Beyond the Coasian Irrelevance: Asymmetric Information

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I. Introduction

- Asymmetric information causes inefficiencies in trade and negotiation

  - Private information enables an agent to command rents
  - Efficient trade may not occur because it may not generate the sufficient surplus to finance the rents

- Allocation of property rights affects the agents' capacity to require rents, and thus matters.

- We wish to draw implications on

  - How to establish new property rights (e.g., auctions v. non-market)?

  - How to reallocate rights (e.g., takings, voluntary sale)?

  - How to protect the rights (e.g., property v. liability rules)?

- We do so with a simple model of unit trade.

II. Preliminaries

- n agents.

- Decision: Agent i "consumes" $x_i \in [0,1]$.

- Type: Agent i has type θ_i distributed over $[\underline{\theta}_i, \overline{\theta}_i] = \Theta_i \subset \mathbb{R}_+$, according to a cdf F_i with density f_i.

- Payoff: $U_i = \theta_i x_i + t_i$, where $t_i \in \mathbb{R}_+$ transfer.

- Allocation: $x \in X \subset [0,1]^n$.

  - Feasible set, X, to be specified (compact).

- Efficiency:

$$W(\theta) := \max_{x \in X} \sum_i \theta_i x_i,$$

  $x^*(\theta)$: the associated maximizer.

Define:

$$W_{-i}(\theta) := \sum_{j \neq i} \theta_j x_j^*(\theta).$$

- Reservation payoff: $r_i(\theta_i) \geq 0$,

- Assume $\sum_i r_i(\theta_i) \leq W(\theta)$.

- *Property rights*: $r_i(\theta_i) = \theta_i \omega_i$, where

  - $\omega_i \in [0, 1]$ is i's *entitlement*,

  - $\omega = (\omega_1, ..., \omega_n)$ is *entitlement allocation*.

Interpretations

- Two environments:

  - Private good: $X = \Delta^n := \{x \in [0,1]^n : \sum_i x_i \leq 1\}$.
  - Public good: $X = \mathcal{X}^n := \{x \in [0,1]^n : x_i = 1 - x_1, \forall i \neq 1\}$.

1. Two-person Bargaining (Myerson-Sattherwaite): $n = 2$, $X = \Delta^2$, and $\omega = (0,1)$.

2. Auctions (Myerson): $X = \Delta^n$, and $\omega = (0,...,0)$.

3. Partnership dissolution (CGK): $X = \Delta^n$, and $\omega \in \Delta^n$.

4. Development/public project: $X = \mathcal{X}^n$. Project by a developer/government (agent 1) yields the value of θ_1 but requires a contiguous land owned by $n-1$ sellers $i = 2, ..., n$.

- Voluntary sale: $\omega = (0, 1, ..., 1)$.
- Taking: $\omega = (1, 0, ..., 0)$. (More precisely, it is an option by the taker.)

5. Pollution (Mailath-Postlewaite, Neeman): Firm (agent 1) chooses a level of activity x_1, which yields $\theta_1 x_1$ for the firm but harms residents by $\theta_i x_1$; $X = \mathcal{X}^n$.

- Polluter's right: $\omega = (1, 0, ..., 0)$
- Pollutee's right: $\omega = (0, 1, ..., 1)$.

6. Property rule vs. Liability Rule (Ayres-Talley): $n = 2$, $X = \Delta^2$.
Agent 1 is a potential infringer, and agent 2 is an owner.

- Property rule: $\omega = (0, 1)$
- Liability rule:
  $r_1(\theta_1) = \max\{0, \theta_1 - D\}$;
  $r_2(\theta_2) = D \Pr\{\theta_1 \geq D\} + \theta_2 \Pr\{\theta_1 < D\}$,
  where D: damages.

III. Analytical Framework

- Without loss of generality, consider a DRM $(x, t) : \Theta \mapsto X \times \mathbb{R}^n$.

- Mechanism (x, t) is *feasible* if

(IR) $\mathbb{E}_{\theta_{-i}}[\theta_i x_i(\theta) + t_i(\theta)] \geq r_i(\theta_i)$,
(IC) $\mathbb{E}_{\theta_{-i}}[\theta_i x_i(\theta) + t_i(\theta)] \geq \mathbb{E}_{\theta_{-i}}[\theta_i x_i(\theta'_i, \theta_{-i}) + t_i(\theta'_i, \theta_{-i})]$,
(BB) $\mathbb{E}[\sum_i t_i(\theta)] \leq 0$.

Note: In some context, feasibility may require (BB) in ex post equality.

Lemma 1 *(Payoff equivalence) If* (x, t) *satisfies (IC), then the interim payoff of agent* i *with* θ_i *is*

$$\int_{\theta_i^0}^{\theta_i} \mathbb{E}_{\theta_{-i}}[x_i(a, \theta_{-i})]da + \text{ a constant},$$

for any $\theta_i^0 \in \Theta_i$.

Proof. Envelope theorem (e.g., Milgrom-Segal).

- Implementing Efficiency:

Suppose mechanism (x^*, t^*) implements efficiency.

- Critical type:

$$\widehat{\theta}_i \in \arg\min_{\theta_i} \mathbb{E}_{\theta_{-i}}[\theta_i x_i^*(\theta) + t_i^*(\theta)] - r_i(\theta_i).$$

- VCG Mechanism: (x^*, t^V), where

$$t_i^V(\theta) := W_{-i}(\theta) - W(\widehat{\theta}_i, \theta_{-i}).$$

- VCG mechanism satisfies (IC) in dominant strategies:

$$\begin{aligned} & \theta_i x_i^*(\tilde{\theta}_i, \theta_{-i}) + t_i^V(\tilde{\theta}_i, \theta_{-i}) \\ = & \sum_i \theta_i x_i^*(\tilde{\theta}_i, \theta_{-i}) - W(\widehat{\theta}_i, \theta_{-i}) \\ \leq & \sum_i \theta_i x_i^*(\theta_i, \theta_{-i}) - W(\widehat{\theta}_i, \theta_{-i}) \\ = & W(\theta) - W(\widehat{\theta}_i, \theta_{-i}). \end{aligned}$$

- VCG mechanism yields net budget surplus of

$$\mathcal{S}(\theta) := W(\theta) - \sum_i [W(\theta) - W(\widehat{\theta}_i, \theta_{-i})] - \sum_i r_i(\theta_i).$$

Meaning: What is left over after paying information rents and the reservation payoffs for the critical types.

Theorem 1 *(Williams, Krishna-Perry) The efficient outcome is implementable if and only if the VCG mechanism yields nonnegative expected net budget surplus, i.e.,* $\mathbb{E}[\mathcal{S}(\theta)] \geq 0$.

Proof idea: “If” part: The only issues is that in some cases, (BB) is required in ex post equality. Not a problem.

Lemma: Given risk neutrality, any mechanism that is feasible with ex ante budget surplus can be made to be feasible with any aggregate budget surplus smaller in expectation. (Do AGV.)

“Only if” part: Payoff equivalence.

IV. Results

IV.1 **Establishing a Property Right**

- Private good environment: One unit to be assigned to one of n potential agents (i.e., $X = \Delta^n$).

  - Does the initial ownership matter?
  - If so, how should government assign it? Lottery or Auctions (e.g., second-price)?

(1) Auction: $\omega_A = (0, ..., 0)$.

- Critical type: $\hat{\theta}_i = \underline{\theta}_i$ for $i = 1, ..., n$.
- Budget surplus from VCG: For any θ, let $m \in \arg\max_k \{\theta_k\}$.

$$\begin{aligned}
\mathcal{S}(\theta) &= \sum_i W(\underline{\theta}_i, \theta_{-i}) - (n-1)W(\theta) \\
&= \sum_i \max\{\underline{\theta}_i, \max_{j \neq i} \theta_j\} - (n-1) \max_k \{\theta_k\} \\
&= \max\{\underline{\theta}_m, \max_{j \neq m} \theta_j\} + \sum_{i \neq m} \max\{\underline{\theta}_i, \max_{j \neq i} \theta_j\} - (n-1)\theta_m \\
&\geq \max\{\underline{\theta}_m, \max_{j \neq m} \theta_j\} \geq 0.
\end{aligned}$$

Hence, ω_A yields an efficient allocation, say via a Vickrey auction.

(2) Lottery+Ex post Bargaining: Suppose 1 is assigned the property right. $\omega_1 := (1, 0, ..., 0)$.

- Critical types: For $\overline{\theta}_1$ for agent 1, for the others, $\underline{\theta}_j$, $j = 2, ..., n$.

- Assume $\overline{\theta}_1 \geq \overline{\theta}_j$, for all but at most one $j \neq 1$. $\underline{\theta}_1 < \overline{\theta}_j$ for at least one $j \neq 1$. (cf. Myerson-Satterthwaite assumption).

• Budget surplus from VCG: For any θ,

$$\begin{aligned}
\mathcal{S}(\theta) &= W(\bar{\theta}_1, \theta_{-1}) + \sum_{i \neq 1} W(\underline{\theta}_i, \theta_{-i}) - (n-1)W(\theta) - \bar{\theta}_1 \\
&= \max_{j \neq 1}\{\theta_j - \bar{\theta}_1, 0\} + \sum_{i \neq 1} \max\{\underline{\theta}_i, \max_{j \neq i} \theta_j\} - (n-1) \max_k\{\theta_k\} \\
&= \begin{cases} 0 & \text{if } m = 1 \\ \max\{\theta_m - \bar{\theta}_1, 0\} - (\theta_m - \max\{\underline{\theta}_m, \max_{j \neq m} \theta_j\}) & \text{if } m \neq 1 \end{cases} \\
&\leq 0 [< 0 \text{ for a positive measure of } \theta].
\end{aligned}$$

• No feasible bargaining yields an efficient allocation (Generalization of Myerson-Satterthwaite).

• Property right assignment matters! Auctions are better than a lottery.

(3) Lottery + ex ante negotiation): What if the agents negotiate prior to the lottery drawing. Equivalent to the equal-share partnership. $\omega = (\frac{1}{n}, ..., \frac{1}{n})$.

- Assume symmetric bidders: $\underline{\theta}_i = \underline{\theta}_j$, $\overline{\theta}_i = \overline{\theta}_j$, and $F_i = F_j =: F$.

- Critical types: For each agent, $\hat{\theta}$ satisfies $F(\hat{\theta})^{n-1} = \frac{1}{n}$.

- Budget surplus:

$$\mathcal{S}(\theta) = \sum_i W(\widehat{\theta}, \theta_{-i}) - (n-1)W(\theta) - \widehat{\theta}$$

$$\begin{aligned}
&= \sum_i \max\{\widehat{\theta}, \max_{j \neq i} \theta_j\} - (n-1)\theta_m - \widehat{\theta} \\
&= \sum_i \max\{\frac{n-1}{n}(\widehat{\theta} - \theta_m), (\max_{j \neq i} \theta_j) - \frac{\widehat{\theta} + (n-1)\theta_m}{n}\} \\
&\geq \left(\frac{n-1}{n}\right)(\widehat{\theta} - \theta_m) + \sum_{i \neq m} \max\{\left(\frac{n-1}{n}\right)(\widehat{\theta} - \theta_m), (\max_{j \neq i} \theta_j) - \frac{\widehat{\theta} + (n-1)\theta_m}{n}\} \\
&\geq \left(\frac{n-1}{n}\right)(\widehat{\theta} - \theta_m) + (n-1)\theta_m - (n-1)\frac{\widehat{\theta} + (n-1)\theta_m}{n} \\
&= \left(\frac{n-1}{n}\right)(\widehat{\theta} - \theta_m) + \left(\frac{n-1}{n}\right)(\theta_m - \widehat{\theta}) \\
&= 0.
\end{aligned}$$

Hence, lottery works as well if all agents are symmetric and negotiate ex ante.

• What if the parties are not symmetric. The above argument depends only on the fact that the critical type is the same for all agents.

• Consider a lottery $\omega_L = (s_1, s_2, ...s_n) \in \Delta^n$ such that $\forall i$, $\prod_{j \neq i} F_j(\widehat{\theta}) = s_i$, for some $\widehat{\theta}$. Then, the same conclusion follows.

• Anything we can say about the relative probabilities? If $F_i(\cdot) < F_j(\cdot)$, then

$$s_i = \frac{\prod_k F_k(\widehat{\theta})}{F_i(\widehat{\theta})} > \frac{\prod_k F_k(\widehat{\theta})}{F_j(\widehat{\theta})} = s_j.$$

That is, the lottery must favor the one with higher valuation. In fact, bureaucratic procedure may have this effect.

IV.2 **Pollution, Takings, Property v. Liability Rules**

• Public good environment: $X = \mathcal{X}^n$. Recall agent 1 realizes the value of θ_1 from obtaining contiguous land owned by $n-1$ sellers, each valuing his property θ_i, $i = 2, ..., n$. .

• Assume $\overline{\theta}_1 = \sum_{i \neq 1} \overline{\theta}_i$ and $\underline{\theta}_1 = \sum_{i \neq 1} \underline{\theta}_i$, which makes analysis simple.

• Alternative interpretations: agent 1 is a potential taker-polluter-infringer.

(1) Voluntary Sale (Pollutee's right): $\omega = (0, 1, ..., 1)$.

- Critical types: For $\underline{\theta}_1$ for agent 1, for the others, $\overline{\theta}_j$, $j = 2, ..., n$.

- Budget surplus from VCG: For any θ,

$$\mathcal{S}_\infty(\theta) = W(\underline{\theta}_1, \theta_{-1}) + \sum_{i \neq 1} W(\overline{\theta}_i, \theta_{-i}) - (n-1)W(\theta) - \sum_{i \neq 1} \overline{\theta}_i$$

$$= \max\{\sum_{j \neq 1} \theta_j, \underline{\theta}_1\} + \sum_{i \neq 1} \max\{\sum_{j \neq 1, i} \theta_j, \theta_1 - \overline{\theta}_i\} - (n-1) \max\{\sum_{j \neq 1} \theta_j, \theta_1\}$$

$$\begin{cases} < 0 & \text{if } \theta_1 > \sum_{i \neq 1} \theta_i \\ = 0 & \text{if } \theta_1 \leq \sum_{i \neq 1} \theta_i. \end{cases}$$

- Large number of sellers:

Assume: All sellers are symmetric, $\overline{\theta}_j = \overline{\theta}$ and $\underline{\theta}_j = \underline{\theta}$ with average $\mathbb{E}[\theta_j] = \theta^e$, for $j = 2, ..n$.

As $n \to \infty$,

$$\frac{\mathcal{S}^n_\infty(\theta)}{n} \to \begin{cases} \theta^e - \overline{\theta} & \text{if } \lim_{n\to\infty} \frac{\theta_1^n}{n} > \theta^e \\ 0 & \text{if } \lim_{n\to\infty} \frac{\theta_1^n}{n} < \theta^e. \end{cases}$$

Hence, inefficiency never disappears. ("Hold-out" problem).

(2) Taking (Polluter's right): $\omega_1 = (1, 0, ..., 0)$.

- Critical types: For $\overline{\theta}_1$ for agent 1, for the others, $\underline{\theta}_j$, $j = 2, ..., n$.

- Budget surplus: For any θ,

$$\begin{aligned}
\mathcal{S}_0(\theta) &= W(\overline{\theta}_1, \theta_{-1}) + \sum_{i \neq 1} W(\underline{\theta}_i, \theta_{-i}) - (n-1)W(\theta) - \overline{\theta}_1 \\
&= \max\{\sum_{j \neq 1} \theta_j - \overline{\theta}_1, 0\} + \sum_{i \neq 1} \max\{\underline{\theta}_i + \sum_{j \neq 1, i} \theta_j, \theta_1\} \\
&\quad -(n-1) \max\{\sum_{j \neq 1} \theta_j, \theta_1\} \\
&\begin{cases} < 0 & \text{if } \theta_1 < \sum_{i \neq 1} \theta_i \\ = 0 & \text{if } \theta_1 \geq \sum_{i \neq 1} \theta_i. \end{cases}
\end{aligned}$$

- Large number of sellers:

As $n \to \infty$,

$$\frac{\mathcal{S}^n_\infty(\theta)}{n} \to \begin{cases} \underline{\theta} - \theta^e & \text{if } \lim_{n\to\infty} \frac{\theta_1^n}{n} < \theta^e \\ 0 & \text{if } \lim_{n\to\infty} \frac{\theta_1^n}{n} > \theta^e. \end{cases}$$

Again, inefficiency persists.

Which of the two regimes, (1) or (2), depends on the distribution of θ_1 relative to θ^e. If θ_1 is likely to be high relative to θ^e, then (2) is better, and vice versa.

(3) Taking (with Compensation): A more realistic approach. Assume $n = 2$. Suppose the buyer has to compensate the seller by D.

[Another interpretation: This is a liability rule whereby the infringer has an option to infringe by paying damages D.]

- Critical types:

\- Agent 1: "take" iff $\theta_1 \geq D$, so $r_1(\theta_1) = \max\{\theta_1 - D, 0\}$.

$$\widehat{\theta}_1 \in \arg \min_{\theta_1 \in [\underline{\theta}, \overline{\theta}]} \int_{\underline{\theta}}^{\theta_1} F_2(a)da - \max\{\theta_1 - D, 0\}.$$

The objective function is quasi-concave, so $\widehat{\theta}_1$ is either $\underline{\theta}$ or $\overline{\theta}$. Jumps down from $\overline{\theta}$ to $\underline{\theta}$ as D rises past a threshold

$$D^* = \overline{\theta} - \int_{\underline{\theta}}^{\overline{\theta}} F_2(a)da = \mathbb{E}[\theta_2].$$

\- Agent 2: $r_2(\theta_2) = F_1(D)\theta_2 + (1 - F_1(D))D$.

$$\widehat{\theta}_2 \in \arg \min_{\theta_2 \in [\underline{\theta}, \overline{\theta}]} \int_{\underline{\theta}}^{\theta_2} F_2(a)da - F_1(D)\theta_2 - (1 - F_1(D))D.$$

So, $\widehat{\theta}_2 = D$.

- Analysis of the liability rule:

i) For $D < \mathbb{E}[\theta_2] = D^*$: $\widehat{\theta}_1 = \overline{\theta}$ and $\widehat{\theta}_2 = D$, so

$$\begin{aligned}\mathcal{S}_D(\theta) &= \max\{\theta_2, \overline{\theta}\} + \max\{\theta_1, D\} - \max\{\theta_1, \theta_2\} - (\overline{\theta} - D) - D \\ &= \max\{\theta_1, D\} - \max\{\theta_1, \theta_2\}.\end{aligned}$$

Increasing in D.

ii) For $D > \mathbb{E}[\theta_2] = D^*$: $\widehat{\theta}_1 = \underline{\theta}$ and $\widehat{\theta}_2 = D$, so

$$\begin{aligned}\mathcal{S}_D(\theta) &= \max\{\theta_2, \underline{\theta}\} + \max\{\theta_1, D\} - \max\{\theta_1, \theta_2\} - D \\ &= \theta_2 - D + \max\{\theta_1, D\} - \max\{\theta_1, \theta_2\}.\end{aligned}$$

Decreasing in D.

i) + ii) $\Rightarrow$ $\mathcal{S}_D(\theta)$ maximized at $D = D^* = \mathbb{E}[\theta_2]$.

By Jensen's inequality, for any θ_1,

$$\mathbb{E}_{\theta_2}[\max\{\theta_1, \mathbb{E}[\theta_2]\}] = \max\{\theta_1, \mathbb{E}[\theta_2]\} < \mathbb{E}_{\theta_2}[\max\{\theta_1, \theta_2\}].$$

Hence, $\mathbb{E}[\mathcal{S}_{D^*}(\theta)] < 0$.

- Summary: A liability rule is better than either buyer's right or seller's right.

But a liability rule cannot achieve an efficient outcome.

(4) Taking (with Uncertain Ruling): Suppose any taking attempt is challenged and there is uncertainty how the court rules. In fact, suppose the court randomizes between (1) and (2). The result is then pretty much the same IV-1(3), so efficiency is possible.

V **Conclusion** To be completed.