Beyond the Coasian Irrelevance: Wealth Constraints

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Che and Gale (2006)

- Binding financial constraints (or “wealth constraints”) makes the initial assignment of properties relevant, and yield some useful implications on non-market assignment and restriction on transfer rights.
Issues

- Can we justify non-market assignment of initial ownership?
- If so, how should we structure the assignment scheme?
- What are the tradeoffs for limiting transfer rights (or alienability).
Existing Justifications for Nonmarket Mechanisms

- Redistributive goals (e.g., Wijkander, Weitzman)
- Second-best argument: Market equilibrium not Pareto Optimal because of some distortion (e.g., Guesnerie-Roberts)

None of these consider resale; not concerned about the assignment of initial ownership.
Existing Justifications for Inalienability

- Externalities + Transaction costs (Calabresi and Melamed)
  - Barring polluters
  - Moralism
  - Paternalism
Preview of Results

When agents are wealth constrained,

- Nonmarket assignment (of transferable goods) can be justified on allocative efficiency grounds.
- Favoring the poor in the assignment is desirable, justifying need-based programs.
- Identify the cost of “alienability” in the form of “speculation activities”; Limiting it may be justified in some cases.
- Asymptotic Coase theorem.
Model

- A unit mass of risk-neutral buyers who each demand one unit, and mass $m \geq 0$ of non-buyers (“rest of the population”).
- A buyer has a type, $(w, \nu)$
  \[
  w = \text{wealth} \in [0,1] \sim G(w)
  \]
  \[
  \nu = \text{valuation} \in [0,1] \sim F(\nu)
  \]
  (profit or consumption value)
- A non-buyer has the same $w$ distr and $\nu = 0$.
- Quasilinear preferences
- The (indivisible) good is supplied elastically at zero marginal cost, up to industry capacity, $S \in (0,1)$. 
Welfare Criterion

- **Utilitarian efficiency**: Total realized value (or average value realized per unit)
  - Ex ante perspective ("Vickrey test"): 
    \[ \text{What would an individual choose should she have an equal chance of landing in the shoes of each member of the society?} \]

- **First-best benchmark**: buyers with \( v \geq v^* \) are served, where \( v^* \) satisfies \( S = 1 - F(v^*) \).

- **Average value realized** = \( \mathbb{E}[v | v \geq v^*] \).
First-best Allocation
Three Mechanisms

- **A competitive market**: *(resale right doesn’t matter).*

- **Nonmarket (random) assignment without resale**: price is capped and a lottery assigns the good; not allowed to resell.

- **Nonmarket with resale**: same as above except resale is permitted after assignment.

*Not a mechanism design exercise (cf. Che and Gale (1999)).*
Examples

- **Housing in Korea:** Nonmarket with resale previously; competitive market now.
- **School choice:** market and nonmarket; no resale.
- **Military recruitment under draft:**
  - Nonmarket without resale (Vietnam);
  - Nonmarket with resale (Civil war).
- **Government resources:** all three used.
- **Immigration visas:** Nonmarket without resale.
Competitive Market

- Demand = number of buyers \textit{willing and able} to pay the price
  \[ D(p) = (1 - F(p))(1 - G(p)) \]

- Supply = \( S \)

- Equilibrium price: \( p_e \) satisfying \( D(p_e) = S \).
Competitive Market Equilibrium

- Average value realized: $E[v | v \geq p_e]$. 
Nonmarket without Resale

- The price is capped at $q < p_e$ and excess demand is assigned randomly (i.e., a lottery, with one entry per participating agent).

- Buyers with $(w, v) \geq (q, q)$ participate in the rationing and are successful with probability $S/[(1 - F(q))(1 - G(q))]$. 
Nonmarket without Resale

\[ \text{Average value realized: } E[\nu | \nu \geq q] < E[\nu | \nu \geq p_e]. \]
Nonmarket (random) assignment without Resale

- Random assignment without resale is less efficient than the market.

- Intuition: Random assignment allows buyers with low wealth to consume, but also those with low valuations.
Nonmarket (random) with Resale

- Price is capped at $q < p_e$ and excess demand is rationed randomly; *resale is permitted*.

- Suppose the resale price, $r$, is higher than $q$ (if not, there would not be rationing).

- All buyers and even “non-buyers” with $w \geq q$ will participate in rationing.
Nonmarket (random) assignment with Resale

- All buyers with \((w, v) > (q, 0)\) participate; each gets the good with probability
  \[
  \rho(q, m) = \frac{S}{(1 + m)(1 - G(q))}
  \]
  \[\rightarrow 0 \text{ as } m \rightarrow \infty\]

- Resale Market:
  - **Demand side:** Unsuccessful buyers purchase at the resale price, \(r\), if \((w, v) \geq (r, \hat{r})\).
  - **Supply side:** Successful buyers/non-buyers with \(v < r\) sell.

- **Measure of buyers:** \([1 - F(r)][1 - G(r)](1 - \rho(q, m))\).
- **Measure of sellers:** \(S \cdot (F(r) + m)/(1 + m)\).
Resale Market Equilibrium

\[ [1 - F(r)][1 - G(r)](1 - \rho(q, m)) = S \cdot (F(r) + m)/(1 + m). \]
\[ \Leftrightarrow D(r) = S - \rho(q, m)[1 - F(r)](G(r) - G(q)) \]
\[ \Rightarrow \text{equilibrium resale price: } r^*(q, m) > p_e. \]

- **Average value:** \( \mathbb{E}[v \mid v \geq r^*] > \mathbb{E}[v \mid v \geq p_e]. \)

- **Lower q and lower m raise the average value realized.**

- **As } m \rightarrow \infty, r^* \rightarrow p_e. \text{ (will generalize to the Asymptotic Coase theorem)**
Random/Resale versus the Market

There would be excess demand if $r^* = p_e \Rightarrow r^* > p_e$. 
Rationing/Resale versus the Market
Intuition

- Coase theorem doesn’t apply if individuals are wealth constrained.

- Allocating the good to the poor improves efficiency since only the wealthy can buy on the resale market.

- Random assignment with resale does a better job than the market in allocating ownership to the poor.

- Can do even better than random assignment if resale is permitted (e.g., need-based programs, affirmative action...)

- Speculation limits this benefit and virtually wipes it out if there are many potential speculators.
A More General Assignment Rule

- **Separability:** A type-$(v, w)$ buyer is assigned the good with probability $x(w, v) = a_x(w)b_x(v)S$.

- **Non-concentration:** The fraction of supply assigned to any set of agents (buyer and non-buyers) participating is of the same order as the proportion of its measure to the total measure of individuals participating in the assignment.

- Example (type contingent rule):
  
  $x(w, v)/x(w', v') = $ some positive constant whenever $(w, v), (w', v')$ both participate
Merit- and Need-dominance

- An assignment rule, $x$, *merit-dominates* an assignment rule, $y$, if $x$ assigns a higher probability to high-valuation buyers than $y$ does ($FOSD$).

- An assignment rule, $x$, *need-dominates* $y$ if $x$ is more likely to allocate the good to low-wealth buyers than $y$ is ($FOSD$).
Nonmarket without Resale

- For any cap \( q < p_e \), only buyers with \((w, v) \geq (q, q)\) participate. Non-buyers never do.

- If \( x \) [strictly] merit-dominates \( y \), then \( x \) yields [strictly] higher value than \( y \).

- Any assignment rule merit-dominated by the merit-blind rule is strictly less efficient than the market.

- There exists a (non-concentrating) assignment rule strictly more efficient than the market.
Discrete Example

Assume $S = \frac{1}{2}$.

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- **Efficient Allocation:** Only buyers with $\nu = 2$ get the good.
- **Competitive Market:** Only high wealth get the good with $p = 1$.
  $\Rightarrow$ Average value realized: $3/2$.
- **Nonmarket w/o Resale:** Set $q = w_L$; A buyer with $\nu = 2$ is twice as likely to get the good as $\nu = 1$, who is in turn twice as likely to get the good as $\nu = 0$ (if all participate). Of course, $\nu = 0$ never participate.
  $\Rightarrow$ Average value realized: $5/3 > 3/2$. 
Nonmarket with Resale

- The assignment rule, $x$, relatively merit-dominates $y$ if, for any $v' > v$, $b_x(v')/b_x(v) \geq b_y(v')/b_y(v)$.

- $x$ is *meritorious* if it relatively merit-dominates a merit-blind rule (i.e., $b_x(v') \geq b_x(v)$ for all $v' > v$).
Nonmarket with Resale – cont’d

- For any cap $q < p_e$, all buyers and non-buyers with $w \geq q$ participate.

- Any meritorious assignment rule produces a strictly more efficient allocation than the competitive market.

- Lowering the price cap increases efficiency, given a meritorious assignment technology.
Benefit of Need-based Assignment

- If \( x \) relatively merit-dominates and need-dominates \( y \), then \( x \) has a (weakly) higher total realized value than \( y \) does.

- *Full efficiency may be possible if the good is allocated to the poorest buyers.*
Efficiency of Need-based Assignment

- Need-based assignment
- Efficient allocation
Restricting Transfer Rights

- **Asymptotic Coase theorem**: Any non-concentrating assignment rule (i.e., ownership distribution) leads to the same allocation (as the market), as $m \to \infty$.

- **Prohibiting resale** (i.e., inalienability) is desirable for some non-concentrating assignment rule if $m > M$ for some $M > 0$. 
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- **Nonmarket w/o Resale:** Same assignment rule; recall
  $\Rightarrow$ Average value realized: $\frac{5}{3}$.
- **Nonmarket w/ Resale:**
  $\Rightarrow$ Average value realized: $(5+2m)/(3+2m)+(1+2m)/2(3+2m) \rightarrow \frac{3}{2} < \frac{5}{3}.$
Extensions

- A Dual System with Nonmarket and a Market
- Regulation of Resale
- Pre-payment Resale
- Direct Subsidy vs. Nonmarket with Resale
- Social Cost of Speculation
- Elastic Supply
Conclusions

- Even imperfect nonmarket assignment improves efficiency when buyers are wealth-constrained, *if resale is permitted*.
- Restricting resale may be desirable if the assignment rule is sufficiently meritorious and speculation potential is severe.
- With some assignment rules, allowing resale may be beneficial:
  - Health care
  - Government auctions
  - Immigration visas
  - School Choice
- Lack of resale not an evidence of efficiency; nor its presence a sign of inefficiency.