Supplementary Notes for "Decentralized College Admissions"

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The notes consist of three sections. Section A and Section B provide formal analyses of "restricted applications" and "information acquisition and evaluation costs," respectively. Section C provides empirical evidences for the aggregate uncertainty and non-monotonicity on Hanyang's admission strategies.

A Restricted Applications

A student's taste $y \in \mathcal{Y} \equiv [0,1]$ is drawn according to a distribution that depends on the underlying state. For a given s, let $K(y|s)$ be the distribution of y with density function $k(y|s)$, which is continuous and obeys (strict) monotone likelihood ratio property: for any $y' > y$ and $s' > s$,

$$\frac{k(y'|s')}{k(y|s')} > \frac{k(y'|s)}{k(y|s)}, \tag{A.1}$$

meaning that a student's value for A is likely to be high when s is high. We further assume that there exists $\delta > 0$ such that $\left|\frac{k_y(y|s)}{k(y|s)}\right| < \delta$ for any $y \in [0,1]$ and $s \in [0,1]$, which indicates that students' tastes change moderately relative to the state. Each student with taste y forms a posterior belief about the state s, given by the following conditional density:

$$l(s|y) := \frac{k(y|s)}{\int_0^1 k(y|s)ds}.$$

Before proceeding, we make the following observations: First, for each student, applying to a college dominates not applying at all. Second, since a student does not know the score and the student's preference is independent of the score, the student's application depends solely on the preference. Third, since each student's preference depends on the state, the mass of students applying to each college varies across states. Let $n_i(s)$ be the mass of students who apply to college $i = A, B$ in state s.

Next, consider colleges' admission strategies. Since colleges face no enrollment uncertainty, it is optimal to admit all students up to a cutoff: for $i = A, B$,

$$\hat{v}_i(s) := \inf \{v \in [0,1] \,|\, n_i(s)(1 - G(v)) \leq \kappa\}.$$

If $n_i(s) \geq \kappa$ in state s, then college i will set its cutoff so as to admit students up to its capacity. Otherwise, it will admit all applicants.

Consider students' application decisions. Fix any strategy $\sigma : \mathcal{Y} \to [0,1]$ which specifies a probability of applying to A for each $y \in \mathcal{Y}$. The mass of applicants to each college is

$$n_A(s|\sigma) := \int_0^1 \sigma(y)k(y|s)\,dy$$

and $n_B(s|\sigma) = 1 - n_A(s|\sigma)$. A student with taste y expects to be admitted by college $i = A, B$ with probability

$$P_i(y|\sigma) \equiv \mathbb{E}[1 - G(\hat{v}_i(s)) \,|\, y, \sigma] = \int_0^1 q_i(s|\sigma)l(s|y)ds,$$

where $q_i(s|\sigma) := \min\big\{\kappa/n_i(s|\sigma), 1\big\}$. This probability depends on the student's preference y since it is correlated with the underlying state. Note that a student with taste y will apply to A if and only if $yP_A(y|\sigma) \geq (1-y)P_B(y|\sigma)$.

We now provide characterizations of students' application behavior. Lemma A1 shows that students follow a cutoff strategy in any equilibrium, given a moderate value of δ, and Theorem A1 shows that equilibrium involves strategic application by students if one school is more popular than the other.

Lemma A1. *Suppose $\delta \leq \frac{1}{2}$. In any equilibrium, there exists a cutoff $\hat{y}$ such that students with $y \geq \hat{y}$ apply to A and those with $y < \hat{y}$ apply to B. And such an equilibrium exists.*

Proof. Define $T(y|\sigma) := y\,P_A(y|\sigma) - (1-y)\,P_B(y|\sigma)$ and fix any σ. To prove the optimality of the cutoff strategy, we show that $T'(y|\sigma) > 0$ for any y. Note that

$$\begin{aligned} T'(y|\sigma) &= P_A(y|\sigma) + P_B(y|\sigma) + yP'_A(y|\sigma) - (1-y)P'_B(y|\sigma) \\ &\geq y\big(P_A(y|\sigma) + P'_A(y|\sigma)\big) + (1-y)\big(P_B(y|\sigma) - P'_B(y|\sigma)\big) \\ &= y\int_0^1 q_A(s|\sigma)\big(l(s|y) + l_y(s|y)\big)ds + (1-y)\int_0^1 q_B(s|\sigma)\big(l(s|y) - l_y(s|y)\big)ds. \end{aligned}$$

Observe that

$$l(s|y) + l_y(s|y) = \frac{k(y|s)}{\int_0^1 k(y|s)ds}\left(1 + \frac{k_y(y|s)}{k(y|s)} - \frac{\int_0^1 k_y(y|s)ds}{\int_0^1 k(y|s)ds}\right) > \frac{k(y|s)}{\int_0^1 k(y|s)ds}(1 - 2\delta),$$

where the inequality holds since

$$\frac{k_y(y|s)}{k(y|s)} > -\delta \quad \text{and} \quad \frac{\int_0^1 k_y(y|s)ds}{\int_0^1 k(y|s)ds} = \frac{\int_0^1 \frac{k_y(y|s)}{k(y|s)} k(y|s)ds}{\int_0^1 k(y|s)ds} < \delta$$

because $\left|\frac{k_y(y|s)}{k(y|s)}\right| < \delta$. Similarly,

$$l(s|y) - l_y(s|y) = \frac{k(y|s)}{\int_0^1 k(y|s)ds}\left(1 - \frac{k_y(y|s)}{k(y|s)} + \frac{\int_0^1 k_y(y|s)ds}{\int_0^1 k(y|s)ds}\right) > \frac{k(y|s)}{\int_0^1 k(y|s)ds}(1-2\delta),$$

where the inequality holds since $\left|\frac{k_y(y|s)}{k(y|s)}\right| < \delta$. We thus have $T'(y|\sigma) > 0$ whenever $\delta \geq \frac{1}{2}$.

It remains to show that there exists an equilibrium in cutoff strategy. Let $\hat{y}$ be a cutoff. Then, $n_A(s|\hat{y}) = \int_{\hat{y}}^1 k(y|s)dy = 1 - K(\hat{y}|s)$. Hence,

$$P_A(y|\hat{y}) = \int_0^1 \min\left\{\frac{\kappa}{1-K(\hat{y}|s)}, 1\right\} l(s|y)ds \text{ and } P_B(y|\hat{y}) = \int_0^1 \min\left\{\frac{\kappa}{K(\hat{y}|s)}, 1\right\} l(s|y)ds.$$

Now, let $T(y|\hat{y}) := yP_A(y|\hat{y}) - (1-y)P_B(y|\hat{y})$. Note that

$$T(0|\hat{y}) = -P_B(0|\hat{y}) = -\int_0^1 \min\left\{\frac{\kappa}{K(\hat{y}|s)}, 1\right\} l(s|0)ds < 0,$$

where the inequality holds since $\min\left\{\frac{\kappa}{K(\hat{y}|s)}, 1\right\} > 0$ and $l(s|0) \geq 0$, with strict inequality for a positive measure of states. Similarly, $T(1|\hat{y}) > 0$. By the continuity of $T(\cdot|\hat{y})$, there is a $\tilde{y}$ such that $T(\tilde{y}|\hat{y}) = 0$. Moreover, such a $\tilde{y}$ is unique since $T'(y|\hat{y})\big|_{y=\tilde{y}} > 0$.

Next, let $\tau : [0,1] \to [0,1]$ be the map from $\hat{y}$ to $\tilde{y}$, which is implicitly defined by $T(\tau(\hat{y})|\hat{y}) = 0$ according to the implicit function theorem (since $T'(y|\hat{y})\big|_{y=\tilde{y}} > 0$). Since $P_A(y|\cdot)$ is nondecreasing and $P_B(y|\cdot)$ is nonincreasing $\hat{y}$, $\tau(\cdot)$ is decreasing. Hence, there is a fixed point such that $\tau(\hat{y}) = \hat{y}$, and so there is $\hat{y}$ such that $T(\hat{y}|\hat{y}) = 0$. ■

Theorem A1. *Suppose $\mu_A(s) > \frac{1}{2}$ $\left(\mu_A(s) = \frac{1}{2}\right)$ for almost all s. Then, $\hat{y} \in (\frac{1}{2}, 1)$ $\left(\hat{y} = \frac{1}{2}\right)$, where $\hat{y}$ is the equilibrium cutoff.*

Proof. We first show $\hat{y} < 1$. If $\hat{y} = 1$, then $n_A(s|1) = 1 - K(1|s) = 0$ and so $P_A(y|\hat{y}) = 1$ for any y. Hence, $T(1|1) = P_A(1|1) = 1$, which contradicts the fact that $T(\hat{y}|\hat{y}) = 0$. Next, we show that $\hat{y} > \frac{1}{2}$ whenever $\mu_A(s) > \frac{1}{2}$. Suppose to the contrary $\hat{y} \leq \frac{1}{2}$. Then, it holds that $\frac{1}{2} < \mu_A(s) = 1 - K(\frac{1}{2}|s) \leq 1 - K(\hat{y}|s)$, so $K(\hat{y}|s) < 1 - K(\hat{y}|s)$. Therefore,

$$P_A(y|\hat{y}) - P_B(y|\hat{y}) = \int_0^1 \min\left\{\frac{\kappa}{1-K(\hat{y}|s)}, 1\right\} l(s|y)ds - \int_0^1 \min\left\{\frac{\kappa}{K(\hat{y}|s)}, 1\right\} l(s|y)ds \leq 0. \tag{A.2}$$

Hence, if $\hat{y} < \frac{1}{2}$, then

$$T(\hat{y}|\hat{y}) = \hat{y}P_A(\hat{y}|\hat{y}) - (1-\hat{y})P_B(\hat{y}|\hat{y}) < \tfrac{1}{2}\big(P_A(\hat{y}|\hat{y}) - P_B(\hat{y}|\hat{y})\big) \leq 0, \tag{A.3}$$

where the first inequality holds since $\hat{y} < \frac{1}{2}$. Thus, $T(\hat{y}|\hat{y}) < 0$, a contradiction. Suppose now $\hat{y} = \frac{1}{2}$. Notice that since $K(\hat{y}|s) < 1 - K(\hat{y}|s)$, we have $K(\frac{1}{2}|s) < \frac{1}{2} < 1-\kappa$, where the the second inequality holds since $\kappa < \frac{1}{2}$. So, $\kappa/(1-K(\frac{1}{2}|s)) < 1$. Therefore, the last inequality of (A.2) becomes strict, and hence $T(\hat{y}|\hat{y}) = \frac{1}{2}\big(P_A(\frac{1}{2}|\frac{1}{2}) - P_B(\frac{1}{2}|\frac{1}{2})\big) < 0$, a contradiction again.

Lastly, $\hat{y} = \frac{1}{2}$ whenever $\mu_A(s) = \frac{1}{2}$. If $\hat{y} < \frac{1}{2}$, then $\frac{1}{2} = \mu_A(s) = 1 - K(\frac{1}{2}|s) < 1 - K(\hat{y}|s)$, so we have $K(\hat{y}|s) < 1 - K(\hat{y}|s)$. By (A.2) and (A.3), we reach a contradiction. If $\hat{y} > \frac{1}{2}$, then $\frac{1}{2} = \mu_A(s) = 1 - K(\frac{1}{2}|s) > 1 - K(\hat{y}|s)$ and so $K(\hat{y}|s) > 1 - K(\hat{y}|s)$. We then have $P_A(y|\hat{y}) - P_B(y|\hat{y}) \geq 0$ and

$$T(\hat{y}|\hat{y}) = \hat{y}P_A(\hat{y}|\hat{y}) - (1-\hat{y})P_B(\hat{y}|\hat{y}) > \tfrac{1}{2}\big(P_A(\hat{y}|\hat{y}) - P_B(\hat{y}|\hat{y})\big) \geq 0,$$

where the first inequality holds since $\hat{y} > \frac{1}{2}$. Thus, $T(\hat{y}|\hat{y}) > 0$, a contradiction. ■

The intuition behind Theorem A1 is clear. If A is more popular than B, then A becomes more difficult to get in than B, all else equal. Hence, students who prefer B ($y \leq \frac{1}{2}$) will definitely apply to B. But, even students who mildly prefer A (i.e., y is greater than but close to $\frac{1}{2}$) will apply to B instead of A.

Let us now consider fairness and welfare properties of the equilibrium outcome. First, the equilibrium is unfair. Justified envy arises in that (i) students who happen to have applied to a more popular college for a given state may be unassigned even though their scores are good enough for the other college; and (ii) students who prefer but avoid an ex ante more popular college get into an ex ante less popular college, but they could have gotten into the former when it becomes ex post less popular. Second, a college may be undersubscribed in equilibrium so that its capacity is not filled even though there are unassigned, acceptable students. By assigning those students to unfilled seats of that college, students and the college will be both better off. Thus, the equilibrium outcome is inefficient.

Theorem A2. *The outcome of the restricted applications is unfair. Suppose $K(\hat{y}|s) < \kappa$ for a positive measure of states. Then, college B suffers from under-subscription, and the outcome is Pareto inefficient.*

Proof. For the first part of the theorem, observe that for a given s, justified envy arises whenever $\hat{v}_A(s) \neq \hat{v}_B(s)$. Suppose to the contrary $\hat{v}_A(s) = \hat{v}_B(s)$ for almost all s. Recall that equilibrium admission cutoffs satisfy

$$G(\hat{v}_A(s)) = \max\left\{1 - \frac{\kappa}{1-K(\hat{y}|s)}, 0\right\} \quad \text{and} \quad G(\hat{v}_B(s)) = \max\left\{1 - \frac{\kappa}{K(\hat{y}|s)}, 0\right\}.$$

Since $G(\cdot)$ is strictly increasing, if $\hat{v}_A(s) = \hat{v}_B(s)$, then we must have either $n_i(s) < \kappa$ for all $i = A, B$ (so that $\hat{v}_A(s) = \hat{v}_B(s) = 0$) or $n_A(s) = n_B(s) \geq \kappa$. Note, however, that we cannot have $n_i(s) < \kappa$ for all i in equilibrium, since this means that all applicants are admitted by either college, which contradicts the fact that $2\kappa < 1$. Next, suppose $n_A(s) = n_B(s) \geq \kappa$. This implies that $K(\hat{y}|s) = \frac{1}{2}$ for all s (recall that $n_A(s) = 1 - K(\hat{y}|s)$ and $n_B(s) = K(\hat{y}|s)$). However, by (A.1), we have $K(\hat{y}|s') < K(\hat{y}|s)$ for all $s' > s$. Therefore, we reach a contradiction again.

For the second part of the theorem, recall that for given $\hat{y}$ in equilibrium, the mass of students applying to B is $K(\hat{y}|s)$. Thus, if there is a positive measure of states in which $K(\hat{y}|s) < \kappa$, then college B faces under-subscription in such states, implying that the equilibrium outcome is inefficient. ■

B DA with Costly Learning of Preferences by Students

Consider DA with only common measure. In order to study the information acquisition behavior of students, we need to introduce cardinal utilities of the students: in state s, a fraction $\mu_i(s)$ of students gets utility u from college i and $u'(< u)$ from j, where $i, j = A, B$ and $i \neq j$. Students do not know their preferences but can learn about them at small cost $c > 0$. We assume that the cost is sufficiently small so that if a student is certain of gaining admissions from both colleges, then he will incur the cost. (As in the paper, students know their own v).

Assume $\mathbb{E}[\mu_A(s)] > \frac{1}{2}$ without loss of generality (the case in which $\mathbb{E}[\mu_A(s)] = \frac{1}{2}$ is similar but requires separate analysis). We look for an equilibrium in which students with v incur the cost to learn their preferences if and only if $v \geq \hat{v}$ for some cutoff $\hat{v}$; we later show such an equilibrium exists. In this equilibrium with cutoff $\hat{v}$, the strategyproofness of DA means that those who incur the cost rank the colleges according to their actual realized preferences and those who do not incur the cost rank them according to their prior, which means they rank A ahead of B. It then follows that a fraction $\mu_A(s)(1 - G(\hat{v})) + G(\hat{v})$ of students ranks A above B, and the remaining fraction $(1 - \mu_A(s))(1 - G(\hat{v}))$ of students ranks B above A.

Following Azevedo and Leshno (2014), a DA allocation in each state s is characterized by the cutoffs $(\hat{v}_A(s; \hat{v}), \hat{v}_B(s; \hat{v}))$ for the colleges such that each student is assigned to the most preferred college (according to his ranking, described above) among the colleges whose cutoff is below his score. Clearly, the outcome maintains the feature that only top 2κ in v are assigned to either college, which implies that $\min\{\hat{v}_A(s; \hat{v}), \hat{v}_B(s; \hat{v})\} = \overline{v}$, where $\overline{v}$ satisfies $1 - G(\overline{v}) = 2\kappa$. Obviously, students with $v < \overline{v}$ will not be assigned anywhere. This in turn implies that no student with $v \leq \overline{v}$ has any incentive to incur the learning cost. Hence, from now on, we restrict attention to $\hat{v} > \overline{v}$.

To be precise, $(\hat{v}_A(s;\hat{v}), \hat{v}_B(s;\hat{v}))$ satisfies $D_i(\hat{v}_i(s;\hat{v}); \hat{v}_j(s;\hat{v})) = \kappa$, for $i, j = A, B$ and $i \neq j$, where D_i is the "demand" for college i:

$$D_A(\tilde{v};\hat{v}_B) := \begin{cases} \mu_A(s)(1-G(\tilde{v})) & \text{if } \tilde{v} \geq \hat{v}, \\ \mu_A(s)(1-G(\hat{v})) + \max\{\mu_B(s)(G(\hat{v}_B(s)) - G(\hat{v})), 0\} + G(\hat{v}) - G(\tilde{v}) & \text{if } \tilde{v} < \hat{v}, \end{cases}$$

and

$$D_B(\tilde{v};\hat{v}_A) := \begin{cases} \mu_B(s)(1-G(\tilde{v})) & \text{if } \tilde{v} \geq \hat{v}, \\ \mu_B(s)(1-G(\hat{v})) + \max\{\mu_A(s)(G(\hat{v}_A(s) - G(\hat{v})), 0\} & \\ \quad + \min\{G(\hat{v}_A(s;\hat{v})), G(\hat{v})\} - G(\tilde{v}) & \text{if } \tilde{v} < \hat{v}. \end{cases}$$

To see this, consider D_A. (D_B can be understood similarly.) The mass of students with $v \geq \hat{v}$ and ranking A above B is $\mu_A(s)(1-G(\hat{v}))$. If $\mu_A(s)(1-G(\hat{v})) \geq \kappa$, then only the top κ students among them are assigned to A, so $\mu_A(s)(1-G(\hat{v}_A(s;\hat{v}))) = \kappa$. If $\mu_A(s)(1-G(\hat{v})) < \kappa$, then all of such students are assigned to A, and the students with $v \geq \hat{v}$ and ranking B above A are also assigned to A whenever they are rejected by B (if exist). And then, the remaining seats are filled by the top students below $\hat{v}$. Hence, $\hat{v}_A(s;\hat{v})$ satisfies

$$\mu_A(s)(1-G(\hat{v})) + \max\{\mu_B(s)(G(\hat{v}_B(s)) - G(\hat{v})), 0\} + G(\hat{v}) - G(\hat{v}_A(s;\hat{v})) = \kappa.$$

Consider now a student with v. Suppose the student incurs the cost and finds himself preferring i. Then, he will be assigned to i and get u if $v \geq \hat{v}_i(s;\hat{v})$ or will be assigned to j and get u' if $\hat{v}_i(s;\hat{v}) > v \geq \overline{v}$. Denote the student's payoff by $\hat{u}(v;\hat{v}) - c$, where

$$\hat{u}(v;\hat{v}) := \int_0^1 \Big(\mu_A(s)\big(\mathbb{1}_{\{v\geq \hat{v}_A(s;\hat{v})\}}u + \mathbb{1}_{\{\hat{v}_A(s)>v\geq\overline{v}\}}u'\big) + \mu_B(s)\big(\mathbb{1}_{\{v\geq \hat{v}_B(s;\hat{v})\}}u + \mathbb{1}_{\{\hat{v}_B(s)>v\geq\overline{v}\}}u'\big)\Big)ds.$$

Suppose the student does not incur the cost. Since the student ranks A above B, he will be assigned to A and get $\mu_A(s)u + \mu_B(s)u'$ if $v \geq \hat{v}_A(s)$, or will be assigned to B and get $\mu_A(s)u' + \mu_B(s)u$ if $\hat{v}_A(s) > v \geq \overline{v}$. Thus, the student's payoff is

$$\tilde{u}(v;\hat{v}) := \int_0^1 \Big(\mathbb{1}_{\{v\geq \hat{v}_A(s;\hat{v})\}}(\mu_A(s)u + \mu_B(s)u') + \mathbb{1}_{\{\hat{v}_A(s)>v\geq\overline{v}\}}(\mu_A(s)u' + \mu_B(s)u)\Big)ds$$

Next, let $\Delta_u(v;\hat{v}) := \hat{u}(v;\hat{v}) - \tilde{u}(v;\hat{v})$ and observe that

$$\Delta_u(v;\hat{v}) = \int_0^1 \mu_B(s)\Big(u\big(\mathbb{1}_{\{v\geq \hat{v}_B(s;\hat{v})\}} - \mathbb{1}_{\{\hat{v}_A(s;\hat{v})>v\geq\overline{v}\}}\big) - u'\big(\mathbb{1}_{\{v\geq \hat{v}_A(s;\hat{v})\}} - \mathbb{1}_{\{\hat{v}_B(s;\hat{v})>v\geq\overline{v}\}}\big)\Big)ds$$

$$= \int_0^1 \mu_B(s)\,(u-u')\left(\mathbb{1}_{\{v \geq (\hat{v}_A(s;\hat{v}) \vee \hat{v}_B(s;\hat{v}))\}} - \mathbb{1}_{\{(\hat{v}_A(s;\hat{v}) \wedge \hat{v}_B(s;\hat{v})) > v \geq \overline{v}\}}\right) ds$$
$$= \int_0^1 \mu_B(s)\,(u-u')\,\mathbb{1}_{\{v \geq (\hat{v}_A(s;\hat{v}) \vee \hat{v}_B(s;\hat{v}))\}}\, ds$$

where the last equality holds since $\hat{v}_A(s;\hat{v}) \wedge \hat{v}_B(s;\hat{v}) = \overline{v}$. Note that

$$\Delta_u(\hat{v};\hat{v}) = \int_0^1 \mu_B(s)\,(u-u')\,\mathbb{1}_{\{\hat{v} \geq (\hat{v}_A(s;\hat{v}) \vee \hat{v}_B(s;\hat{v}))\}}\, ds$$
$$= \int_0^1 \mu_B(s)\,(u-u')\,\mathbb{1}_{\left\{1-\frac{\kappa}{1-G(\hat{v})} < \mu_A(s) < \frac{\kappa}{1-G(\hat{v})}\right\}}\, ds,$$

where the last equality holds since $\mu_i(s)(1-G(\hat{v})) < \kappa$ for $i = A, B$ if and only if $\hat{v} \geq \hat{v}_i(s;\hat{v})$ for $i = A, B$. Note also that $\Delta_u(\overline{v},\hat{v}) = 0$ and $\Delta_u(\hat{v};\hat{v})$ is strictly increasing in $\hat{v}$. Hence, there exists $\hat{v}^*(> \overline{v})$ such that $\Delta_u(\hat{v}^*;\hat{v}^*) = c$. We observe that $\hat{v}^* < \max_s \hat{v}_A(s;\hat{v}^*)$, or else $\mathbb{1}_{\left\{1-\frac{\kappa}{1-G(\hat{v}^*)} < \mu_A(s) < \frac{\kappa}{1-G(\hat{v}^*)}\right\}} = 0$ for almost all s, violating the equilibrium condition.

Since $\Delta_u(v;\hat{v})$ is nondecreasing in v, students with $v \geq \hat{v}^*$ will incur the cost. For students with $v < \hat{v}^*$,

$$\Delta_u(v;\hat{v}^*) = \int_0^1 \mu_B(s)\,(u-u')\,\mathbb{1}_{\{\hat{v}^* > v \geq (\hat{v}_A(s;\hat{v}^*) \vee \hat{v}_B(s;\hat{v}^*))\}}\, ds$$
$$= \int_0^1 \mu_B(s)\,(u-u')\,\mathbb{1}_{\left\{1-\frac{\kappa}{1-G(\hat{v}^*)} < \mu_A(s) < \frac{\kappa-(G(\hat{v}^*)-G(v))}{1-G(\hat{v}^*)}\right\}}\, ds$$
$$< c,$$

showing that those students do not incur the cost. Hence, the students' cutoff strategy with cutoff $\hat{v}^*$ forms an equilibrium indeed.

We highlight several properties of DA allocation in this case:

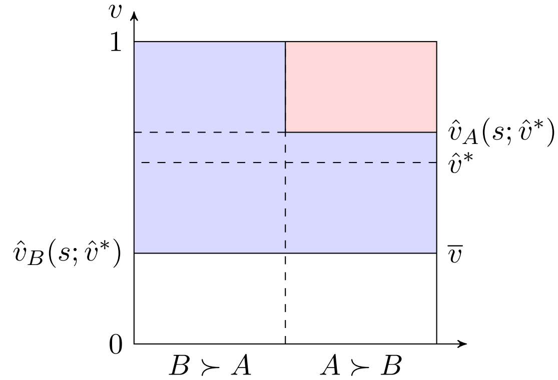
1. As with the baseline case, neither college over-enrolls or under-enrolls, and the outcome is jointly optimal for the colleges in the sense that the top 2κ students are assigned to colleges.

2. The fact that $\hat{v}^* \in (\overline{v}, \max_s \hat{v}_A(s;\hat{v}^*))$ means that there are states in which $\hat{v}^* < \max\{\hat{v}_A(s;\hat{v}^*), \hat{v}_B(s;\hat{v}^*)\}$ (depicted in Case 1 and 2-2 below) so that students with $v \in (\hat{v}^*, \hat{v}_A(s;\hat{v}^*))$ have no choice between the two colleges and should not have incurred the learning cost but they do; and there may arise states in which $\hat{v}^* > \max\{\hat{v}_A(s;\hat{v}^*), \hat{v}_B(s;\hat{v}^*)\}$ (depicted in Case 2-2 below) so that students with $v \in (\max\{\hat{v}_A(s;\hat{v}^*), \hat{v}_B(s;\hat{v}^*)\}, \hat{v}^*)$ do have a choice between the two colleges yet do not incur the learning cost. In short, the information acquisition behavior of students is inefficient.

3. The informational inefficiency in turn implies that justified envy and allocational inefficiency may arise. It remains true, however, that no student who is unassigned has justified envy.

☐ **Allocations.**

1. Suppose $\mu_A(s)(1 - G(\hat{v}^*)) \geq \kappa$ in equilibrium.

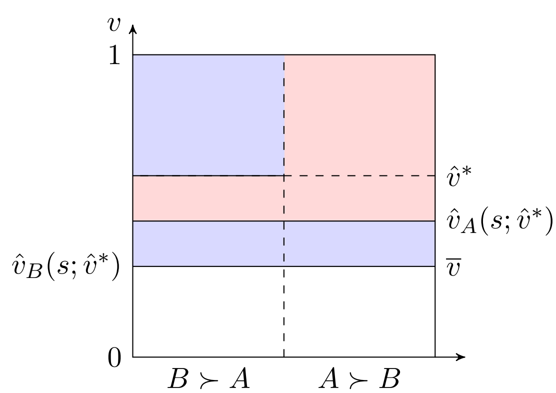


College A admits the top κ students among those with $v \geq \hat{v}^*$ and ranking it above B, so $\hat{v}_A(s;\hat{v}^*)$ satisfies $\mu_A(s)(1 - G(\hat{v}_A(s;\hat{v}^*))) = \kappa$. College B admits students from the top except those who are admitted by A, so $\hat{v}_B(s;\hat{v}^*) = \overline{v}$ satisfies

$$(1 - \mu_A(s))(1 - G(\hat{v}^*)) + \mu_A(s)(G(\hat{v}_A(s;\hat{v}^*)) - G(\hat{v}^*)) + G(\hat{v}^*) - G(\overline{v}) = \kappa.$$

2. Suppose $\mu_A(s)(1 - G(\hat{v}^*)) < \kappa$ in equilibrium. There are two cases as follows:

2-1. $(1 - \mu_A(s))(1 - G(\hat{v}^*)) < \kappa$.



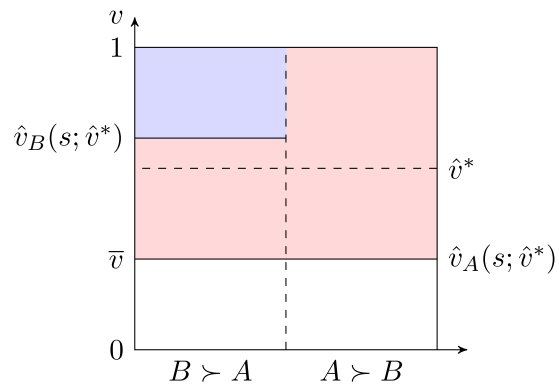
College A admits all students with $v \geq \hat{v}^*$ and raking it above B, and it also admits those with $v < \hat{v}^*$ to fill the remaining seats. Hence, $\hat{v}_A(s;\hat{v}^*)$ satisfies

$$\mu_A(s)(1 - G(\hat{v}^*)) + G(\hat{v}^*) - G(\hat{v}_A(s;\hat{v}^*)) = \kappa.$$

College B admits all students with $v \geq \hat{v}^*$ and raking it above A. It fills the remaining seats with students scoring below $\hat{v}^*$ except those who are admitted by A. Hence, $\hat{v}_B(s; \hat{v}^*) = \overline{v}$ satisfies

$$(1 - \mu_A(s))(1 - G(\hat{v}^*)) + G(\hat{v}_A(s; \hat{v}^*)) - G(\overline{v}) = \kappa.$$

2-2. $(1 - \mu_A(s))(1 - G(\hat{v}^*)) \geq \kappa$.



College B fills its capacity with the students scoring above $\hat{v}^*$ and raking it above A, so its cutoff $\hat{v}_B(s; \hat{v}^*)$ satisfies $(1 - \mu_A(s))(1 - G(\hat{v}_B(s; \hat{v}^*))) = \kappa$. College A admits students from the top except those who are admitted by B. Hence, $\hat{v}_A(s; \hat{v}^*) = \overline{v}$ satisfies

$$\mu_A(s)(1 - G(\hat{v}^*)) + (1 - \mu_A(s))(G(\hat{v}_B(s; \hat{v}^*)) - G(\hat{v}^*)) + G(\hat{v}^*) - G(\overline{v}) = \kappa.$$

C Empirical Evidence

C.1 Testing Aggregate Uncertainty

Table C.1 provides summary statistics of 34 US colleges in our sample. Next, Table C.2 summarizes the p-values for each colleges of Fisher's exact test and the Chi-square test.

C.2 Admissions from Hanyang University

We provide summaries of students' CSAT scores for the Department of Business (DoB) and the Department of Mechanical Engineering (DME) in Hanyang. Table C.3 shows that for DoB, the average scores of the top 10% admittees in the $i+1$-th round is higher than that of the bottom 10% in the ith round for each year and for all rounds. Table C.4 for DME exhibits a similar nonmonotonicity from the first to third rounds for each year.

Table C.1: Summary Statistics

Year	# of admitted	# of enrolled	Yield rate
2011	4548.88	1643.88	.41
	[4695.54]	[1599.02]	[.15]
2012	4595.97	1643.65	.40
	[4770.93]	[1626.04]	[.14]
2013	4709.76	1616.41	.39
	[4763.40]	[1582.49]	[.14]

Note: Standard deviations are in brackets.

Table C.2: Results for (1) the exact test and (2) the Chi-square test

	p-value			p-value	
College	(1)	(2)	College	(1)	(2)
Babson C	0.9005	0.8994	Kenyon C	0.8041	0.8012
Barnard C	0.6169	0.6159	Lafayette C	0.8732	0.8719
Bates C	0.0797	0.0798	Middlebury C	0.0066	0.0065
Boston	0.0013	0.0013	Olion C of Engineering	0.5233	0.5317
Brown	0.0012	0.0012	Princeton	9.59E-12	8.31E-12
CalTech	0.0577	0.0584	Rensselaer Polytech	0.0291	0.0285
Carnegie Mellon	0.4988	0.4988	Scripps C	0.6519	0.6511
Claremont McKenna C	0.0003	0.0003	St. Lawrence	0.0589	0.0587
C of Holy Cross	2.20E-16	2.20E-16	Stanford	2.31E-06	2.50E-06
C of William&Mary	0.2232	0.2227	U Chicago	2.20E-16	2.20E-16
Copper Union	0.9532	0.9512	U Maryland	0.4440	0.4438
Dartmouth C	0.2218	0.2217	U Michigan	0.0277	0.0277
Dickinson C	0.4713	0.4727	U Penn	0.3662	0.3665
Elon	0.6875	0.6872	U Rochester	0.0001	0.0001
George Washington	0.0313	0.0309	USC	2.42E-11	2.31E-11
Georgia Tech	0.0022	0.0022	U Wisconsin	0.0008	0.0008
Johns Hopkins	0.1791	0.1799	Vanderbilt	0.7578	0.7576

Table C.3: CSAT scores: Department of Business

Year	Round	Admittees	Average CSAT Scores		
			Total	≥90th Percentile	≤10th Percentile
2011	1	60	95.368	95.764	95.131
	2	5	95.140	95.197	95.094
	3	7	95.124	95.216	95.054
	4	2	95.129	95.199	95.060
	≥5	5	95.088	95.211	95.021
2012	1	69	97.872	98.076	97.598
	2	6	97.556	97.713	97.450
	3	8	97.382	97.460	97.306
	4	3	97.272	97.350	97.213
	≥5	14	97.142	97.507	96.961
2013	1	37	97.383	97.608	97.239
	2	5	97.223	97.290	97.176
	3	7	97.220	97.321	97.131
	4	3	97.146	97.169	97.134
	≥5	4	97.112	97.146	97.091

Note: The total score of CSAT is normalized by 100.

Table C.4: CSAT scores: Department of Mechanical Engineering

Year	Round	Admittees	Average CSAT Scores		
			Total	≥90th Percentile	≤10th Percentile
2011	1	48	91.059	92.074	90.529
	2	7	90.336	90.563	90.104
	3	2	90.097	90.119	90.076
	4	3	89.931	89.971	89.871
	≥5	5	89.706	89.847	89.630
2012	1	50	95.414	95.799	95.102
	2	4	95.114	95.176	95.064
	3	1	95.093	95.093	95.093
	4	2	95.011	95.031	94.991
	≥ 5	8	94.945	94.981	94.887
2013	1	49	94.757	95.317	94.460
	2	4	94.502	94.631	94.414
	3	3	94.559	94.626	94.513
	4	2	94.432	94.450	94.414
	≥ 5	6	94.346	94.390	94.287

Note: The total score of CSAT is normalized by 100.

References

Azevedo, Eduardo M. and Jacob D. Leshno. 2014. "A Supply and Demand Framework For Two-Sided Matching Market." Manuscript, Department of Economics and Public Policy, University of Pennsylvania.