



Available online at www.sciencedirect.com

SCIENCE @ DIRECT®

Games and Economic Behavior 46 (2004) 383–397

GAMES and
Economic
Behavior

www.elsevier.com/locate/geb

Asymmetric information about rivals' types in standard auctions

Jinwoo Kim^a and Yeon-Koo Che^{b,*}

^a Department of Economics, University of Southern California, Los Angeles, CA 90081, USA

^b Department of Economics, University of Wisconsin, Madison, WI 53706, USA

Received 19 December 2001

Abstract

We study auctions in which bidders may know the types of some rival bidders but not others. This asymmetry in bidders' knowledge about rivals' types has different effects on the two standard auction formats. In a second-price auction, it is weakly dominant to bid one's valuation, so the knowledge of rivals' types has no effect, and the good is allocated efficiently. In a first-price auction, bidders refine their bidding strategies based on their knowledge of rivals' types, which yields an inefficient allocation. We show that the inefficient allocation in the first-price auction translates into a poor revenue performance. Given a standard regularity condition, the seller earns higher expected revenue from the second-price auction than from the first-price auction, whereas the bidders are better off from the latter.

© 2003 Elsevier Inc. All rights reserved.

JEL classification: C70; D44; D82

1. Introduction

Much of the existing auction literature assumes that bidders possess symmetric knowledge regarding rival bidders' types. For instance, the standard independent private values (IPV) models assume that bidders are ignorant of the rival bidders' types, knowing only their distributions (e.g., Riley and Samuelson (1981) and Myerson (1981)). In a Bertrand model, bidders are assumed to know the realized types of all opponents. The models with affiliation allow bidders to refine their assessment of the rival bidders' types

* Corresponding author.

E-mail address: yche@facstaff.wisc.edu (Y.-K. Che).

based on their signals, but treatment of a given bidder's knowledge about other bidders is often symmetric (see Milgrom and Weber (1982)).

In practice, bidders' knowledge about their rivals is unlikely to be symmetric. Bidders tend to know more about some rival bidders than others. For instance, in a procurement auction contested by domestic firms and foreign firms, domestic firms are likely to know more about the technical capabilities of their domestic rivals than those of foreign rivals. In auctions for government assets, such as mineral, timber harvesting, and spectrum rights, bidders often consist of incumbent firms with a long history of operation in the industry and relative newcomers in the area. It is then presumably easier for a bidder to estimate the preferences and technological abilities of the old firms than those of the new ones. A similar distinction may exist with respect to the institutional buyers and noninstitutional buyers in art auctions as well as treasury auctions.

We develop a model that accommodates such asymmetry in the bidders' knowledge of their rivals' types. Specifically, our model considers, in a symmetric IPV setting, an arbitrary partition of bidders into "knowledge groups" such that bidders know the valuations of their rivals within the same group, while they know only the distribution of types for bidders outside the group. This model offers a simple way of describing the aforementioned asymmetry in a bidder's knowledge of his rivals. Aside from the dichotomous nature of a bidder's information about his rivals, our model is general. In particular, our model allows for an arbitrary partition structure, which includes the two standard assumptions—full information and no information about other bidders—as special cases.

We study how the asymmetric knowledge of a bidder's rivals affects the performance of standard auctions. While the weak dominance argument continues to work for the second-price auction (which is equivalent to an ascending-bid auction in the IPV setting), the standard equilibrium analysis for a first-price auction does not follow, due to two features of our model:

- (1) an equilibrium bidding strategy depends not just on one's own valuation but also on those of his group members; and
- (2) given the arbitrary partition structure, the auction becomes generally asymmetric since the group sizes may be different.

Second, we compare first- and second-price (or equivalently, ascending-bid) auctions in terms of allocative efficiency and expected revenue. We find that the second-price auction dominates the first-price auction on both accounts. The efficiency dominance of the second-price auction can be explained as follows. In a second-price auction (or equivalently ascending bid auction), it is a weakly dominant strategy to bid one's own valuation, regardless of the knowledge partition. Hence, absent a reserve price, the second-price auction allocates the good efficiently. By contrast, an equilibrium bidding strategy in a first-price auction depends on a bidder's knowledge of his rivals' valuations, since the latter affects the degree to which a given bidder shades his bid. Since the realized types of rival bidders vary across groups, a bidder with the same valuation but facing different valuations of the within-group rivals will bid differently, which results in

inefficient allocation of the good, even when the valuations of the goods are symmetrically distributed.

Given the inefficient allocation arising from first-price auctions, the revenue equivalence theorem would normally imply that a second-price auction revenue-dominates a first-price auction. Our nonstandard informational assumption, however, makes the revenue equivalence theorem less than immediate. Our main analysis consists in reestablishing this step to show that the aforementioned revenue ranking holds in our model.

To our knowledge, the current paper is the first to study how bidders' knowledge of their rivals' types—in particular, its asymmetry—affects auctions. A flavor of this feature is present in an auction with interdependent valuations since a bidder's signal contains information about other bidders' valuations (see Milgrom and Weber (1982), Maskin (1992), Ausubel (1997), Esö and Maskin (2000), Dasgupta and Maskin (2000), Perry and Reny (2002), and Jehiel and Moldovanu (2001)). Despite the similarity, there are several important distinctions between this literature and the current paper. First, a bidder's signal affects her valuation as well as, and typically more than, her opponents' valuations in the interdependent valuation models,¹ so these models are inadequate to study the effects of bidders' knowledge about their opponents' valuations. Our model addresses the latter issue more effectively by separating a bidder's valuation and his information about rival bidders. Second, the extent to which a bidder's signal impacts on others' valuations (hence, a bidder's knowledge of his rivals) is symmetric in some of these models (see Milgrom and Weber (1982) and Ausubel (1997), for instance), whereas asymmetric knowledge about the rivals' types is the central focus of the current paper. Third, many interdependent valuation models assume one-dimensional signals,² whereas in our model a bidder observes multidimensional signals, one for his valuation and others for rivals' valuations. This distinction matters since a one-dimensional signal often leads to an efficient allocation in a symmetric first-price auction (see Milgrom and Weber (1982)), whereas an inefficient allocation arises in our model with multidimensional signals.³ Finally, these papers are primarily concerned about allocative efficiency, whereas we study both efficiency and revenue. The current paper's focus on the private-value setting separates the issue of "knowledge of rivals' types" as a distinct problem of its own, which is relevant in many auction environments, and also yields an unambiguous revenue comparison of the standard auctions.

The rest of the paper is organized as follows. In the next section, we set up the model. Section 3 then characterizes equilibria arising from the standard auctions and compares them in terms of allocative efficiency and revenue. Section 4 concludes.

¹ This property is reflected in the single-crossing property assumed in these models (see, for example, Maskin (1992), Esö and Maskin (2000), Dasgupta and Maskin (2000), Perry and Reny (2002)).

² Exceptions are Jehiel and Moldovanu (2001) and Esö and Maskin (2000), who allow for multidimensional signals.

³ See Jehiel and Moldovanu (2001) for the difficulty multidimensional signals pose for general mechanism design. See also Jehiel et al. (1999) for a similar point made in a private value setting.

2. Model

A seller has a single indivisible good to sell to $n \geq 3$ risk neutral bidders. Bidders have independent private values. Specifically, bidder i 's valuation v_i is drawn from the interval $[0, 1]$, following a common distribution function $F(\cdot)$ whose density function $f(\cdot)$ is bounded away from zero. F is assumed to be common knowledge. Letting N denote the set of all bidders, the profile of bidders' valuations, $\mathbf{v} := (v_i)_{i \in N} \in \mathbf{V} := [0, 1]^n$, has the joint density $f_N(\mathbf{v}) := \prod_{i \in N} f(v_i)$.⁴

We depart from standard IPV models (represented by Myerson (1981) and Riley and Samuelson (1981)) by allowing some bidders to know about each other's valuation. To express this idea formally, we impose a partition structure \mathcal{G} on the set N , where \mathcal{G} is the set of disjoint groups (or disjoint sets) with each group consisting of bidders whose realized valuations are common knowledge among themselves. That is, for each group $G \in \mathcal{G}$, the profile of its members' valuations, $\mathbf{v}_G := (v_i)_{i \in G}$, is common knowledge among those bidders. Note that *every realization* of \mathbf{v}_G is commonly known to bidders in G while only the *distribution* of \mathbf{v}_G is known to bidders in the other groups. A bidder is referred to as a *group leader* if he has the highest valuation in the group. For later use, we let $\mathbf{v}_{G \setminus i} := (v_j)_{j \in G \setminus i} \in \mathbf{V}_{G \setminus i}$ and $\mathbf{v}_{-G} := (v_j)_{j \in N \setminus G} \in \mathbf{V}_{-G}$ denote the profile of valuations for group G except for bidder i and the valuation profile for all bidders except for group G , respectively. The joint density for the vector \mathbf{v}_G is denoted by $f_G(\mathbf{v}_G) := \prod_{i \in G} f(v_i)$, and the joint densities for the vector $\mathbf{v}_{G \setminus i}$ and \mathbf{v}_{-G} are similarly denoted by $f_{G \setminus i}$ and f_{-G} , respectively. Also, the partition structure is assumed to be common knowledge and exogenous (i.e., fixed prior to the realization of \mathbf{v}).

Throughout, we consider two standard auctions, first-price and second-price auction with zero reserve price.⁵ As is well known, in a second-price auction, all bidders bid simultaneously, and the highest bidder wins the good and pays the highest losing bid. Given the IPV assumption, second-price auctions are equivalent to English auctions. Ties are broken arbitrarily. In a first-price auction, all bidders bid simultaneously, and the highest bidder wins and pays his bid. Unlike a second-price auction, a tie-breaking rule is important in guaranteeing existence of an equilibrium in this format. For instance, in a standard Bertrand game played by two firms with heterogeneous costs, a Nash equilibrium exists only when the ties are broken in favor of the lower-cost firm. For the same reason, we assume that

- (1) a tie is broken in favor of a bidder with a higher valuation if there are multiple highest bidders; and

⁴ Throughout the paper, bold face letters are used to denote vectors.

⁵ These auction formats are not optimal from the revenue perspective in our setting. The seller can generate higher revenue by employing some "message games" (in addition to auctions) to exploit the common knowledge of types. We focus on the standard auctions for several reasons: First, these auctions are most frequently used. Second, as will be seen later, focusing on the two formats entails no loss if the goal of mechanism design is allocative efficiency. Third, analyzing these auction formats can yield general insights beyond the particular information structure assumed here (while the cross-reporting mechanism appears to require the particular partition structure). The assumption of zero reserve price is made purely for simplicity, and all subsequent results extend in the natural way (i.e., with no change in the comparisons) if a binding reserve price is introduced.

- (2) if there are multiple highest bidders with the same valuation, then the object is assigned randomly with equal probability among those bidders.

While this tie-breaking rule is endogenous, it can be implemented by performing an auxiliary second-price auction with bidders who submitted the highest bid in first-price auction.⁶ In both games, we look for a Nash equilibrium in weakly undominated strategies.

Our model includes as special cases two extreme partition structures. In the one case, every set in the partition is a singleton, so every bidder knows only his valuation. This is the standard assumption made in the auction literature. The other case has one grand set in the partition, which means that bidders know all other bidders' types. The resulting game is precisely the Bertrand game. As is well known, revenue equivalence holds for these two partition structures.

Proposition 0. *If $|G| = 1$ for all $G \in \mathcal{G}$ or there is a $G \in \mathcal{G}$ such that $|G| = n$, then first-price and second-price auctions allocate the good efficiently and are revenue equivalent.⁷*

Proof. For the former case, the result follows from Myerson (1981) or Riley and Samuelson (1981). For the latter, a group leader bids the second-highest valuation (and wins the auction) in equilibrium,⁸ which results in the same outcome as the weakly dominant equilibrium of the second-price auction does. \square

To focus on more interesting cases, we impose the following assumption in the sequel.

Assumption 1. There is a group G such that $1 < |G| < n$.

Apart from this assumption, we do not impose any restriction on the partition structure. Naturally, different groups may contain different numbers of bidders, so the partition may be asymmetric. For instance, if there are four bidders with $N = \{1, 2, 3, 4\}$, our model encompasses three different possibilities: two groups of two (e.g., $\{\{1, 2\}, \{3, 4\}\}$), one group of three and one group of one (e.g., $\{\{1, 2, 3\}, \{4\}\}$), two groups of one and one group of two (e.g., $\{\{1\}, \{2\}, \{3, 4\}\}$), while Proposition 0 covers the two special cases: $\{\{1, 2, 3, 4\}\}$ and $\{\{1\}, \{2\}, \{3\}, \{4\}\}$.

3. Equilibrium characterization and revenue implications

In a second-price auction, a bidder has a weakly dominant strategy to bid his own valuation. The optimality of this strategy does not depend on the information about rivals'

⁶ See Maskin and Riley (2000) for a similar assumption about tie breaking. Further, our tie-breaking rule can be justified as producing a limiting equilibrium of a game in which bidders must bid in a discrete space and a random tie-breaking rule is used.

⁷ Throughout, $|\cdot|$ denotes the cardinality of a set.

⁸ There are a continuum of payoff-relevant equilibria in which the bidder with the second highest valuation bids his valuation with probability large enough for the high bidder to bid the same. In all such equilibria, the bidder with the highest valuation bids the second highest valuation, yielding the same outcome.

types. Hence, the equilibrium as well as the expected revenue does not depend on the information partition. In particular, the efficient allocation arises from the second-price auction. The equilibrium for first-price auction is not as immediate, however, so we focus on this auction format in the remainder.

3.1. Characterization of equilibrium in a first-price auction

In the case of a first-price auction, two features of our model make the standard existence and characterization of equilibrium inapplicable. First, each bidder observes the entire profile of valuations of his group members, so this creates a multi-dimensional signal problem. Second, since we assume an arbitrary knowledge partition, the environment is generally asymmetric. Nevertheless, a pure strategy equilibrium can be shown to exist. Since the proof follows largely that of Reny (1999), it is omitted.⁹ Here, we characterize several properties of the equilibrium.

Proposition 1. (i) *In equilibrium, a group leader bids at least a second-highest valuation among bidders in the same group and wins against all bidders in the group.*

(ii) *Any equilibrium is essentially pure in the sense that each group leader adopts a pure strategy with probability one.*

(iii) *Given Assumption 1, the equilibrium allocation is inefficient with positive probability.*

Proof. See Appendix A. \square

This proposition implies that the allocation is efficient within each group in any equilibrium and that competition is effectively among group leaders. Further, the extent of the bid-shading by a group leader is limited by the second highest valuation of his group. This means that a group leader facing within-group rivals with higher valuations will bid more aggressively than a group leader facing ones with lower valuations, even though the former leader has a lower valuation than the latter leader. This feature generates an inefficient allocation, as illustrated in the following example.

Example 1. Suppose that there are four bidders in two equal-sized groups: $\mathcal{G} = \{\{1, 2\}, \{3, 4\}\}$. Suppose also that each bidder draws his valuation uniformly from $[0, 1]$. It is then a (symmetric) equilibrium for each bidder to bid $\min\{v_i, \max\{\frac{2}{3}v_i, v_j\}\}$ when his valuation is v_i and that of the other bidder in the group is v_j . In this equilibrium, a group leader with valuation v_i adopts an unconstrained bid $B(v_i) = \frac{2}{3}v_i$, unless it is less than the valuation of the other bidder in his group.¹⁰ That is, a leader behaves much

⁹ The proof can be found in Kim and Che (2001) (<http://www.ssc.wisc.edu/~yche/chekim6.pdf>) and is also available upon request.

¹⁰ That this is an equilibrium can be seen as follows. When $v_i \geq v_j$, bidder i chooses a bid $b \geq v_j$ to maximize $y_G(b)(v_i - b)$, where $y_G(b)$ is the probability of winning against the other group of bidders following their equilibrium strategies. It can be verified that $y_G(b) = 2b \min\{1, 3b/2\} - b^2$. Hence, $y_G(b)(v_i - b)$ is increasing with b for $b < \frac{2}{3}v_i$ and decreasing for $b > \frac{2}{3}v_i$, which implies $\max\{\frac{2}{3}v_i, v_j\}$ is indeed an equilibrium.

as in a standard first-price auction (i.e., singleton partitions), except that he behaves like a Bertrand player against his within-group rivals. Note also that the unconstrained bid, $B(v_i) = \frac{2}{3}v_i$, adopted by the group leader is the same as the equilibrium bid that would be employed in a standard first-price auction game (i.e., singleton knowledge partitions) with *only three* players. Essentially, the group leader acts as if he faces no competition from the lower valuation bidder in his group as long as $\frac{2}{3}v_i \geq v_j$. Clearly this will reduce the competition, all else equal. On the other hand, whenever $\frac{2}{3}v_i < v_j$, the second highest valuation acts as a constraint, thus raising the intensity of competition.

These equilibrium strategies are clearly seen to generate an inefficient allocation. Suppose

$$v_1 > v_3 > v_4 > \frac{2}{3}v_1 > \max\left\{\frac{2}{3}v_3, v_2\right\},$$

which occurs with a positive probability. Bidders 1 and 3 are group leaders who bid $\frac{2}{3}v_1$ and v_4 , respectively, in equilibrium. Since $\frac{2}{3}v_1 < v_4$, bidder 3 then wins the object, which is inefficient since $v_1 > v_3$. As explained in the above paragraph, a low v_2 allows bidder 1 to shade more than bidder 3, who is constrained by a high v_4 , so the latter outbids the former.

3.2. Revenue implications

The allocative inefficiency found above can lead to a revenue characterization, if the Revenue Equivalence theorem would hold. In particular, the theorem would enable us to characterize the expected revenue from an auction solely by the allocation it induces in equilibrium.

To be specific, suppose that, in an auction equilibrium, bidder i wins the good with probability $x_i(\mathbf{v})$ and makes an expected payment $t_i(\mathbf{v})$, when bidders have valuations, \mathbf{v} . Letting

$$\bar{x}_i(\mathbf{v}_G) := \int_{\mathbf{v}_{-G}} x_i(\mathbf{v}_G, \mathbf{v}_{-G}) f_{-G}(\mathbf{v}_{-G}) d\mathbf{v}_{-G}$$

and defining $\bar{t}_i(\mathbf{v}_G)$ similarly, bidder i 's expected payoff can be expressed as

$$\bar{\pi}_i(\mathbf{v}_G) := v_i \bar{x}_i(\mathbf{v}_G) - \bar{t}_i(\mathbf{v}_G). \tag{1}$$

If the so-called envelope theorem holds, then we also have

$$\bar{\pi}_i(\mathbf{v}_G) = \int_0^{v_i} \bar{x}_i(s, \mathbf{v}_{G \setminus i}) ds \quad \forall \mathbf{v}_G. \tag{2}$$

By substituting (2) into (1), we can then solve for the expected revenue, which, upon integration by parts, can be rewritten as:

$$\int_{\mathbf{V}} \left(\sum_i J(v_i) x_i(\mathbf{v}) \right) f_N(\mathbf{v}) d\mathbf{v}, \tag{3}$$

where

$$J(v_i) := v_i - \frac{1 - F(v_i)}{f(v_i)}.$$

The revenue characterization in (3) would provide an easy method for comparing revenues across different auction formats, given the standard regularity condition:

Assumption 2. $J(\cdot)$ is strictly increasing on the interval $[0, 1]$.

In particular, given Assumption 2, an auction format inducing an efficient allocation would yield strictly higher expected revenue than one inducing an inefficient allocation.¹¹

The key to the revenue characterization in (3) is the envelope theorem—i.e., whether (2) holds. Unfortunately, our nonstandard informational structure makes the standard argument for the theorem inapplicable. To illustrate, let $\bar{z}_i(b; v_i, \mathbf{v}_{G \setminus i})$ and $\bar{\tau}_i(b; v_i, \mathbf{v}_{G \setminus i})$ respectively denote bidder i 's winning probability and expected payment in equilibrium, when he observes \mathbf{v}_G and bids b (which may not be his equilibrium bid) and all others play their equilibrium strategies. The resulting payoff for bidder i can be then expressed as:

$$\pi_i(b; v_i, \mathbf{v}_{G \setminus i}) := \bar{z}_i(b; v_i, \mathbf{v}_{G \setminus i})v_i - \bar{\tau}_i(b; v_i, \mathbf{v}_{G \setminus i}). \quad (4)$$

In equilibrium, bidder i plays his equilibrium strategy $b_i(\mathbf{v}_G)$, so he must win with probability, $\bar{x}_i(\mathbf{v}_G) = \bar{z}_i(b_i(\mathbf{v}_G); \mathbf{v}_G)$, and pay $\bar{t}_i(\mathbf{v}_G) = \bar{\tau}_i(b_i(\mathbf{v}_G); \mathbf{v}_G)$ and hence receive the payoff of $\bar{\pi}_i(\mathbf{v}_G) = \bar{x}_i(\mathbf{v}_G)v_i - \bar{t}_i(\mathbf{v}_G)$, given profile \mathbf{v}_G of group G valuations.

A standard argument for the envelope theorem would establish two properties:

- (a) $\pi_i(b; v_i, \mathbf{v}_{G \setminus i})$ is absolutely continuous in v , which enables one to express it as an integral of its partial derivative with respect to v (see Theorem 1 of Milgrom and Segal (2002), for instance); and
- (b) the partial derivative, whenever it exists, equals bidder i 's probability of winning.

These two properties do not hold in a first-price auction. In its equilibrium, bidder i 's winning probability, $\bar{z}_i(b; v_i, \mathbf{v}_{G \setminus i})$, may depend on his own valuation v_i : If we raise a bidder's valuation, while holding his bid constant, then his winning probability would fall since other bidders in the same group would adjust their strategies. The dependence of \bar{z}_i on v_i makes the standard argument problematic, for neither (a) nor (b) may hold. To see this, reconsider Example 1, in which bidder $j \neq i$ bids $\min\{v_j, \max\{\frac{2}{3}v_j, v_i\}\}$. Figure 1 shows how bidder j 's equilibrium bidding strategy varies with bidder i 's valuation v_i , when j 's valuation remains fixed at v_j .

¹¹ Note that the bidders' surplus is

$$\text{total surplus} - \text{seller's revenue} = \int_{\mathbf{v}} \left(\sum_i \frac{1 - F(v_i)}{f(v_i)} x_i(\mathbf{v}) \right) f_N(\mathbf{v}) d\mathbf{v}.$$

Hence, if $(1 - F(\cdot))/f(\cdot)$ is strictly decreasing on the interval $[0, 1]$, then bidders receive higher expected surplus in an auction that induces an inefficient auction than in an auction that induces an efficient allocation.

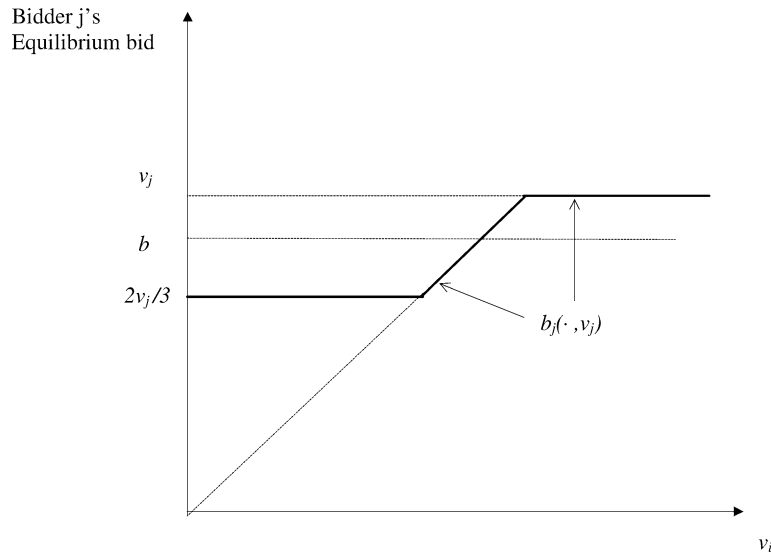


Fig. 1. The effect of an increase in bidder i 's valuation on bidder j 's equilibrium bid.

Given j 's equilibrium bidding strategy, bidder i 's winning probability when bidding b is

$$\bar{z}_i(b; v_i, v_{G \setminus i}) = \begin{cases} 2b \min\{1, 3b/2\} - b^2 & \text{if } v_i < b, \\ 0 & \text{if } v_i \geq b. \end{cases} \quad (5)$$

Clearly, bidder i 's winning probability decreases in v_i *discontinuously*, when his bid is held constant at b . Hence, property (a) does not hold. Property (b) also becomes problematic. To see this, suppose that $\bar{\pi}_i(v_G)$ is differentiable in v_i (which is unclear given the possible discontinuity). Then, the derivative will equal

$$\frac{\partial \bar{\pi}_i(v_G)}{\partial v_i} = \bar{x}_i(v_G) + \left. \frac{\partial \bar{z}_i(b; v_i, v_{G \setminus i})}{\partial v_i} \right|_{b=b_i(v_G)} [v_i - b_i(v_G)]. \quad (6)$$

If the second term does not vanish, property (b) will not hold.

Despite these possible problems, we show below that condition (2) holds in any equilibrium of the first-price auction. The key observations are that the failure of properties (a) and (b) is an out-of-equilibrium phenomenon and that condition (2) requires the two properties to hold only on the equilibrium path. For instance, in the above example, whenever \bar{z}_i falls discontinuously in v_i , we have $v_i = b_i(v_G)$, implying that the second term in (6) indeed vanishes. We present this result below.

Lemma 1 (Envelope theorem). *In any equilibrium of a first-price auction,*

$$\bar{\pi}_i(v_G) = \int_0^{v_i} \bar{x}_i(s, v_{G \setminus i}) ds \quad \forall v_i \in [0, 1].$$

Proof. See Appendix A. \square

Given Lemma 1, we can easily draw several revenue implications. First, recall that the allocation is efficient in a first-price auction when each bidder observes only his own signal (i.e., the singleton partition structure), whereas the allocation is inefficient when bidders have asymmetric information about their rivals' types (i.e., the partition satisfying Assumption 1). Hence, it follows immediately from (3) that, given Assumption 2, the expected revenue is lower when bidders have asymmetric knowledge about rivals' types. Second, recall that the expected revenue is the same in a second-price auction for all information structures and that revenue equivalence holds between the first- and second-price auctions, given the singleton partition structure (see Proposition 0). The above observation then implies that the expected revenue is lower in a first-price auction than in a second-price auction, if the bidders have asymmetric information about their rivals' types.

Proposition 2. *Given Assumptions 1 and 2, a first-price auction generates strictly lower expected revenue than a second-price auction. If we further assume that $(1 - F(\cdot))/f(\cdot)$ is strictly decreasing on the interval $[0, 1]$, bidders receive greater surplus in a first-price auction than in a second-price auction.*

Several remarks are in order. First, recall that a first price auction generates the same revenue in the two extreme partition structures: full information and no information about rivals' types. Proposition 2 then implies that the revenue (as well as total surplus) from the first-price auction changes nonmonotonically as the bidders learn more about their rivals: The revenue falls as bidders' information shifts from "no information about rivals" to "information about some rivals," and it rises back to the original level as the bidders acquire full information about their rivals.

Second, even though the reduced revenue for the first-price auction can be attributed to the inefficient allocation caused by bidders' (asymmetric) knowledge, the revenue loss does not coincide with the efficiency loss. Proposition 2 informs us that, given a mild regularity condition, the revenue loss is greater than the efficiency loss, which means that the bidders are collectively better off from acquiring (asymmetric) knowledge of their rivals' types.

Third, our revenue result can be also interpreted in terms of the two effects discussed in Example 1. As was seen there, the knowledge of rivals' types reduces the effective number of competitors a group leader faces. This effect blunts competition. At the same time, the knowledge of the rivals' types may raise the leader's bid in a way not possible in the standard case. Our revenue result suggests that the former effect dominates the latter effect, so the net effect is to reduce the competition. The same intuition suggests that, if the partition structure is changed to increase the former effect (more members within a group), holding fixed the total number of bidders, then the overall competition would fall. While this point cannot be shown generally, it is illustrated in the next example.

Example 2. Consider a first-price auction with 6 bidders, each with valuation drawn uniformly and independently from $[0, 1]$. Consider two partition structures: two groups of three bidders and three groups of two bidders. In the former partition, a bidder i 's equilibrium bidding function is $\min\{v_i, \max\{\frac{3}{4}v_i, v_j\}\}$, and it is $\min\{v_i, \max\{\frac{4}{5}v_i, v_j\}\}$ in the latter partition, where v_j is the highest rival information in his group. Comparison of the unconstrained bidding functions, $\frac{3}{4}v_i$ and $\frac{4}{5}v_i$, reveals that the competition-reduction

effect is more severe when the group size is bigger, holding fixed the total number of bidders. This is intuitive since a group leader can afford to bid lower when he knows that there are more bidders with lower valuations. Meanwhile, the bigger the group is, the more likely is v_j to take a large value, thus more likely to constrain the leader's bid. Consistent with our analytical result, the former effect turns out to dominate the latter effect: The expected revenue is 0.6998 in the former partition and 0.7067 in the latter partition.

4. Concluding remarks

We have shown that the bidders' asymmetric knowledge about their rivals' types adversely affects the first-price auction, both in terms of allocative efficiency and expected revenue, thus favoring the second-price auction relative to the first-price auction. We conclude by commenting on some extensions.

Correlated private values. If bidders' valuations are correlated but private, then the revenue/efficiency ranking found in this paper still holds. In particular, the weak dominance property of the second-price auction continues to hold, and produces an efficient allocation. Likewise, Proposition 1 continues to hold, so an inefficient allocation arises in any equilibrium of a first-price auction. As in the current model, this inefficiency implies a poor revenue performance of the first-price auction, relative to the second-price auction. Furthermore, the well-known linkage effect reinforces the revenue dominance of the second price auction (see Milgrom (1989)).

Our model is not immediately generalizable to the affiliated/interdependent valuations case since the within-group knowledge of valuations will render allocations inefficient even in the second price auction. While the linkage effect will continue to favor the second-price auction, the precise comparison remains unclear.

Costs of learning rivals' types. In practice, the knowledge of rivals' types may not be freely available but may rather require a costly learning process. Costly learning of rivals' types can reinforce the results of the current paper. To fix the idea, suppose that bidders can learn the types of his within-group rivals at some small cost $c > 0$, while it is prohibitively costly to learn the types of rivals outside his group. The presence of such learning cost will not change the revenue comparison. In a second-price auction, the weak dominance property implies that the bidders have no incentive to learn their rivals' types. By contrast, bidders will have incentives to learn their (within-group) rivals' types in the first-price auction. Learning rivals' types enables a bidder to refine his assessments of his rivals' strategies and tailor his own strategy based on the refined assessments. While this feature presents some modeling challenges since different types of bidders will have differing incentives to learn their rivals' types,¹² some types of bidders will likely learn in

¹² For instance, a bidder with zero valuation will never learn. Our conjecture is that, for a sufficiently small c there exists a threshold level of valuation for each bidder such that in equilibrium that bidder learns if and only if his valuation exceeds that threshold value. Our inefficiency result will continue to hold, given this outcome. Further, some learning costs will be expended. This conceptual problem can be avoided, at some loss of realism, if one models the learning of rivals' types as taking place before or at the same time as learning one's own valuation, with the learning decision unobserved to the rivals.

equilibrium if c is sufficiently small. Since the learning cost adds to the social costs, the inferiority of a first-price auction will then persist and may even be more severe.

Within-group collusion. It is natural to suspect that the mutual knowledge of valuations within a group may facilitate collusion among group members. This will simply mean in our framework that all bidders except for group leaders drop out of competition in each auction format. Then, competition will be simply among group leaders without any within-group challenge. If there are K groups, then it becomes a K -bidder auction in which group $G \in \mathcal{G}$ leader draws his valuation from cdf $F^{|G|}$. Clearly, if the group size is the same across the groups, then efficient allocation will be attained under both formats, and their revenue equivalence will be restored.¹³ If the group sizes are heterogeneous, then the ensuing auctions become standard asymmetric auctions. Incidentally, Marshall et al. (1994) considered asymmetric auctions in which bidders' valuations are drawn from a cdf of the form, F^{m_i} , for some integer m_i for each $i \in N$. Interestingly, their numerical analysis reveals that a first-price auction revenue dominates the second-price auction, when F takes a uniform distribution. Whether this reversal of ranking holds more generally (in the case of collusion) remains an issue.

Imperfect knowledge of rivals' types. An avenue of extension is to relax the dichotomous nature of a bidder's knowledge. In the current model, each bidder either knows his rival's type completely (if his rival is inside his group) or not at all (if his rival is outside his group). If a bidder were to obtain imperfect signals about his rivals' types, this would not affect the comparison in terms of allocative efficiency. These signals will affect the bidders' strategies in a way that will disrupt efficient allocation in the first-price auction, while the weak dominance argument will continue to imply that these signals will have no impact on the bidding behavior in the second-price auction. The revenue comparison becomes nontrivial, however, since the envelope argument does not hold.¹⁴ Generalization along this line thus awaits further research.

Acknowledgments

We thank Ian Gale, Phil Haile, Don Hausch, Phil Reny, Larry Samuelson, and an Associate Editor, and the seminar participants at University of Wisconsin for helpful comments.

Appendix A

Proof of Proposition 1. (i) Fix any bidder $i \in G$ and let $m(i) \in \arg \max_{j \in G \setminus i} v_j$ be the highest valuation bidder, excluding bidder i , in the same group, and let $v_{m(i)}$ be his valuation. Fix an equilibrium. Let $b_i(v_G)$ and \underline{b}_i respectively denote an arbitrary

¹³ It is easy to see that the revenues will be lower with this type of collusion than without the collusion.

¹⁴ The first-price auction creates a linkage effect somewhat reminiscent of the one arising in the affiliated value setting. The presence of this effect makes the overall ranking ambiguous.

selection of bidder i 's equilibrium bids and their infimum, given the valuation profile, \mathbf{v}_G , of group G . Since we restrict the equilibrium strategies to be undominated, we must have $b_i(\mathbf{v}_G) \leq v_i$. To prove the statement, it suffices to show that, whenever $v_i > v_{m(i)}$, $\underline{b}_i \geq v_{m(i)}$ and i beats all bidders in G . Note first that bidder i must receive a strictly positive (expected) payoff in equilibrium (since he can bid slightly higher than $v_{m(i)}$ and win with positive probability). If $\underline{b}_i < v_{m(i)}$, then bidder $m(i)$ must also earn a strictly positive payoff in equilibrium. We can show that both bidder i and bidder $m(i)$ cannot earn strictly positive payoffs. For both bidders to earn positive payoffs, their infimum must coincide and each must put a mass point there. But then it pays either one of them to raise the mass point slightly, which will increase the probability of winning discontinuously while lowering his payoff conditional on winning only slightly. Hence, we have a contradiction. This implies that $\underline{b}_i \geq v_{m(i)}$ if $v_i > v_{m(i)}$ (or else both i and $m(i)$ would earn strictly positive payoffs). Bidder i will also win against all other members in the group in that case, due to our tie-breaking rule.

(ii) Let $b_G(\mathbf{v}_G)$ be an arbitrary selection from the support of a group G leader's (possibly mixed) equilibrium bids when the valuation profile of group G members is \mathbf{v}_G . Consider two valuation profiles \mathbf{v}_G and $\bar{\mathbf{v}}_G$ with $\mathbf{v}_G \leq \bar{\mathbf{v}}_G$ and $v_G^1 < \bar{v}_G^1$, where v_G^1 (respectively \bar{v}_G^1) denote the group G leader's valuation, given profile \mathbf{v}_G (respectively $\bar{\mathbf{v}}_G$). Letting $b := b_G(\mathbf{v}_G)$ and $\bar{b} := b_G(\bar{\mathbf{v}}_G)$, we show that $b \leq \bar{b}$; i.e., an arbitrary selection of equilibrium bids is nondecreasing. Since $\bar{b} \geq \bar{v}_{m(i)} \geq v_{m(i)}$ by (i), we are done if $b \leq \bar{v}_{m(i)}$. Hence, assume that $b > \bar{v}_{m(i)}$. This means that, given the profile of $\bar{\mathbf{v}}_G$, the group G leader could win against all of his within group rivals by bidding b , so his winning probability would be simply that of outbidding other group leaders, denoted $y_G(b)$. Likewise, given the profile of \mathbf{v}_G , the group G leader would face the winning probability of $y_G(\bar{b})$ when bidding \bar{b} . Then, incentive compatibility requires $y_G(b)[v_G^1 - b] \geq y_G(\bar{b})[v_G^1 - \bar{b}]$ and $y_G(\bar{b})[\bar{v}_G^1 - \bar{b}] \geq y_G(b)[\bar{v}_G^1 - b]$. Combining these two inequalities gives

$$(\bar{v}_G^1 - v_G^1)(y_G(\bar{b}) - y_G(b)) \geq 0. \tag{A.1}$$

Suppose, to the contrary, that $b > \bar{b}$. Then, since $y_G(\cdot)$ is nondecreasing, we must have $y_G(\bar{b}) = y_G(b) > 0$. But this cannot hold since the group G leader would strictly prefer to bid \bar{b} when \mathbf{v}_G is realized. We therefore conclude that an arbitrary selection from the equilibrium strategies must be nondecreasing. Because there can be only countably many jumps in a nondecreasing and bounded correspondence, the support of the group G leader's equilibrium bids is a singleton for almost every \mathbf{v}_G .

The proof of (iii) (i.e., the inefficiency result) is omitted but available in Kim and Che (2001), <http://www.ssc.wisc.edu/~yche/chekim6.pdf>. \square

Proof of Lemma 1. Recall that in equilibrium bidder i receives

$$\bar{\pi}_i(\mathbf{v}_G) = \bar{x}_i(\mathbf{v}_G)[v_i - b_i(\mathbf{v}_G)]. \tag{A.2}$$

There are two cases.

Case 1 ($v_i \leq v_{m(i)}$). Note first that $\bar{\pi}_i(\mathbf{v}_G) = 0$ if $v_i < v_{m(i)}$, by Proposition 1(i). We now prove that $\bar{\pi}_i(\mathbf{v}_G) = 0$ if $v_i = v_{m(i)}$. Suppose to the contrary that bidder i earns a strictly positive payoff. Then, his infimum bid must be less than v_i but weakly greater than the

infimum bid of bidder $m(i)$. If it is strictly greater, then bidder $m(i)$ can profitably deviate by outbidding i 's infimum bid. If the infimums coincide, then both bidders must put mass points there, which yields a contradiction. Hence, bidder i cannot make a strictly positive payoff, which implies that bidder i makes zero payoff in that case. Since Proposition 1(i) also implies that $\bar{x}_i(v_i, \mathbf{v}_{G \setminus i}) = 0$ for any $v_i < v_{m(i)}$,

$$\bar{\pi}_i(\mathbf{v}_G) = 0 = \int_0^{v_i} \bar{x}_i(s, \mathbf{v}_{G \setminus i}) ds,$$

for any $v_i \leq v_{m(i)}$.

Case 2 ($v_i > v_{m(i)}$). Let $y_G(b)$ denote the probability that bidder i outbids all bidders outside his group when he bids b and all outside bidders follow their equilibrium strategies. (This probability does not depend on bidder i 's valuation since the bidders outside his group do not observe this valuation.) Since no bidder in G other than i bids strictly greater than $v_{m(i)}$, bidder i beats all other bidders in his group by bidding $v_{m(i)}$ or more, given our tie-breaking rule. Hence, bidder i will win with probability $y_G(b)$ by bidding $b \geq v_{m(i)}$; i.e., $\bar{z}_i(b; \mathbf{v}_G) = y_G(b)$ for any $b \geq v_{m(i)}$. Furthermore, by Proposition 1(i), bidder i never bids below $v_{m(i)}$. Hence, it follows that

$$\bar{\pi}_i(\mathbf{v}_G) = \max_{b \geq v_{m(i)}} \bar{z}_i(b; \mathbf{v}_G)[v_i - b] = \max_{b \geq v_{m(i)}} y_G(b)[v_i - b].$$

Further, if $b_i(\mathbf{v}_G)$ is an equilibrium bidding strategy, it must be a solution to the above constrained maximization problem and must satisfy $\bar{x}_i(\mathbf{v}_G) = y_G(b_i(\mathbf{v}_G))$ since, by Proposition 1(ii), $b_i(\mathbf{v}_G)$ is unique for almost every \mathbf{v}_G .

Observe now that a function, $\phi(b, v_i) := y_G(b)[v_i - b]$, has a derivative, $\phi_{v_i}(b, v_i) = y_G(b)$, for all $v_i \geq v_{m(i)}$, and that the derivative is uniformly bounded (by 1). Hence, $\bar{\pi}_i(\mathbf{v}_G)$ is absolutely continuous in v_i and can be expressed as an integral of $y_G(b_i(\mathbf{v}_G)) = \bar{x}_i(\mathbf{v}_G)$. The result then follows since

$$\begin{aligned} \bar{\pi}_i(\mathbf{v}_G) &= \int_{v_{m(i)}}^{v_i} \phi_{v_i}(b_i(\mathbf{v}_G), v_i) ds + \bar{\pi}_i(v_{m(i)}) = \int_{v_{m(i)}}^{v_i} \bar{x}_i(s, \mathbf{v}_{G \setminus i}) ds + \bar{\pi}_i(v_{m(i)}) \\ &= \int_{v_{m(i)}}^{v_i} \bar{x}_i(s, \mathbf{v}_{G \setminus i}) ds + \int_0^{v_{m(i)}} \bar{x}_i(s, \mathbf{v}_{G \setminus i}) ds = \int_0^{v_i} \bar{x}_i(s, \mathbf{v}_{G \setminus i}) ds, \end{aligned}$$

where the first equation follows from Theorem 2 of Milgrom and Segal (2002), the second from $\phi_{v_i}(b_i(\mathbf{v}_G), v_i) = y_G(b_i(\mathbf{v}_G)) = \bar{x}_i(\mathbf{v}_G)$, and the third from the result in Case 1. \square

References

- Ausubel, L., 1997. An efficient ascending-bid auction for multiple objects. Working paper. University of Maryland.
- Dasgupta, P., Maskin, E., 2000. Efficient auctions. *Quart. J. Econ.* 115, 341–388.
- Esö, P., Maskin, E., 2000. Multi-good efficient auctions with multidimensional information. Working paper. Harvard University and MIT.

- Jehiel, P., Moldovanu, B., 2001. Efficient design with interdependent valuations. *Econometrica* 69, 1237–1259.
- Jehiel, P., Moldovanu, B., Stachetti, E., 1999. Multidimensional mechanism design for auctions with externalities. *J. Econ. Theory* 85, 258–293.
- Kim, J., Che, Y.-K., 2001. Asymmetric information about rivals' types in standard auctions. Mimeo.
- Marshall, R.C., Meurer, M., Richard, J.-F., Stromquist, W., 1994. Numerical analysis of asymmetric first price auctions. *Games Econ. Behav.* 7, 193–220.
- Maskin, E., 1992. Auctions and privatization. In: Siebert, H. (Ed.), *Privatization*. Institute für Weltwirtschaften der Universität Kiel, pp. 115–136.
- Maskin, E., Riley, J., 2000. Equilibrium in sealed high bid auctions. *Rev. Econ. Stud.* 67, 413–438.
- Milgrom, P., 1989. Auctions and bidding: a primer. *J. Econ. Perspect.* 3, 3–22.
- Milgrom, P., Segal, I., 2002. Envelope theorems for arbitrary choice sets. *Econometrica* 70, 583–601.
- Milgrom, P., Weber, R., 1982. A theory of auctions and competitive bidding. *Econometrica* 50, 1089–1122.
- Myerson, R.B., 1981. Optimal auction design. *Math. Oper. Res.* 6, 58–73.
- Perry, M., Reny, P., 2002. An efficient auction. *Econometrica* 70, 1199–1212.
- Reny, P.J., 1999. On the existence of pure and mixed strategy Nash equilibria in discontinuous games. *Econometrica* 67, 1029–1056.
- Riley, J.G., Samuelson, W., 1981. Optimal auctions. *Amer. Econ. Rev.* 71 (3), 381–392.