*Public Choice* **92:** 109–126, 1997.



# Rent dissipation when rent seekers are budget constrained *

YEON-KOO CHE[1] & IAN GALE[2]
[1]*Department of Economics, University of Wisconsin, Madison, WI 53706;* [2]*Department of Economics, Georgetown University, Washington, DC 20057 and Antitrust Division, U.S. Department of Justice, Washington, DC 20530, USA*

Accepted 7 March 1995

**Abstract.** In the original Tullock (1975, 1980) game, an individual bidder's probability of winning with a bid b is proportional to $b^R$, where the exponent reflects economies of scale in rent seeking. Different interpretations can be given to these probabilities. First, one may view R as a reflection of the political culture. Alternatively, one may view R as a choice variable for a politician. Intuition suggests that a society with a high tolerance for the selling of political favors and politicians who are receptive to rent seeking would both induce greater rent-seeking expenditures than other societies, all else equal. This paper shows that a lower value of R can actually lead to more rent dissipation than a higher value. This paper also reinforces two points concerning rent seeking. First, the analysis confirms the robustness of under-dissipation of rents, even in the face of entry. Second, it points out the importance of distinguishing between rent-seeking expenditures and rent dissipation. When bidders must borrow, for example, total expenditure may understate rent dissipation.

## 1. Introduction

Many government policies affect both the creation of rents and their allocation. The existence of these rents generates rent seeking by potential recipients. While popular interest in rent seeking has focused on possible distortions of the political process, academic interest has centered on rent dissipation.[1] Specifically, the question addressed is whether the contestants compete away all of the rents. If they do, then the value of the monopoly rents will indicate the value of the (possibly unobserved) rent-seeking expenditures. In this paper, we study rent dissipation in the presence of budget constraints.

Rent seekers may face budget constraints in a number of settings. The rent seeker or "bidder" could be an individual with low wealth who seeks a government license to operate a business. In particular, his wealth could be below the discounted profit stream that the license would generate. The bidder could be a lobbyist who represents a municipality vying for a new military

* The analysis and conclusions of this paper represent those of the authors and do not necessarily represent those of the Department of Justice. We thank an anonymous referee for helpful comments.

base (or trying to keep an existing base open) whose budget is below the value to the municipality of being awarded the base. Similarly, the bidder could represent an under-capitalized start-up firm seeking a government contract. Despite the practical relevance of these scenarios, they have received little attention in the literature on rent seeking.

In the original Tullock (1975, 1980) game, each bidder makes an irrevocable payment (a "bid") in hopes of winning a prize that all bidders value at v. In particular, an individual bidder's expected surplus from bidding $b_i$ is $v\frac{b_i^R}{\sum_1^N b_j^R} - b_i$. The exponent R reflects economies of scale in rent seeking in the sense that a higher value of R means that a given increase in rent-seeking expenditures generates a larger increase in the probability of winning, all else equal. The two cases studied most frequently have been R = 1 and R = $\infty$. (Rowley, 1991, contains a comprehensive list of references.) We refer to the former as "the lottery", and to the latter as "the all-pay auction." The lottery awards the prize randomly, with the probability of receipt being proportional to one's bid, whereas the all-pay auction awards the prize to the highest bidder. In both cases, all bids are forfeited.

Different interpretations can be given to these probabilities. First, one may view R as a reflection of the political culture. In particular, it reflects the extent to which a society rewards influence activities. Under this interpretation, a low value of R means that the society has a low tolerance for rent seeking. In such an environment, a politician may cast votes or award favors based on factors besides rent-seeking expenditures. Second, one may view R as a choice variable for a politician.[2] Clearly, a politician cannot openly hold an auction for her vote on a particular piece of legislation. But she could reward the lobbyist making the highest expenditure by awarding the prize to him. At the same time, she retains the option of sometimes awarding the prize to someone else.

Intuition suggests that a society with a high tolerance for the selling of political favors and politicians who are receptive to rent seeking would both induce greater rent-seeking expenditures than other societies, all else equal. This intuition is supported by the literature on rent seeking without binding budget constraints. When the rent seekers value the prize equally, the all-pay auction dissipates all rents, just as a standard auction would do (Hillman and Riley, 1989; Baye, Kovenock and de Vries, 1993a,b), whereas the lottery exhibits incomplete dissipation.

This paper characterizes the (expected) total expenditures in the presence of budget constraints. In so doing, it demonstrates that the results can be dramatically different when there are budget constraints. Specifically, the lottery often generates more rent dissipation than the all-pay auction (and also the standard auction). The all-pay auction tends to generate a lower total expenditure since high-wealth bidders are essentially able to “preempt” low-wealth bidders. The lottery dampens this ability to preempt, which makes the low-wealth bidders more aggressive.[3] Thus, rent dissipation may be higher in societies where rent-seeking activity meets with less encouragement (i.e., where R = 1). A politician who obscures the appearance of selling favors by allocating the political prize randomly may therefore receive greater campaign contributions, for example.

This paper also reinforces two points concerning rent seeking. First, the analysis confirms the robustness of under-dissipation of rents, even in the face of entry. When bidders value the political prize equally, budget constraints often lead to incomplete dissipation. Moreover, this result can hold when the bidders’ aggregate wealth exceeds the value of the prize. Second, the analysis points out the importance of distinguishing between rent-seeking expenditures and rent dissipation. The two can differ when bidders must borrow, for example, since the opportunity cost of one additional dollar of lobbying expenditures can then exceed one dollar. In that case, total expenditure may understate rent dissipation.

Two earlier papers are noteworthy. Hillman and Katz (1984) looks at risk-averse rent seekers. They assume identical bidders, however, whereas our bidders are heterogeneous. One similarity, though, is that total expenditures can understate rent dissipation in both papers. Linster (1993) models international competition and the formation of alliances as a rent-seeking game. The similarity comes from the presence of a budget constraint. Each country has an exogenously-specified endowment that must be divided between the consumption good and the military good.

The next section lays out the model. We analyze rent seeking with budget constraints in Section 3. Initially, we assume that borrowing is not possible, so wealth constitutes an absolute limit on what a bidder can spend. There may be multiple equilibria in the all-pay auction, but we show that the expected total expenditure is unique. There is a unique equilibrium in the lottery, however, which we characterize completely. We allow access to credit in Section 4. Concluding remarks are contained in Section 5.

## 2. The model

A seller has an indivisible object and she faces $N \geq 2$ bidders. The seller could be a politician who controls the awarding of a government contract, the opening or closing of a military base, or the awarding of a license to produce a particular good, for example. The bidders are lobbying to receive the object. They are risk-neutral, and they each value the object at $v > 0$. We look for Nash equilibria in bidding strategies. That is, bidding behavior must be optimal given the strategies of the other bidders.

We first note the properties of the equilibria when bidders do not face budget constraints. In the all-pay auction, the expected total expenditure is equal to v (Hillman and Riley, 1989; Baye, Kovenock and de Vries, 1993a,b).[4] In the lottery, the total expenditure is v(N−1)/N. Finally, in a standard oral or sealed-bid auction, total expenditure is equal to v.[5]

Thus, in two of the three cases there is complete dissipation of rents. We now consider the impact of budget constraints.

## 3. Budget constraints

Suppose that each bidder i has wealth $w_i$, which is publicly known, and $w_1 > w_2 > ... > w_N > 0$. We assume, for now, that the bidders' access to capital is sufficiently limited that they do not borrow. Thus, wealth is an absolute constraint on what a bidder can spend.

### 3.1. *All-pay auctions*

In an all-pay auction, bidders submit bids simultaneously. The highest bidder wins, but all bidders forfeit their bids. Thus, if a bidder submits a bid b, and the bid wins, his surplus is v − b. If the bid loses, his surplus is −b. We impose the natural restriction that bids must be nonnegative.

The equilibria of the all-pay auction are in mixed strategies, and at least two bidders make non-zero bids with positive probability. If $w_2 \geq v$, then this is essentially identical to the case without budget constraints, and all bidders receive zero expected surplus. If $w_2 < v$, then bidder 1 receives a strictly positive expected surplus, and the expected total expenditure is $w_2$. The proof of the proposition is in Appendix 1.

*Proposition 1*. The expected total expenditure in the all-pay auction is $\min\{v, w_2\}$.

### 3.2. *The lottery*

In the lottery, bidders with low wealth are constrained and bid their wealth, while high-wealth bidders are unconstrained and all make the same bid. To see this, suppose that each bidder j = 2, 3, ..., N submits a bid $b_j$. If bidder 1 submits a bid b, he wins with probability

$$\frac{b}{b + \sum_2^N b_j}.$$

Bidder 1 maximizes his expected surplus,

$$S_1(b) \equiv v\frac{b}{b + \sum_2^N b_j} - b,$$

subject to the constraint that the bid not exceed his wealth. $S_1(\bullet)$ is a concave function, so the optimum is at the unique global maximum (if it lies between zero and $w_1$) or else it is at a boundary (either zero or $w_1$).

The derivative of the expected surplus function is

$$S_1'(b) \equiv v\frac{\sum_2^N b_j}{[b + \sum_2^N b_j]^2} - 1.$$

Writing $\sigma \equiv \sum_2^N b_j$, the optimal bid for bidder 1 is

$$b_1 = \begin{cases} 0 & \text{if } v < \sigma \\ \sqrt{v\sigma} - \sigma & \text{if } \sigma \leq v \leq (w_1 + \sigma)^2/\sigma \\ w_1 & \text{if } v > (w_1 + \sigma)^2/\sigma \end{cases}$$

In the first case, the bids of the other bidders are so high that it is not worthwhile bidding at all. In the second case, bidder 1 is unconstrained, while in the third, his wealth is sufficiently low that he can only bid his wealth.

The first of these cases cannot occur in equilibrium. If $v < \sigma$ in equilibrium, then a bidder $j > 1$ who submits a bid $b_j > 0$ receives expected surplus of

$$S_j(b_j) = v\frac{b_j}{\sum_1^N b_i} - b_j = v\frac{b_j}{\sigma} - b_j < 0,$$

where the second equality follows from the assumption that $b_1 = 0$. This contradicts the optimality of $b_j$. Thus, $v \geq \sigma$. Note also that if $v > \sigma$, the optimal bid for bidder 1 is $b_1 < v - \sigma$, so the aggregate bid is less than v. Applying the same logic to the other bidders, we conclude that the total expenditure is strictly below v in equilibrium, and that every bidder makes a strictly positive bid.

We next show that each bidder's equilibrium bid is a weakly increasing function of his wealth.

*Lemma 1*. The equilibrium bids form a weakly decreasing sequence: $b_1 \geq b_2 \geq ... \geq b_N$.

*Proof*. Take two bidders, say 1 and 2. Suppose, contrary to the hypothesis, that $b_1 < b_2$. This means that $b_1 < w_1$, since $b_2 \leq w_2$. Thus, bidder 1 is unconstrained, so we can write

$$S_1'(b_1) = v\frac{b_2 + \sum_3^N b_j}{[\sum_1^N b_j]^2} - 1 = 0.$$

By contrast,

$$S_2'(b_2) = v\frac{b_1 + \sum_3^N b_j}{[\sum_1^N b_j]^2} - 1 \geq 0,$$

since bidder 2 may or may not be constrained. The assumption that $b_1 < b_2$ contradicts these two inequalities. Thus, we have $b_1 \geq b_2$. The same argument can be used with any two bidders. QED

We now show that there exists a critical level of wealth such that all bidders with higher wealth are unconstrained while those with lower wealth bid their wealth.

*Lemma 2*. There exists a bid b* such that bidders with strictly greater wealth bid b* in equilibrium whereas bidders with wealth less than or equal to b* bid their wealth.

*Proof*. The proof is completed by showing that if a given bidder bids his wealth, then all bidders with lower wealth also bid their wealth. Suppose, to the contrary, that bidder 1 bids $b_1 = w_1$ but bidder 2 bids $b_2 < w_2$, for example. We can now write

$$S_1'(b_1) = v\frac{b_2 + \sum_3^N b_j}{[\sum_1^N b_j]^2} - 1 \geq 0,$$

$$S_2'(b_2) = v\frac{b_1 + \sum_3^N b_j}{[\sum_1^N b_j]^2} - 1 = 0.$$

Satisfaction of these two inequalities requires $b_1 \leq b_2$, which contradicts the hypothesis that $b_1 = w_1$ and $b_2 < w_2$. The same argument holds with any two bidders. Thus, there exists an index i* such that all bidders i < i* bid strictly less than their wealth whereas all i ≥ i* bid their wealth.

We now show that all bidders i < i* make the same bid. Once again, consider bidders 1 and 2. Since they both bid strictly less than their wealth, their bids are at interior maxima. The first-order conditions immediately show that $b_1 = b_2$. QED

Lemma 2 shows that, in any equilibrium, there is a critical bidder, i*, such that bidders i*, i*+1, ..., N bid their wealth, while all others bid strictly less than their wealth. Those who bid less than their wealth all bid b*, where $w_{i*} \leq b^* < w_{i*-1}$. The last step is to show that the equilibrium is unique.

*Lemma 3*. The equilibrium is unique.

*Proof*. The case of N = 2 is straightforward. Now consider N > 2, and suppose that there are two equilibria. In each equilibrium, there is a critical bidder.

Let i* and $i' >$ i* denote the two critical bidders. Lemma 2 shows that all unconstrained bidders make the same bid in a given equilibrium. Let b* and $b'$ denote these unconstrained bids in the two equilibria.

Consider the equilibrium in which bidder $i'$ is critical. Since $i' >$ i* $\geq 1$, there is at least one unconstrained bidder in this equilibrium. We can rule out $i'$ = 2 and i* = 1 since, if all other bidders are constrained, bidder 1 has a unique best response. Thus, $i' > 2$, so at least two bidders are unconstrained in the equilibrium where $i'$ is critical.

Lemma 2 showed that $w_{i*} \leq$ b* $< w_{i*-1}$ and $w_{i'} \leq b' < w_{i'-1}$, which means that b* $> b'$. Thus, bidders i = 1, 2, ..., i* $-$ 1 make a strictly higher bid when bidder i* is critical than when $i'$ is. In addition, bidders i*, i*+1, ..., $i' - 1$ bid their wealth when i* is critical whereas they are unconstrained in the other equilibrium. Finally, bidders $i'$, $i'$+1, ..., N bid their wealth in both equilibria. Thus, the bids are weakly higher for every bidder when i* is the critical bidder, so the aggregate expenditure by one's competitors is weakly higher when i* is the critical bidder.

Now consider the equilibrium where $i'$ is critical. We show that the optimal bid for bidder $i' - 1$ is weakly decreasing in the aggregate expenditure of his rivals. Let $\sigma'$ denote the aggregate bids of all bidders other than bidder $i' - 1$. Since bidder $i' - 1$ is unconstrained and bids $b'$, his first-order condition can be written as

$$v \frac{\sigma'}{[b' + \sigma']^2} - 1 = 0.$$

From this we conclude that

$$v = \frac{[b' + \sigma']^2}{\sigma'} < 4\sigma'.$$

The inequality holds because bidder $i' - 1$ makes the same bid as bidder 1 does, and because all other bids exceed zero in equilibrium, so $b' < \sigma'$.

The first-order condition for bidder $i' - 1$ also shows that

$$\frac{db_{i'-1}}{d\sigma'} < 0 \text{ if and only if } v < 4\sigma'.$$

Since the latter condition holds, and since the aggregate bid of his competitors is higher when i* is critical, his bid should be no higher when i* is the critical bidder than when $i'$ is. But b* $> b'$, which provides a contradiction, so the equilibrium is unique. QED

The analysis above leads to the following proposition.

*Proposition 2.* The total expenditure in the lottery is (i* $-$ 1)b* + $\sum_{i*}^{N} w_j$, where i* satisfies $w_{i*} \leq b* < w_{i*-1}$ and $v\frac{(i*-2)b*+\sum_{i*}^{N} w_j}{[(i*-1)b*+\sum_{i*}^{N} w_j]^2} - 1 = 0$. If $v \geq \frac{[\sum_1^N w_j]^2}{\sum_2^N w_j}$, then all bidders bid their wealth.

Note that all bidders participate, even though they are asymmetric in terms of wealth. By contrast, when bidders have asymmetric valuations, but no budget constraints, the tendency is for some bidders not to participate (Hillman and Riley, 1989).

We can also show that the presence of binding budget constraints lowers the expected total expenditure. The first-order condition for bidder 1 indicates that the equilibrium bids satisfy the following inequality:

$$\sum_1^N b_j \leq v\frac{\sum_2^N b_j}{\sum_1^N b_j} \leq \frac{v(N-1)}{N}.$$

The left-hand expression is the total expenditure while the right-hand expression is the total expenditure in the absence of budget constraints, as noted earlier. The second inequality holds since bidder 1 has the highest bid, by Lemma 1. If some bidders are constrained, then the total expenditure is strictly lower here than it is in the absence of budget constraints.

### 3.3. *Expenditure comparisons*

The all-pay auction generates an expected total expenditure of $\min\{v, w_2\}$, whereas the lottery generates $(i*\text{-}1)b* + \sum_{i*}^{N} w_j \leq v(N-1)/N$. Thus, if $w_2 \geq v$, the all-pay generates a higher expected total expenditure than the lottery. For sufficiently high v, however, the total expenditure is higher in the lottery than in the all-pay auction.

*Proposition* 3. There exists $v* > w_2$ such that the lottery generates a higher expected total expenditure than the all-pay auction if and only if $v > v*$.

*Proof*. We first show that there exists a value of v for which the expected total expenditure is greater in the lottery. Take a value such that

$$v \geq \frac{[\sum_1^N w_j]^2}{\sum_2^N w_j}.$$

Proposition 2 then implies that all bidders are constrained in the equilibrium, so the total expenditure exceeds $w_2$, the expected total expenditure in the all-pay auction in this case.

Now we show that the equilibrium total expenditure in the lottery is increasing in v. Consider bidder j, and define the total expenditure to be $e_j \equiv b_j + \sigma_j$. We can now write bidder j's expected payoff as

$$\Pi_j(e_j;\sigma_j,v) = v\frac{e_j - \sigma_j}{e_j} - (e_j - \sigma_j).$$

The cross partial derivative with respect to $e_j$ and $b_j$ is positive, as is the cross partial with respect to $e_j$ and v. Since $\sigma_j$ is increasing in $e_j$, for $j \neq 1$, the above game exhibits strategic complementarities and satisfies the single-crossing property of Milgrom and Shannon (1994). Theorem 16 of Milgrom and Shannon (1994) implies that the equilibrium total expenditure is increasing in v. In the standard auction, the equilibrium total expenditure equals v for $v \leq w_2$ and it equals $w_2$ otherwise. Thus, there exists $v^* > w_2$ such that the lottery generates a higher expected total expenditure than the all-pay auction if and only if $v > v^*$. QED

In a standard oral auction, the total expenditure is also equal to $\min\{v,w_2\}$. The standard auction extracts revenue from only one bidder, which has a profound impact on total expenditure if the second-highest wealth is far below v. The all-pay extracts revenue from more than one bidder, but the low-wealth bidders are passive. Indeed, only two bidders participate actively. The lottery extracts revenue from more than one bidder, while also allocating the good randomly, which reduces the low-wealth bidders' disincentive to participate. As a consequence, all bidders participate actively in the lottery, and it generates higher total expenditure than the two auctions, for sufficiently high v.

Now consider the impact of entry. Letting N increase without bound, we see that the expression for the total expenditure is unaffected in the all-pay auction and the standard auction. In other words, allowing more and more entrants with low wealth has no impact on total expenditure. By contrast, allowing such entry in the lottery always causes total expenditure to increase.

## 4. Availability of credit

We previously assumed that an absolute wealth constraint limited individual expenditures. An absolute limit is a convenient abstraction, but may not be realistic in many cases. The bidder could be the owner of a business firm, for example, so lobbying takes resources away from other productive activities. The opportunity cost of spending one additional dollar on lobbying exceeds one dollar, once lobbying expenditures exceed a certain threshold. Similarly, the object for sale could be a license that will generate future income against which bidders can borrow. We consider bidders who have access to external financing in this section.

Suppose that credit is available at an interest rate r. (The interest rate could also reflect the opportunity cost of using internal funds. Moreover, the results will be qualitatively similar if the cost of funds varies with the amount borrowed.) As before, bidder i has wealth $w_i$, which is publicly known, and $w_1 > w_2 > ... > w_N > 0$. A bidder with wealth w must borrow $(b - w)$, and repay it next period, in order to bid $b > w$. The cost of bidding b (assuming no discounting) is $w + (b - w)(1+r) = b+(b - w)r$.

### 4.1. *All-pay auctions*

The equilibrium of the all-pay auction with borrowing is similar to the equilibrium without borrowing, although bidder 2's supremum bid changes if $w_2 < v$. The most that bidder 2 would bid in that case is

$$\bar{b} \equiv (v + w_2 r)/(1 + r),$$

since the cost to him of bidding $\bar{b}$ is

$$\bar{b} + (\bar{b} - w_2)r = v.$$

In other words, he is indifferent to bidding $\bar{b}$ if he wins with probability one. We now characterize the expected total expenditure. The proof is in Appendix 2.

*Proposition 4*. If $w_2 \geq v$, then the expected total expenditure in the all-pay auction is v. If $w_2 < v$, then it is $\frac{\bar{b}^2}{v} + \frac{r[2\bar{b}^2 - w_2^2 - min\{w_1^2, \bar{b}^2\}]}{2v}$.

Suppose that $w_2 < v$. As r approaches zero, the expected total expenditure approaches v. The expected total expenditure is not equal to the difference between v and the total bidders' expected surplus (which equals $v - \bar{b}$ if $w_1 \geq \bar{b}$) here. The reason is that part of the rent dissipation is in the form of interest payments. Thus, the expected total expenditure can understate the amount of rent dissipation.

### 4.2. *The lottery*

When credit is available, bidders with low wealth borrow, those with intermediate levels of wealth bid their wealth, and high-wealth bidders are unconstrained. Suppose that each bidder j = 2, 3, ..., N submits a bid $b_j$. If bidder 1 submits a bid b, her expected surplus is

$$S_1(b) \equiv v \frac{b}{b + \sum_2^N b_j} - c(b; w_1),$$

where $c(b;w) \equiv b + max\{0, b-w\}r$ is the cost of bidding b when one's wealth is w. The slope of bidder 1's expected surplus function is

$$S_1'(b) \equiv \begin{cases} v\frac{\sum_2^N b_j}{[b+\sum_2^N b_j]^2} - 1 & \text{if } b < w_1 \\ v\frac{\sum_2^N b_j}{[b+\sum_2^N b_j]^2} - (1+r) & \text{if } b > w_1 \end{cases}$$

Again, let $\sigma \equiv \sum_2^N b_j$. The optimal bid for bidder 1, given the aggregate bid of the other bidders, is then

$$b_1 = \begin{cases} 0 & \text{if } v < \sigma \\ \sqrt{v\sigma} - \sigma & \text{if } \sigma \leq v \leq (w_1+\sigma)^2/\sigma \\ w_1 & \text{if } (1+r)(w_1+\sigma)^2/\sigma \geq v > (w_1+\sigma)^2/\sigma \\ \sqrt{v\sigma/(1+r)} - \sigma & \text{if } v > (1+r)(w_1+\sigma)^2/\sigma \end{cases}$$

The first three cases are the same as before. In particular, the case with b = 0 is impossible in equilibrium. In the final case, bidder 1's wealth is so low that he borrows to bid.

Several properties of the equilibrium carry over from the case with absolute budget constraints. For example, equilibrium bids are a weakly increasing function of wealth.

*Lemma 4*. The equilibrium bids form a weakly decreasing sequence: $b_1 \geq b_2 \geq ... \geq b_N$.

*Proof*. Take two bidders, say 1 and 2. Suppose, to the contrary, that $b_1 < b_2$. There are two cases: $b_1 < w_1$ or $b_1 \geq w_1$. If $b_1 < w_1$, bidder 1 is unconstrained, so

$$S_1'(b_1) = v\frac{b_2 + \sum_3^N b_j}{[\sum_1^N b_j]^2} - 1 = 0.$$

Bidder 2 will bid his wealth or borrow, given the hypothetical, so his bid satisfies the inequality

$$v\frac{b_1 + \sum_3^N b_j}{[\sum_1^N b_j]^2} - 1 \geq 0.$$

As before, the assumption that $b_1 < b_2$ provides a contradiction. Thus, we have $b_1 \geq b_2$.

Now suppose that $b_1 \geq w_1$. In this case, bidder 2 must borrow, so

$$S_2'(b_2) = v\frac{b_1 + \sum_3^N b_j}{[\sum_1^N b_j]^2} - (1+r) = 0.$$

Regardless of what bidder 1 does, optimizing behavior requires that his bid satisfy the inequality

$$v\frac{b_2+\sum_3^N b_j}{[\sum_1^N b_j]^2}-(1+r)\leq 0.$$

The same argument implies that $b_1 \geq b_2$. These arguments can be used with any two bidders. QED

We can now show that there exist two critical levels of wealth when borrowing is possible. All bidders with wealth above the larger critical wealth are unconstrained, those between the two critical wealths bid their wealth, and all others borrow.

*Lemma 5*. There exist bids B* and B** such that bidders with wealth strictly greater than B* all bid B*, bidders with wealth between B* and B** bid their wealth, while bidders with lower wealth borrow and bid B**.

*Proof*. We first show that if a given bidder bids his wealth, then all bidders with lower wealth bid their wealth or borrow. Likewise, if a given bidder borrows, then all bidders with lower wealth borrow.

Consider the case where bidder 1 borrows. Suppose, contrary to the hypothesis, that bidder 2 does not borrow. That is, $b_1 > w_1$, but $b_2 \leq w_2$. We can now write

$$v\frac{b_2+\sum_3^N b_j}{[\sum_1^N b_j]^2}-(1+r)=0,$$

$$v\frac{b_1+\sum_3^N b_j}{[\sum_1^N b_j]^2}-(1+r)\leq 0.$$

The first equation is bidder 1's first-order condition. The second equation reflects the fact that bidder 2 bids no more than his wealth (although he may be indifferent at the margin to borrowing). Satisfaction of these two inequalities requires $b_1 \leq b_2$, which contradicts the assumption. Thus, if a given bidder borrows, all bidders with lower wealth also borrow. Since such bidders are at an interior maximum, their bids must be equal. Call that common bid B**.

Now suppose, contrary to the hypothesis, that bidder 1 bids his wealth, but bidder 2 bids less than his wealth. This is the case covered by Lemma 2.QED

The last step is to show that B* and B** are unique. This follows the same steps as the proof of Lemma 3. Likewise, the determination of B* and B** is a straightforward extension.

The analysis above leads to the following proposition. Let i** denote the minimum index such that all bidders $i \geq$ i** borrow, and let i* denote the maximum index such that all bidders $i < i^*$ bid strictly less than their wealth.

*Proposition 5*. The total expenditure in the lottery is of the form $(i^* - 1)B^* + \sum_{i^*}^{i^{**}-1} w_j + (N - i^{**} + 1)B^{**}$.

### 4.3. *Expenditure comparisons*

There is an analog to Proposition 3 here. Specifically, there exists a critical value $v^* > w_2$ such that, for every $v > v^*$, there is an interest rate $r^*(v)$ with the property that the expected total expenditure is higher in the lottery than in the all-pay auction, if $r > r^*(v)$. The proof follows the same steps as the proof of Proposition 3. Conversely, for a sufficiently small interest rate, the expected total expenditure is lower in the lottery.

The all-pay and the standard auction do not generate the same total expenditure here. Total expenditure in the standard auction is equal to $\min\{v, \bar{b}\}$ when $w_2 < v$. It is then straightforward to show that the all-pay auction can result in lower total expenditure. Suppose that $w_1 \geq \bar{b}$. In both auctions, bidder 1 receives expected surplus of $v - \bar{b}$, while all other bidders receive zero. Thus, rent dissipation is the same in the two auctions. However, part of the rent dissipation in the all-pay auction comes from bidder 2's interest payments, whereas no interest payments are made in the standard auction. This shows that the expected total expenditure is strictly smaller in the all-pay auction.

The situation changes when $w_1 < \bar{b}$. Now bidder 1 must borrow in the standard auction, which can reverse the expenditure rankings. Holding $w_2$ fixed, consider the expected total expenditure in the all-pay auction, as $w_1$ approaches $w_2$. Straightforward calculations show that the limiting expected total expenditure is $\bar{b} + w_2 r(v - w_2)/v(1+r) > \bar{b}$, and the latter term equals the total expenditure in the standard auction. In the standard auction, bidder 1 borrows an amount close to $\bar{b} - w_2$, with probability one. In the all-pay, less borrowing occurs, so less of the rent is dissipated in interest payments.

## 5. Concluding remarks

In this paper, we have compared the outcomes of two rent-seeking games and a standard auction. When bidders face budget constraints, the total expenditure is often higher in the lottery than in the all-pay auction or the standard auction. Bidders are passive in the all-pay auction because they view the high-wealth bidder as the likely winner, so at most two bidders make non-zero bids. When borrowing is not possible, there is (weakly) more rent dissipation in the standard auction than in the all-pay auction. When borrowing is possible, the ranking can go either way. In the lottery, not only are low-wealth bidders more aggressive, but all bidders pay as well.

As noted in the introduction, different interpretations can be given to the parameter that determines the probability of receiving the political prize. If R is a reflection of the political culture, then a society with a low tolerance for rent seeking may have a higher expected total expenditure. If R is seen as a choice variable for politicians, then a politician may gain from sometimes awarding a political prize to someone other than the highest bidder.

Although bidders value the prize equally, dissipation of rent is often far from complete here, even when the bidders' aggregate wealth exceeds the value of the political prize. There is some empirical support for incomplete dissipation. In one recent study, Hazlett and Michaels (1993) looked at the Federal Communications Commission's lotteries of cellular telephone licenses, and found that dissipation of rent was far from complete. Since the lotteries took place over a long period, and later participants were able to see the success of early recipients, it is questionable whether there was a systematic underestimate of valuations.

Laband and Sophocleus (1992), when discussing political lobbying, note that "the visible dollar investments fall way short of total transfers", while also stressing that rent seeking can involve currency other than money. Our case with borrowing highlighted a related point. Total expenditures may understate rent dissipation because of the opportunity cost of funds. This understatement may only reflect a transfer to a third party, as in the case of interest payments, but may reflect a loss of producers' surplus in other cases.

Finally, we also considered entry. The existence of rents typically leads to rent seeking by potential recipients. We showed that entry may have no effect on total expenditure in the all-pay auction and the standard auction, whereas it always increases total expenditure in the lottery. In practice, there are often significant barriers to entry, so these issues are moot. When the Pentagon procures advanced military hardware, for example, there may be few firms capable of production. Likewise, when communities are fighting to save military bases, entry is not an issue.

## Notes

1. Concerns with distortions of the political process were voiced repeatedly during the recent debates in the U.S. House of Representatives and Senate on lobbying reform. See, for example, "House Passes Overhaul Of Lobbying, Bans Gifts; Some Members Say Bill Does Not Go Far Enough", *The Washington Post*, 25 March 1994.
2. Until the relevant legislation (2 U.S.C. §439a) was amended in 1989, members of Congress were able to convert campaign funds to personal use. Business can still be directed to a politician's law firm or other enterprise. These observations suggest that there is at least some appropriability of rent-seeking expenditures.

3. Similar intuition and results hold when bidders value the prize differently but do not face budget constraints. The equilibria are qualitatively different in that case, however. In particular, a bidder with a budget constraint does not act as though his valuation were the smaller of his true valuation and his budget.
4. These equilibria are all in mixed strategies. For a discussion of mixed strategies see Tullock (1987) and the reply in Hillman and Samet (1987).
5. There are many Nash equilibria, only one of which is robust to mistakes by bidders. Whenever we discuss standard auctions, we will consider such "trembling-hand perfect equilibria."

## References


Baye, M., Kovenock, D. and De Vries, C. (1993a). Rigging the lobbying process: An application of all-pay auctions. *American Economic Review* 83: 289–294.

Baye, M., Kovenock, D and De Vries, C. (1993b). The all-pay auction with complete information. Mimeo.

Baye, M., Kovenock, D. and De Vries, C. (1994). The solution to the Tullock rent-seeking game when $R > 2$: Mixed-strategy equilibria and mean dissipation rates. *Public Choice,* forthcoming.

Hazlett, T. and Michaels, R. (1993). The cost of rent-seeking: Evidence from cellular telephone license lotteries. *Southern Economic Journal* 53: 425–435.

Hillman, A. and Katz, E. (1984). Risk-averse rent seekers and the social costs of monopoly power. *Economic Journal* 94: 104–110.

Hillman, A. and Riley, J. (1989). Politically contestable rents and transfers. *Economics and Politics* 1: 17–39.

Hillman, A. and Samet, D. (1987). Characterizing equilibrium rent-seeking behavior. *Public Choice* 54: 85–87.

Laband, D. and Sophocleus, J. (1992). An estimate of resource expenditures on transfer activity in the United States. *Quarterly Journal of Economics* 107: 959–983.

Linster, B. (1993). A rent-seeking model of international competition and alliances. *Defence Economics* 4: 213–226.

Milgrom, P. and Shannon, C. (1994). Monotone comparative statics. *Econometrica* 62: 157–180.

Rowley, C. (1991). Gordon Tullock: Entrepreneur of Public Choice. *Public Choice* 71: 149–169.

Tullock, G. (1975). On the efficient organization of trials. *Kyklos* 28: 745–762.

Tullock, G. (1980). Efficient rent seeking. In J.M. Buchanan, R.D. Tollison and G. Tullock (Eds.), *Toward a theory of the rent seeking society.* College Station: Texas A&M University Press.

Tullock, G. (1987). Another part of the swamp. *Public Choice* 54: 83–84.

## Appendix 1

*Proof of Proposition 1*
We begin with a preliminary lemma showing that no bidder selects an interior bid with positive probability.

*Lemma A1*. No bidder has a mass point at any bid $b \in (0,w_2)$.

*Proof*. A bidder must be indifferent among all bids that he might make in a mixed-strategy equilibrium. Thus, all such bids must return the same expected payoff. Now suppose, contrary to the hypothesis, that a single bidder has a mass point at $b \in (0,w_2)$. There is an interval of the form $(b - \varepsilon,b)$, with $\varepsilon$ small, where no bidder puts positive density. If another bidder put density there, then moving all of that density to b yields a discrete increase in the probability of winning, with only an infinitesimal increase in the payment. (Note that this other bidder's wealth must be at least equal to b in order for him to put positive density in this interval.) But if no other bidder puts density in an interval of this form, then the bidder with the mass point at b would increase his expected surplus by moving the mass lower.

Now suppose that two or more bidders have mass points at $b \in (0,w_2)$. Each bidder would increase his expected surplus individually by moving his mass higher. Such a move is feasible for all of these bidders, unless b is equal to one of the bidder's wealth, in which case there is still at least one bidder able to deviate profitably. Thus, no bidder can have mass at b. QED

We begin the proof of Proposition 1 with the case of $w_2 \geq v$. There may not be a unique equilibrium, but the equilibrium payoffs are unique. In particular, we show that every bidder receives an expected surplus equal to zero. Suppose, to the contrary, that exactly one bidder has a strictly positive expected surplus. This bidder's supremum (i.e., the least upper bound on his bid) must be v. The supremum cannot exceed v, since such a bid yields a negative surplus, even if it wins with probability one. If the supremum is $b^{**} < v$, then at least one other bidder would be able to bid $b^{**}+\varepsilon$, for small $\varepsilon > 0$, and receive a surplus of $v - (b^{**}+\varepsilon) > 0$. Thus, the supremum is $b^{**} = v$. A bid $b = v$ yields an expected surplus no higher than zero, however, which contradicts the premise that the bidder receives a strictly positive expected surplus.

Now suppose that two or more bidders receive strictly positive expected surpluses. This, too, leads to a contradiction. The bidders with non-zero expected surpluses must have the same infimum bid. If not, those with strictly lower infima would lose with probability one when they bid less than the highest infimum, so their expected surpluses cannot be strictly positive. If these bidders have the common infimum $b^* > 0$, then in order for each of them to win with positive probability when bidding $b^*$, they must all have mass points at $b^*$. But Lemma A1 shows that this cannot happen in equilibrium. Thus, they must all have infima equal to zero.

The bidders cannot all have mass points at zero, since it would then be better individually to move the mass higher. If two or more do not have mass at zero, however, then all bidders must receive zero expected surplus, since a bid of zero loses with probability one, and a bid arbitrarily close to zero loses with a probability arbitrarily close to one. If exactly one bidder does not have mass at zero, then that bidder alone receives an expected surplus that is bounded above zero when he makes an infinitesimal bid. But it is impossible for exactly one bidder to have a strictly positive expected surplus, as was shown above. Thus, all bidders receive zero expected surplus, and the expected total expenditure is v, since all rents are dissipated (in expectation).

Now suppose that $v > w_2$. Bidder 1 can guarantee himself a positive expected surplus by submitting a bid above $w_2$. No other bidder can receive a strictly positive expected surplus, by the arguments given above. Thus, bidder 1 must bid $w_2$ or above with positive probability. If not, bidder 2 could receive an expected surplus of $v - w_2 > 0$ by bidding $w_2$. Likewise, bidder 2's supremum bid must be $w_2$ or else bidder 1 would not bid $w_2$.

No other bidder will make a non-zero bid. Suppose, to the contrary, that the highest supremum from bidders 3, 4, ..., N comes from bidder j and equals $\tilde{b} > 0$. Clearly, $\tilde{b} \leq w_j < w_2$. A bid of $\tilde{b}$ must yield bidder j an expected surplus of zero. (By the discussion above, it cannot be strictly higher, and optimizing behavior means that it cannot be strictly lower.) We now show that a bid $\tilde{b} + \varepsilon$, for small $\varepsilon > 0$, yields bidder 2 a strictly positive expected surplus, which provides a contradiction.

Let $G_i(b)$ denote the probability that bidder i's bid is less than or equal to b. By Lemma A1, there cannot be mass points at $\tilde{b}$. Bidder j wins with a bid of $\tilde{b}$ if bidders 1 and 2 both have bids below $\tilde{b}$. Since all other bidders bid less than $\tilde{b}$ with probability one, and since bidder j receives zero expected surplus, we have

$$vG_1(\tilde{b})G_2(\tilde{b}) - \tilde{b} = 0.$$

At the same time, bidder 2 wins with a bid of $\tilde{b} + \varepsilon$ if bidder 1 bids less than $\tilde{b} + \varepsilon$. Such a bid yields an expected surplus equal to

$$vG_1(\tilde{b} + \varepsilon) - (\tilde{b} + \varepsilon),$$

which exceeds zero since $G_2(\tilde{b}) < 1$. This contradicts the result that bidder 2's expected surplus must equal zero. Thus, only bidders 1 and 2 make non-zero bids.

In the unique equilibrium, bidder 1 puts point mass of $(v - w_2)/v$ on $w_2$ and distributes the remainder uniformly on $(0,w_2)$.[1] Bidder 2 puts point mass of $(v - w_2)/v$ on zero, and distributes the remainder uniformly on $(0,w_2)$. All others bid zero. The expected total expenditure is therefore $w_2$. QED

## Note

1. Strictly speaking, bidder 1 puts his mass infinitesimally above $w_2$. If there is a smallest currency unit, then bidder 1 puts his mass on the smallest unit strictly above $w_2$. The game we are studying is the continuous limit of this discrete game.

## Appendix 2

*Proof of Proposition 4*

If $w_2 \geq v$, then the arguments in the proof of Proposition 1 show that all bidders must receive zero expected surplus and only bidders whose wealth is at least equal to v make non-zero bids. Thus, the expected total expenditure is equal to v.

Now suppose that $w_2 < v$. We consider the subcase of $w_1 \geq \bar{b}$ first. Bidder 1 can guarantee himself a strictly positive expected surplus by submitting a bid above $\bar{b}$. Following the arguments in the proof of Proposition 1, bidder 1 puts density 1/v on the interval $(0,w_2]$, and (1+r)/v on $(w_2, \bar{b})$ in equilibrium. Bidder 2 puts mass of $(v - \bar{b})/v$ on zero, and distributes the remainder uniformly on $(0, \bar{b})$. All other bidders bid zero.

Bidder 1 receives an expected surplus of $v - \bar{b}$ from all bids that he might submit, while bidder 2 receives an expected surplus of zero from all of his. The expected total expenditure is

$$\int_0^{w_2} \frac{b}{v} db + \int_{w_2}^{\bar{b}} \frac{(1+r)b}{v} db + \int_0^{\bar{b}} \frac{b}{v} db$$
$$= \frac{\bar{b}^2}{v} + \frac{r(\bar{b}^2 - w_2^2)}{2v}.$$

If $w_1 < \bar{b}$, bidder 1 may borrow. Bidder 2 now puts mass of $[v - \bar{b} - (\bar{b} - w_1)r]/v$ on zero, density 1/v on the interval $(0,w_1)$, and density (1+r)/v on the interval $(w_1, \bar{b})$. The expected total expenditure is

$$\int_0^{w_2} \frac{b}{v} db + \int_{w_2}^{\bar{b}} \frac{(1+r)b}{v} db + \int_0^{w_1} \frac{b}{v} db + \int_{w_1}^{\bar{b}} \frac{(1+r)b}{v} db$$
$$= \frac{\bar{b}^2}{v} + \frac{r\bar{b}^2}{v} - \frac{rw_2^2}{2v} - \frac{rw_1^2}{2v}.$$

This completes the proof. QED