectively reduces the specificity of her investment *at the margin*, thus reducing the exposure of her marginal investment return to expropriation. Such a contract thus improves her incentives to make the specific investment. Alternative contracts can be ranked along this logic: If a contract reduces the investor's marginal specificity exposure more than another contract, the former contract will protect her investment return better than the latter, thus inducing a higher level of investment.[5] By the same token, if two contracts entail precisely the same *marginal* specificity, their effect on investment incentives will be precisely the same, even when they differ in terms of the investors' exposure to *absolute* specificity that they induce. Hence, the absolute degree of specificity investors are exposed to does not matter in the existing models. In our dynamic model, absolute specificity will matter for a sufficiently large δ.

This point can be illustrated via a simplified example. Suppose party 1 has a binary choice: either to "not invest" or to "invest". Investment costs her $C > 0$ but enables the parties to realize a gross surplus of ϕ_I through trade, whereas "not invest" costs nothing but yields an opportunity to realize ϕ_N. Assume that $\phi_I - C > \phi_N$ (so investment is efficient). Each party has equal probability of becoming the proposer in bargaining. In case of bargaining breakdown, the parties collect their respective contract payoffs, $\psi_1(m)$ and $\psi_2(m)$, which, as presumed above, depend on the contract in place, m, but *not* on whether party 1 invests or not. (That is, the contract does not affect marginal specificity.)

Suppose $\delta = 0$, just as in the static models. Then, the contract payoffs are realized unless the parties agree to trade in the first period, so, given choice $j = I, N$, party 1 will receive $\psi_1(m) + \frac{1}{2}(\phi_j - \psi_1(m) - \psi_2(m)) = \frac{1}{2}(\phi_j - \psi_2(m) + \psi_1(m))$ from negotiation. Consequently, she will invest if and only if

$$(1) \qquad \tfrac{1}{2}(\phi_I - \psi_2(m) + \psi_1(m)) - C \geq \tfrac{1}{2}(\phi_N - \psi_2(m) + \psi_1(m)) \quad \Longleftrightarrow \quad \tfrac{1}{2}\phi_I - C \geq \tfrac{1}{2}\phi_N,$$

regardless of the contract in place, m. As argued above, contracts affecting only absolute specificity thus have no effect on the outcome if $\delta = 0$.

[5]For example, the celebrated prescription of Grossman-Hart-Moore (GHM) that complementary assets should be owned together, can be understood in this light. The common ownership of assets lowers the owner's marginal specificity exposure at the expense of raising the non-owner's. If complementarity of assets were to mean, as was assumed by GHM, that the former's gain exceeds the latter's loss, then common ownership would result in better aggregate incentives than separate ownerships.

If $\delta \approx 1$, however, contracts affecting only absolute specificity do matter. In this case, "invest" is supported in an equilibrium if and only if

$$(2)\qquad \tfrac{1}{2}(\phi_I - \psi_2(m) + \psi_1(m)) - C \geq \psi_1(m) \quad \Longleftrightarrow \quad \tfrac{1}{2}\phi_I - C \geq \tfrac{1}{2}(\psi_2(m) + \psi_1(m)).$$

This can be seen as follows. Suppose party 1 invests. Then, since no further investment is possible, the ensuing subgame coincides with the standard random-proposer bargaining game with a fixed surplus (and probabilistic breakdown). That game has a unique (subgame perfect) equilibrium in which party 1 receives the payoff in the LHS of (2). Since she can guarantee the payoff of $\psi_1(m)$ by not investing and perpetually inducing rejection, (2) is necessary to support "invest" in equilibrium. The condition is also sufficient to support "invest" if $\delta \approx 1$. Suppose party 1 adopts the strategy of "investing whenever she hasn't invested before." Her payoff from investing is again the LHS of (2), but her deviation payoff is reduced (relative to the RHS of (1)) by the prospect that she will invest next period in case of disagreement. If no agreement is reached, the pie will grow due to party 1's investment. Since the investment cost will be borne solely by the investor, the prospect of the investment causes party 2 to demand more credibly to settle, leaving less for the investor when she deviates. Given (2), the "investment" strategy thus becomes self-enforcing.[6]

The condition on the right in (2) shows that absolute specificity matters, and in particular that a lower aggregate contract payoff is more likely to support "invest." This means contracts that would increase the parties' vulnerability to holdup, rather than those protecting them from it, can be desirable. This insight will be seen to provide novel predictions on well known contracts, such asset ownership structures, exclusivity agreements, and trade contracts.

Several papers have developed somewhat similar insights, though in differing modelling contexts. Halonen (2002) shows in a repeated-game model that joint ownership of an asset

[6]More precisely, suppose party 1 deviates and chooses "not invest." Since she *will invest* next period if it arrives, party 2's continuation payoff following no agreement – and, therefore his reservation utility – is $\delta\frac{1}{2}(\phi_I - \psi_1 + \psi_2) + (1-\delta)\psi_2$. Thus, party 1's payoff from "not invest" is at most

$$\max\left\{\phi_N - \delta\tfrac{1}{2}(\phi_I - \psi_1 + \psi_2) - (1-\delta)\psi_2,\, \delta[\tfrac{1}{2}(\phi_I - \psi_2 + \psi_1) - C] + (1-\delta)\psi_1\right\}.$$

The first payoff is received if party 1 offers party 2's continuation payoff, which he will accept; the second payoff is received if party 1 offers a lower amount (which party 2 will reject) or if party 2 becomes the proposer (in which case he will offer party 1's net continuation payoff, $\delta[\frac{1}{2}(\phi_I - \psi_2 + \psi_1) - C] + (1-\delta)\psi_1$, which she will accept). Given (2) and that investment is efficient, for $\delta \approx 1$, both terms are dominated by party 1's "invest" payoff — the LHS of (2).

strictly dominates single ownership for intermediate values of players' discount factor, δ, since the former can make the repeated game punishment more severe. Baker et al. (2001, 2002) also demonstrate that the absolute payoff levels can affect the efficiency ranking of different ownership structures in a repeated trade setting. The repeated trade opportunities assumed in these papers make the folk theorem of repeated games applicable, which implies that an efficient outcome is sustainable as $\delta \to 1$, irrespective of the underlying organizational arrangements. In this sense, the organizational issues become irrelevant for a sufficiently large δ in these papers. By contrast, the parties have a single trading opportunity in our model (just as in the standard holdup problem), which makes the folk theorem inapplicable. Indeed, the organizational issues remain relevant when $\delta \approx 1$ in our model. Matouschek (2004) studies the effects of ex ante contracts on the ex post trading (in)efficiencies when the parties have two-sided asymmetric information (*à la* Myerson-Sattherthwaite), but have no ex ante investments. Contracts inducing low disagreement payoffs can facilitate efficient agreement (but prove more costly when agreement fails). Most closely related to the current paper is its prequel, Che and Sákovics (2004). That paper studied a dynamic holdup model much like the current one, but *without* the possibility of breakdown – and therefore, effectively *in the absence of ex ante contracts*. The roles of ex ante contracts are the focus of the current study.

The rest of the paper is organized as follows. Section 2 describes the model, establishes several benchmarks and reviews the results of the (existing) static contract models. Section 3 analyzes the equilibria in our dynamic contract model and establishes the effects of contracts. Section 4 then applies the results to several well-known contracts/organizations. Section 5 concludes.

2 The model

2.1 Description of the model

Two risk-neutral parties, 1 and 2, can create a surplus from trade. The joint surplus from trade at a given date increases with the aggregate investment party 1 ("investor") has accumulated. If the parties agree on a trade decision, $q \in Q$, with the cumulative investment, $k \in K$, by party 1 (measured in the cost of investment), then it creates the joint surplus of $\Phi(q, k)$ (gross of the investment cost). The set of feasible trades, Q, is a compact subset of $\mathbb{R}^m$ for some $m \in \mathbb{N}$, and contains a null trade q_0 that yields a zero surplus, i.e., $\phi(q_0, \cdot) \equiv 0$. We assume that $K = [0, \bar{k}]$ for some $\bar{k} > 0$.[7] As will become clear, it is useful to focus on the efficient trade decision conditional on the level of investment,

$$\phi(k) := \max_{q \in Q} \Phi(q, k),$$

which we assume is well defined. Further, we assume that the efficient surplus function, $\phi(\cdot)$, is strictly increasing, strictly concave and differentiable.

Investments and trade can occur at any discrete period, $t = 1, 2, ...$, in the shadow of a contract, $m \in \mathcal{M}$, signed at period $t = 0$, for some arbitrary set of contracts $\mathcal{M}$. The subsequent time line follows the extensive form sketched in the introduction. To be more precise, each period $t \geq 1$ (if it is reached) consists of two stages: first investment and then bargaining. In the investment stage, party 1 makes a non-negative incremental investment, which adds to the existing stock. Once it is made, the investment is sunk and its level becomes a common (yet unverifiable) knowledge to the parties. In the bargaining stage, a party is chosen randomly to offer to his partner a share of the surplus that would result from trade at that point. We assume that party 1 is chosen with probability $\alpha \in (0, 1)$, and party 2 is chosen with the remaining probability, $1 - \alpha$. If the offer is accepted, then trade takes place according to the agreed-upon terms, and the game ends. If the offer is rejected, then, with probability $\delta \in [0, 1)$, the game moves to the next period without trade, and the same process is repeated; i.e., party 1 can make an incremental investment, which is followed by a new bargaining round with a random proposer. With probability $1 - \delta$, bargaining breaks down irrevocably, and the two parties collect their *contract payoffs*, $\psi_1(k; m)$ and $\psi_2(k; m)$, respectively,[8] when the cumulative investment up to that point is k.

Our model is general enough accommodate a broad set of circumstances in terms of the underlying environment and the allowed contracts/organizations. In particular, several well-known contract problems are included in our model.[9]

[7] All of our results remain unchanged, although they become cumbersome to present, if K is discrete.

[8] In some occasion, we will refer to these payoffs will be also referred to as "status quo payoffs."

[9] In principle, a more sophisticated contract, e.g., one requiring exchanges of messages, can be incorporated into our model, with ψ_i interpreted as the equilibrium payoff of party i in that contract (sub)game. Of course, there is the issue of how these latter payoffs are determined and what contract payoffs are feasible. These are difficult questions to address even in the static model (as is well known from the debates on the incomplete contract paradigm). The more complex extensive form makes our dynamic model even less suitable for analyzing, let alone likely to offer any new insight on, these problems. For these reasons, we do not address the question of optimal contracts, limiting attention instead to several well known contracts and organization

Example 1 ***(Ownership contracts)*** *The asset ownership model of Grossman-Hart-Moore can be represented with m denoting a partition of total physical assets, $\mathcal{A}$, into assets owned by party 1, A_1^m, and the assets owned by party 2, A_2^m, and $\mathcal{M}$ denoting the set of all such partitions. Suppose that, given allocation $m \in \mathcal{M}$, the two parties realize the surplus of $\pi_1(k; A_1^m)$ and $\pi_2(k; A_2^m)$ from individual production, respectively, when their team production fails. This model is subsumed in our model by setting the contract payoffs $\psi_1(k; m) := \pi_1(k; A_1^m)$ and $\psi_2(k; m) := \pi_2(k; A_2^m)$.*

Example 2 ***(Trade contracts)*** *A seller (party 1) delivers a good to a buyer (party 2). If the seller invests k, trading $q \in [0, \bar{q}] =: Q$ units of the good generates a gross surplus of $v(q, k)$ to the buyer but requires the seller to incur the production cost of $c(q, k)$. Prior to the investment, the parties sign a contract requiring the exchange of $\hat{q} \in Q$ units for the price of $\hat{t} \in \mathbb{R}$. Edlin and Reichelstein (1996) studied such a contract, which can be indexed by $m = (\hat{q}, \hat{t})$. This model conforms to our model with $\Phi(q, k) := v(q, k) - c(q, k)$, $\psi_1(k; \hat{q}, \hat{t}) = \hat{t} - c(\hat{q}, k)$, and $\psi_2(k; \hat{q}, \hat{t}) = v(\hat{q}, k) - \hat{t}$.*

Example 3 ***(Exclusivity contracts)*** *Party $i = 1, 2$ has the outside opportunity to trade with a third party and realize $\pi_i^O > 0$, respectively. (These payoffs do not depend on k when investments are not transferable to the external trades.) Suppose that at $t = 0$ the parties can sign an exclusivity agreement that would prohibit such external trades. Segal and Whinston (2000) studied whether such a contract can protect returns to specific investments. Our model can be used to reexamine this issue by setting $\mathcal{M} = \{EX,\ NEX\}$ and $\psi_1(k; EX) = \psi_2(k; EX) \equiv 0$ and $\psi_1(k; NEX) \equiv \pi_1^O, \psi_2(k; NEX) \equiv \pi_2^O$.*

As Example 3 illustrates, the absence of a contract is a special case of our model. Clearly, the status quo payoffs are not necessarily lower in such a case, particularly when compared with an exclusivity contract.[10] Throughout, we make the following assumptions, which are natural with the above examples.

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forms.

[10]In Che and Sákovics (2004), the status quo payoffs are assumed to be zero. But this is just a normalization, and the reader should not interpret the current paper as assuming that the status quo payoffs will indeed rise as the parties sign some ex ante contract. As the exclusivity example illustrates, a contract may increase or decrease the parties' status quo payoffs.

Assumption 1 ***(Regularity)*** *For each $m \in \mathcal{M}$, (a) $\psi_i(\cdot; m)$ is nonnegative, nondecreasing and twice differentiable; (b) $\phi(\cdot) - z\psi_2(\cdot; m)$ and $\psi_1(\cdot; m)$ are strictly concave for all $z \in [0, 1]$;*[11] *and (c) $\psi_1(k; m) - k$ is strictly decreasing in k.*

Assumption 2 ***(Specificity)*** *For each $m \in \mathcal{M}$ and $k \in K$, $\phi(k) - \psi_1(k; m) - \psi_2(k; m)$ is strictly positive and nondecreasing in k.*

Assumption 1 implies that parties' contract-induced disagreement payoffs are nondecreasing in their investments and satisfy other technical conditions. Assumption 1-(c) implies that party 1 will not invest unless there is internal trade between the partners. It simplifies our equilibrium characterization. Assumption 2 means that the parties' investments are specific in the sense that they generate higher total surplus when the parties trade efficiently than when they disagree, both in the *absolute* and *marginal* senses. These assumptions are sensible in the context of the above examples. For instance, the disagreement outcome under any ownership may be inefficient since non-owners may not exert sufficient human capital input (as is assumed in GHM); a trading contract may not specify the efficient trade level (as with Edlin and Reichelstein (1996) and Che and Hausch (1999)); and external trading does not yield as much surplus as the efficient internal trading both under exclusive and nonexclusive regimes (see Segal and Whinston (2000)). Some applications will require relaxing Assumptions 1(c) and Assumption 2, in which case a remark will be made to note the implication of the relaxation.

Assumption 2 implies that the disagreement outcome can never be efficient, so the social optimum can be characterized independently of the contract in place. The first-best is achieved when the seller chooses

$$k^* = \arg\max_{k \in K} \phi(k) - k,$$

and the parties trade immediately to realize $\phi(k^*)$. We assume that $k^* \in (0, \bar{k})$, which implies that k^* satisfies $\phi'(k^*) = 1$.

[11] A sufficient condition is that both ϕ and ψ_2 are concave but the former is more concave; i.e., $\phi''(\cdot) \leq \psi_2''(\cdot; m)$. Existing models often assume that $\psi_2'(\cdot; m) \equiv 0$, in which case the assumption follows from the strict concavity of ϕ.

2.2 Benchmark outcomes

Before proceeding, it is useful to establish the equilibrium outcome of the static game as a benchmark, and replicate the main insight from the traditional contract literature in our model. We do so in a way that will facilitate comparison with the subsequent dynamic results. Consider the following payoff function of party 1 under contract m:

$$(3) \qquad U_\delta(k;m) := \alpha[\phi(k) - (1-\delta)\psi_2(k;m)] + (1-\alpha)(1-\delta)\psi_1(k;m) - (1-(1-\alpha)\delta)k,$$

and let $k_\delta(m) := \arg\max_{k \in K} U_\delta(k;m)$ denote its unique maximizer, which is well defined by Assumption 1(b).

The highest investment level implementable by m is will be seen later to equal $k_\delta(m)$. For now, we observe that the payoff function corresponds to two important benchmarks when δ takes extreme values. First, if $\delta = 1$, then $U_\delta(k;m) = \alpha[\phi(k) - k]$, the (re-scaled) social objective function. Hence, $k_1(m) = k^*$ regardless of the contract m in place. Next, if $\delta = 0$, then $U_\delta(k;m) = \alpha[\phi(k) - \psi_2(k;m)] + (1-\alpha)\psi_1(k;m) - k$. This is precisely party 1's equilibrium payoff in the (standard) static model, where all the investment is done at once, followed by (asymmetric) Nash bargaining.[12] Hence, its maximizer, $k_0(m)$, represents the equilibrium investment induced by contract m in the static model. Finally, we can observe how $k_\delta(m)$ varies with δ.

Lemma 1 *Given Assumption 1(a)-(b), $k_\delta(m)$ is nondecreasing in δ, and approaches to $k_0(m)$ as $\delta \to 0$ and to k^* as $\delta \to 1$. If Assumption 1(c) holds additionally, then $k_{\delta'}(m) > k_\delta(m)$ for $\delta' > \delta$, whenever $k_{\delta'} > 0$.*

Proof. See the Appendix.

As mentioned in the Introduction, in the static models, the value of contracts lies in reducing the investor's marginal specificity exposure. For instance, if contract m causes party 1's contract payoff, $\psi_1(k;m)$, to increase strictly with k, then his exposure to holdup at the margin, $\phi'(k) - \psi_1'(k;m)$, will be reduced (compared with the null contract case). It is thus useful to characterize how contracts compare in terms of the extent to which they reduce each party's marginal specificity exposure. Let us write $\psi_i(k;m) = \int_0^k \psi_i'(\tilde{k};m)d\tilde{k} + \bar{\psi}_i(m)$, where $\bar{\psi}_i(m) := \psi_i(0;m)$ for $i = 1, 2$.

[12] In the special case when $\alpha = \frac{1}{2}$, it coincides with the NBS.

Definition 1 *(a) A contract m' is (weakly)* ***M-specificity reducing on*** m ***for party*** $i = 1, 2$*, or* $m' \succ_i^M (\succeq_i^M)\, m$*, if* $\psi_i'(k; m') > (\geq)\psi_i'(k; m)$ *for all* $k \in K$*.*

(b) Contracts m and m' are ***M-specificity invariant for party*** $i = 1, 2$*, or* $m \sim_i^M m'$ *if* $m \succeq_i^M m'$ *and* $m \preceq_i^M m'$*. The contracts are* ***M-specificity invariant****, or* $m \sim^M m'$*, if* $m \sim_i^M m'$ *for* $i = 1, 2$*.*

If contracts m and m' are M-specificity invariant, then their associated contract payoffs differ only in absolute magnitude, i.e., $\psi_i(k; m') - \psi_i(k; m) = \bar{\psi}_i(m') - \bar{\psi}_i(m)$ for all $k \in K$. In the static setting, the investment levels are partially characterized by the order on M-specificity, as is shown next.

Proposition 1 *In the static model ($\delta = 0$), if* $m' \succeq_1^M m$ *and* $m' \preceq_2^M m$*, then* $k_0(m') \geq k_0(m)$*. The inequality is strict if (in addition)* $k_0(m') > 0$ *and if either* $m' \succ_1^M m$ *or* $m \succ_2^M m'$*.*

Proof. Observe that

$$U_0'(k; m') - U_0'(k; m) = (1 - \alpha)[\psi_1'(k; m') - \psi_1'(k; m)] - \alpha[\psi_2'(k; m') - \psi_2'(k; m)] \geq 0,$$

if $m' \succeq_1^M m$ and $m' \preceq_2^M m$. Since the payoff function has a unique maximizer, given Assumption 1, the inequality implies that $k_0(m') \geq k_0(m)$. Further, the inequality is strict if, in addition, $k_0(m') > 0$ and if $m' \succ_1^M m$. The latter condition implies that

$$U_0'(k_0(m'); m) = \alpha[\psi_2'(k_0(m'); m') - \psi_2'(k_0(m'); m)] - (1 - \alpha)[\psi_1'(k_0(m'); m') - \psi_1'(k_0(m'); m)] < 0.$$

Since $k_0(m') > 0$, then $k_0(m) \neq k_0(m')$, implying that $k_0(m') > k_0(m)$. ∎

This result shows that a shift from a contract, m, to another, m', improves party 1's incentive for investment, if the shift lowers her marginal specificity exposure (i.e., $\phi'(k) - \psi_1'(k; m') \leq \phi'(k) - \psi_1'(k; m)$) *and* raises party 2(= non-investor)'s. This insight underlies the GHM's asset ownership prescriptions, as mentioned before.[13] Meanwhile, if two contracts are

[13] To illustrate, suppose as in Hart (1995) that there are two assets, $A = \{a_1, a_2\}$. An ownership structure, m, is then represented by a partition of A, (A_1^m, A_2^m), where $A_i^m, i = 1, 2$ refers to the asset(s) party i owns under ownership structure m. Suppose as with GHM, that the parties' contract payoffs in case of disagreement depend on the assets they own (and possibly, party 1's investment k), given by $\pi_i(k; A_i^m)$. There are three alternative structures: (1) separate ownership or non-integration: $m_N := (\{a_1\}, \{a_2\})$, (2) common ownership by party 1: $m_1 := (\{a_1, a_2\}, \emptyset)$, and (3) common ownership by party 2: $m_2 := (\emptyset, \{a_1, a_2\})$. Given our one-sided investment model, then the notion of *strictly complementary* assets coincide with an M-specificity order: $m_1 \succeq_1^M m_N$ (see Hart (1995): conditions (2.2) and (2.3) (p. 37)). Then, Proposition 1 implies that $k_0(m_1) > k_0(m_N)$, thus replicating a crucial result of GHM's asset ownership theory.

M-specificity invariant, they yield the same incentive for the investment.

Corollary 1 *In the static model, M-specificity invariant contracts lead to the same investment: if* $m \sim^M m'$*, then* $k_0(m') = k_0(m)$.

In other words, differences in absolute specificities do not matter. This result underlies Segal and Whinston (2000)'s claim of irrelevance of exclusivity: if investments are not transferable to external trades ($\psi_1'(k; m) \equiv 0$), then any two contracts are M-specificity invariant and therefore whether the parties have an exclusivity agreement does not affect the incentives for specific investment (since it affects the contract payoffs only in the *absolute* magnitude).

It will be seen later that absolute specificity *does* matter in our dynamic model. Specifically, the aggregate level of specificity $\phi(0) - \bar{\psi}_1(m) - \bar{\psi}_2(m)$ may affect the level of sustainable investment. It is thus useful to compare contracts by the associated level of absolute specificity.

Definition 2 *Contract* m' *is (weakly)* ***A-specificity reducing on*** *contract* m*, or* $m' \succ^A (\succeq^A) m$*, if* $\bar{\psi}_1(m') + \bar{\psi}_2(m') > (\geq) \bar{\psi}_1(m) + \bar{\psi}_2(m)$.

3 Effects of Contracts in the Dynamic Environment

3.1 The equilibrium concept

In our model, the stake of bargaining in any given period depends only on the aggregate investment accumulated up to that point. Consequently, the cumulative investment constitutes the only payoff-relevant part of the history. A *Markov strategy* profile specifies for each period party 1's (incremental) investment choice as a function of the cumulative investment up to the *last period*, and, for the bargaining stage, a price offer and a response rule for each party — a function mapping from an offer to {accept, reject} — both as functions of the *current-period* cumulative investment. We look for subgame perfect equilibria in Markov strategies, called *Markov Perfect equilibria* (MPE).

Formally, we can characterize an equilibrium corresponding to contract $m \in \mathcal{M}$, by a pair of *value functions*, $(\sigma_m, \beta_m) : \mathbb{R}_+ \mapsto \mathbb{R}^2$, that map from the *current period* cumulative investment into the continuation values for parties 1 and 2, respectively, evaluated immediately before the selection of a proposer in that period; and by an *investment strategy*, $y_m : K \mapsto K$, with $y_m(k) \geq k$, that maps party 1's previous-period cumulative investment into her current-period cumulative investment.

Given an ex ante contract $m \in \mathcal{M}$, a Markov Perfect equilibrium is characterized by a triple, $\langle \sigma_m, \beta_m, y_m \rangle$, satisfying that: for each $k \in K$,

$$\begin{aligned} \sigma_m(k) = \alpha \max\{&\phi(k) - \delta\beta_m(y_m(k)) - (1-\delta)\psi_2(k;m), \\ &\delta[\sigma_m(y_m(k)) - (y_m(k) - k)] + (1-\delta)\psi_1(k;m)\} \\ &+(1-\alpha)\{\delta[\sigma_m(y_m(k)) - (y_m(k)-k)] + (1-\delta)\psi_1(k;m)\}, \end{aligned} \tag{4}$$

$$\begin{aligned} \beta_m(k) = (1-\alpha)\max\{&\phi(k) - \delta[\sigma_m(y_m(k)) - (y_m(k)-k)] - (1-\delta)\psi_1(k;m), \\ &\delta\beta_m(y_m(k)) + (1-\delta)\psi_2(k;m)\} \\ &+\alpha\{\delta\beta_m(y_m(k)) + (1-\delta)\psi_2(k;m)\}, \end{aligned} \tag{5}$$

$$y_m(k) \in \arg\max_{\tilde{k} \in [k, \bar{k}]} \{\sigma_m(\tilde{k}) - (\tilde{k} - k)\}. \tag{6}$$

Party 1's value function is explained as follows. Given an existing investment stock of k, party 1 is chosen with probability α to make an offer. She can either make an offer which is unacceptable to party 2, in which case she obtains her net continuation value, $\delta[\sigma_m(y_m(k)) - (y_m(k) - k)] + (1-\delta)\psi_1(k;m)$.[14] Or else, she can make the lowest offer acceptable to party 2, which equals his continuation value, $\delta\beta_m(y_m(k)) + (1-\delta)\psi_2(k;m)$. In the latter case, party 1 collects $\phi(k) - \delta\beta_m(y_m(k)) - (1-\delta)\psi_2(k;m)$. She chooses the more profitable of the two alternatives. With probability $1-\alpha$, party 1 becomes the responder. Based on the above reasoning, she will earn her continuation value, $\delta[\sigma_m(y_m(k)) - (y_m(k)-k)] + (1-\delta)\psi_1(k;m)$, no matter how party 2 resolves his trade-off. Party 2's value function has a similar explanation. The last equation, (6), states that party 1 chooses her best incremental investment, taking into account the current stock and the future evolution of the game. Clearly, these conditions are necessary for Markov Perfection. They are also sufficient, since there is no profitable (possibly non-Markovian) deviation, given the single-period deviation principle.

3.2 The characterization of implementable investments

We now characterize the set of MPE. Of particular interest is the level of investment sustainable under a given contract. We say that a cumulative investment, $k \in K$, is *implementable*

[14] The game continues with probability δ in which case she collects $\sigma_m(y_m(k)) - (y_m(k) - k)$ and the bargaining is terminated with probability $1-\delta$ in which case she collects her contract payoff, $\psi_1(k;m)$.

by contract m if there exists an MPE with $y_m(\cdot)$ in which trade occurs after reaching k. In our dynamic model, multiple levels of investments are typically supportable by different investment dynamics. In order to identify the entire set of implementable investments, we need to characterize party 1's continuation payoff, $\sigma_m(k)$, at each k. This task is not trivial, however, due to the multiplicity of equilibria. Unlike the pure bargaining game, where the continuation payoff can be uniquely pinned down, a range of continuation payoffs corresponding to different investment dynamics can be sustainable, and characterizing the former requires getting a handle on all implementable investment dynamics. The following lemma provides several observations.

Lemma 2 *Fix any MPE with contract m satisfying Assumptions 1 and 2.*
(i) If $\hat{k} \in K$ is implementable by m, then party 1 chooses $\hat{k}$ and trade occurs in the first period, and

$$\sigma_m(\hat{k}) - \hat{k} \leq U_0(\hat{k}; m). \tag{7}$$

(ii) For any $k \in K$,

$$\sigma_m(k) - k \geq U_\delta(k; m) - \delta[\alpha\psi_2(y_m^\infty(k)) - (1-\alpha)\psi_1(y_m^\infty(k)) + (1-\alpha)y_m^\infty(k)], \tag{8}$$

where $y_m^\infty(k) := \lim_{t\to\infty} y_m^t(k)$.
(iii) For any $k \in K$, $y_m^\infty(k) \leq \max\{k, k_\delta(m)\}$.

Part (i) states that party 1's equilibrium payoff (under any contract) cannot exceed the payoff she would receive in the static equilibrium. That upper bound is attained if the investment strategy has $y(\hat{k}) = \hat{k}$, i.e., no further investment is specified after reaching $\hat{k}$ (in which case a pure bargaining game ensues just as in the static game, and hence the same payoff). Whether party 1 will actually choose $\hat{k}$ depends on what payoff she will receive when deviating from $\hat{k}$, in particular on the worst sustainable punishment that can be levied against the deviator, which is characterized in part (ii). The RHS of (8) describes the payoff party 1 will receive when the strategy has her jump to $y^\infty(k)$, the highest investment level sustainable under that strategy, next period if no agreement is reached in the current period. It turns out that sending party 1 to the highest implementable investment level (should disagreement occur) constitutes the worst sustainable punishment. One can then use (i) and (ii) to identify the highest implementable investment level: It is the highest level that party 1 wishes to choose given the threat of any downward deviation being punished by a jump to *that* level next period. That level turns out to be $k_\delta(m)$.

We can establish a couple of necessary conditions for MPE based on this lemma. By definition, once $k_\delta(m)$ is reached no further investment is possible, so a pure bargaining game will ensue, leading to a unique continuation payoff $\sigma_m(k_\delta(m)) = U_0(k_\delta(m); m) + k$ in that equilibrium. Since party 1 can guarantee this payoff by choosing $k_\delta(m)$, any implementable investment should make party 1 no worse than that payoff. Hence, if $\hat{k} \in K$ is implementable by m, by Lemma 2-(i), we must have

$$(IC) \qquad U_0(\hat{k}; m) \geq U_\delta(k_\delta(m); m).$$

Next, suppose party 1 deviates by not investing and perpetually inducing rejection. Then she will obtain

$$(1-\delta)[\bar{\psi}_1(m) + \delta\bar{\psi}_1(m) + \delta^2\bar{\psi}_1(m) + ...] = \bar{\psi}_1(m).$$

If $\hat{k}$ is implementable by m, we must have

$$(IR) \qquad U_0(\hat{k}; m) \geq \bar{\psi}_1(m).$$

Below, we refer to the highest investment level satisfying (IR), $k_{IR}(m) := \sup\{k \leq \bar{k} \mid U_0(k; m) \geq \bar{\psi}_1(m)\}$.

We have shown that (IC) and (IR) are necessary for any cumulative investment k implementable by m. It turns out that they are sufficient as well.

Theorem 1 *Given Assumptions 1 and 2, any cumulative investment $\hat{k}$ is implementable by a contract m if and only if*

$$\hat{k} \in K_\delta(m) := \{k \in K \mid U_0(k; m) \geq \max\{\bar{\psi}_1(m), U_0(k_\delta(m); m)\}\}.$$

When $\hat{k} \in K_\delta(m)$ is implemented, party 1 chooses k and trade occurs all in the first period.

Proof. See the Appendix.

As can be seen in the proof (in the Appendix), any investment in K_δ is implemented by a strategy that calls for no further investment following the target investment being reached, but calls for the worst punishment of having party 1 go up to $k_\delta(m)$ following any deviation.

The set of implementable investments, $K_\delta(m)$, can be made more transparent with the aid of Figures 1(a) and 1(b).

[Insert Figures 1(a) and 1(b) around here.]

Since $U_0(\cdot; m)$ is concave (by Assumption 1), the set $K_\delta(m)$ is an interval. Its largest element, denoted $\bar{k}_\delta(m)$, must satisfy $\bar{k}_\delta(m) = \min\{k_\delta(m), k_{IR}(m)\} < k^*$, for all $\delta < 1$. As can be seen from Figure 1(a), $\bar{k}_\delta(m) = k_\delta(m)$ if (IR) is not binding, whereas $\bar{k}_\delta(m) = k_{IR}(m)$ if (IR) is binding. Likewise, the lowest sustainable investment, denoted $\underline{k}_\delta(m)$, must also satisfy $U_0(\underline{k}_\delta; m) = \max\{\bar{\psi}_1(m), U_0(k_\delta(m); m)\}$. Recall that $U_0(k; m)$ is maximized at $k = k_0(m)$. Hence, (IR) holds strictly at $k = k_0(m)$ given Assumption 2. Furthermore, if $k_0(m) \in (0, \bar{k})$ and Assumption 1(a)-(c) holds, then $k_\delta(m) > k_0(m)$ for any $\delta > 0$, so we must have $U_0(k_0(m); m) > \max\{\bar{\psi}_1(m), U_0(k_\delta(m); m)\}$. This means that $\underline{k}_\delta(m) < k_0(m) < \bar{k}_\delta(m)$ whenever $\delta > 0$. In other words, our dynamic game can sustain investments that are strictly higher and those that are strictly lower than the investment level sustainable in the static equilibrium.

Despite the multiplicity, the set of MPE's in our dynamic model has some desirable continuity features. For instance, it attains the static equilibrium outcome as a unique MPE when $\delta = 0$: Since $k_\delta(m) \to k_0(m)$ as $\delta \to 0$, $K_\delta(m)$ collapses to a unique equilibrium investment, $k_0(m)$.

Finally, since $\bar{k}_\delta(m) \leq k^*$ and $\phi(\cdot)$ is concave, the highest sustainable investment, $\bar{k}_\delta(m)$, generates the highest net joint surplus for the parties. If the parties have credible ways to coordinate on selecting the equilibrium, the highest equilibrium investment will be selected (assuming that any distributional issue can be addressed by the exchange of side payments). Focusing on the best equilibrium also isolates the new predictions made available from the current dynamic model. In particular, we can interpret the existing predictions as coming from our dynamic model but with a particular selection. In this sense, focusing on the best equilibrium serves to effectively complement the existing literature. For this reason, we will focus on the highest implementable investment, $\bar{k}_\delta(m)$, throughout the rest of the paper.

3.3 The effects of a contract shift

To study how the choice of a contract $m \in \mathcal{M}$ affects the highest implementable investment, we need to examine both $k_{IR}(m)$ and $k_\delta(m)$, since $\bar{k}_\delta(m) = \min\{k_{IR}(m), k_\delta(m)\}$. Since (IR) is not binding at $k = k_0(m)$, however, there exists $\hat{\delta} > 0$ such that, for all $\delta \leq \hat{\delta}$, (IR) is not binding; i.e., $\bar{k}_\delta(m) = k_\delta(m) \leq k_{IR}(m)$. Hence, for a sufficiently low δ, it suffices to examine how $k_\delta(m)$ varies with contracts. Consider a shift from contract m to contract m'. Observe that

$$\begin{aligned} U_\delta(k;m') - U_\delta(k;m) &= (1-\delta)\{(1-\alpha)[\psi_1(k;m') - \psi_1(k;m)] - \alpha[\psi_2(k;m') - \psi_2(k;m)]\} \\ &= (1-\delta)[U_0(k;m') - U_0(k;m)]. \end{aligned}$$

It follows that the effect of a contract shift on k_δ is much the same as that on k_0. This shows that, while the dynamic model allows a higher investment to be sustained than the static model, the comparative statics associated with a contract shift remains unchanged, *for a sufficiently low* δ. In this sense, the results found the extant contract literature are robust. We thus have the following generalization of Proposition 1:

Proposition 2 *For any two contracts $m, m' \in \mathcal{M}$, there exists $\hat{\delta} > 0$ such that, for any $\delta \leq \hat{\delta}$, if $m' \succeq_1^M m$ and $m \succeq_2^M m'$, then $\bar{k}_\delta(m') \geq \bar{k}_\delta(m)$. (The inequality is strict if, in addition, $\bar{k}_\delta(m') > 0$ and either $m' \succ_1^M m$ or $m \succ_2^M m'$.) In particular, if $m \sim^M m'$, then $\bar{k}_\delta(m') = \bar{k}_\delta(m)$.*

What happens for a large δ? We next show that our dynamic contract model does make a difference in this case. In such a case, the (IR) constraint may be binding, which introduces a different type of consideration in terms of contract design. In fact, there is a sense in which comparison of alternative contracts is meaningful only if (IR) is binding. To see this, fix any contract $m \in \mathcal{M}$, and suppose that $k_{IR}(m) \geq k^*$ so that (IR) can never be binding. Then, since $k_\delta(m) \leq k^* \leq k_{IR}(m)$, we have $\bar{k}_\delta(m) \to k^*$ as $\delta \to 1$ (i.e., the first-best outcome is attainable arbitrarily closely). This result holds regardless of the contract (or its absence[15]) as long as the (IR) holds at k^* with that contract. In that sense, contracts become irrelevant for $\delta \approx 1$, unless (IR) binds under some contracts.

Proposition 3 ***(Irrelevance of (IC) in the Limit)*** *For any $m \in \mathcal{M}$ with $k_{IR}(m) \geq k^*$, $\bar{k}_\delta(m) \to k^*$ as $\delta \to 1$.*

To the extent that one cares about the implementability of the best equilibrium, this proposition underscores the significance of (IR) in comparative contract research when $\delta \approx 1$. To see whether the (IR) constraint introduces a new insight absent in the existing literature, again fix a contract m and write the (IR) condition as

$$\alpha[\phi(k) - \psi_2(k;m)] + (1-\alpha)\psi_1(k;m) - k \geq \bar{\psi}_1(m), \tag{9}$$

[15]In Che and Sákovics (2004), we show asymptotic efficiency in a model without breakdown (with discounting), corresponding to the absence of a contract.

which can be rewritten as:

$$\alpha\phi(k) + (1-\alpha)\int_0^k \psi_1'(\tilde{k};m)d\tilde{k} - \alpha\int_0^k \psi_2'(\tilde{k};m)d\tilde{k} - \alpha[\bar{\psi}_1(m) + \bar{\psi}_2(m)] \geq 0. \tag{10}$$

Clearly, changes in M-specificities have the same qualitative effect on the (IR) constraint as on k_δ, for an increase in ψ_1' and a decrease in ψ_2' relax the constraint. There is a new effect, however: *absolute* specificity matters for (IR). In particular, the higher the absolute specificity, the more relaxed the constraint is; i.e., a decrease in $\bar{\psi}_2 + \bar{\psi}_1$ relaxes the constraint. The effect of a contract shift can be clearly seen by how it affects the LHS of (10). Consider as before a shift from contract m to contract m'. The resulting increase in the LHS is written as:

$$\begin{aligned}(1-\alpha)\int_0^k [\psi_1'(\tilde{k};m') - \psi_1'(\tilde{k};m)]d\tilde{k} - \alpha\int_0^k [\psi_2'(\tilde{k};m') - \psi_2'(\tilde{k};m)]d\tilde{k} \\ -\alpha[\bar{\psi}_1(m') + \bar{\psi}_2(m') - \bar{\psi}_1(m) - \bar{\psi}_2(m)].\end{aligned} \tag{11}$$

Inspecting (11) yields our main general result.

Proposition 4 *For any two contracts m, m', if $m' \succeq_1^M m$, $m' \preceq_2^M m$, and $m' \preceq^A m$, then $\bar{k}_\delta(m') \geq \bar{k}_\delta(m)$. The latter inequality is strict if one of the first two orders is strict and (IR) is not binding for both contracts, or the last order is strict and (IR) is binding for either contract.*

Proof. As noted, if $m' \succeq_1^M m$ and $m' \preceq_2^M m$, then $k_\delta(m') \geq k_\delta(m)$. Also, if $m' \succeq_1^M m$ and $m' \preceq_2^M m$, and $m' \preceq^A m$, then $k_{IR}(m') \geq k_{IR}(m)$. The last statement follows from that fact that $\bar{k}_\delta(\tilde{m}) = \min\{k_{IR}(\tilde{m}), k_\delta(\tilde{m})\}$, for $\tilde{m} = m, m'$. ▌

The new effect becomes most clear when two contracts are M-specificity invariant.

Corollary 2 ***(Contract Payoffs Minimization Principle)*** *Suppose that $m' \sim^M m$. Then, $\bar{k}_\delta(m') \geq \bar{k}_\delta(m)$ if and only if $m' \preceq^A m$, or equivalently,*

$$\bar{\psi}_1(m') + \bar{\psi}_2(m') \leq \bar{\psi}_1(m) + \bar{\psi}_2(m). \tag{12}$$

If (12) is strict and (IR) is binding for either m or m', then $\bar{k}_\delta(m') > \bar{k}_\delta(m)$.

This corollary has some striking implications for several contracts, as will be seen in the next section.

4 Applications

This section applies Proposition 4 and Corollary 2 to various contracting/organizational settings. In some cases, we will focus on M-specificity invariant contracts, i.e., the ones that expose the investor to the same degree of marginal specificity. All such contracts yield the same outcome in the static models (see Corollary 1). Yet, these contracts may entail different outcomes in our dynamic model if the investor is exposed to different degrees of *absolute* specificity (see Corollary 2). Hence, these contracts make our novel contributions most transparent. Moreover, M-specificity invariant contracts are realistic in many settings. For instance, if the investments are completely specific and their values are not realized without the participation of both parties' human capital, then the contract payoffs arising from different ownership structures will not vary with the parties' specific investment, so they will differ only in the absolute magnitude. Likewise, if the investments are not transferable to the external trades, whether the internal partners have exclusivity agreements or not will affect the contract payoffs only in absolute magnitude. In these cases, we derive some surprising new results.

4.1 Ownership contracts

To begin, suppose there are two assets, $A = \{a_1, a_2\}$. An ownership structure, m, is then represented by a partition of A, (A_1^m, A_2^m), where $A_i^m, i = 1, 2$ refers to the asset(s) party i owns under ownership structure m. Suppose, as with GHM, that the parties' contract payoffs in case of disagreement depend on the assets they own (and possibly, party 1's investment k), given by $\pi_i(k; A_i^m)$. There are three alternative structures: (1) separate ownership or nonintegration: $m_N := (\{a_1\}, \{a_2\})$, (2) common ownership (or integration) by party 1: $m_1 := (\{a_1, a_2\}, \emptyset)$, and (3) common ownership (or integration) by party 2: $m_2 := (\emptyset, \{a_1, a_2\})$. As mentioned earlier, this setup is accommodated in our model by setting $\psi_i(k; m) := \pi_i(k; A_i^m)$ for $i = 1, 2$. Suppose these alternative ownership structures are M-specificity invariant. As mentioned before, specificity invariance may arise because the investment is totally specific, so the entire return to the investment is lost when there is disagreement: i.e., $\pi_i(k; A_i^m) = \pi_i(0; A_i^m)$ for all k.

Of special interest in the GHM model is the case in which the assets are complementary. Complementarity of assets is defined there in terms of effects alternative ownership structures have on marginal specificities (see footnote 13). Since we want to be able to talk about complementarity even under M-specificity invariance, we define an alternative, perhaps more natural, notion of complementarity:

Definition 3 *Assets are* ***absolutely complementary*** *if*

$$\bar{\psi}_2(m_N) + \bar{\psi}_1(m_N) < \min\{\bar{\psi}_1(m_1) + \bar{\psi}_2(m_1), \bar{\psi}_1(m_2) + \bar{\psi}_2(m_2)\}.$$

In words, this simply means that, absent investment effects, total asset values are higher when they are owned together than when they are owned separately. Clearly, given absolutely complementary assets, the contract payoffs minimization principle (Corollary 2) yields the following implication.

Proposition 5 *If the alternative ownership contracts are M-specificity invariant and the assets are absolutely complementary, then separate ownership implements a [strictly] higher investment than either form of common ownership [if individual rationality were binding at the common ownership].*

This result contrasts with the important GHM prescription that (marginally) complementary assets should always be owned together. Clearly, the difference stems from the fact that the IR constraint plays an important role in our dynamic setting. As mentioned above, the GHM result is based on, and driven by, the effects ownership structures have on how the contract payoffs depend on the investment. While the above result assumes that the contract payoffs do not vary with investment, it will continue to hold when some degree of such co-movement is allowed.

Remark 1 ***(Joint ownership)*** *A similar argument can be used to justify joint ownership of assets, which is often viewed as suboptimal in the static literature. Suppose, as is often argued by authors, that a joint ownership of assets entails even less efficient utilization of assets than a common ownership of the assets, for instance because conflicting interests result in ineffective decision making under joint ownership.*[16] *This simply means that*

$$\bar{\psi}_2(m_J) + \bar{\psi}_1(m_J) < \min\{\bar{\psi}_1(m_1) + \bar{\psi}_2(m_1), \bar{\psi}_1(m_2) + \bar{\psi}_2(m_2)\},$$

[16] For instance, Hart (1995, pp48) describes joint ownership as a regime in which both parties have veto power over the use of assets. According to this view, the joint ownership will be similar in effect to a mutual exclusivity agreement, discussed below.

where m_J refers to the joint ownership regime. Then, our contract payoffs minimization principle will imply that the joint ownership can support higher investment than any common ownership, provided that the joint ownership is M-specificity invariant to the other ownership structures. A similar implication can be drawn on the value of joint ownership relative to separate ownership, if the former entails less efficient use of assets than the latter.

Remark 2 ***(Does asset ownership motivate investment?)*** *The above analysis casts doubt on the robustness of some incentivizing effects of asset ownership found in the existing literature. The GHM view was that transferring an asset ownership to a party* unambiguously motivates *his specific investments. Meanwhile, the static models based on the outside option specification of bargaining hold that, given some plausible condition, the ownership transfer* unambiguously demotivates *the asset gainer's investment. Our dynamic model affirms neither view. Given absolute complementarity of assets, transfer of an asset to party 1* motivates *his investment, starting from a common ownership by party 2, but* demotivates *his investment when transfer occurs starting from separate ownership.*

4.2 Exclusivity contracts

A long-held belief was that a right to prohibit one's partner from trading with a third party — the so-called exclusivity agreement — protects the return of specific investments made by the recipient of that right, thus promoting such an investment. A recent paper by Segal and Whinston (2000) challenges the wisdom of this belief. They observe that, if investments are specific and non-transferable to external trade, such an agreement can affect the parties' contract payoffs only in absolute magnitude but not in the way investments affect those payoffs. Hence, exclusivity and non-exclusivity are M-specificity invariant. As is clear from Corollary 1, then an exclusivity agreement would have no effect on the outcome in such a circumstance according to the traditional models (see Segal and Whinston (2000), Proposition 1).

In our dynamic model, however, a change in the absolute level of the contract payoffs matters, since it affects the individual rationality constraints. To see this, suppose as in Example 3 that the parties' contract payoffs are $\pi_1^O > 0$ and $\pi_2^O > 0$ if there is no exclusivity agreement, whereas their contract payoffs are zero under the exclusivity agreement. It thus follows that

$$\bar{\psi}_1(EX) + \bar{\psi}_2(EX) \equiv 0 < \pi_1^O + \pi_2^O \equiv \bar{\psi}_1(NEX) + \bar{\psi}_2(NEX).$$

Given the M-specificity invariance between the two regimes, the next result follows immediately from the contract payoffs minimization principle:

Proposition 6 *An exclusivity agreement implements a [strictly] higher investment [if the individual rationality were binding under non-exclusivity].*

This result shows that, if the individual rationality constraint were binding for a party, awarding him the exclusivity right can protect his specific investment, which contrasts with the irrelevance result by Segal and Whinston (2000). It is worth noting that exclusive dealing relaxes the individual rationality constraints by making the status quo outcome (= failure of internal trade) less attractive for the parties. It is also important to note that this effect arises not only for the party who offers the exclusivity protection but also for the recipient of such a protection (who may not offer the same protection to the other party in return).[17]

4.3 Trade contracts

Trade contracts specify the terms of ex post trade between the parties. Whether trade contracts — renegotiable if the parties so choose — can solve the holdup problem has been the focus of several recent works (see Chung 1991; Edlin and Reichelstein, 1996; Che and Hausch, 1999; and Segal and Whinston, 2002). The answer, according to them, depends on the nature of the investment. If the investment is selfish in the sense that the investor directly benefits from her investment ($c_k < 0$ and $v_k = 0$ in our case), then Edlin and Reichelstein (1996) showed that a simple trade contract specifying a fixed quantity of trade, $\hat{q}$, and a transfer payment $\hat{t}$ (to be made by the buyer [party 2] to the seller [party 1]) can implement the first best outcome. Rephrased in terms of our framework, the main idea behind the efficiency result is that the trade contract, with an appropriately chosen $\hat{q}$, can eliminate the investor's marginal specificity exposure entirely if the investment is *selfish*. By contrast, if the investment is cooperative in the sense that the direct beneficiary of the investment is the investor's partner ($c_k = 0$ and $v_k > 0$ in our case), then no trade contract can reduce the investor's exposure to marginal specificity. Rather it reduces her partner's marginal specificity, so the contract can only worsen underinvestment, as shown in Che and Hausch (1999).

[17] de Meza and Selvaggi (2003) obtain a similar result, using a three-way bargaining game with the possibility of reallocating/subcontracting the production assignment (in case of non-exclusivity). Their result and ours complement each other toward a positive role exclusivity may play in promoting specific investments.

The effects of trade contracts can be reexamined in our dynamic model. As will be seen, some flavor of the standard results carries over, but there are some new elements. To begin, consider a simple trade contract indexed by $(\hat{q}, \hat{t})$. The contract payoffs associated with this contract are $\psi_1(k; \hat{q}, \hat{t}) = \hat{t} - c(\hat{q}, k)$ and $\psi_2(k; \hat{q}, \hat{t}) = v(\hat{q}, k) - \hat{t}$. Clearly, trade contracts with different $\hat{q}$'s will not be M-specificity invariant. It is useful to consider the cases of selfish and cooperative investments separately.

4.3.1 Selfish investments

If the investments are purely selfish, then we have $v_k \equiv 0$ while $c_k < 0$. Suppose, as with Edlin and Reichelstein, that the parties sign a contract specifying $\hat{q} = q^*(k^*) := \arg\max_{q\in Q} v(q, k^*) - c(q, k^*)$. ($\hat{t}$ does not influence the outcome and thus can be arbitrary.) Note that, with this contract,

$$\begin{aligned} U_\delta(k; \hat{q}, \hat{t}) &= \alpha[\phi(k) - (1-\delta)\psi_2(k; \hat{q}, \hat{t})] + (1-\alpha)(1-\delta)\psi_1(k; \hat{q}, \hat{t}) - (1-(1-\alpha)\delta)k \\ &= \alpha[\phi(k) - (1-\delta)(v(q^*, k) - \hat{t})] + (1-\alpha)(1-\delta)(\hat{t} - c(q^*, k)) - (1-(1-\alpha)\delta)k. \end{aligned}$$

Differentiating this function with respect to k, using $v_k \equiv 0$, yields

$$U'_\delta(k; \hat{q}, \hat{t}) = [1 - (1-\alpha)\delta](\phi'(k) - 1).$$

It then follows that $k_\delta(\hat{q}, \hat{t}) = k^*$, with $\hat{q} = q^*(k^*)$. Since $U_\delta(k; q^*, \hat{t})$ is maximized at $k = k^*$, regardless of δ, $U_0(k; q^*, \hat{t})$ is also maximized at $k = k^*$. Since $U_0(\cdot; q^*, \hat{t})$ is strictly concave, this means that only $k = k^*$ satisfies

$$U_0(k; \hat{q}, \hat{t}) \geq U_0(k_\delta(\hat{q}, \hat{t}); \hat{q}, \hat{t}).$$

Hence, the only investment level implementable by a contract with $\hat{q} = q^*$, if there is any, would be $k = k^*$. Indeed, it is straightforward to show that k^* can be implemented in equilibrium by the investment strategy[18]

$$y(k) = \begin{cases} k^* & \text{if } k \in [0, k^*], \\ k & \text{if } k \in (k^*, \bar{k}] \end{cases}$$

[18]The payoff at k given the strategies is:

$$\sigma(k) - k = \begin{cases} \max\{U_\delta(k) - \delta[\alpha\psi_2(k^*)) - (1-\alpha)\psi_1(k^*) + (1-\alpha)k^*], \delta[U_0(k^*) + k] - k\} & \text{if } k \in [0, k^*], \\ U_0(k) & \text{if } k \in (k^*, \bar{k}]. \end{cases}$$

Ignoring the payoff term associated with delay (which is always less than $U_0(k^*)$), the payoff function is quasi-concave and attains a maximum at k^*. Hence the described strategies are an equilibrium.

Since the set of MPE investments $K_\delta(\hat{q}, \hat{t})$ is a singleton $\{k^*\}$, the chosen trade contract implements the first-best outcome as a unique equilibrium.

Proposition 7 *With selfish investments, a trade contract* $(\hat{q}, \hat{t})$ *with* $\hat{q} = q^*(k^*)$ *implements the first-best outcome as a unique MPE.*

This result reaffirms the efficiency result obtained by Edlin and Reichelstein (1996) in the static setting. This efficiency result has an added dimension in our dynamic setting, though, as it suggests the role of the trade contract as *eliminating* bad equilibria as much as attaining the efficient one: While the efficient outcome may be *an* equilibrium under different contracts or even without any contract if δ is close to 1 (as suggested in the previous section), it admits less efficient outcomes as well, which the current contract eliminates.

4.3.2 Cooperative investments

If the investments are purely cooperative, then $c_k \equiv 0$ while $v_k > 0$. We examine how trade contracts with different $\hat{q}$ will affect the outcome in this case. Since $\bar{k}_\delta(m) = \min\{k_\delta(m), k_{IR}(m)\}$, we investigate the effects on each of the two terms. First, the effect on k_δ can be studied by examining

$$U_\delta(k; \hat{q}, \hat{t}) = \alpha[\phi(k) - (1-\delta)(v(\hat{q}, k) - \hat{t})] + (1-\alpha)(1-\delta)(\hat{t} - c(\hat{q}, k)) - (1-(1-\alpha)\delta)k.$$

Differentiating this function with respect to k, using $c_k \equiv 0$, gives

$$U'_\delta(k; \hat{q}, \hat{t}) = \alpha\phi'(k) - \alpha(1-\delta)v_k(\hat{q}, k) - [1-(1-\alpha)\delta]. \tag{13}$$

Since $v_k(q, k) > 0$ for all $k > 0$ and $v_k(0, k) \equiv 0$, this derivative is maximized when $\hat{q} = 0$. It follows that $k_\delta(\hat{q}, \hat{t})$ is maximized when $\hat{q} = 0$. This observation is consistent with that of Che and Hausch (1999) in that no non-trivial contract can improve the incentives for specific investment. However, this does not prove that trade contracts are worthless. Even with $\hat{q} > 0$, $k_\delta(\hat{q}, \hat{t}) \to k^*$ as $\delta \to 1$, just as observed in Proposition 3. Hence, whether a contract is valuable for $\delta \approx 1$ depends crucially on whether the contract can relax the IR constraint.

With a trade contract and cooperative investment, the (IR) constraint can be rewritten as follows:

$$\alpha[\phi(k) - \psi_2(k; \hat{q}, \hat{t})] + (1-\alpha)\psi_1(k; \hat{q}, \hat{t}) - k \geq \bar{\psi}_1(\hat{q}, \hat{t})$$

$$\iff \alpha\phi(k) - k \geq \alpha[v(\hat{q}, k) - c(\hat{q}, k)]. \tag{14}$$

Clearly, the RHS of (14) vanishes with $\hat{q} = 0$. If $k_{IR}(0,0) \geq k^*$, so that the IR constraint is never binding in the absence of contracts. Then, any trade contract would be worthless, since $\bar{k}_\delta(0,0) = k_\delta(0,0) > k_\delta(\hat{q},\hat{t}) \geq \bar{k}_\delta(\hat{q},\hat{t})$ for any $(\hat{q},\hat{t})$ with $\hat{q} > 0$.

Suppose now $k_{IR}(0,0) < k^*$, so that the IR constraint would be binding in the absence of contracts for some large δ. In this case, a contract specifying a quantity that would create a negative surplus, i.e., quantity in the set,

$$Q_\delta^- := \{q' \in Q \mid v(q', k_{IR}(0,0)) - c(q', k_{IR}(0,0)) < 0\},$$

can relax the (IR) constraint. This set will be nonempty if Q includes an excessive large level of quantity so that any trade will propose net loss, regardless of k. Of course, even when a contract with $\hat{q} \in Q_\delta^-$ exists, it may not implement a higher investment than the null contract, since $k_\delta(0,0) > k_\delta(\hat{q},\hat{t})$ for $\hat{q} > 0$. As mentioned earlier, the latter inequality becomes unimportant as $\delta \to 1$, since the second term in (13) vanishes. That is, as $\delta \to 1$, (IR) is all that matters. If $k_{IR}(0,0) < k^*$, then (IR) is binding in the absence of contracts. In this case, a trade contract specifying $\hat{q} \in Q_1^-$ relaxes the constraint *and* it can implement a higher investment than the null contract.

Proposition 8 *A trade contract is worthless if $k^* \leq k_{IR}(0,0)$ or if Q_1^- is empty. If $k_{IR}(0,0) < k^*$ (or equivalently $\alpha\phi(k^*) - k^* < 0$) and Q_1^- is nonempty, then a trade contract specifying $\hat{q} \in Q_1^-$ strictly dominates the null contract for δ sufficiently close to 1.*

This result modifies the conclusion of Che and Hausch (1999) in an important way: A trade contract generating a negative surplus can be strictly welfare improving in the dynamic setting. While such a contract may appear unappealing, it can improve the incentives by making credible a larger punishment than would be feasible without that contract.

5 Conclusion

We have shown that allowing for a simple and plausible investment dynamics in a holdup model produces much different implications on the design of important contracts and organizations than have been suggested in the literature. The novel theme in our prediction is that the incentives for specific investments depend not just on how a contract affects the investor's exposure to the specificity *at the margin* — the focus on the recent contract/organization literature — but more importantly on how the contract affects the investor's exposure to *absolute* specificity. The absolute specificity was never a concern in the static models since the individual rationality constraint is never binding there, but it is an important consideration in our dynamic model since the steeper incentives provided by investment dynamics may cause the latter constraint to be binding.

A shift of emphasis from marginal specificity to absolute specificity takes us back to the original "transaction cost analysis" (TCA) authors (Klein at el., 1978; Williamson 1979, 1985), who were largely concerned about the absolute level of specificities parties are subject to as the source of inefficiencies and the rationale for organizational interventions. While we agree that the absolute specificity matters, our specific predictions differ from these authors as well. Our theory predicts that contracts that would exacerbate the parties' vulnerability to holdup — rather than those protecting them from holdup (as proposed by the TCA authors) — can be desirable. As discussed in the paper, this view throws a more positive light on a variety of "hostage taking" or "hands-tying" arrangements such as exclusivity agreements, joint ownership of assets, and trade contracts compelling parties to trade excessive amounts. These contracts/organization forms can perform well in our dynamic model since they can create a strong equilibrium punishment for deviation.

The fact that our predictions are largely based on the absolute level of quasi-rents could also make them more empirically testable. As Whinston (2002) points out, the GHM theory is difficult to test, since the (marginal) effects of investment on the disagreement payoffs are difficult to estimate, especially since most feasible levels of investment are not made in equilibrium. By contrast, hypotheses pertaining to the effects of absolute specificities can be tested without observing payoff consequences of all investment choices, especially when investments are totally specific.

6 Appendix: Proofs

Proof of Lemma 1: Given Assumption 1(a)-(b), $U_\delta(k;m)$ has increasing differences in $(k;\delta)$ and has a unique maximizer. Hence, we have $k_\delta(m) \le k_{\delta'}(m)$ for $\delta' > \delta$. The limiting properties hold by Berge's theorem of maxima since U_δ is continuous and has a unique maximizer. Given Assumption 1(a)-(c), for any $k \in K$

$$\begin{aligned} U'_\delta(k;m) &= \alpha[\phi'(k) - (1-\delta)\psi'_2(k;m)] + (1-\alpha)(1-\delta)\psi'_1(k;m) - [1-(1-\alpha)\delta] \\ &< \alpha[\phi'(k) - (1-\delta')\psi'_2(k;m)] + (1-\alpha)(1-\delta')\psi'_1(k;m) - [1-(1-\alpha)\delta'] = U'_{\delta'}(k;m), \end{aligned}$$

where the strict inequality holds since $\psi'_1(k;m) < 1$ by Assumption 1(c) and since $\psi'_2(k;m) \ge 0$. Since $U'_1(k^*;m) = 0$, this inequality means, with $\delta' = 1$, $U'_\delta(k^*;m) < 0$, which means that $k_\delta(m) < k^* < \bar{k}$ for all δ. Now consider any $\delta' > \delta$ such that $k_{\delta'}(m) > 0$. Since $k_{\delta'}(m) < \bar{k}$ from the above argument, we must have $0 = U'_1(k_{\delta'}(m);m) > U'_\delta(k_{\delta'}(m);m)$, which implies that $k_\delta(m) \ne k_{\delta'}(m)$. Given the first statement of the lemma, we then have $k_\delta(m) < k_{\delta'}(m)$, as needed. ▌

Proof of Lemma 2: The proof consists of several steps:

Step 1: *In any MPE with contract m satisfying Assumptions 1 and 2, we have*

$$\sigma_m(k) \ge U_\delta(k;m) + k - \delta[\alpha\psi_2(y_m^\infty(k)) - (1-\alpha)\psi_1(y_m^\infty(k)) + (1-\alpha)y_m^\infty(k)],$$

for any $k \in K$. If the parties agree to trade after reaching k in MPE, then

$$\sigma_m(k) \le U_0(k;m) + k.$$

Proof. For notational simplicity, we suppress the dependence on m. Observe first that

$$\begin{aligned} &\sigma(k) \\ \ge\ & \alpha\phi(k) - \alpha\delta\beta(y(k)) - \alpha(1-\delta)\psi_2(k) + (1-\alpha)\delta[\sigma(y(k)) - (y(k)-k)] + (1-\alpha)(1-\delta)\psi_1(k) \\ =\ & \alpha\phi(k) - (1-\delta)[\alpha\psi_2(k) - (1-\alpha)\psi_1(k)] - (1-\alpha)\delta(y(k)-k) \\ & -\alpha\delta[\delta\beta(y^2(k)) + (1-\delta)\psi_2(y(k))] + (1-\alpha)\delta[\delta[\sigma(y^2(k)) - (y^2(k)-y(k))] + (1-\delta)\psi_1(y^2(k))] \end{aligned}$$

$$
\begin{aligned}
&= \alpha\phi(k) - (1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}[\alpha\psi_2(y^{t-1}(k)) - (1-\alpha)\psi_1(y^{t-1}(k))] - (1-\alpha)\sum_{t=1}^{\infty}\delta^t(y^t(k) - y^{t-1}(k)) \\
&\quad + \lim_{t\to\infty}\delta^t[(1-\alpha)\sigma(y^t(k)) - \alpha\beta(y^t(k))] \\
&= \alpha[\phi(k) - (1-\delta)\psi_2(k)] + (1-\alpha)(1-\delta)\psi_1(k) + (1-\alpha)\delta k \\
&\qquad -(1-\delta)\sum_{t=1}^{\infty}\delta^t[\alpha\psi_2(y^t(k)) + (1-\alpha)(y^t(k) - \psi_1(y^t(k)))], \\
&= U_\delta(k) + k - (1-\delta)\sum_{t=1}^{\infty}\delta^t[\alpha\psi_2(y^t(k)) + (1-\alpha)(y^t(k) - \psi_1(y^t(k)))],
\end{aligned}
$$

where the first inequality follows since the trade may not be optimal, the first equality holds by substituting the value functions (whether trade occurs or not at $y(k)$), the second equality is obtained by following the procedure recursively and collecting terms, and the penultimate equality follows by rearranging the summation series and since $\beta(\cdot)$ and $\sigma(\cdot)$ are both finite (each lying in $[0, \phi(\bar{k})]$).

To prove the first claim, note that the bracketed term in the last line is nondecreasing in $y^t(k)$, by Assumption 1(a) and (c). Thus, since $y^t(k) \leq y^\infty(k)$, the last line cannot be less than

$$
\begin{aligned}
&U_\delta(k) + k - (1-\delta)\sum_{t=1}^{\infty}\delta^t[\alpha\psi_2(y^\infty(k)) + (1-\alpha)(y^\infty(k) - \psi_1(y^\infty(k)))] \\
= \; &U_\delta(k) + k - \delta[\alpha\psi_2(y^\infty(k)) + (1-\alpha)(y^\infty(k) - \psi_1(y^\infty(k)))],
\end{aligned}
$$

as needed. To prove the second claim, notice that the first inequality (of the above string) holds with equality if trade occurs immediately at k. Next, recall that the bracketed term in the last line is nondecreasing in $y^t(k)$. Thus, since $y^t(k) \geq k$, the last line — and therefore $\sigma(k)$ — is no greater than

$$
\begin{aligned}
&U_\delta(k) + k - (1-\delta)\sum_{t=1}^{\infty}\delta^t[\alpha\psi_2(k) + (1-\alpha)(k - \psi_1(k))] \\
= \; &U_\delta(k) + k - \delta[\alpha\psi_2(k) + (1-\alpha)(k - \psi_1(k))] \\
= \; &U_0(k) + k.
\end{aligned}
$$

▮

Step 2: *From any initial level of investment, in an MPE with contract m satisfying Assumptions 1 and 2, trade occurs in the first period.*

Proof. Suppose trade does not occur immediately following $k = y(0) \geq 0$. (The dependence on the contract in place, m, will be suppressed throughout.) Then, we have from (4) that

$$\sigma(k) = \delta[\sigma(y(k)) - (y(k) - k)] + (1 - \delta)\psi_1(k). \tag{15}$$

Next, we show that $\sigma(k) = \sigma(y(k)) - (y(k) - k) = \psi_1(k)$. On the one hand, incentive compatibility implies that

$$\sigma(y(k)) - (y(k) - k) \geq \sigma(k)$$

(since $y(k)$ is an equilibrium investment path from k), on the other hand, it also implies that

$$\sigma(k) - k \geq \sigma(y(k)) - y(k)$$

(since $k = y(0)$ and $y(k)$ is also feasible starting from zero investment). These two inequalities give

$$\sigma(k) = \sigma(y(k)) - (y(k) - k),$$

which implies, when substituted into (15), that $\sigma(k) = \sigma(y(k)) - (y(k) - k) = \psi_1(k)$, as was to be shown.

We consider two cases. Suppose first $k = y(0) > 0$. If party 1 deviates from $k = y(0)$ by not investing at all and perpetually inducing rejection then she will obtain

$$(1 - \delta)\left[\psi_1(0) + \delta\psi_1(0) + \delta^2\psi_1(0) + ...\right] = \psi_1(0).$$

Since $\sigma(k) - k = \psi_1(k) - k < \psi_1(0)$ by Assumption 1(c), the deviation is profitable. Hence, we have a contradiction.

Suppose now $y(0) = 0$. Since $y(y(0)) = 0$, if no trade occurs at $y(0) = 0$, then $\beta(0) = \delta\beta(0) + (1 - \delta)\psi_2(0)$, implying that $\beta(0) = \psi_2(0)$. Hence,

$$\sigma(0) + \beta(0) = \psi_1(0) + \psi_2(0).$$

Meanwhile, it follows from (4) and (5) that

$$\sigma(0) + \beta(0) \geq \phi(0).$$

We then have a contradiction since, by Assumption 2, $\phi(0) > \psi_1(0) + \psi_2(0)$. ▮

Step 3: *In any MPE with contract m satisfying Assumptions 1 and 2,* $y_m^\infty(k) \leq \max\{k, k_\delta(m)\}$.

Proof. Suppose not. Then, there must exist $k \in K$ such that $\hat{k} := y(k) > \max\{k, k_\delta\}$. (The dependence on m is suppressed to economize on notation.) Since $\hat{k}$ is an equilibrium choice starting from k, we must have

$$\sigma(\hat{k}) - \hat{k} \geq \sigma(k') - k', \forall k' \geq k.$$

Let

$$k^o := \sup\{k' | \sigma(k') - k' = \sigma(\hat{k}) - \hat{k}\}.$$

By definition, for each $\epsilon > 0$, there exists an $k_\epsilon \leq k^o$ within ϵ distance from k^o, such that $\sigma(k_\epsilon) - k_\epsilon = \sigma(\hat{k}) - \hat{k}$. Since $\sigma(k'') - k'' < \sigma(\hat{k}) - \hat{k}$ for all $k'' > k^o$, $y^\infty(k_\epsilon) \leq k^o$. Further, this must hold for any $\epsilon > 0$, so we must have $y^\infty(k') \leq k^o$ for any $k' < k^o$.

Meanwhile, for $\epsilon > 0$ sufficiently small, trade will occur after reaching k_ϵ (since $k_\epsilon \to k^o$ as $\epsilon \to 0$ and $y(k_\epsilon) \leq k^o$, as noted above). Hence, by Lemma Step 1,

$$U_0(k_\epsilon) \geq \sigma(k_\epsilon) - k_\epsilon \geq U_\delta(k_\epsilon) - \delta[\alpha\psi_2(y^\infty(k_\epsilon)) + (1-\alpha)(y^\infty(k_\epsilon) - \psi_1(y^\infty(k_\epsilon)))].$$

Now, take the limit of all sides as $\epsilon \downarrow 0$, using the fact that $\lim_{\epsilon\downarrow 0} k_\epsilon = k^o$ and that $y^\infty(k_\epsilon) \leq k^o$, which implies that $\lim_{\epsilon\downarrow 0} y^\infty(k_\epsilon) = k^o$. Then, the left-most and the right-most sides converge to $U_0(k^o)$, implying that $\lim_{\epsilon\downarrow 0}\{\sigma(k_\epsilon) - k_\epsilon\} = U_0(k^o)$. Since, by the definition of k_ϵ, $\lim_{\epsilon\downarrow 0}\{\sigma(k_\epsilon) - k_\epsilon\} = \sigma(\hat{k}) - \hat{k}$, we must have $\sigma(\hat{k}) - \hat{k} = U_0(k^o)$.

Suppose now that, starting at k, party 1 deviates to $k' \in [\max\{k, k_\delta\}, \hat{k})$ instead of investing up to $\hat{k} = y(k)$. We have

$$\begin{aligned}
\sigma(k') - k' &\geq U_\delta(k') - \delta[\alpha\psi_2(y^\infty(k')) + (1-\alpha)(y^\infty(k') - \psi_1(y^\infty(k')))] \\
&\geq U_\delta(k') - \delta[\alpha\psi_2(k^o) + (1-\alpha)(k^o - \psi_1(k^o))] \\
&> U_0(k^o) \\
&= \sigma(\hat{k}) - \hat{k},
\end{aligned}$$

where the first inequality follows from Step 1, the second inequality follows since $y^\infty(k') \leq k^o$ as noted above, the strict inequality follows since $k^o \geq \hat{k} > k' \geq k_\delta$,[19] and the last equality follows from the argument established above. The inequality proves that the deviation to

[19]Note that

$$U_0(k^o) - \delta[\alpha\psi_2(k^o) + (1-\alpha)(k^o - \psi_1(k^o))] = U_\delta(k^o) < U_\delta(k'),$$

since $k^o > k' \geq k_\delta$.

$k' < \hat{k}$ is profitable, contradicting the initial hypothesis that $y(k) > \max\{k, k_\delta\}$ for some $k \in K$. ▌

Steps 1 and 2 yield parts (i) and (ii), whereas Step 3 has proven part (iii). ▌

Proof of Theorem 1: We have already shown in the text that $k \in K_\delta$ if k is implementable by m. Hence, it suffices to show the converse. Let $\bar{k}_\delta(m) := \max K_\delta(m)$, and consider the following investment strategy:

$$\hat{y}_m(k) = \begin{cases} \hat{k} & \text{if } k \in [0, \hat{k}], \\ \bar{k}_\delta(m) & \text{if } k \in (\hat{k}, \bar{k}_\delta(m)] \\ k & \text{if } k \in (\bar{k}_\delta(m), \bar{k}]. \end{cases}$$

With this strategy, party 1 initially invests to $\hat{k} = y_m(0)$ and never increases further (i.e., $y_m(y_m(0)) = \hat{k}$).

Substituting this investment strategy into (4) and (5) and solving for party 1's payoff, we obtain

$$\begin{aligned} &\hat{\sigma}(k) - k \\ = &\begin{cases} \max\{U_\delta(k) - \delta[\alpha\psi_2(\hat{k})) - (1-\alpha)\psi_1(\hat{k}) + (1-\alpha)\hat{k}], \delta[U_0(\hat{k}) + k] + (1-\delta)\psi_1(k) - k\} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } k \in [0, \hat{k}], \\ \max\{U_\delta(k) - \delta[\alpha\psi_2(\bar{k}_\delta) - (1-\alpha)\psi_1(\bar{k}_\delta) + (1-\alpha)\bar{k}_\delta], \delta[U_0(\bar{k}_\delta) + k] + (1-\delta)\psi_1(k) - k\} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } k \in (\hat{k}, \bar{k}_\delta] \\ U_0(k) \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \text{if } k \in (\bar{k}_\delta(m), \bar{k}]. \end{cases} \end{aligned}$$

(The dependence on m is suppressed to economize on space.)

Note first that trade follows immediately after $\hat{k}$ is reached, as claimed. That is, $\hat{\sigma}(\hat{k}) = U_0(\hat{k}) + \hat{k} \geq \delta(U_0(\hat{k}) + \hat{k}) + (1-\delta)\psi_1(\hat{k})]$ since $\phi(k) \geq \psi_1(k) + \psi_2(k)$ by Assumption 2. Observe next that likewise, $\hat{\sigma}(k) - k = U_0(k; m)$ at $k = \bar{k}_\delta(m)$. In particular, $\hat{k}$ and $\bar{k}_\delta(m)$ constitute local maxima, with the former being the global maximum: party 1's payoff at $k = \hat{k}$ dominates her payoff at any $k \in K$, and her payoff at $k = \bar{k}_\delta(m)$ dominates her payoff at any $k \in (\hat{k}, \bar{k}]$. It then follows that $\hat{k} \in \arg\max_{\tilde{k} \in [k, \bar{k}]}\{\hat{\sigma}(\tilde{k}) - \tilde{k}\}$ for any $k \leq \hat{k}$, and $\max\{\bar{k}_\delta(m), k\} \in \arg\max_{\tilde{k} \in [k, \bar{k}]}\{\hat{\sigma}(\tilde{k}) - \tilde{k}\}$ for any $k > \hat{k}$. Hence, the aforementioned strategy, $\hat{y}_m(\cdot)$, is optimal, given $\hat{\sigma}(\cdot)$.

The last statement follows from Lemma 2-(i). ▌

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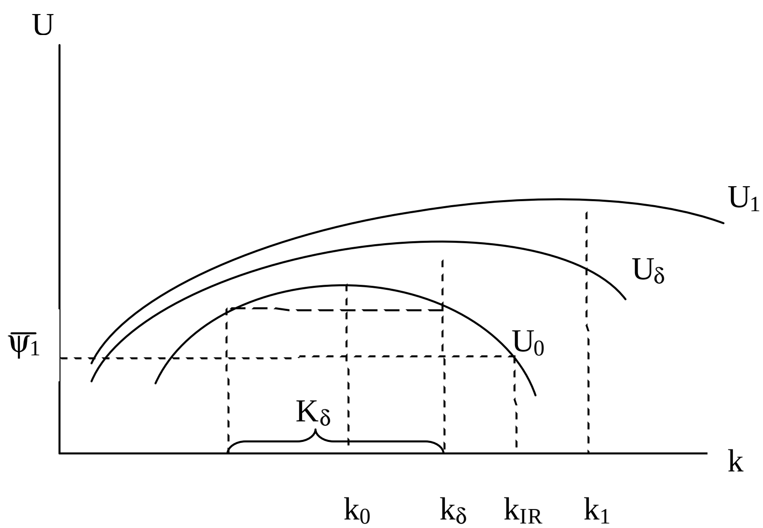

Figure 1a: K_δ when IR is not binding

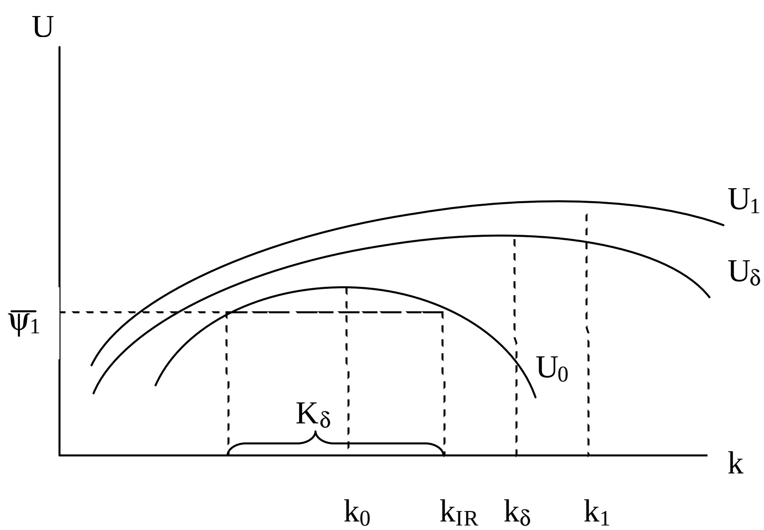

Figure 1b: K_δ when IR is binding