Market versus Non-Market Assignment of Ownership*

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ABSTRACT: We compare different methods of assigning ownership of a good when some agents are wealth-constrained. When a good is sold at the market-clearing price, a high-wealth individual may buy it but a low-wealth (or illiquid) individual may not, even if the latter individual would have the higher valuation given equal wealth. Schemes that assign the good randomly may yield higher welfare than the competitive market would—if the recipients of the good are allowed to resell. Need-based schemes that favor the poor are particularly desirable. The ability to resell is critical to the results, but resale induces speculators to participate, so regulation of resale may be beneficial.

KEYWORDS: non-market assignment, rationing, resale, speculation.

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When ye are passed over Jordan into the land of Canaan ... ye shall divide the land by lot for an inheritance among your families ... every man's inheritance shall be in the place where his lot falleth ... (Num. 33:51-54)

1 Introduction

Suppose that a government wishes to privatize a scarce resource such as public land or radio spectrum. Selling it at the market-clearing price is a natural way to assign ownership, but non-market assignment methods such as first-come-first-served or a lottery also have a long history, as the passage above indicates, and their use remains widespread. Land was assigned on a first-come-first-served basis during the 1889 *Oklahoma Land Rush*. Emissions rights, fishing rights, and import quotas are often grandfathered based on historical data. Children are accepted into public schools based on where they live, which school their siblings attend, or simply the outcome of a lottery. Substantial numbers of new housing units are subject to binding price caps, with the units assigned by lottery.[1] In addition, lotteries are used to assign immigration visas and jury duty, and to select conscripts for military service. Organs are assigned to transplant patients by a priority rule that depends on factors such as the recipient's age and the severity of the condition. More broadly, court systems can be seen as (re)assigning property rights via non-market methods.

We study how non-market methods of assigning ownership compare with the market. The term *non-market methods* refers to schemes that employ institutions other than an unregulated market to assign ownership. This includes the possibility of assigning the good without using a price mechanism at all, as well as selling the good at a price other than the market-clearing price. It also includes two-step procedures with a resale market in the second stage.

While the exact method of assigning ownership should not matter as long as resale is allowed and agents have unlimited wealth,[2] in many important settings individuals face

[1]In Singapore, most citizens live in units sold by the government at prices that are well below the market. Some 82 per cent of Singapore's citizens live in "public housing flats," and about 95 per cent of those residents own their units. (See "Building Homes, Shaping Communities," at http://www.mnd.gov.sg/, accessed on Nov. 13, 2008.) The price cap is as low as half of the price on the resale market (Tu and Wong, 2002). The same fraction is given by Green, Malpezzi and Vandell (1994) for Korea. See also Kim (2002).

[2]This is the well-known Coase theorem (1960). The Coase Theorem is invoked frequently when new assignment schemes are proposed. One example concerns the Federal Communications Commission (FCC) spectrum license auctions. Opponents of the FCC's favored design argued that the design would not affect the ultimate allocation so revenue maximization should be the only goal. See the discussion in Milgrom (2004).

significant liquidity or wealth constraints.

Individuals and firms may face liquidity constraints for a variety of reasons. If the value that recipients derive from the good is largely non-monetary, as in the case of consumption goods, then an agent must rely on her own endowment to acquire it. If the value is monetary, she may borrow using the good as collateral. Yet, the value may be private information, prospective investors may not have good estimates of the value, and the collateral could devalue due to inadequate care by the agent. Many corporate and household assets fit into this latter category. Particularly relevant are human capital assets such as education and the right to immigrate since it is difficult to predict whether someone will develop marketable job skills or entrepreneurial skills, and it is difficult to collateralize such skills. These adverse selection and moral hazard problems may limit agents' access to capital markets for the purpose of financing the purchase.

When liquidity or wealth constraints arise, the good may not ultimately accrue to those who would value it most highly; if they do not acquire the good initially, they may be unable to purchase it on the resale market because of these constraints. More importantly, different methods of assigning the good may yield different *ultimate* allocations. In particular, selling the good at the market-clearing price may not guarantee an efficient allocation. For instance, if a productive asset or license to produce is sold to firms it may go to the firm that is better capitalized or more liquid instead of the one with the better business plan. In that sense, the market may not allocate the good efficiently.

We study how non-market methods of assigning ownership compare with the market in such an environment. To this end, we consider a model in which a good is assigned to a mass of agents who have quasilinear utilities with respect to the good and money, and who differ in their valuations of the good and their wealth constraints. The object could be a productive asset such as a license to operate a business (e.g., takeoff-and-landing slots, taxi medallions), to exploit resources (e.g., hunting or fishing rights), or to export goods, in which case the valuation reflects the monetary payoff that the asset will generate. Or, the object could be a consumption good such as housing or health care, in which case the valuation reflects the utility from consuming the good.

We compare the allocations resulting from market and non-market assignments in terms of Utilitarian welfare. In our context, Utilitarian welfare can be represented by the total value realized by agents in the given allocation. Utilitarian welfare is justified by Pareto efficiency if agents have quasilinear preferences and the surplus from the good is transferable. Then, if one allocation generates higher total value than another, there exists a side-payment scheme that makes the former Pareto superior to the latter. The surplus from the good would be transferable if the surplus comprises monetary returns, as with corporate assets or human capital. We focus on such goods as our canonical example.[3]

Our analysis yields several results.

☐ Desirability of non-market assignment:

As mentioned above, selling the good at the market-clearing price may yield inefficiencies if agents are liquidity constrained. A high-valuation agent with low wealth may not get the good while someone with higher wealth but a lower valuation may. It turns out that selling at a below-market price and using random rationing yields a welfare-superior allocation *if and only if resale is allowed*. Random rationing provides the good to some low-wealth individuals and low-valuation individuals who would not get it in the competitive market. If resale is permitted, the low-valuation recipients will resell while low-wealth high-valuation recipients will not. As a consequence, more high-valuation agents will get the good ultimately than in the competitive market.

☐ Desirability of need-based assignment:

The benefits of non-market assignment depend in part on the information available. Information on earnings and asset holdings can be used to favor the poor (*need-based assignment*), which is beneficial for the same reasons that random assignment is: more of the good is assigned initially to low-wealth high-valuation individuals. We show that shifting the initial assignment toward the agents who are more likely to have low wealth improves welfare, and that the maximally need-based assignment may attain the first-best outcome—the Utilitarian optimum—if the wealth signal is perfectly accurate.

☐ Speculation and desirability of restricting resale:

When a good is assigned at a below-market price and is transferable, this induces resale at a higher price. The opening of a resale market has two countervailing effects. On the one hand, the resale market mitigates the misallocation resulting from the initial non-market assignment. This raises efficiency. At the same time, it invites into the initial assignment process pure speculators who participate for the sole purpose of reselling. Their participation reduces the probability that low-wealth high-valuation agents get the good, which erodes the benefit of non-market assignment recognized above. In fact, the benefits of non-market

[3] If the surplus from the good is nonmonetary, as with consumption goods, Utilitarian welfare remains a reasonable criterion. The necessity of making interpersonal comparisons disappears under an ex ante perspective: An individual would maximize a Utilitarian welfare function if she acted under a veil of ignorance about her preferences, knowing only that "she has an equal chance of landing in the shoes of each member of the society" (Vickrey, 1945). Also see Harsanyi (1953, 1955).

assignment may vanish as the number of potential speculators grows without bound. In such cases, restricting or prohibiting resale may be desirable.

☐ Desirability of in-kind subsidies: The non-market assignment schemes employed here involve an in-kind subsidy since recipients of the good pay a below-market price. Although in-kind subsidies are often replicable by pure cash subsidies, we recognize one respect in which in-kind subsidies are superior to cash subsidies. An in-kind subsidy can be designed (via restrictions on resale) to limit speculation; the same cannot be done with a cash subsidy. We show that if speculation is significant, there exists a budget-balanced non-market assignment scheme with restricted resale that dominates any budget-balanced pure cash-subsidy mechanism.

Several specific conclusions can be drawn from the results. First, non-market assignment schemes may outperform an unregulated market. Despite the widespread use of non-market methods, their efficiency properties are not well-appreciated so their superior performance here provides a rationale for the use of such methods when wealth differences loom large. These observations should not be interpreted as a criticism of the fundamental merits of markets, however, since the non-market schemes succeed *only* in conjunction with a competitive resale market. Put differently, non-market assignment schemes complement markets.

Second, our findings provide a new basis for schemes that favor the poor. Need-based schemes are common in college admissions, subsidized housing programs, and license auctions. While these programs are often motivated by redistributive goals, our results suggest an efficiency rationale. Transferability of benefits is necessary for the desirable performance here, but it is often prohibited by such programs, so welfare may rise if they were to allow transferability.

Third, allowing resale may attract speculators whose participation undermines efficiency and redistribution goals. It may then be desirable to restrict transferability. Restrictions on transferability are typically justified by paternalistic arguments or concerns about fairness (e.g., those who wish to sell an organ might not make rational decisions or only the wealthiest patients will get a transplant), whereas the argument here is based on efficiency considerations.

The remainder of this paper is organized as follows. Section 2 lays out the model, and it describes the competitive market allocation. Section 3 characterizes the outcome when the good is subject to a price cap and the available supply is assigned randomly. We study need-based assignment and its optimal design in Section 4. Section 5 discusses the problems associated with speculation and how regulation may mitigate them. Other issues and related work are discussed in Section 6 and Section 7, respectively, with concluding remarks in Section 8.

2 The Model

2.1 Primitives

An indivisible good is supplied at a constant marginal cost of c, up to a fixed supply, $S \in (0, 1)$. The good could be initially owned by the government or by private suppliers, as long as the government can control the assignment process. Inelastic supply is a reasonable assumption for many resources or rights such as radio spectrum, health care services, business licenses, and public education, at least in the short term. The main results will be seen to hold with elastic supply (see Subsection 4).

There is a mass $1 + m$ of agents who will each demand zero or one unit of the good, with $m \geq 0$. They also consume a divisible numeraire called "money." Each agent has two attributes: her endowment of money or *wealth*, w; and her *valuation* of the good, v. She is privately informed of her *type*, (w, v). The attributes w and v are distributed independently over $[0, 1]^2$, with non-zero density for almost every (w, v) in the support. Independence is assumed largely for analytical ease; the results are robust to introducing (even large) correlations between w and v. Further, independence helps to isolate the role that each attribute plays.[4]

Wealth is distributed according to the cumulative distribution function (cdf) $G(w)$. When it comes to valuations, agents fall into two categories. A unit mass has $v \in [0, 1]$ distributed according to the cdf $F(v)$. The remaining m have $v = 0$, meaning that they derive no direct utility from the good. These agents may still participate for speculative reasons so we refer to them as *pure speculators*.

The agents have quasilinear utility. Specifically, a type-(w, v) agent gets utility $vx + w - p$ if she consumes the good with probability $x \in [0, 1]$ and pays $p \leq w$. We envision a situation in which capital markets are imperfect to such a degree that agents cannot borrow against the value of the good (even though the surplus can be monetized). Hence, an agent with $w < v$ is *wealth-constrained* in that she is unable to pay as much as she is willing to pay. As will be seen in Subsection 6.3, our main result holds in a more general setup in which agents

[4]If the poor are more likely to have high valuations than the wealthy, favoring the poor would be desirable simply because low wealth serves as a proxy for a high valuation. Assuming independence avoids this confounding of effects.

simply have different marginal utilities of wealth.[5] We assume the good to be "scarce" in that the measure of agents willing and able to pay the marginal cost exceeds the supply; that is, $[1 - F(c)][1 - G(c)] > S$.

As mentioned, we evaluate social welfare by the *Utilitarian* welfare function. Utilitarian welfare is a compelling measure of efficiency in our model since any Utilitarian inefficient allocation is Pareto improvable via appropriate side payments or financing arrangements. The typical allocation in our model, including that of the competitive market, will be inefficient since such financing arrangements are not feasible, due to a missing (capital) market.

The scarcity of the good means that the entire supply will be assigned under the mechanisms that we consider. Hence, Utilitarian welfare can be effectively represented by the sum of realized valuations, which we call *total value*.[6] Welfare is maximized by providing the good to the S agents with the highest valuations. Let $v^* > 0$ denote the critical valuation such that $1 - F(v^*) = S$. When all S individuals with valuations of v^* and above acquire the good, the total value is

$$V^* := \int_0^1 \int_{v^*}^1 v dF(v) dG(w) = \int_{v^*}^1 v dF(v) = S\phi(v^*),$$

where

$$\phi(z) := \frac{\int_z^1 v dF(v)}{1 - F(z)}$$

is the expectation of an agent's valuation, conditional on exceeding z.

2.2 Assignment Schemes

The paper compares assignment schemes that differ in terms of how the good is assigned initially, and whether it is transferable subsequently. We will compare the performance of three alternatives: (1) *a competitive market*, (2) *a non-market assignment scheme without transferability*, and (3) *a non-market assignment scheme with transferability.* A competitive market scheme can be implemented if a government agency acts as a Walrasian auctioneer or employs a multi-unit auction.[7] In a non-market assignment scheme, the good is initially

[5]Combining wealth constraints with quasilinear preferences has an analytical advantage. Income effects arise only when $w < v$, so Utilitarian efficiency can be measured by the total value of the good consumed, less costs.

[6]The wealth distribution does not enter because the marginal utility of wealth is constant and equal for all agents.

[7]The competitive market outcome can also be replicated when lobbyists offer bids to a government official, who makes inferences about their merits (based on the bids) and then assigns the good. See the discussion

assigned according to a specific rule. For instance, the government could impose a price cap and employ a priority rule of its choosing. The recipients of the good are allowed to sell it in (3), but not in (2). Note that when the good in question is a service, resale is still possible since the *right* to the service can be exchanged.

The three regimes are observed widely in the markets for a variety of goods and services. They are all observed in the housing market, for example. The regimes are employed (or could be employed) in settings including government provision of goods and services, as well as government assignment of rights:

- **Fugitive Property, Entitlements, and Government Resources:** Fugitive property — a good or resource whose ownership is not yet established — can be assigned to the individual who claims it first (*the rule of first possession*) or to the individual who owns property tied to it (*tied ownership*).[8] These methods correspond to non-market assignment with transferability. The Oklahoma Land Rush is also an example. Government land in Oklahoma was sold by auction in 1906, which corresponds to a competitive market. Emissions permits provide examples of a non-market assignment scheme with transferability since they are typically assigned based on the history of emissions, and they are frequently transferable.[9] Many countries assign transferable fishing rights, often according to historical catch levels.[10]

- **Education:** Suppose that students will be assigned to two schools based solely on place of residence, and the number of students who actually prefer school A exceeds its capacity. The valuation now represents the premium that a student is willing to pay for the right to attend A. Since the nominal price of attending the school is zero, the preference for A will be capitalized in housing prices. This corresponds to the market regime. If admission to school A is determined by lottery, this is a non-market assignment scheme without transferability. The final regime arises if a lottery awards *transferable* vouchers that confer the right to attend A.

- **Health Care:** Organ transplants are often arranged through a non-market assignment scheme without transferability. One could also employ an assignment scheme with transferability in which patients waiting for a transplant may trade their places in the

of Esteban and Ray (2005) in Section 7.

[8] For instance, a landowner has the right to natural gas deposits underneath the land.

[9] The Kyoto Protocol is an example. See http://unfccc.int/resource/docs/convkp/kpeng.pdf, accessed October 30, 2006. Raymond (2003) also describes that process as well as those used to assign a range of public property.

[10] See Shotton (2001).

queue. Finally, a competitive market would provide organs to those willing and able to pay the market-clearing price.

- **Military Recruitment:** An all-volunteer army corresponds to the competitive market. A draft lottery is effectively a non-market assignment scheme without transferability. A draft with tradable deferments represents an assignment scheme with transferability. This is essentially what transpired during the U.S. Civil War when conscripts avoided service in the Union Army by paying non-draftees to take their places.

In the remainder of this section we focus on a perfectly competitive market.

2.3 A Competitive Market

A competitive market operates according to the standard textbook description: Demand and supply are formed, and the price adjusts to clear the market. There is no supply at any price $p < c$. At any $p \geq c$, the entire supply, S, is available. On the demand side, the measure of the agents willing and able to pay p is

$$D(p) := [1 - G(p)][1 - F(p)].$$

Since $D(c) > S$, the market clears at the price $p^e > c$ such that

$$D(p^e) = [1 - G(p^e)][1 - F(p^e)] = S. \quad (1)$$

Since $1 - F(v^*) = S$, we have $[1 - G(v^*)][1 - F(v^*)] < S$, so $p^e < v^*$. This means that the equilibrium allocation does not maximize welfare.[11]

In Figure 1, the welfare-maximizing allocation would give the good to all agents in region $A + B$ whereas the market assigns it to those in $B + C$. In that sense, the market favors high-wealth low-valuation agents (region C) over low-wealth high-valuation agents (region A).

[11] If no agents were wealth-constrained (i.e., $w \geq v$ for all), the competitive market equilibrium would maximize welfare.

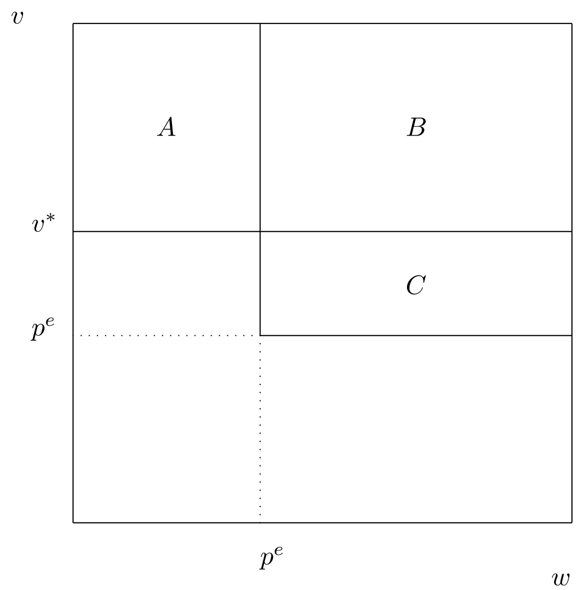

Figure 1: Benchmark Allocations

The market yields total value of

$$V^e := \int_{p^e}^1 \int_{p^e}^1 vdF(v)dG(w) = [1 - G(p^e)] \int_{p^e}^1 vdF(v) = S\phi(p^e) < S\phi(v^*) = V^*.$$

The second equality holds since $[1 - G(p^e)][1 - F(p^e)] = S$, by (1), and the inequality holds since $p^e < v^*$ and ϕ is a strictly increasing function.

Two points are worth making. First, the inefficiency of the competitive market is attributable to the capital market imperfection.[12] If the capital markets were perfect so that the agents could borrow up to their valuation, the first-best outcome would arise. Second, the inefficiency will not be mitigated by opening another market. Suppose that a resale market opens. If the agents do not anticipate that it will open, the resale market will not trigger further sales since the initial market transactions realize all mutually beneficial trades that are feasible. Individuals who purchase the good have $v \geq p^e$ so they would only sell at prices exceeding p^e, but there would be no additional demand at such prices.

Now suppose that agent do anticipate that a resale market will open. If the resale market were to be *active*, the prices would need to be equal in the two markets; otherwise, individuals

[12] If the value of the good were not transferable, as may be the case with consumption goods, then the competitive market allocation will be Pareto efficient (but not Utilitarian efficient) in the sense that no financing arrangements will change the outcome.

would have an incentive to switch from one market to the other. Either way, the ultimate allocation is the same as above.

3 Analysis with Random Assignment

We now analyze the simplest non-market assignment scheme: The price is capped at $\overline{p} \in [c, p^e)$ and those who demand it at $\overline{p}$ all have an equal probability of receiving the good. We refer to this scheme as *random assignment*. It is particularly easy to implement since no knowledge of individuals' preferences or wealth is required. For simplicity, we assume that each individual may participate in the assignment scheme only once.

3.1 Random Assignment without Transferability

We begin with the case in which resale is not permitted. This could arise because the supplier mandates it or because there are legal restrictions. Then, only agents whose valuation and wealth both exceed $\overline{p}$ will attempt to acquire the good. In particular, pure speculators will not participate.

The participants each receive the good with probability

$$\frac{S}{[1 - F(\overline{p})][1 - G(\overline{p})]}.$$

The expected valuation for such agents is $\phi(\overline{p})$, and the aggregate quantity is S, so random assignment gives a total value of $S\phi(\overline{p})$. Since $\overline{p} < p^e$, we have $S\phi(\overline{p}) < S\phi(p^e)$, meaning that the allocation is inferior to the competitive market allocation. Moreover, its performance gets worse as the cap is lowered.

A shift from the market regime to random assignment alters the set of recipients in two ways. First, it allows some agents with $(w, v) \in [\overline{p}, p^e) \times [p^e, 1]$ to receive the good. (Since $w < p^e$, these agents are unable to buy the good in the market regime.) The redistribution to those with low wealth is welfare-neutral because of the independence of wealth and valuations. Second, the shift to random assignment allows those with valuations $v \in [\overline{p}, p^e)$ to receive the good with positive probability. This latter effect clearly lowers total value.

Although independence of wealth and valuations simplified the comparison, it is not crucial. That is, one can get similar results even with negative correlation between wealth and valuations.[13]

3.2 Random Assignment with Transferability

We now assume that agents are able to resell. The good is assigned uniformly to those who demand it at the price $\overline{p}$, and the agents who obtain the good may sell it in a competitive resale market. We will see that the equilibrium resale price, $r_{\overline{p}}$, exceeds the cap so any agent who receives the good can pocket $r_{\overline{p}} - \overline{p} > 0$ by reselling. Hence, anyone who is able to pay $\overline{p}$ will participate, including pure speculators. The $[1+m][1-G(\overline{p})]$ participants will each receive the good with probability

$$\rho(\overline{p}; m) := \frac{S}{[1+m][1-G(\overline{p})]}.$$

Suppose that the resale price is $r > \overline{p}$. Resale demand at that price comprises the agents who did not receive the good initially but who are willing and able to pay r:

$$RD(r) := [1-F(r)][1-G(r)][1-\rho(\overline{p}; m)]. \tag{2}$$

Now consider resale supply. If an agent with valuation v keeps the good, she will receive utility of $v + (w - \overline{p})$; reselling gives utility of $r + (w - \overline{p})$. It is optimal to resell if $v < r$ so resale supply equals the amount of the good initially assigned to agents with $v < r$:

$$RS(r) := S\left(\frac{F(r)+m}{1+m}\right). \tag{3}$$

Equating resale demand and supply yields:

$$\begin{aligned} [1-F(r)][1-G(r)][1-\rho(\overline{p}; m)] &= S\left(\frac{F(r)+m}{1+m}\right) \\ \Rightarrow D(r) &= S - \rho(\overline{p}; m)[1-F(r)][G(r)-G(\overline{p})]. \end{aligned} \tag{4}$$

The product on the last line is the measure of agents who are unable to pay r but will not resell the good if they receive it. These individuals are in region A' in Figure 2. Since this term is positive, comparison of (4) with (1) makes it clear that $r > p^e$. Assigning the

[13] This may appear to favor random assignment relative to the market, but that need not be the case. Increased density for low-wealth high-valuation types will certainly favor random assignment, but an increased density for high-wealth low-valuation agents will worsen welfare. As long as there are enough of the latter types, random assignment will perform worse than the market.

good to some individuals in A' reduces the available supply in the resale market, causing the equilibrium resale price to be higher than the competitive equilibrium price, p^e.

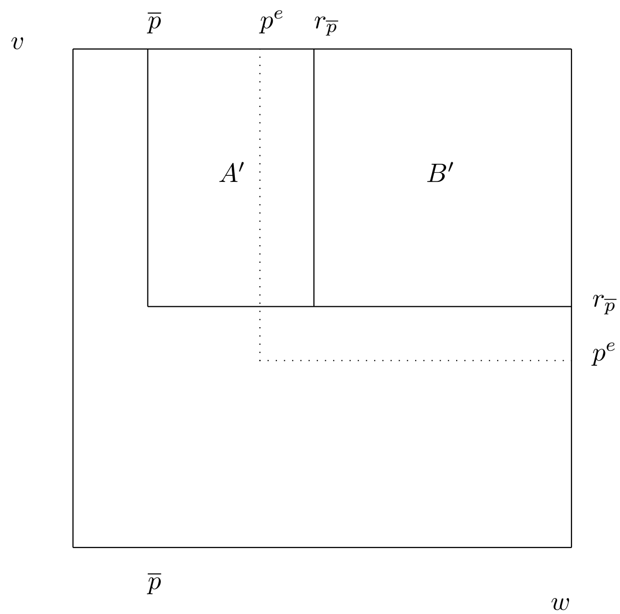

Figure 2: Random Assignment with Transferability

Suppose, to the contrary, that the resale price were $r \leq p^e$. By (1), $D(r) \geq S$, so the agents who are willing and able to pay r (region B') would exhaust S by themselves. (They are either assigned the good and keep it, or they buy it on the resale market.) In addition, some agents in region A' are assigned the good and do not resell it. Since there will be excess demand on the resale market when $r \leq p^e$, the equilibrium resale price must exceed p^e. A consequence is that $S\phi(r_{\overline{p}}) > S\phi(p^e) = V^e$, given independence of v and w. For any $w \geq \overline{p}$, the expected value of v, conditional on exceeding $r_{\overline{p}}$, is $\phi(r_{\overline{p}})$, so the total value is $S\phi(r_{\overline{p}})$. In other words, random assignment with transferability yields strictly higher welfare than either random assignment without transferability or the competitive market.[14]

This result follows from a simple difference between market and random assignment. Starting from the market assignment, random assignment shifts the good towards the poor and away from the wealthy. Since the wealthy can buy back from the market but poor lacks that ability, the shifting of assignment toward the latter enhances the overall welfare. The high valuation poor will keep the good, whereas low-valuation poor will resell the good

[14]High-valuation agents with $w < \overline{p}$ do not get the good, so $r_{\overline{p}} < v^*$, which means that welfare is not maximized.

so their receipt of the good does not harm welfare, all else equal. This means that resale is crucial for this superior welfare performance; in fact, without resale there is no welfare justification for assigning the good randomly.

A similar logic applies to reductions in the price cap. As $\overline{p}$ falls, more of the good accrues to the poor. The ultimate allocation improves since $r_{\overline{p}}$ rises as $\overline{p}$ falls, so welfare is highest when the cap is c, the lowest level at which supply is available. The formal results are now given.

Proposition 1. *Random assignment without transferability yields lower welfare than the competitive market, and welfare falls as the price cap, $\overline{p} > c$, falls. Random assignment with transferability yields higher welfare than the competitive market, and welfare rises as the price cap falls.*

Random assignment and resale are beneficial, but speculation reduces the benefits by lowering the quantity assigned to low-wealth agents. In particular, $\rho(\overline{p}; m) \to 0$ as $m \to \infty$, meaning that speculators acquire almost the entire supply. As a consequence, the resale market mimics the competitive market outcome. This implies that the benefits from random assignment and resale disappear as $m \to \infty$. We return to this point later.

4 Need-based Assignment with Resale

A uniform assignment scheme requires no information about agents' characteristics. When information is available, additional possibilities arise. For instance, a student may get priority in admission to a school based on factors correlated with his valuation such as test scores. This is a *merit-based* scheme. The awarding of *need-based* scholarships provides an example of assignment based on wealth.[15] It is obvious that merit-based schemes would be desirable from a welfare standpoint. We now show the less-obvious property that need-based assignments schemes have the same effect of improving welfare.

To model need-based assignment schemes formally, we suppose that there is a verifiable signal, s, that is distributed over $[0, 1]$ according to the density $h(s|w)$. We assume that a lower value of the signal is more likely when an agent has low wealth:

$$(SMLRP) \qquad \frac{h(s|w)}{h(s'|w)} > \frac{h(s|w')}{h(s'|w')}, \ \forall (s, w) > (s', w').$$

[15] Likewise, many government transfer programs in the U.S. are means-tested. See Currie and Gahvari (2008).

The good is assigned at price $\overline{p} < p^e$, based only on the signal, s, and the recipients can resell the good. Now define an arbitrary assignment rule, $x : [0,1] \to [0,1]$, a measurable function mapping from the signal to the probability of initial assignment. Given an initial price $\overline{p} < p^e$, any agent with $(w,v) \geq (\overline{p}, 0)$ wishes to participate. Then, x is *feasible* if the number who receive the good equals the supply:

$$(1+m)\int_{\overline{p}}^{1}\left(\int_{0}^{1} x(s)h(s|w)ds\right) g(w)dw = S.$$

Let $X_{\overline{p}}$ be the set of all feasible assignment rules at price $\overline{p}$.

We say that an assignment rule $y \in X_{\overline{p}}$ *need-dominates* $x \in X_{\overline{p}}$ if agents with a lower s are more likely to receive the good under y than under x; formally, *there exists* $\hat{s} \in [0,1]$ *such that* $y(s) \gtreqless x(s)$ *if* $s \lesseqgtr \hat{s}$. An assignment rule x is *need-based* if x need-dominates the random assignment rule $\bar{x} \in X$ with $\bar{x}(s) = \rho(\overline{p}, m) = \frac{S}{(1+m)(1-G(\overline{p}))}$ for all s. We also define the *maximally need-based assignment* to be $x^* \in X_{\overline{p}}$ with $x^*(s) = 1$ for $s \leq \hat{s}^*$ and $x^*(s) = 0$ for $s > \hat{s}^*$, for some $\hat{s}^* \in [0,1]$. The following result is obtained.

Proposition 2. *An assignment rule* $y \in X_{\overline{p}}$ *yields higher welfare than an assignment rule* $x \in X_{\overline{p}}$ *if the former need-dominates the latter.*

As discussed in the previous section, an assignment that favors the poor does better in welfare terms since it improves the chances that high-valuation low-wealth individuals obtain the good, without lowering the chances for the high-valuation high-wealth agents, who can buy the good on the resale market.

Two corollaries follow. First, since random assignment with resale dominates the competitive market, the above proposition implies the following.

Corollary 1. *A need-based assignment rule yields higher welfare than the competitive market assignment.*

Next, the proposition identifies the optimal assignment.

Corollary 2. *A maximally need-based assignment* x^* *generates the highest welfare among all feasible assignment rules in* $X_{\overline{p}}$.

If the signal is accurate, the first-best may be obtainable. To see this, assume that s is a perfect signal of w (equivalently, assume that wealth is observable). Now consider the maximally need-based assignment rule under which agents with w^* or below receive the good for free, and nobody else receives the good, where $(1+m)G(w^*) = S$. These individuals comprise region $A + B$ in Figure 3.

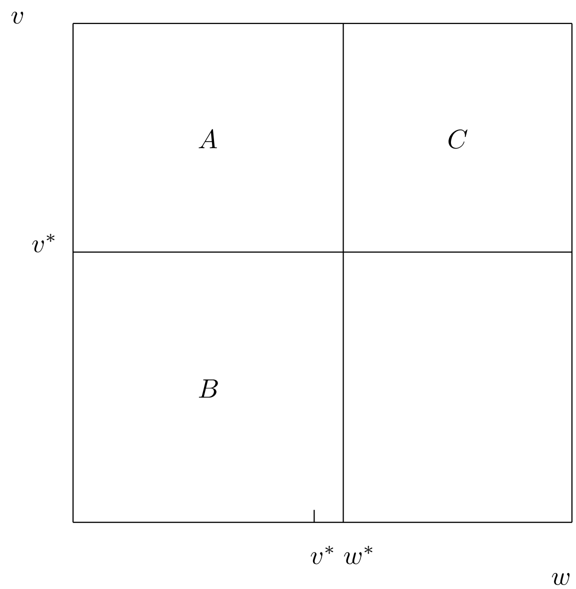

Figure 3: Efficiency of Need-Based Assignment

If $v^* \leq w^*$, which holds when S is sufficiently large, this rule maximizes welfare since the agents with $v \geq v^*$ (region $A + C$ in Figure 3) end up with the good in equilibrium.[16] All who do not get the good initially have $w \geq w^* \geq v^*$ so they are able to purchase on the resale market at the equilibrium price, $r = v^*$. Note that this benefit from need-based screening does not depend on the independence of v and w at all.

5 Speculation and Regulation of Resale

When a good is offered for sale at a below-market price, allowing resale presents a tradeoff. Although resale improves efficiency, all else equal, it also invites speculation.[17] This speculation reduces the probability that the good is assigned to those with high valuations but low wealth, thereby diminishing the benefits of non-market assignment. It may then be desirable

[16]When the resale price is r, supply will be $RS(r) = S\frac{F(r)+m}{1+m}$. Those not assigned the good are willing to buy on the resale market if $v \geq r$, so resale demand is $RD(r) = [1 - F(r)][1 - G(\max\{r, w^*\})]$. Since $RD(r) \gtreqless RS(r)$ if $r \lesseqgtr v^*$, the equilibrium resale price is v^*.

[17]When the FCC used a lottery to assign cellular telephone licenses, it received nearly 400,000 applications. The application fee was zero initially, and only \$230 in 1993 (Kwerel and Williams, 1993). The use of lotteries to assign housing in Korea engendered so much speculation that it was blamed for volatility in housing prices (Malpezzi and Wachter, 2005).

to regulate resale.[18]

We first show that the benefits from non-market assignment disappear as speculation grows. Specifically, we show that the welfare generated by a broad class of non-market assignment methods becomes arbitrarily close to that of competitive market as the number of pure speculators become sufficiently large. Assume that there exists a signal, σ, on which the assignment of the good can depend. Let σ be distributed over a support Σ according to the density $k(\sigma|w,v)$. The signal distribution is arbitrary except that we assume that its likelihood ratio is bounded:

$$(BLRP) \quad \text{There exists } L>0 \text{ such that } \frac{k(\sigma|w,v)}{k(\sigma|w',v')} < L, \forall \sigma \in \Sigma, (w,v), (w',v') \in [0,1]^2.$$

The condition means that the signal is not perfectly informative as no particular value of σ enables the planner to distinguish any two types. This assumption is reasonable in many settings; for example, no standardized test can perfectly distinguish students' abilities, and it can be difficult to learn individuals' financial information.

Since we shall consider how the assignment outcome varies with the mass of speculators, m, we index the problem by m. Fix any m. Given any price $\overline{p} \leq p^e$, we consider an arbitrary (measurable) *assignment rule*, $x^m : \Sigma \to [0,1]$, that is feasible:

$$(1+m)\int_{\overline{p}}^{1}\left(\int_{\Sigma} x^m(\sigma)k(\sigma|w,v)d\sigma\right)g(w)dw = S.$$

Let $\mathcal{X}^m$ be the set of all feasible assignment rules at the price $\overline{p}$. Obviously, our random assignment as well as all need-based assignment rules are elements of $\mathcal{X}^m$. The main observation then follows.

Proposition 3. *Given (BLRP), any assignment rule $x^m \in \mathcal{X}^m$ with unrestricted resale yields an outcome that becomes arbitrarily close to that of competitive market as $m \to \infty$; that is, the measure of type-(w,v) agents whose eventual allocations differ from their allocations under the competitive market approaches zero as $m \to \infty$.*

Proof: See the Appendix.

This result is reminiscent of the Coase theorem in that the initial assignment does not matter much; however, the ultimate allocation is not efficient here.[19] When there is substantial participation by pure speculators, most of the supply will be resold, thereby mimicking

[18] Speculation may also have a direct welfare cost. The price caps on new housing in Korea were criticized for encouraging speculation and diverting resources away from other productive investment activities.

[19] Jehiel and Moldovanu (1999) also find that different ownerships result in the same but inefficient alloca-

the competitive market. It may then be desirable to discourage speculation by regulating resale. Regulation could take the form of a blanket prohibition on resale.[20] More generally, recipients of the good may be prevented from profiting from resale until a certain date.[21] Consistent with these practices, we consider a regulation that prohibits resale for a fixed period of time.

We now use a discrete version of our model to demonstrate that regulation can be desirable.

EXAMPLE 1. *Suppose a good generates a flow surplus of v to a type-(w, v) agent, for a lifespan normalized to one. Let the supply be $S = \frac{1}{2}$. The numbers (measures) of agents of different types are given by the following table:*

	$w = w_L$	$w = w_H$
$v = v_H$	$\frac{a}{2}$	$\frac{a}{2}$
$v = v_L$	$\frac{1-a}{2}$	$\frac{1-a}{2}$
$v = 0$	$\frac{m}{2}$	$\frac{m}{2}$

Assume that $w_H > v_H > v_L > w_L > 0$. Also assume that $a \leq \frac{1}{2}$.

In a competitive market, a price less than or equal to w_L would attract all agents with $v \geq v_L$, leading to excess demand. Thus, the equilibrium price exceeds w_L and only the agents with $w = w_H$ obtain the good. The average realized value is

$$av_H + (1 - a)v_L.$$

Now suppose the good is assigned randomly at the price $\overline{p} \leq w_L$, and recipients are prohibited from reselling until time $z \in [0, 1]$. If the agent pays $\overline{p}$ for the good and then resells it for r at time z, she gets utility of $zv + r + (w - \overline{p})$. Holding onto the good for the entire period yields $v + (w - \overline{p})$, so it is optimal to resell if and only if $r \geq (1 - z)v$. It can be

tion; but the main source of inefficiency in their model is externalities that one's ownership imposes on the other agents.

[20] For instance, in the 3G spectrum auctions in the U.K., resale of licenses was prohibited. See Klemperer (2004). Although firms can get around the ban via a corporate merger, doing so is often costly.

[21] Some owners of subsidized housing units face the latter form of constraint. See 42 U.S.C. 12875 for a discussion of restrictions in the "Housing Opportunities for People Everywhere" (HOPE) program in the U.S. When a housing unit is sold within six years, the seller may not keep any "undue profit," meaning that the seller must disgorge proceeds exceeding the original price, adjusted for inflation. Similarly, if a *designated entity* sells a spectrum license during the first five years, it must reimburse the FCC for the entire bidding credit plus interest. See Federal Communications Commission 47 CFR Part 1 [WT Docket No. 05-211; FCC 06-52].

seen that the equilibrium resale price is $r = (1-z)v_L$. If $r > (1-z)v_L$, only individuals with $v = v_H$ will keep the good or demand it on the resale market. Since the measure of agents with v_H is $\frac{a}{2}$, there will be excess supply on the resale market. If $r < (1-z)v_L$, there will be excess demand since all agents with v_L or v_H will demand the good if they did not receive it initially. Thus, $r = (1-z)v_L$ in equilibrium.

Pure speculators will all participate if $r > \overline{p}$, which implies $1 - z > \frac{w_L}{v_L}$ or $z < 1 - \frac{w_L}{v_L}$. In that case, each participant has probability $\frac{1}{2(m+1)}$ of obtaining the good. Conditional on $z < 1 - \frac{w_L}{v_L}$, the average realized value is maximized by $z = 0$ (i.e., immediate resale). In that case, the resale market clears at the price $r = v_L$, and the average realized value is[22]

$$\left(a\frac{2m+3}{2(m+1)}\right)v_H + \left(1 - a\frac{2m+3}{2(m+1)}\right)v_L, \tag{5}$$

which is higher than under the market.

Now consider $z \geq 1 - \frac{w_L}{v_L}$. Then, pure speculators are deterred so only agents with $v > 0$ participate. Those of type (w_L, v_H) and (w_H, v_H) get the good initially or in the resale market. In this case, average realized value is maximized when $z = 1 - \frac{w_L}{v_L}$. Then, $r = w_L$ and the average realized value equals

$$z(av_H + (1-a)v_L) + (1-z)\left(2av_H + (1-2a)v_L\right). \tag{6}$$

Observe this is higher than the average realized value generated by the market. Comparing (6) with (5), we see that restricting resale is desirable if and only if $m \geq \frac{z - \frac{1}{2}}{1-z} = \frac{v_L - 2w_L}{2w_L}$.

Our focus has been on the efficiency effects of non-market assignment schemes, but this example also illustrates why resale may be regulated in social programs motivated by *redistributive goals*. The option to resell clearly increases the value of entitlement for those selected to receive the good. *Ex ante*, though, allowing resale invites speculation and reduces a low-wealth agent's probability of receipt. Since the latter effect may dominate the former effect (which is the case in this example), the target group may be better off ex ante from regulation of resale.

[22]The measure $\frac{a}{2}$ with $(w, v) = (w_H, v_H)$ get the good initially through the assignment scheme, or via the resale market, while the measure $\frac{a}{4(m+1)}$ with (w_L, v_H) get it initially.

6 Discussion

In this section we examine several additional issues and discuss the robustness of our results.

6.1 In-kind versus Cash Subsidies

The non-market assignment schemes employed here involve an in-kind subsidy since they make the good available at a below-market price. A natural alternative is a cash-subsidy scheme. Instead of assigning the good at a below-market price to any group of agents, one can assign a cash subsidy to the same agents so that they can buy the good on the unregulated market. The needed cash subsidy can be then financed using the proceeds from selling the good at the market price. Indeed, any non-market assignment scheme with unrestricted resale appears to be replicable in this fashion by some budget-balanced cash subsidy program.[23]

There is an important sense in which cash subsidies cannot replicate in-kind subsidies, however. While a non-market assignment scheme (i.e., an in-kind subsidy) can be designed so that speculation activities can be discouraged through restrictions on post-assignment resale, the same is not feasible with cash subsidies. We show via an example that a non-market assignment scheme with an appropriate restriction on resale can actually yield higher welfare than any cash-subsidy scheme if there are sufficiently many potential speculators.

Example 2. *Revisit Example 1 with $a = \frac{1}{2}$. Assume for simplicity that the random assignment is the only feasible assignment technology for both in-kind and cash subsidy. (That is, no signal about agents' types are available.) Consider first our non-market assignment of the good. If the good is assigned with a price cap of $\overline{p} = \hat{w}$ and resale is prohibited until $z = 1 - \hat{w}$, then a total value $\hat{V}^n = \frac{3}{4} + \frac{1}{8}\hat{w}$ is realized (by the analysis in Example 1).*

Now suppose that a cash subsidy is assigned (again by the uniform assignment). For a cash subsidy to have an effect, the recipient of the subsidy must be able to pay at least \$1 since there is a measure $\frac{1}{2}$ of agents with (w,v) = (2,2) or (2,1) who are willing and able to pay

[23]To see this, consider any random assignment rule with a price cap, $\overline{p}$, and suppose that it entails an equilibrium resale price of r. Now let a cash-subsidy scheme be employed instead. Suppose that the profits from sales of the good are recycled to agents. In particular, let the *anticipated* net proceeds be distributed to agents before the good is sold, using the same random assignment scheme. This means that $r - \overline{p}$ is awarded to each of S agents. (In order to replicate the in-kind subsidy scheme, it is necessary that only agents with $w \geq \overline{p}$ receive the subsidy. This could be achieved by requiring that they post a bond of $\overline{p}$.) This policy endows each successful type-(w, v) agent with the ability to pay at least r. The ensuing competitive market clears at the price r, so it yields the same allocation as would the original non-market assignment scheme with transferability.

that much. Suppose, then, that a subsidy $s \geq 1 - \hat{w}$ is given to a measure ℓ, $0 < \ell \leq 1 + m$, of randomly chosen agents. Budget balance means that the aggregate subsidy cannot exceed the net proceeds from the sale, implying

$$(1 - \hat{w})\ell \leq \frac{1}{2}(\overline{p} - c) \leq \frac{2-c}{2} \Rightarrow \ell \leq \frac{2-c}{2(1-\hat{w})}$$

(since the price cannot exceed \$2). All agents will participate, so the probability that a given agent receives the cash subsidy is no greater than

$$\frac{\ell}{1+m} \leq \frac{2-c}{2(1-\hat{w})(1+m)}.$$

At most $\frac{2-c}{8(1-\hat{w})(1+m)}$ high-valuation low-wealth agents get the subsidy, so the total value resulting from any budget-balanced cash subsidy is bounded above by

$$2 \times \left(\frac{1}{4} + \frac{2-c}{8(1-\hat{w})(1+m)}\right) + 1 \times \left(\frac{1}{4} - \frac{2-c}{8(1-\hat{w})(1+m)}\right) = \frac{3}{4} + \frac{2-c}{8(1-\hat{w})(1+m)}.$$

If $m > \frac{2-c}{(1-\hat{w})\hat{w}} - 1$, this upper bound is less than $\hat{V}^n = \frac{3}{4} + \frac{1}{8}\hat{w}$, the total value attainable under random assignment with restricted resale.

This example shows that an in-kind subsidy, with regulation of resale, does strictly better than *any* budget-balanced cash subsidy, for sufficiently large m, in the case of random assignment.[24] We conclude that the ability to control speculation distinguishes an in-kind subsidy scheme from a cash-subsidy scheme: the former may outperform the latter.[25]

6.2 Elastic Supply

Our analysis has assumed that supply is perfectly inelastic; in practice, however, it is often responsive to price. We now consider the case of elastic supply and show that the results are similar if the elasticity is not too great.

Return to the original demand structure and assume that there are no pure speculators (i.e., $m = 0$). Suppose that the good is supplied competitively according to a twice-

[24]The example assumes random assignment for both in-kind and cash subsidies. The same logic applies to other more general assignment rules, however. One can imagine, as before, an arbitrary assignment rule that depends on some observable signal of the agent. The same point then holds, given that the same assignment rule is used to assign the in-kind subsidy and the cash subsidy.

[25]Currie and Gahvari (2008) discuss several scenarios under which in-kind subsidies may dominate lump-sum cash subsidies. They assume that resale is *prohibited*, whereas resale is needed to get that result here.

differentiable, strictly convex, aggregate cost function, $c(\cdot)$. When price equals p, supply is $S(p) \in \arg\max_{q \geq 0} pq - c(q)$, which implies $c'(S(p)) = p$. Also, $S(\cdot)$ is increasing and differentiable. The competitive equilibrium price, p^e, satisfies $D(p^e) = S(p^e)$.

Suppose, next, that the good is assigned randomly, with its price capped at $\overline{p} < p^e$. If the good is transferable, all individuals with $w \geq \overline{p}$ will participate. A resale equilibrium exists. Following (4), the equilibrium price, $r_{\overline{p}}$, satisfies

$$D(r_{\overline{p}}) = S(\overline{p}) - \rho(\overline{p})[1 - F(r_{\overline{p}})][G(r_{\overline{p}}) - G(\overline{p})]. \tag{7}$$

Welfare comprises the valuations of all who consume the good less the costs of production:

$$\hat{W}(\overline{p}) := S(\overline{p})\phi(r_{\overline{p}}) - c(S(\overline{p})). \tag{8}$$

Capping the price and randomly assigning the good presents a tradeoff here since the quantity supplied falls. Moreover, the resulting deadweight loss is not negligible, even if the cap is just below the market price, since the agents losing access may be wealth-constrained, with valuations well above p^e. Nonetheless, if supply is sufficiently inelastic, random assignment continues to outperform the competitive market.

Proposition 4. *If* $\frac{S'(p^e)}{S(p^e)} < \frac{f(p^e)}{F(p^e)}$, *there exists a price cap,* $\overline{p} < p^e$, *such that random assignment with transferability yields higher welfare than the competitive market does.*

Proof: See the Appendix.

6.3 General Preferences

We previously considered agents with quasilinear utility. Since they had equal marginal utility of money, the allocation of money had no *direct* impact on welfare. We now show that non-market assignment with transferability may dominate the market, even though the allocation of money does affect welfare directly and every agent's wealth exceeds her valuation.

As before, an agent has a valuation $v \in [0, 1]$, which is distributed according to F. Each agent again has a zero endowment of the good, which is supplied at zero cost up to $S \in (0, 1)$. The wealth, w, is now distributed over $[1, \infty)$ according to G.

An agent of type (w, v) receives utility of $vx + \eta(w - \pi)$ if she obtains the good with probability x and makes the (net) payment π. The function $\eta : \Re_+ \mapsto \Re_+$ satisfies $\eta'(\cdot) \geq 1$

and $\eta''(\cdot) \leq 0$, with $\eta''(y) < 0$ for some $y > 0$. Since $\eta'(\cdot) \geq 1$, no one is willing to spend more than v, and $w \geq 1 \geq v$ means that agents are *not* wealth-constrained.[26] Wealth still matters, however, because an agent with higher wealth has a lower marginal utility of money. Thus, a high-wealth agent is willing to pay more for the good than a low-wealth agent is, all else equal.

Two regimes will be compared: *the competitive market* and *random assignment with transferability*. Welfare maximization again favors assigning the good to those with high valuations, all else equal. It also favors putting more of the payment burden (which might be necessary to finance supply of the good) on agents with higher wealth. Random assignment with transferability does better on both accounts.

In a competitive market with a price of p, an agent with (w, v) would buy the good if and only if

$$v + \eta(w - p) \geq \eta(w). \tag{9}$$

Let $\Omega^+(p)$ be the associated set of these types, and let $\delta(p) := \Pr\{(w, v) \in \Omega^+(p)\}$ be the measure. Since δ is strictly decreasing, there is a unique equilibrium price, $p^e < 1$, with $\delta(p^e) = S$. High-wealth individuals have a lower shadow value of money, so they are more likely to buy the good than those with low wealth, all else equal. The payment burden is not allocated efficiently either, as purchasers all pay the same price.

Now consider random assignment with transferability. Suppose that the price is capped at $\overline{p} < p^e$ and the good is assigned randomly (i.e., with equal probability). The resale price will exceed $\overline{p}$, and $w \geq 1 > \overline{p}$, so everyone participates and each of them receives the good with probability $\rho = \frac{S}{1+m}$. Resale demand is $(1 - \rho)\delta(r)$ if the resale price is $r \geq p^e$, and a recipient of the good will resell if and only if

$$v + \eta(w - \overline{p}) < \eta(w + r - \overline{p}). \tag{10}$$

Let $\Omega^-(r, \overline{p})$ denote the set of *potential* resellers (i.e., agents who would resell the good for r if they got it), with $\sigma(r, \overline{p})$ its measure. The latter function has partial derivatives satisfying $\sigma_1 > 0$ and $\sigma_2 \geq 0$, respectively, inside the relevant region. The resale supply at r is given by $\rho\sigma(r, \overline{p})$.

Let $r(\overline{p})$ denote the equilibrium resale price, which satisfies

$$(1 - \rho)\delta(r) = \rho\sigma(r, \overline{p}). \tag{11}$$

[26] The assumption also avoids the uninteresting case in which one can improve welfare simply by transferring money from buyers to sellers.

It is clear that $r(\overline{p}) > p^e$ if and only if $\overline{p} < p^e$.[27] The lower price, $\overline{p}$, raises the chances that low-wealth agents obtain the good. In addition, their payment is lower than in the competitive market equilibrium. The burden is now absorbed partly by sellers, who receive a lower price, and partly by the unsuccessful participants with high wealth, who pay more than the competitive market price to purchase on the resale market. These changes may render random assignment with transferability superior to the market.

PROPOSITION 5. *There exists $\overline{p} < p^e$ such that random assignment with a price cap of $\overline{p}$ and resale yields higher welfare than the competitive market does.*

Proof: See the Appendix.

7 Related Literature

The current paper furthers the now well-known theme that agents' wealth constraints impact allocative efficiency. Che and Gale (1998) studied standard auctions when bidders differ in their valuations and wealth and showed that standard auctions differ in terms of allocative efficiency and seller's expected revenue. Che and Gale (2000) solved for the mechanism that maximizes a seller's expected revenue when a buyer has private information about her valuation and wealth. They found that the optimal mechanism may contain a menu of lotteries indexed by the probability of sale and the entry fee. The insight that it is beneficial to make a good "divisible" by selling it in probability units is also relevant here.[28] In fact, random assignment in the current paper implements a similar idea. While finding a mechanism that maximizes efficiency (subject to budget balance) is beyond the scope of the current paper, we conjecture that it will share features of the revenue-maximizing mechanism found in that paper. We leave the pursuit of such a mechanism for future research.

Gali and Fernandez (1999) study how a market and a tournament match workers to inputs when the workers differ in ability and wealth. Both schemes provide efficient matching when capital markets are perfect, but inefficiencies arise with imperfect capital markets, in which case the tournament does relatively better. Esteban and Ray (2006) consider a government that awards licenses to produce. A government concerned about efficiency

[27] If $\overline{p} = p^e$ and $r = p^e$, the marginal types identified by (9) and (10) are identical, so $\sigma(p^e, p^e) = 1 + m - \delta(p^e)$, and (11) is satisfied at $r = p^e$. If $\overline{p}$ is lowered, σ falls, so we must have $r(\overline{p}) > p^e$.

[28] At the same time, divisibility alone does not solve the problem completely in that selling the good in probability units on the market does not maximize welfare. Moreover, in practice, agents may be loss-averse, so they may be unwilling to pay positive entry fees without being guaranteed receipt of the good. This will make the lottery approach impractical.

assigns licenses based on lobbying expenditures since lobbying is a signal of productivity; however, wealthier sectors find it *less costly* to lobby, which jams the productivity signal. The resulting allocation corresponds to the market regime here. Esteban and Ray focus on how allocative efficiency changes with the underlying wealth distribution. The current paper also finds a signal-jamming effect, but it focuses on a different issue; namely, how alternative assignment schemes perform.[29]

There is also a related literature that rationalizes market intervention based on welfare criteria different from those used here. Weitzman (1977) took as a benchmark the allocation of goods that would prevail if all agents had the mean income. He then showed that an equal allocation of goods may be closer to the benchmark than the market allocation is. Sah (1987) compared different regimes from the perspective of the members of the poorest group. By contrast, we focused on total value realized, i.e., the Utilitarian welfare.

8 Concluding Remarks

This paper has studied the welfare implications of utilizing non-market methods to assign ownership of a good. We have shown that non-market schemes may outperform market assignment, if resale is permitted. Welfare rises when the good is put into the hands of low-wealth high-valuation agents who lack the means to buy at the market price. Speculation tempers those benefits, however. Restricting transferability may therefore be desirable when potential speculators are numerous

The results apply to the assignment of public resources and entitlements such as rights to exploit natural resources, as well as immigration visas and exemptions from civic duty such as military service or jury duty. The results apply to government-led industrialization processes in many developing economies as well.[30]

Many goods are already assigned using non-market assignment schemes, and the results here have implications for how these existing schemes can be improved. In particular, the introduction of transferability may offer benefits to programs that do not currently permit

[29]Our results yield an interesting implication of their setup: If the government is unconcerned about welfare, lobbying will not arise, so the resulting situation corresponds to random assignment here. Thus, if resale of licenses is allowed, the latter assignment method dominates the allocation generated by a government that is responsive to productivity signals.

[30]The Korean industrialization process was marked by licensing policies that targeted industries and firms for export quotas, trade protection and other privileges (Amsden, 1989). During a period dubbed the "licence raj," the Indian government controlled large areas of economic activity through the awarding of rights and "permissions" (Esteban and Ray, 2006).

it.[31] The U.S. assigns 50,000 permanent resident visas per year by lottery.[32] Becker (1987) proposed selling visas to a pool of qualified applicants. A simple alternative is to retain the lottery system but permit recipients to sell their visas to other qualified applicants (e.g., others in the original pool). Our results suggest that this change could yield an improvement relative to the current system and the Becker proposal.[33]

A lottery could be used to assign transferable educational vouchers as well. With the pool of recipients and the transfer process regulated to discourage speculation, such a system may select students more efficiently than would a system of local attendance zones or random assignment of non-transferable vouchers. In the same vein, one could imagine a military draft with tradable deferments.[34] Allowing patients who are waiting for organ transplants to trade their places in the queue via a regulated transfer procedure might even be desirable. Other objectives or institutional details may loom large in each of these cases, but the results here argue for consideration of non-market assignment schemes and transferability. Our analysis has also ignored transaction costs. When these costs are substantial, one must weigh the benefits of non-market assignment against the transaction costs that may be incurred.

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[31] The resale rules can also be adapted to control speculation and to accommodate other institutional constraints.

[32] Eligibility for the *Diversity Immigrant Visa Program* is restricted to individuals from countries with low rates of immigration to the U.S. See http://www.travel.state.gov/pdf/dv07FinalBulletin.pdf, accessed July 8, 2006.

[33] Other factors may enter a policymaker's welfare function. For instance, visa sales would generate revenue for the Treasury.

[34] Tobin (1970) noted that the same conclusion can be reached on the basis of egalitarian concerns. He first pointed out that the all-volunteer army was "just a more civilized and less obvious way of ... allocating military service to those eligible young men who put the least monetary value on their safety and on alternative uses of their time." He added that the difference between the two schemes is who pays — general taxpayers or the individuals who wish to avoid military service.

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Appendix: Proofs

PROOF OF PROPOSITION 2: The proof consists of several steps.

Step 1: *For each $x \in X_{\overline{p}}$, there exists a unique equilibrium in the resale market, with a price $r_x^* \in (\overline{p}, 1)$.*

Proof: We first write the condition for the equilibrium resale price. There cannot be an equilibrium at a resale price $r < \overline{p}$, for resale demand would clearly exceed resale supply. Following the logic of Section 3, for an arbitrary resale price, $r \geq \overline{p}$, resale demand is

$$RD_x(r) := \int_r^1 \int_r^1 \left(\int_0^1 [1 - x(s)]h(s|w)ds \right) dF(v)dG(w),$$

and the resale supply is given by

$$RS_x(r) := S - \int_{\overline{p}}^1 \int_r^1 \left(\int_0^1 x(s)h(s|w)ds \right) dF(v)dG(w).$$

Observe that $RD_x(\cdot)$ is decreasing and $RS_x(\cdot)$ is increasing, both are continuous, and they satisfy $RD(1) = 0 < RS_x(1) = S$ and $RD_x(\overline{p}) = [1 - F(\overline{p})][1 - G(\overline{p})] > S > RS_x(\overline{p})$. Hence, there exists a unique equilibrium resale price, $r_x^* \in (\overline{p}, 1)$, that satisfies $RD_x(r_x^*) = RS_x(r_x^*)$. $\|$

Step 2: *If $\int_0^1 [y(s) - x(s)]h(s|w)ds \geq 0$, then $\int_0^1 [y(s) - x(s)]h(s|w')ds > 0$ for $w' < w$.*

Proof: Recall that $y(s) \gtreqless x(s)$ if $s \lesseqgtr \hat{s}$. We then have

$$\begin{aligned}
& \int_0^1 [y(s) - x(s)]h(s|w')ds \\
= \; & \int_0^1 [y(s) - x(s)] \frac{h(s|w')}{h(s|w)} h(s|w)ds \\
> \; & \int_0^1 [y(s) - x(s)] \frac{h(\hat{s}|w')}{h(\hat{s}|w)} h(s|w)ds \\
= \; & \frac{h(\hat{s}|w')}{h(\hat{s}|w)} \left(\int_0^1 [y(s) - x(s)]h(s|w)ds \right) \\
> \; & 0,
\end{aligned}$$

where the first inequality follows since (SMLRP) implies that $[y(s)-x(s)][\frac{h(s|w')}{h(s|w)} - \frac{h(\hat{s}|w')}{h(\hat{s}|w)}] > 0$ for any $s \neq \hat{s}$, and the last follows from the hypothesis. ‖

Step 3: *For any $r > \overline{p}$,*

$$\int_{\overline{p}}^{r} \left(\int_{0}^{1} [y(s) - x(s)]h(s|w)ds \right) g(w)dw > 0.$$

Proof. Fix any $r > \overline{p}$. Suppose, to the contrary, that

$$\int_{\overline{p}}^{r} \left(\int_{0}^{1} [y(s) - x(s)]h(s|w)ds \right) g(w)dw \leq 0. \tag{12}$$

Then, by Step 2, we must have

$$\int_{r}^{1} \left(\int_{0}^{1} [y(s) - x(s)]h(s|w)ds \right) g(w)dw < 0. \tag{13}$$

Summing (12) and (13), we obtain

$$\int_{\overline{p}}^{1} \int_{0}^{1} [y(s) - x(s)]h(s|w)dsg(w)dw < 0,$$

which is a contradiction since both x and y are feasible so the left-hand side must be equal to zero. ‖

Step 4: *The equilibrium resale price r_y^*, which follows assignment according to y, exceeds the equilibrium price r_x^*, which follows assignment according to x.*

Proof. For any $r \geq r_y^*$,

$$\begin{aligned}
& RD_x(r) - RS_x(r) \\
= \; & D(r) - S + \int_{r}^{1} \int_{\overline{p}}^{r} \left(\int_{0}^{1} x(s)h(s|w)ds \right) g(w)dwf(v)dv \\
< \; & D(r) - S + \int_{r}^{1} \int_{\overline{p}}^{r} \left(\int_{0}^{1} y(s)h(s|w)ds \right) g(w)dwf(v)dv \\
= \; & RD_y(r) - RS_y(r) \\
\leq \; & 0,
\end{aligned}$$

where the first inequality follows from Step 3, and the last follows from the fact that $r \geq r_y^*$. We thus conclude that $r_x^* < r_y^*$. ‖

Step 5: *Assignment y yields higher welfare than assignment x.*

Proof. We first write the social welfare under arbitrary $x \in X_{\overline{p}}$:

$$
\begin{aligned}
W^*(x) &:= \int_{\overline{p}}^{r_x^*} \int_{r_x^*}^{1} v \left(\int_0^1 x(s)h(s|w)ds \right) f(v)dvg(w)dw + \int_{r_x^*}^{1} \int_{r_x^*}^{1} vf(v)dvg(w)dw \\
&= \left(\int_{r_x^*}^{1} vf(v)dv \right) \left(\int_{\overline{p}}^{r_x^*} \int_0^1 x(s)h(s|w)dsg(w)dw \right) + \left(\int_{r_x^*}^{1} vf(v)dv \right) \left(\int_{r_x^*}^{1} g(w)dw \right) \\
&= \phi(r_x^*)(1 - F(r_x^*)) \left(\int_{\overline{p}}^{r_x^*} \int_0^1 x(s)h(s|w)dsg(w)dw + 1 - G(r_x^*) \right) \\
&= \phi(r_x^*)S,
\end{aligned}
$$

where the first equality holds since the good is consumed by agents with $(w, v) \in [\overline{p}, r_x^*] \times [r_x^*, 1]$ with probability $x(s)$ (that is, only when assigned initially) and by agents with $(w, v) \in [r_x^*, 1]^2$ with probability 1, and the last equality follows from the fact that the measure of agents who obtain the good ultimately must equal S given equilibrium resale price r_x^*.

Then, since y need-dominates x, by Step 4, $r_y^* > r_x^*$. Finally, $\phi(\cdot)$ is strictly increasing so

$$W^*(y) = \phi(r_y^*)S > \phi(r_x^*)S = W^*(x),$$

proving the result. $\|$ ∎

Proof of Proposition 3: Consider any $x^m \in \mathcal{X}^m$. It suffices to show that r^m, the equilibrium resale price following the initial assignment according to x^m, converges to p^e. To this end, write the resale excess demand at $r \geq \overline{p}$ as:

$$RD_x(r) - RS_x(r) = D(r) - S + \int_r^1 \int_{\overline{p}}^{r} z^m(w, v)g(w)dwf(v)dv,$$

where

$$z^m(w, v) := \int_\Sigma x^m(\sigma)k(\sigma|w, v)d\sigma.$$

By $(BLRP)$, for any $(w, v), (w', v') \in [0, 1]^2$, we must have

$$
\begin{aligned}
z^m(w, v) &= \int_\Sigma x^m(\sigma)k(\sigma|w, v)d\sigma \\
&< \int_\Sigma x^m(\sigma)Lk(\sigma|w', v')d\sigma = Lz^m(w', v').
\end{aligned}
$$

Then, for each (w, v),

$$z^m(w,v) \; < \; \frac{\int_{\overline{p}}^1 \int_0^1 Lz^m(w',v')f(v')dv'g(w')dw' + m\int_{\overline{p}}^1 Lx^m(w',0)g(w')dw'}{(1+m)[1-G(\overline{p})]}$$
$$= \frac{LS}{(1+m)[1-G(\overline{p})]}, \tag{14}$$

where the inequality follows from $z^m(w,v) < Lz^m(w',v')$ and the fact that $\int_{\overline{p}}^1 \int_0^1 f(v')dv'g(w')dw' + m\int_{\overline{p}}^1 g(w')dw' = (1+m)[1-G(\overline{p})]$.

For any given $r > p^e$, resale excess demand is

$$\begin{aligned} RD_x(r) - RS_x(r) &= D(r) - S + \int_r^1 \int_{\overline{p}}^r z^m(w,v)g(w)dwf(v)dv \\ &< D(r) - S + \frac{LS(1-F(r))}{1+m} \\ &\to D(r) - S < 0 \qquad \text{as } m \to \infty. \end{aligned}$$

The first inequality follows from (14) and the last follows from $D(p^e) = S$ and $r > p^e$. This argument proves that the equilibrium resale price $r^m \to p^e$ as $m \to \infty$. ∎

Proof of Proposition 4: It suffices to show that $\hat{W}'(p^e) < 0$ here, which implies that lowering the cap increases welfare. To that end, fix $\overline{p} \leq p^e$ and rewrite welfare as:

$$\hat{W}(\overline{p}) = \{\rho(\overline{p})[G(r_{\overline{p}}) - G(\overline{p})] + [1 - G(r_{\overline{p}})]\} \int_{r_{\overline{p}}}^1 vdF(v) - c(S(\overline{p})).$$

Since $r(p^e) = p^e$, we have

$$\begin{aligned} \hat{W}'(p^e) &= -(1-G(p^e))f(p^e)r'(p^e)p^e - (1-\rho(p^e))g(p^e)r'(p^e)\int_{p^e}^1 vdF(v) \\ &\quad -\rho(p^e)g(p^e)\int_{p^e}^1 vdF(v) - c'(S(p^e))S'(p^e) \\ &= -(1-G(p^e))f(p^e)r'(p^e)p^e - F(p^e)[1-F(p^e)]g(p^e)r'(p^e)\phi(p^e) \\ &\quad -[1-F(p^e)]^2 g(p^e)\phi(p^e) - S'(p^e)p^e, \end{aligned} \tag{15}$$

where the second equality holds since $\rho(p^e) = S(p^e)/[1-G(p^e)] = D(p^e)/[1-G(p^e)] = 1 - F(p^e)$, $\phi(z) = \int_z^1 vdF(v)/[1-F(z)]$, and $c'(S(p^e)) = p^e$.

Totally differentiating both sides of (7) and using $\rho(p^e) = 1 - F(p^e)$ yields

$$r'(p^e) = -\frac{S'(p^e) + g(p^e)[1 - F(p^e))]^2}{g(p^e)(1 - F(p^e))F(p^e) + f(p^e)(1 - G(p^e))}.$$

Substituting this into (15) and collecting terms, we get

$$\begin{aligned}\hat{W}'(p^e) &= -\frac{[\phi(p^e) - p^e](1 - F(p^e))g(p^e)[D(p^e)f(p^e) - S'(p^e)F(p^e)]}{g(p^e)(1 - F(p^e))F(p^e) + f(p^e)(1 - G(p^e))} \\ &= -\frac{[\phi(p^e) - p^e](1 - F(p^e))g(p^e)[S(p^e)f(p^e) - S'(p^e)F(p^e)]}{g(p^e)(1 - F(p^e))F(p^e) + f(p^e)(1 - G(p^e))}.\end{aligned}$$

Hence, $\hat{W}'(p^e) < 0$ if and only if $\frac{S'(p^e)}{S(p^e)} < \frac{f(p^e)}{F(p^e)}$. ∎

Proof of Proposition 5: We prove the result using a bound on aggregate utility. Fix any $\overline{p} < p^e$, and suppose that the equilibrium resale price is $r(\overline{p}) \geq p^e$. The first step is to determine the agents' aggregate utility, given a resale price of $r(\overline{p})$, under the assumption that agents are *constrained* to behave in such a way that the competitive equilibrium allocation arises. Unsuccessful agents with $(w, v) \in \Omega^+(p^e)$ would purchase on the resale market at the price $r(\overline{p})$ and successful agents with $(w, v) \notin \Omega^+(p^e)$ would resell the good at $r(\overline{p})$. Then, the competitive allocation would arise, and the agents would receive aggregate utility of

$$\begin{aligned}\Gamma(\overline{p}) &= \rho \int_{\Omega^+(p^e)} (v + \eta(w - \overline{p}))dF(v)dG(w) + (1 - \rho) \int_{\Omega^+(p^e)} (v + \eta(w - r(\overline{p})))dF(v)dG(w) \\ &\quad + (1 - \rho) \int_{\Omega^-(p^e, p^e)} \eta(w)dF(v)dG(w) + \rho \int_{\Omega^-(p^e, p^e)} \eta(w + r(\overline{p}) - \overline{p})dF(v)dG(w).\end{aligned}$$

The first product pertains to high-valuation agents who get the good through the initial assignment; the second represents the ones who purchase on the resale market. The third product covers the low-valuation agents who never get the good; the fourth represents the ones who are assigned the good but resell it. Note that when $\overline{p} = p^e$, we have $r(\overline{p}) = p^e$ and $\Gamma(\overline{p})$ is precisely the same aggregate utility as is generated by a competitive market.

The derivative of the aggregate utility with respect to $\overline{p} < p^e$ is

$$\begin{aligned}\Gamma'(\overline{p}) &= -\rho\delta(p^e)\mathbb{E}[\eta'(w - \overline{p})|\Omega^+(p^e)] - r'(\overline{p})(1 - \rho)\delta(p^e)\mathbb{E}[\eta'(w - r(\overline{p}))|\Omega^+(p^e)] \\ &\quad - (1 - r'(\overline{p}))\rho\sigma(p^e, p^e)\mathbb{E}[\eta'(w + r(\overline{p}) - \overline{p})|\Omega^-(p^e, p^e)].\end{aligned}$$

Totally differentiating (11) with respect to $\overline{p}$ and invoking the implicit function theorem, we obtain

$$-r'(\overline{p}) = \frac{\rho\sigma_2(r(\overline{p}),\overline{p})}{\rho\sigma_1(r(\overline{p}),\overline{p}) - (1-\rho)\delta'(r(\overline{p}))} \geq 0.$$

It is not difficult to see that $\sigma_2(p^e,p^e) \leq -\delta'(p^e)$.[35] Hence,

$$0 \leq -r'(p^e) \leq \frac{\rho\sigma_2(p^e,p^e)}{-(1-\rho)\delta'(p^e)} \leq \frac{\rho}{1-\rho},$$

with one of the inequalities being strict. Substituting in, we obtain

$$\begin{aligned}
\Gamma'(p^e) &= -\rho\delta(p^e)\mathbb{E}[\eta'(w-p^e)|\Omega^+(p^e)] - r'(p^e)(1-\rho)\delta(p^e)\mathbb{E}[\eta'(w-p^e)|\Omega^+(p^e)] \\
&\quad -(1-r'(p^e))\rho\sigma(p^e,p^e)\mathbb{E}[\eta'(w)|\Omega^-(p^e,p^e)] \\
&= -(\rho + r'(p^e)(1-\rho))\delta(p^e)\mathbb{E}[\eta'(w-p^e)|\Omega^+(p^e)] \\
&\quad -(1-r'(p^e))\rho\sigma(p^e,p^e)\mathbb{E}[\eta'(w)|\Omega^-(p^e,p^e)] \\
&< -(\rho + r'(p^e)(1-\rho))\delta(p^e) - (1-r'(p^e))\rho\sigma(p^e,p^e) \\
&= -(\rho + r'(p^e)(1-\rho))S - (1-r'(p^e))\rho(1+m-S) \\
&= -\rho(1+m) = -S.
\end{aligned}$$

The lone inequality follows because $-r'(p^e) \leq \frac{\rho}{1-\rho}$, so $r'(p^e)(1-\rho) \geq -\rho$; $\eta'(w) \geq [>]\ 1$ [for a positive measure of w]; and $\eta(\cdot)$ is strictly concave. The second-to-last equality follows since $\sigma(p^e,p^e) = 1+m-\delta(p^e)$ and $\delta(p^e) = S$.

The string of inequalities implies that $\Gamma(\overline{p}) + S\cdot\overline{p} > \Gamma(p^e) + S\cdot p^e$, for some $\overline{p} < p^e$, whereas the competitive market yields total utility of $\Gamma(p^e)+S\cdot p^e$. At the same time, random assignment with a price cap of $\overline{p} < p^e$ will give agents aggregate utility that is strictly above $\Gamma(\overline{p})$, since all are weakly better off and some are strictly better off at the resale equilibrium with $r(\overline{p})$ than with the constrained behavior. Hence, welfare is strictly higher with random assignment and the price cap, $\overline{p}$. ∎

[35]Note, first, that $\Omega^+(p^e) = \Theta\backslash\Omega^-(p^e,p^e)$, where $\Theta := [1,\infty)\times[0,1]$. Next, observe that

$$\Omega^+(p^e-\epsilon) \supset \Theta\backslash\Omega^-(p^e,p^e-\epsilon)$$

for any $\epsilon > 0$. These two facts imply $-\delta'(p^e) \geq \sigma_2(p^e,p^e)$.