European Economic Review 43 (1999) 947–957

EUROPEAN ECONOMIC REVIEW

Multi-Dimensional Screening

Mechanism design with a liquidity constrained buyer: The 2×2 case[1]

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Abstract

This paper studies the implications of buyers' liquidity constraints for the optimal selling strategy. The possibility that a buyer faces a binding liquidity constraint affects the seller's strategy in a nontrivial way. Specifically, when a seller has one unit of a good to sell to a buyer with a quasilinear utility function, the 'no-haggling' result indicates that textbook monopoly pricing is optimal, absent liquidity constraints. Introducing a potentially binding liquidity constraint vitiates the no-haggling result, and can make it strictly beneficial for the seller to use nonlinear pricing, to commit to a declining price sequence, or to require the buyer to post a cash bond. © 1999 Elsevier Science B.V. All rights reserved.

JEL classification: C70; D42; D45

Keywords: Mechanism design; Liquidity constraint

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[1] This paper is adapted from our paper, entitled 'The optimal mechanism for selling to budget-constrained consumers', which considers a continuum of buyer types. The current paper illustrates the points made in that paper for a discrete 2×2 case.

0014-2921/99/$ – see front matter
PII: S0014-2921(98)00107-X

1. Introduction

The existing literature on price discrimination is concerned with screening consumers based on their willingness to pay, which implicitly assumes that consumers' liquidity constraints are not binding. In practice, liquidity constraints are important in many situations. The mere fact that many consumers pay substantial financing charges to credit card companies confirms the presence of liquidity constraints in many transactions. Consumers' liquidity constraints are especially relevant for durable goods such as houses and cars, since these items are expensive relative to the disposable income of many households, and because consumers' access to capital markets is often imperfect.

The current paper studies the optimal selling strategy when a buyer may be liquidity constrained. We demonstrate, using a discrete 2×2 case, that the possibility of a liquidity constraint affects the optimal selling strategy in a non-trivial way. When a seller has a single indivisible good to sell to a buyer with a quasilinear utility function, the 'no-haggling' result holds. That is, posting a single price (i.e., textbook monopoly pricing) is an optimal strategy for the seller (Harris and Raviv, 1981; Riley and Zeckhauser, 1983; Stokey, 1979). This result implies that it would never be strictly beneficial for a seller to offer a menu of contracts involving different quantities or lotteries, or to commit to a sequence of prices that decline over time.[2] We show that the no-haggling result does not hold when the buyer may be liquidity constrained. Specifically, the presence of a binding liquidity constraint can rationalize the use of nonlinear pricing, commitment to a declining price sequence, or the use of a cash bond requirement for the buyer.

2. Model

A seller has one unit of a good, which she values at zero. The quantity $q \in [0, 1]$ can be interpreted as the fraction of a unit that the buyer obtains, if the good is divisible, or the probability that the buyer obtains one unit of the good, if the good is indivisible. Alternatively, one can interpret q as representing quality (which is observable to the buyer). The buyer is risk-neutral, and he values $q \in [0, 1]$ units of the good at $v_1 q$ or $v_2 q$, where $v_2 > v_1 > 0$.

The buyer has a budget of either w_1 or w_2, which is the maximum that he can spend. In the standard models, it is implicitly assumed that $w_1 = w_2 \geq v_2$. In that case, the optimal method of sale is to offer $q = 1$ at a price of $p = v_1$ or

[2] Thus, a declining intertemporal price sequence can only be explained by a seller's inability to commit to the monopoly price. See Gul et al. (1986).

$p = v_2$, whichever is more profitable (Harris and Raviv, 1981; Riley and Zeckhauser, 1983; Stokey, 1979).

To make budget constraints an interesting possibility here, we assume:

Assumption 1. $0 \le w_1 < v_2 < w_2$.

The second inequality means that the buyer cannot pay v_2 if he has a budget of w_1. The last inequality means that the buyer is not constrained if he has the high budget, regardless of his valuation. From now on, a buyer with (v_i, w_j) will be referred to as a type-*ij* buyer. The probability that the buyer is a type *ij* is $\pi_{ij} \ge 0$, and $\sum_{i,j \in \{1,2\}} \pi_{ij} = 1$. There are two mutually exclusive and exhaustive cases:

- Positive correlation (or monotone likelihood ratio property):

$$\frac{\pi_{21}}{\pi_{11}} \le \frac{\pi_{22}}{\pi_{12}}.$$

- Negative correlation:

$$\frac{\pi_{21}}{\pi_{11}} > \frac{\pi_{22}}{\pi_{12}}.$$

In the positive correlation case, having a high budget makes it (weakly) more likely that the buyer has a high valuation. The opposite is true in the negative correlation case. (Note that we include the case of independence as part of the positive correlation case.)

By the revelation principle, the seller cannot do better than to offer a direct-revelation mechanism $\langle q_{ij}, t_{ij} \rangle_{i,j \in \{1,2\}}$, where q_{ij} and t_{ij} denote the quantity obtained and the lump-sum payment made, respectively, when the buyer reports type *ij*. The possibility of random payments is excluded, which will be justified later.

Any feasible mechanism must satisfy certain constraints. First, since the buyer has the option of not participating, thereby getting zero surplus, the following condition must be satisfied:

$$v_i q_{ij} - t_{ij} \ge 0, \quad \forall i, j \in \{1, 2\}. \tag{IR}$$

Second, the payment must not exceed the buyer's budget:

$$t_{ij} \le w_j, \quad \forall i, j \in \{1, 2\}. \tag{BC}$$

Finally, incentive compatibility conditions must be met. We consider two different versions, depending on whether the seller can require the buyer to post a cash bond. Suppose that a cash bond requirement can be imposed. Such a requirement can prevent the buyer from exaggerating his budget, so the only nontrivial constraint is to prevent the high-budget type from mimicking the low-budget type:

$$v_i q_{ij} - t_{ij} \geq v_i q_{kl} - t_{kl}, \quad \forall i, j, k, l \in \{1, 2\} \text{ such that } j \geq l. \tag{IC}$$

While a cash bond requirement may appear unrealistic, it can be replaced by a financial disclosure requirement. The latter requirement would have the same effect, since a buyer can conceal his wealth but cannot exaggerate it. Disclosure requirements are often used in real estate transactions, for example.

There are also transactions in which such requirements are not used, so it is of interest to consider the case without a cash bond requirement. Incentive compatibility is stronger without the requirement:

$$v_i q_{ij} - t_{ij} \geq v_i q_{kl} - t_{kl}, \quad \forall i, j, k, l \in \{1, 2\} \text{ such that } t_{kl} \leq w_j.$$

This latter constraint requires that the buyer have no incentive to choose any other contract that he can afford, not just the contracts intended for buyers with (weakly) lower budgets. Clearly, the former set of contracts includes the latter, given (BC). Since (IC′) requires comparison with more contracts, it is at least as strong as (IC). One of our interests is in determining when there is no loss of generality in restricting attention to (IC′).

The seller's problem is to maximize her expected revenue, $\sum_{i,j \in \{1,2\}} \pi_{ij} t_{ij}$, subject to these feasibility constraints. We will first solve the problem for the more restrictive set of mechanisms satisfying (IC′). In Section 4, we solve the problem allowing for cash bonds, so that (IC) is imposed.

3. Optimal restricted mechanism

The basic methodology involves the transformation of the mechanism design problem into a nonlinear pricing problem. We shall argue that the seller can implement the optimal mechanism by offering a convex, piecewise-linear pricing function, $T(q)$, with slopes in $\{v_1, v_2\}$, and with $T(0) = 0$. Pick any feasible mechanism satisfying (IR), (BC) and (IC′), and consider the contracts for the high-budget types (i.e., type-$i2$ buyers, $i = 1, 2$). Now consider the indifference curve of each type that passes through its own contract. (The indifference curves are straight lines.) Since each of these types can mimic the other, these contracts must be on the upper envelope of the two indifference curves, which is a convex, piecewise-linear curve with slopes in $\{v_1, v_2\}$ (see Fig. 1.)

Suppose that the seller offers all contracts in this upper envelope, and assume that she can induce the buyer to choose the contract most favorable to her if the buyer is indifferent. Then, each high-budget type will choose a contract that will

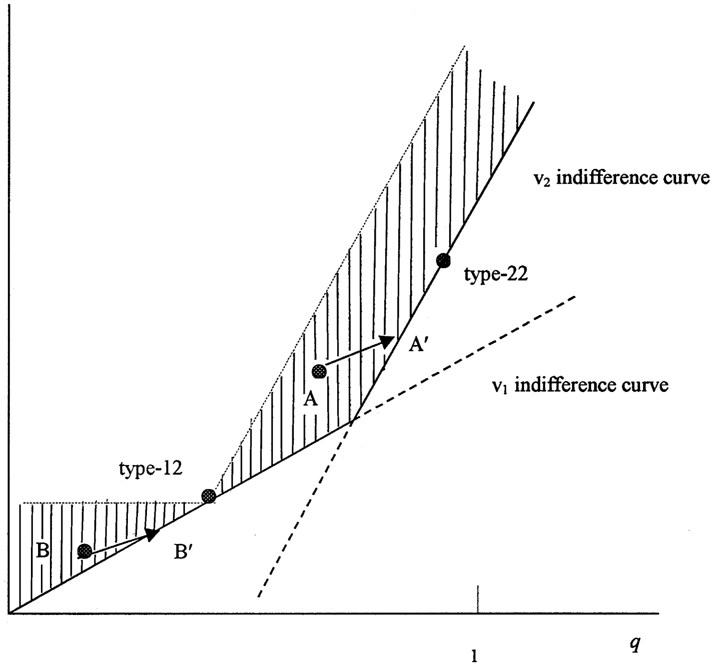

Fig. 1. Feasible pricing function.

pay the seller at least the amount that she would get under the original mechanism.

We now turn to the low-budget types. In particular, consider a type-21 buyer. Under the original mechanism, the buyer's contract will lie in either hatched area (say point A or B); otherwise a type 22 or type 12 would mimic. In either case, when the seller replaces the original mechanism with the upper envelope, a type 21 prefers to move to A' or B', for example, depending on the original contract, or to a contract that involves paying more than under the original contract. A similar argument establishes that a type-11 buyer will now pay weakly more as well. Thus, since every type pays (weakly) more, the seller will do at least as well as before by offering the pricing function whose graph is given by the upper envelope.

This argument has established that we can focus on convex, piecewise-linear pricing functions with slopes in $\{v_1, v_2\}$, without loss of generality. The additional restriction that $T(0) = 0$ is easily obtained. To see this, note that (IR) for the original mechanism implies that $T(0) \leq 0$. But if $T(0) < 0$, the seller could move the entire function upward and do no worse in terms of revenue. Based on this argument, we now look for the optimal nonlinear pricing function of the form described above. Specifically, we consider

$$T(q) \equiv \begin{cases} v_1 q & \text{if } q \in [0, \hat{q}], \\ v_1 \hat{q} + v_2(q - \hat{q}) & \text{if } q \in (\hat{q}, 1], \end{cases}$$

for some $\hat{q} \in [0, 1]$.

When facing such a pricing function, a buyer with (v_i, w_j) maximizes $v_i q - T(q)$ subject to $T(q) \le w_j$. It is optimal (and feasible) for the buyer to increase q as long as $v_i \ge T'(q)$ and $T(q) < w_j$. Accordingly, a type-12 buyer will pick $q = \hat{q}$, a type 11 will pick $q = \min\{\hat{q}, T^{-1}(w_1)\}$, a type 22 will pick $q = 1$, and a type 21 will pick $q = \min\{1, T^{-1}(w_1)\}$. It follows that the seller's expected revenue is

$$R(\hat{q}) \equiv \min\{v_1\hat{q}, w_1\}\pi_{11} + v_1\hat{q}\pi_{12} + \min\{v_1\hat{q} + v_2(1 - \hat{q}), w_1\}\pi_{21} + [v_1\hat{q} + v_2(1 - \hat{q})]\pi_{22}.$$

The optimization problem for the seller has now been reduced to finding $\hat{q} \in [0, 1]$ to maximize $R(\hat{q})$. It is useful to characterize its derivative in the following three cases, which are exhaustive:

$$R'(q) \equiv \begin{cases} v_1(\pi_{11} + \pi_{12}) - (v_2 - v_1)\pi_{22} \\ \qquad \text{if } \hat{q} \in \left[0, \min\left\{\dfrac{w_1}{v_1}, \dfrac{v_2 - w_1}{v_2 - v_1}\right\}\right], \\ v_1(\pi_{11} + \pi_{12}) - (v_2 - v_1)(\pi_{21} + \pi_{22}) \\ \qquad \text{if } w_1 > v_1 \text{ and } \hat{q} \in \left(\dfrac{v_2 - w_1}{v_2 - v_1}, 1\right], \\ v_1\pi_{12} - (v_2 - v_1)\pi_{22} \\ \qquad \text{if } w_1 \le v_1 \text{ and } \hat{q} \in \left(\dfrac{w_1}{v_1}, 1\right]. \end{cases}$$

Inspecting this derivative yields the seller's optimal strategy. Furthermore, by studying the behavior of the different types, we can uncover the optimal contracts to offer in the original mechanism design problem. The following proposition summarizes the results.

Proposition 1. Suppose that $w_1 > v_1$. (i) *If* $v_2/v_1 \le 1/(\pi_{21} + \pi_{22})$, *then the optimal* $\hat{q}$ *is* $\hat{q}^* = 1$, *and the buyer chooses* $\langle q_{11}, t_{11}\rangle = \langle q_{12}, t_{12}\rangle = \langle q_{21}, t_{21}\rangle = \langle q_{22}, t_{22}\rangle = \langle 1, v_1\rangle$. (ii) *If* $1/(\pi_{21} + \pi_{22}) < v_2/v_1 \le (1 - \pi_{21})/\pi_{22}$, *then* $\hat{q}^* = (v_2 - w_1)/(v_2 - v_1)$, *and the buyer chooses* $\langle q_{11}, t_{11}\rangle = \langle q_{12}, t_{12}\rangle = \langle (v_2 - w_1)/(v_2 - v_1), v_1[(v_2 - w_1)/(v_2 - v_1)]\rangle$, $\langle q_{21}, t_{21}\rangle = \langle q_{22}, t_{22}\rangle = \langle 1, w_1\rangle$. (iii) *If* $v_2/v_1 > (1 - \pi_{21})/\pi_{22}$, *then* $\hat{q}^* = 0$, *and the buyer chooses* $\langle q_{11}, t_{11}\rangle = \langle q_{12}, t_{12}\rangle = \langle 0, 0\rangle$, $\langle q_{21}, t_{21}\rangle = \langle (w_1/v_2), w_1\rangle$, *and* $\langle q_{22}, t_{22}\rangle = \langle 1, v_2\rangle$.

Now, suppose that $w_1 \le v_1$. (iv) *If* $v_2/v_1 \le 1 + (\pi_{12}/\pi_{22})$, *then* $\hat{q}^* = 1$, *and the buyer chooses* $\langle q_{11}, t_{11}\rangle = \langle q_{21}, t_{21}\rangle = \langle (w_1/v_1), w_1\rangle$, $\langle q_{12}, t_{12}\rangle = \langle q_{22}, t_{22}\rangle = \langle 1, v_1\rangle$. (v) *If* $1 + (\pi_{12}/\pi_{22}) < v_2/v_1 \le (1-\pi_{21})/\pi_{22}$, *then* $\hat{q}^* = w_1/v_1$, *and the buyer chooses* $\langle q_{11}, t_{11}\rangle = \langle q_{12}, t_{12}\rangle = \langle q_{21}, t_{21}\rangle = \langle (w_1/v_1), w_1\rangle$, *and* $\langle q_{22}, t_{22}\rangle = \langle 1, w_1 + v_2(1-(w_1/v_1))\rangle$. (vi) *If* $v_2/v_1 > (1-\pi_{21})/\pi_{22}$, *then* $\hat{q}^* = 0$, *and the buyer chooses* $\langle q_{11}, t_{11}\rangle = \langle q_{12}, t_{12}\rangle = \langle 0, 0\rangle, \langle q_{21}, t_{21}\rangle = \langle (w_1/v_2), w_1\rangle$, *and* $\langle q_{22}, t_{22}\rangle = \langle 1, v_2\rangle$.

Proof. Observe that $R'(\cdot)$ is nonincreasing, so $R(\cdot)$ is concave. The threshold values of v_2/v_1 are obtained by setting each row of $R'(\cdot)$ equal to zero. In each subcase, the optimal value of $\hat{q}$ is obtained by inspecting the signs of $R'(\cdot)$ and choosing the supremum quantity at which the derivative is nonnegative. The consumer choices are determined from their optimal responses above. Q.E.D.

Example 1. Assume $\pi_{11} = \pi_{12} = \pi_{21} = \pi_{22} = 1/4$. Then, the optimal pricing function is linear with slope v_1 if $v_2/v_1 \le 2$, and it is linear with slope v_2 if $v_2/v_1 \ge 3$. Otherwise, it is nonlinear, with a kink at $\hat{q}^* = \min\{w_1/v_1, (v_2 - w_1)/(v_2 - v_1)\}$.

The optimal pricing function is linear, with a slope of v_1 if v_2/v_1 is sufficiently low (cases (i) and (iv)), and with a slope of v_2 if v_2/v_1 is sufficiently high (cases (iii) and (vi)). At first glance, these cases appear very similar to the cases without binding liquidity constraints, where the optimal strategy is to set a single per-unit price. Except for (i), however, the low-budget type is quantity rationed here. Thus, one might see contracts with different quantities, albeit with the same per-unit price.

The other cases stand in greater contrast. In particular, there is an intermediate range of v_2/v_1 (cases (ii) and (v)) for which the pricing function has a kink: the slope jumps from v_1 to v_2 at an interior point. The nature of price discrimination is different in these two cases. In case (v), the low-budget types are quantity rationed and face a low per-unit price, while the high-valuation high-budget type faces the higher per-unit price (at the margin). In case (ii), there is discrimination based on valuations; i.e., the low-valuation types face a per-unit price of v_1, while the high-valuation types face a higher per-unit price.

It is interesting that, although analogous schemes are available in the case without binding liquidity constraints, the seller does not exploit them. To see why this occurs, consider the behavior of the seller when the constraints do not bind. When the seller raises the high-valuation type's payment, for example, she may lower the low-valuation type's quantity ($\hat{q}^*$ in our model) to prevent the high-valuation type from taking the low-type's contract. The net effect enters the seller's payoff in a linear fashion, so the optimal strategy is either to eliminate the low type's contract altogether or to allow complete pooling at the price v_1.

To see that the argument above does not apply with binding liquidity constraints, consider the conditions for (ii) to occur. The left inequality can be rewritten as

$$v_1(\pi_{11} + \pi_{12}) < (v_2 - v_1)(\pi_{21} + \pi_{22}),$$

which means that the revenue gain from the high-valuation types, when raising their payment, exceeds the revenue loss from the low-valuation types caused by meeting the incentive compatibility constraint, *assuming that the high-valuation type's payment does not exceed* w_1. In other words, absent the liquidity constraints, it would be more profitable to sell only to the type-2j buyers at the price v_2 than to sell to all types at v_1. Once the type-2j buyer's payment hits w_1, however, additional increases in the payment for obtaining $q = 1$ can generate additional revenue from a type-22 buyer only. The second inequality says that such an increase is not profitable:

$$v_1(\pi_{11} + \pi_{12}) \geq (v_2 - v_1)\pi_{22}.$$

This kind of discontinuity in the tradeoff facing the seller is responsible for the kinked pricing function.

There is an interesting bargaining interpretation of the optimal pricing function in case (ii) as well. Suppose that the seller can commit to a sequence of prices over discrete rounds, and can choose the length of the rounds. The traditional literature shows that, for any period length and discount factor, the optimal strategy with commitment is to stick to the same price (thereby selling in the first round only). Here, by contrast, if $1/(\pi_{21} + \pi_{22}) < v_2/v_1 \leq (1 - \pi_{21})/\pi_{22}$, committing to a declining price sequence can be optimal. The seller can choose the length of the first two rounds so that both types' discount factor for the elapse of one round equals $\delta = (v_2 - w_1)/(v_2 - v_1)$, and she can commit to $p_1 = w_1$ and $p_2 = p_3 = \cdots = v_1$. The high-valuation type will buy one unit in the first period and pay w_1. The low-valuation type will wait until the second period, buy one unit (which is equivalent to $q = \delta = \hat{q}^*$, from the perspective of the first period), and pay v_1, the present discounted value of which is $\delta v_1 = \hat{q}^* v_1$, as described in (ii) of the proposition. Thus, this intertemporal pricing scheme implements the optimal mechanism.

4. Unrestricted mechanisms: The use of cash bonds

We now solve for the optimal selling mechanism in the unrestricted set of feasible mechanisms, with (IC′) replaced by the weaker version (IC). This expansion of the feasible set can be accomplished by requiring the buyer to post a cash bond. Assuming that the buyer does not have access to funds other than his budget, the cash bond requirement effectively prevents the low-budget type from mimicking the high-budget type.

To solve for the optimal mechanism, we again focus on the corresponding price functions. Che and Gale (1997) show (using arguments similar to those in the previous section) that we can restrict attention to a pair of pricing functions, $T(q, w_j)$, one for each budget type, which are nondecreasing and convex in the first argument, and satisfy $T(0, w_j) = 0$ and $T(\cdot, w_1) \geq T(\cdot, w_2)$.[3] The last condition comes from the fact that the high-budget types can always choose the low-budget types' contracts, but not the reverse. Using the arguments in the previous section, we can further restrict attention to functions that are piecewise linear in the first argument, with a slope that jumps from v_1 to v_2 at some $\hat{q}_i \in [0, 1]$. The requirement that $T(\cdot, w_1) \geq T(\cdot, w_2)$ is then equivalent to $\hat{q}_1 \leq \hat{q}_2$.

If $\hat{q}_1 = \hat{q}_2$, then the two pricing functions coincide, making the bond requirement superfluous. Thus, we need only consider cases with $\hat{q}_1 < \hat{q}_2 \leq 1$. The expected revenue from type-$i2$ buyers is $\hat{q}_2\pi_{12}v_1 + [\hat{q}_2\pi_{22}v_1 + (1 - \hat{q}^2)\pi_{22}v_2]$. Fixing $\hat{q}_1 < 1$, the seller will only select $\hat{q}_2 > \hat{q}_1$ if this revenue is increasing in $\hat{q}_2$, which implies $v_2/v_1 < 1 + (\pi_{12}/\pi_{22})$. This is the first necessary condition for cash bonds to yield higher profit. When it holds, the seller optimally selects $\hat{q}_2 = 1$.

We now turn to the type-$i1$ buyers, and search for cases where the seller will select $\hat{q}_1 < 1$. If $w_1 \leq v_1$, the seller can extract the entire budget by setting $\hat{q}_1 = 1$, so we need $w_1 > v_1$ to ensure that $\hat{q}_1 < 1$. When $w_1 > v_1$, the seller will select $\hat{q}_1$ such that $T(1, w_1) \leq w_1$. Suppose, to the contrary, that $T(1, w_1) > w_1$. Then, the seller could raise $\hat{q}_1$, get more revenue from the type-11 buyer, and still get w_1 from the type-21 buyer. Given that the seller selects $\hat{q}_1$ such that $T(1, w_1) \leq w_1$, the revenue from the type-$i1$ buyer is $\hat{q}_1\pi_{11}v_1 + [\hat{q}_1\pi_{21}v_1 + (1 - \hat{q}_1)\pi_{21}v_2]$. To get $\hat{q}_1 < 1$, we need this revenue expression to be decreasing in $\hat{q}_1$, which means $1 + (\pi_{11}/\pi_{21}) < v_2/v_1$. This is the final necessary condition.

The discussion above shows when the two pricing functions differ. Since the constraints are looser with cash bonds, we obtain the following result.

Proposition 2. The use of cash bonds increases the seller's expected revenue if and only if

$$1 + \frac{\pi_{11}}{\pi_{21}} < \frac{v_2}{v_1} < 1 + \frac{\pi_{12}}{\pi_{22}} \tag{1}$$

and $w_1 > v_1$. *When these conditions are satisfied, the optimal (unrestricted) mechanism offers* $\langle q_{11}, t_{11}\rangle = \langle (v_2 - w_1)/(v_2 - v_1), v_1(v_2 - w_1)/(v_2 - v_1)\rangle, \langle q_{21}, t_{21}\rangle = \langle 1, w_1\rangle$, *and* $\langle q_{12}, t_{12}\rangle = \langle q_{22}, t_{22}\rangle = \langle 1, v_1\rangle$.

[3] The problem thus involves designing a two-dimensional mechanism on a two-dimensional type space. For a more general treatment of this problem, see Armstrong (1996), Rochet and Choné (1998), and Jehiel et al. (1998).

Proof. The above discussion confirmed the necessary conditions. We now show that the suggested mechanism is the unique solution when (IC) applies. Consider q_1 and q_2 such that $q_1 \leq q_2$. The discussion above shows that, fixing q_1, the seller's expected revenue is maximized by $q_2 = 1$. Likewise, fixing q_2, and given the constraint that $q_1 \leq q_2$, the seller's expected revenue is maximized by $q_1 = \min\{q_2, (v_2 - w_1)/(v_2 - v_1)\}$. (Note that $q_1 = (v_2 - w_1)/(v_2 - v_1)$ gives $T(1, w_1) = w_1$.) It follows that the unique solution has $\hat{q}_1 = (v_2 - w_1)/(v_2 - v_1) < 1$ and $\hat{q}_2 = 1$. A standard revealed preference argument shows that the seller's revenue is higher when the looser constraint ((IC) rather than (IC′)) applies, as it leads to a different solution. Q.E.D.

To gain more intuition, it is useful to consider the conditions described in Proposition 2. The inequalities in Eq. (1) can be rewritten as

$$v_1 < v_2 \frac{\pi_{21}}{\pi_{11} + \pi_{21}} \tag{2}$$

and

$$v_1 > v_2 \frac{\pi_{22}}{\pi_{12} + \pi_{22}}. \tag{3}$$

Inequality (2) says that, conditional on the seller knowing that a buyer has a low budget, the seller would rather charge v_2 and sell only to the high-valuation type than charge v_1 and sell to all low-budget types, assuming that the low-budget buyer is unconstrained. (In reality, the constraint binds since the seller can only extract $w_1 < v_2$ from the high-valuation type, so she sells to the low-valuation type as well.) On the other hand, inequality (3) says that, conditional on the seller knowing that a buyer has a high budget, the seller would prefer to charge v_1 and sell to both types rather than charge v_2 and sell only to the high-valuation type. Thus, inequality (1) means that, if the seller could observe the buyer's budget, she would charge more to the low-budget types, absent a binding liquidity constraint.

It is not surprising that the use of cash bonds helps the seller in such a circumstance, as the cash bond requirement allows the seller to discriminate against the low-budget types. The requirement cannot be used to discriminate against the high-budget types, however, since they can take any contract that a low-budget type can choose. Naturally, these two inequalities require that the negative correlation property be satisfied: $\pi_{21}/\pi_{11} > \pi_{22}/\pi_{21}$. Hence, if the probabilities exhibit (weak) positive correlation, then the cash bond requirement is superfluous and the mechanism identified in Proposition 1 is optimal in this unrestricted set of mechanisms as well.[4]

[4] For this reason, the restriction to nonrandom payments is without loss of generality.

5. Summary

This paper has shown that the possibility of consumer liquidity constraints changes the optimal selling strategy in nontrivial ways. In doing so, we have developed a rationale for the use of practices such as nonlinear pricing, commitment to a declining price sequence, and cash bond requirements, some of which can only be explained when liquidity constraints are explicitly recognized.

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