Joint Liability and Peer Monitoring under Group Lending

Yeon-Koo Che*

July 2, 2002

Abstract

This paper studies an incentive rationale for the use of group lending as a method of financing liquidity-constrained entrepreneurs. The joint liability feature associated with group lending lowers the liquidity risk of default but creates a free-riding problem. In the static setting, the free-riding problem dominates the liquidity risk effect under a plausible condition, thus making group lending unattractive. When the projects are repeated infinitely many times, however, the joint liability feature provides the group members with a credible means of exercising peer sanction, which can make the group lending attractive, relative to individual lending.

JEL Numbers: C72, D80, G32 and L14.

Key words: Group lending, free riding and peer monitoring.

1 Introduction

Group lending has received much attention recently as an effective means of financing liquidity-constrained entrepreneurs who lack collateralizable assets. While differing in the ways group

*Department of Economics, University of Wisconsin-Madison, Madison, WI 53706, USA. The author is grateful to Ilya Segal (Editor) and three referees for their extremely detailed comments, which led to substantial improvement of the paper. He also thanks Loic Sadoulet for a useful conversation. Part of this research was conducted while the author was visiting the Institut d'Anàlisi Econòmica, CSIC. The author wishes to acknowledge the hospitality extended by the institute during the visit as well as the financial support provided by the Spanish Ministry of Education and by the National Science Foundation Grant, SBR9730472.

lending schemes work in different cases, their common feature is that a group of borrowers as a whole are liable for repayment of their collective loans. The rationales for this joint liability contract have been the subject of many recent studies. For instance, authors have recognized (1) adverse selection (see Ghatak (2000), Armendáriz de Aghion and Gollier (2000), and Sadoulet (1998)), (2) limited enforcement (Besley and Coate (1995), Armendariz de Aghion (1999), Rai and Sjöström (2000)) and (3) moral hazard (see Stiglitz (1990), Varian (1990), Banerjee, Besley, and Guinnane (1994), and Conning (1996), Spagnolo (1999)), as possible reasons for the emergence of joint liability contracts. [1]

Much of this literature, particularly in the latter two categories, has singled out peer monitoring as an important benefit arising from joint liability contracts. It has been argued that peer monitoring among group members can prevent members' shirking in their productive efforts, their poor project selection (Stiglitz (1990) and Varian (1990)) and/or their strategic default (Besley and Coate (1995) and Rai and Sjöström (2000)). While this literature recognizes the connection between the joint liability feature and the incentive for peer monitoring, it does not explore whether group members will indeed have incentives to sanction their peers when necessary. It is often assumed that that the members can coordinate their project choices and productive efforts through complete contracting, which seems unrealistic in many circumstances. In particular, the peer sanction behavior is left largely unexplained. Rather, it is typically *assumed* that group members have access to some exogenous penalty device and that they apply it whenever it is warranted.

The current paper endogenizes the punishment behavior by introducing repeated interaction among group members. Self-enforcing punishment behavior is well known from the repeated game literature. What is novel here is the way in which design of the lending format influences the dynamic interaction of the group members. In particular, we show that peer sanction can be accomplished under group lending even without the group members having access to an explicit penalty device. Specifically, we will show that the joint liability contract itself creates the possibility of peer sanction through the members' effort decisions. A group member's shirking can hurt (or punish) the other members by raising their payment burdens.

[1] See Ghatak and Guinnane (1999) for an excellent survey of the literature.

I shall argue that this built-in penalty mechanism associated with the joint liability contract can make group lending attractive, in comparison with individual lending, in the repeated environment. The joint liability feature is crucial in achieving this outcome since other contract forms (e.g., individual lending) cannot generate a similar self-enforcing dynamic punishment.

To this end, we develop a model along the tradition of moral hazard.[2] We abstract from strategic default possibilities by assuming that the project returns of the entrepreneurs are observable to the lenders (e.g., at a sufficiently low audit cost). Instead, the incentive problem stems from the entrepreneurs' *unobservable* effort decisions and their liquidity constraints. Entrepreneurs make efforts that can increase the expected returns of the projects. If entrepreneurs were not liquidity constrained, they would self finance the projects, thus becoming residual claimants of their project returns. Hence, no incentive problem would arise. Liquidity constraints of the entrepreneurs, however, necessitate giving away positive shares of their project returns in exchange for the initial funding. The sharing of the returns reduces their incentives for efforts, so the agents tend to make too little efforts.

The current paper studies whether group lending is desirable, particularly in comparison with individual loans, in alleviating this incentive problem. We consider both a static setting in which the project is performed only once and a dynamic setting in which the project is performed repeatedly. The findings of the current paper are summarized as follows.

In the static setting, the joint liability feature associated with group lending affects the incentive problem in two ways. Since the additional return generated by a group member is used to repay the other members' loans (with positive probability), each member does not have as much incentive in generating his return as he would had he been individually liable for his loan. This free-riding problem tends to lower the members' incentive for effort, all else equal. On the positive side, group members pool their resources and hence share their idiosyncratic project risks in repayment, which lowers the probability of a default (compared with individual lending). Consequently, a lender will lower her risk premium, which tends to improve their incentives, all else equal, since each entrepreneur internalizes (at least part of)

[2]In this sense, the current work is similar in its modeling approach to Stiglitz (1990) and Varian (1990) but differs from the other work, whose primarily concern is strategic default; i.e., whether a borrower has an incentive to withhold his return and not pay.

his return with a greater probability. Despite this seeming trade-off, we find that the free-rider effect dominates the liquidity effect, thus making group lending undesirable, provided that the effort raises the project return in the sense of the monotone likelihood ratio property. In particular, extending the result of Innes (1990), we show that an individual loan dominates all return-sharing schemes, individual or group, that pay the investor a (weakly) monotonic share of the agents' project returns.

When the group members operate their projects repeatedly, however, the free-rider problem associated with group lending can actually alleviate the incentive problems of the agents and therefore increase their credit-worthiness. We show this result without introducing an *ad hoc* penalty technology for the group members (such as ostracizing members who do not perform well). Rather, the joint liability feature itself makes it credible for members to penalize others through their effort decisions. Under group lending, a member's shirking (= the lowering of his effort) increases the payment burden of his peers, and thereby negatively affecting their payoffs. This means that a group member can be penalized by other members' shirking. If the group members observe the effort decisions of their peers, then they can use this punishment strategy to improve their incentives and thereby to enhance their ex ante credit-worthiness. It is shown that group lending can provide (weakly) better incentives than individual lending, given a mild condition, and furthermore that group lending can eliminate the incentive problem altogether (i.e., achieves the first-best outcome) if there are sufficiently many members who are sufficiently patient and can observe one another's effort.

To my knowledge, the static model of this paper is the first that studies the trade-off between the liquidity-risk effect and the free-riding effect of group lending in a general environment.[3] Our dynamic model is related to the repeated game literature but, unlike standard repeated game models, our payoff structure is generated endogenously through contract design. For instance, Besley and Coate (1995) studies a repeated game played between a lender and a group of borrowers, to address the latter's strategic default incentive. The nature of repeated interaction does not change with the chosen lending format there, and the social

[3] Stiglitz (1990) considers a similar issue in a special model with two entrepreneurs and binary return structure (See also Ghatak and Guinnane (1999)). As will be seen below, the binary return structure is special and not representative of a more general model.

sanction behavior is treated as exogenous. In this regard, Che and Yoo (2001) is closer to the current paper. They study the effect of contract design on the repeated game played by multiple agents. The focus of this latter work is labor contracting, though, and the current paper deals with a richer environment.[4]

The rest of the paper is organized as follows. Section 2 describes the basic model. Section 3 compares individual and group lending in the static setting. Section 4 studies the repeated setting. Section 6 concludes.

2 Static Model

There are $n \geq 2$ risk neutral agents, each endowed with the same, potentially productive project. The project requires a lump-sum investment of K to start and its success depends on the agent's entrepreneurial effort, $e \in \mathcal{R}_+$, measured in monetary units. Specifically, the project generates a return $y \in [0, \overline{y}]$ according to a cdf $F(y|e)$ where $e \in \mathcal{R}_+$ is his effort. The cdf has density, $f(y|e)$, which is strictly positive and continuously differentiable for all $(y, e) \in (0, \overline{y}) \times \mathcal{R}_{++}$. (This permits a possibility that a zero effort yields zero expected return.) We assume that a higher effort generates a higher return in the sense of stochastic dominance: i.e., $F_e(y|e) < 0$ for any $(y, e) \in (0, \overline{y}) \times \mathcal{R}_+$. The main result of this section requires a stronger version of this assumption, Monotone Likelihood Ratio Property:

Condition MLRP: *For any $y' > y$ both in $(0, \overline{y})$ and $e' > e$,*

$$\frac{f(y'|e')}{f(y|e')} > \frac{f(y'|e)}{f(y|e)}.$$

To ensure a unique interior solution, we further assume that $F_{ee}(y|e) > 0$ for any $(y, e) \in (0, \overline{y}) \times \mathcal{R}_+$, and that $\lim_{e \downarrow 0} F_e(y|e) = -\infty$ and $\lim_{e \uparrow \infty} F_e(y|e) = 0$ for any $y \in (0, \overline{y})$. Through-

[4]The current model allows for a continuum of outcomes and a finitely many effort levels for each agent, whereas Che and Yoo assume a binary outcome and a binary effort. Furthermore, the debt contracts studied here introduce new features absent in the Che and Yoo model. See also Martimort (1999), which studies contract design in a repeated *adverse selection* environment and Spagnolo (1999), which studies the linking of social interaction and production interaction in a repeated setting.

out, we use $E_z[\cdot|\mathbf{e}]$ to denote an expectation taken over a random variable z, given the effort profile, $\mathbf{e}$. If e is light-faced, it will indicate either an individual's effort choice of e or all agents choosing the same effort, e.

The agent's expected payoff, gross of the investment cost, is then

$$v(e_i) := E_{y_i}[y_i|e_i] - e_i = \int_0^{\overline{y}} [1 - F(y_i|e_i)]dy_i - e_i.$$

Given $F_{ee} > 0$, $v(\cdot)$ is strictly concave. Hence, there is a unique maximizer, $e^* > 0$, characterized by the first-order condition:

$$-\int_0^{\overline{y}} F_e(y|e)dy = 1 \tag{1}$$

We assume that $v(e^*) > K$, so the project is productive given the first-best effort choice.

The agents are liquidity constrained, so the investment fund must be financed by an investor(s). We assume that the realized project returns are verifiable but that the efforts are unobservable to the investor(s). In many circumstances, group members reside in the same neighborhood, so it is likely that the agents can observe one another's effort. In this latter case, it is clearly to the benefit of the principal to require cross reporting of the mutual observations.[5] Here we assume that such communication is prohibitively costly.[6] Given this restriction, an investor's contract simply specifies a payment from each agent as a function of all project returns. Let $\mathcal{N} := \{1, ..., n\}$ denote the set of agents, y_i agent i's project return, and $\mathbf{y} = (y_1, ..., y_n)$ the vector of all the projects. Then, a contract, $\mathbf{p} = (p_1, ..., p_n)$, is profile of payment functions, where $p_i : [0, \overline{y}]^n \to \Re$. We assume that the function is integrable and satisfies several conditions. Since each agent is liquidity constrained and the investor cannot be liable beyond his investment,

$$(LL) \qquad 0 \le p_i(\mathbf{y}) \le y_i \quad \forall i \in \mathcal{N}, \forall \mathbf{y}.$$

In addition, we impose the Innes (1990)'s monotonicity constraint:

$$(M) \qquad p_i(\mathbf{y}) \text{ is nondecreasing in } y_i \quad \forall i, \forall \mathbf{y}.$$

[5] See Rai and Sjöström (2000), which consider a mechanism that involves cross reporting of the agents. They consider a strategic default model in which the returns are unverifiable but mutually observable to the agents. No moral hazard (effort decision) problem arises in their model.

[6] This assumption is fully justified in the current section if the agents cannot observe one another's efforts. Our repeated version of the model requires peer monitoring of the agents' efforts, though.

The monotonicity constraint can be justified if the agent can secretly borrow to take a lower payment offer available only for a higher realized return, which will effectively eliminate the nonmonotonic portion of the payment function (see Innes (1990) for an alternative justification). We say $\mathbf{p}$ a *feasible* contract if it satisfies (LL) and (M).

The set of all feasible contracts includes a wide range of financial contracts, including some well-known ones. For instance, the set includes as special cases individual debt contracts as well as group debt contracts.

- *Individual loan:* An individual loan requires a borrower to pay a fixed amount or else his entire return is seized. According to this rule, given a return profile, $\mathbf{y}$, agent i pays

$$p_i^1(\mathbf{y}; r^1) = \min\{y_i, r^1\},$$

  for some (gross) interest charge, $r^1 > 0$. Clearly, such a $\mathbf{p}$ satisfies (LL) and (M).

- *Group loan:* A group loan requires a group of borrowers to be jointly liable for repayment of their collective loans. In our notation, a group loan specifies p_i that satisfies $\sum_{i=1}^{n} p_i(\mathbf{y}) = \min\{\sum y_i, nr^n\}$ for some per-capita (gross) interest charge $r^n > 0$. This specification does not pin down the way in which the payment burden is shared among the agents when total return exceeds nr^n. A natural sharing rule would involve setting an internal interest rate so that each agent pays the internal rate if his project return exceeds the rate, or else he defaults (internally) and his entire return is seized to pay the group debt. Specifically, for any return profile, $\mathbf{y}$, a payment from agent i is given by:

$$p_i^n(\mathbf{y}; r^n) := \min\{\rho(\mathbf{y}; r^n), y_i\}, \tag{2}$$

  where the internal rate, $\rho(\mathbf{y}; R)$, is determined to meet the aggregate payment requirement, i.e., to satisfy

$$\rho(\mathbf{y}; r^n) = \sup\left\{ z \le nr^n \,\middle|\, \sum_{j=1}^{n} \min\{z, y_j\} \le nr^n \right\}.$$

  Note that $p_i^n(\mathbf{y}; r^n) = y_i$ if $y_i < \rho(\mathbf{y}; r^n)$, but once $y_i \ge \rho(\mathbf{y}; r^n)$, ρ (and therefore p_i^n) remains constant with an increase in y_i. Hence, $p_i^n(\mathbf{y}; r^n)$ specified in this way satisfies both (LL) and (M). Henceforth, a group loan will refer to this particular sharing rule.[7] Note that this payment function collapses to that of the individual loan if $n = 1$. For later use, we define the expected payment for an agent when all other agents choose effort, e, and his realized return is y_i:

$$P^n(y_i; r^n, e) := E_{\mathbf{y}_{-i}}[p_i^n(\mathbf{y}; r^n)|e] = E_{\mathbf{y}_{-i}}[\min\{\rho(\mathbf{y}; r^n), y^H\}|e].$$

The set of admitted contracts also includes other financial contracts, such as equity contracts (both in individual and group forms).

Fix a feasible contract, $\mathbf{p}$, and suppose that all agents except i pick efforts $\mathbf{e}_{-i} \in \mathcal{R}_+^{n-1}$ and agent i chooses e_i. Then, agent i receives payoff:

$$\pi_i(e_i, \mathbf{e}_{-i}; \mathbf{p}) := E_{\mathbf{y}}[y_i - p_i(\mathbf{y})|\mathbf{e}] - e_i.$$

This contract *implements* an effort profile $\mathbf{e} := (e_1, ..., e_n)$ when it is a Nash equilibrium choice; i.e.,

$$(IC) \qquad \pi_i(e_i, \mathbf{e}_{-i}; \mathbf{p}) \geq \pi_i(e_i', \mathbf{e}_{-i}; \mathbf{p}) \quad \forall i, e_i'.$$

Suppose that a contract $\mathbf{p}$ implements efforts $\mathbf{e}$. Since an investor offering a contract should not incur a loss, we must have, for each i,

$$(PC - R) \qquad E_{\mathbf{y}}[p_i(\mathbf{y})|\mathbf{e}] \geq R,$$

for some $R \geq K$. Imposing this constraint for different values of R enables one to capture different scenarios in terms of the underlying market structure and of the relative bargaining power of the investors. For instance, a large R value will correspond to investors with strong market power, and a small R will correspond to investors with weak market power.

[7] Many other reasonable sharing rules satisfy our feasibility condition. Another reasonable rule is the proportional rule, according to which each agent makes a payment proportional to his project return. That is,

$$p_i(\mathbf{y}) = \min\left\{1, \frac{nr^n}{\sum_{j=1}^n y_j}\right\} y_i.$$

Clearly, this rule also satisfies both (LL) and (M).

Remark 1 *For later use, we digress to consider a (possibly group) debt contract with a per-capita interest rate of r^n for $n \geq 1$. In particular, for $n \geq 2$, suppose that all agents, except agent i, choose the same effort e and that agent i chooses e_i. Then, agent i receives from the loan contract a payoff:*

$$\Pi^n(e_i, e; r^n) := \int_0^{\overline{y}} [y_i - P^n(y_i; r^n, e)] dF(y_i|e_i) - e_i.$$

A per-capita charge, r^n, then breaks even for the lender, given that all agents choose e, if

$$\int_0^{\overline{y}} [y_i - P^n(y_i; r^n, e)] dF(y_i|e) = K.$$

This rate is called henceforth a competitive rate or a break-even rate.

In the case of an individual loan, $P^1(y_i; r^1, e) = \min\{r^1, y_i\}$, so

$$\Pi^1(e_i, e; r) = \int_{r^1}^{\overline{y}} (y_i - r^1) dF(y_i|e_i) - e_i = \int_{r^1}^{\overline{y}} [1 - F(y_i|e_i)] dy_i - e_i,$$

where the second equality follows from integration by parts. Since there is no payoff interdependence, the equilibrium, denoted $e^1(r^1)$, is given by the maximizer of this payoff. Since $F_{ee} > 0$, the latter is well defined and is characterized by a first-order condition:

$$-\int_{r^1}^{\overline{y}} F_e(y_i|e_i) dy_i - 1 = 0. \tag{3}$$

Note that the left side of (3) is strictly smaller than that of (1) when $e_i = e^*$, for any $r^1 > 0$. Given $F_{ee} > 0$, it clearly follows that $e^1(r^1) < e^*$ for any $r^1 > 0$.

In principle, there may not exist any feasible contract that satisfies both (IC) and $(PC-R)$ for an arbitrary R. We assume that at least one contract, an individual loan, satisfies both for $R = K$; i.e., there exists $\hat{r}$ such that

$$E[\min\{y, \hat{r}\}|e^1(\hat{r})] \geq K.$$

Any such $\hat{r}$ must exceed $K > 0$, or else the inequality cannot hold. The above remark then implies that $e(\hat{r}) < e^*$.

We say that a contract, $\mathbf{p}$, is *optimal for $R \geq nK$*, if it solves

$$[S-R] \qquad \max_{\mathbf{p},\mathbf{e}} \sum_{i=1}^{n} \pi_i(\mathbf{e}; \mathbf{p})$$

subject to

$$(LL), (M), (IC), \text{ and } (PC - R).$$

As mentioned above, considering the optimal contract for different values of R enables one to produce an equilibrium prediction robust to the particular conditions of the lending market.[8] The next result, which is an extension of Innes (1990), shows that an individual loan contract is indeed optimal for all values of R.

Proposition 1 *Given Condition MLRP, an individual loan contract is optimal for for any $R \geq K$ for which $[S - R]$ has a solution. The implemented effort level in the optimal individual loan is less than the first-best level.*

Proof. This result is obtained as a simple extension of Theorem 1 of Innes (1990). We show first that, for any contract satisfying the constraints, there exists an individual loan contract that can make all agents weakly better off. To this end, fix any pair $(\mathbf{p}, \mathbf{e})$ satisfying $(LL), (M), (IC)$ and $(PC - R)$ for $R \geq K$. For each i, consider his expected payment when his project return is y_i:

$$P_i(y_i; \mathbf{e}) := E_{\mathbf{y}_{-i}}[p_i(\mathbf{y})|\mathbf{e}].$$

By (LL) and (M), we must have $0 \leq P_i(y_i; \mathbf{e}) \leq y_i$ and $P_i(y_i; \mathbf{e})$ is nondecreasing in y_i. Then, Lemma 2 of Innes (1990) proves that an individual debt contract with $r > 0$ satisfying $E_y[\min\{y, r\}|e_i] = P_i(y; \mathbf{e})$ induces a weakly higher effort. With this increase, $(PC - R)$ continues to hold, and the agent i's payoff can only increase. Applying the result for all agents, a set of individual debt contracts can make all agents (at least weakly) better off. Since $r > 0$, $e(r) < e^*$, proving the last statement. Q.E.D.

In particular, this result implies that group sharing of payment burden, as exemplified by group loans cannot strictly dominate the individual loan contracts. The proposition implies that the free-riding problem arising from group lending outweighs the beneficial liquidity effect it may create. Several examples illustrate this intuition.

[8] The previous version of the paper, Che (2000), considered a model in which multiple investors offer in a Bertrand style. Such a model corresponds to $R = nK$.

Example 1: A binary return structure

A binary return structure has been commonly adopted in the corporate finance literature. As will be seen in this example and the next, the binary model is special and does not reflect the predictions of more general models.

Suppose that each agent has a project which generates a high return, y^H, ("success") and a low return $y^L(< y^H)$ ("failure") with probabilities $\alpha(e)$ and $1-\alpha(e)$, respectively. Assume that $\alpha(\cdot)$ is increasing.[9] We then obtain the equivalence of the two lending formats.

Proposition 2 *Given a binary return structure, a group loan can implement the same effort and yield the same expected payment to the investor as any individual loan.*

The same observation is made for the case of $n = 2$ in the existing literature (see Ghatak and Guinnane (1999)). The current result is more general since we allow for any $n \geq 2$ and for an arbitrary $\alpha(\cdot)$.[10]

As will be seen by the next example, this equivalence result depends crucially on the binary signal. Moreover, group lending may admit additional, bad, equilibrium that arises due to a coordination failure among the agents — one in which agents' shirking reinforces one another's shirking.[11] To illustrate this point, consider following return profile.

e_i \ y_i	0	$\frac{3}{2}$
0	1	0
$\hat{e}$	$\frac{1}{4}$	$\frac{3}{4}$

The first column lists an agent's effort choices, the first row displays possible returns, and each of the rest cells displays the probability that a given return is realized under a given effort choice. Throughout all examples, we assume that $K = 1$ and that r^n is chosen so that the lender just breaks even (i.e., the supply side of the loan market is competitive).

[9]This binary signal structure immediately satisfies Condition MLRP.

[10]Ghatak and Guinnane consider α as a choice variable but assumes the cost of choosing α to be quadratic. Our model here can be seen as assuming an arbitrary cost function.

[11]It turns out that agents' payoffs are supermodular in their efforts under group lending. See Che (2000) for the details.

A lender can break even in individual lending by charging 4/3, assuming that the agent picks $e_i = \hat{e}$. Given that $r^1 = 4/3$, the net surplus for the agent is $(3/4)(3/2 - 4/3) = 1/8$ if he picks $e_i = \hat{e}$ and zero if he picks $e_i = 0$, respectively. If $\hat{e} < 1/8$, then the agent will indeed pick $\hat{e}$, and the project is financed.

Now consider group lending. As shown in Proposition 2, the same effort choice can arise under group lending as well. But there exists another, bad, equilibrium. Consider a group of size $n = 2$. The break-even per-capita charge, denoted $r^2(\hat{e})$, is still greater than $1(= K)$, because of the default risk in the bad state. Suppose now that one member picks the low effort $e = 0$, (which generates zero return). If the other agent picks $\hat{e}$, then the return in the good state is 3/2 which does not cover the group charge, $2r^2(\hat{e}) > 2$, so he receives zero surplus. Thus, as long as $\hat{e} > 0$, it is an equilibrium that both agents shirk. Given this latter behavior, the lender incurs a loss, so it is an equilibrium for no group loan to be offered.

Example 2: An MLRP case

We now present an example where an individual loan implements a strictly higher effort than any group loan in any equilibrium. Consider the following return profile.

e_i \ y_i	0	2	4
0	$\frac{1}{2}$	$\frac{1}{2}$	0
$\hat{e}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Notice that the MLRP holds for the project return with respect to the effort decision. Consider individual lending first. Assuming that the agent picks $\hat{e}$ ("work"), the loan will be defaulted with probability 1/3 (in the zero return state). So, the break-even interest rate is $r^1(\hat{e}) = 3/2$. The agent's net expected surplus (net of the interest payment) will be 1 if he works and 1/4 if he shirks ($e = 0$).

Now consider a group loan with $n = 2$. The pooling of the resources reduces the default chance, so, assuming that both agents work, the (per-capita) break-even interest rate turns out to be $r^2(\hat{e}) = 7/6$, which is less than the break-even charge under individual lending. The net return for each member is again 1 when he works and $5/18 > 1/4$ when he shirks. Thus, if $13/18 < \hat{e} \leq 3/4$, an agent will work under individual lending but not under group lending.

The reason for this difference is explained as follows. Under individual lending, an agent is asked to pay 3/2 whenever his return exceeds this amount. Under group lending, an agent is expected to pay more when his return is 4 than when it is 2: his expected payment is $14/9 > 3/2$ in the former case and $13/9 < 3/2$ in the latter case. The reason is that, when his return is 2, his liability is effectively limited to that amount if his partner has zero return, so he pays 2 which is less than the group liability; but, when his return is 4, he has to pay the entire group charge of $2 \times (7/6) > 2$. That is, a relatively low return insulates an agent from the group liability. Since group lending favors the agent for the relatively lower return realization, the agent has a stronger incentive to shirk under group lending.

Example 3: A non-MLRP case.

Consider the following example, which is the same as Example 2, except when the agent shirks.

e_i \ y_i	0	2	4
0	$\frac{2}{3}$	0	$\frac{1}{3}$
$\hat{e}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Now the MLRP fails, and the intuition provided at the end of Example 2 now favors group lending. The (per-capita) break-even interest charge (assuming that the agents work) is the same as before for both formats since the return profile for working remains unchanged. The deviation payoff (exclusive of effort cost) is now 5/6 for individual lending and 22/27 ($< 5/6$) for group lending. Hence, the incentive for shirking is now stronger with individual lending. The reason for this reversal is the same as before. Group lending favors the lower return than higher return, relatively speaking, which in the current example disfavors the shirking under group lending relatively more than under individual lending. If $1/6 < \hat{e} \leq 5/27$, then the agent will work only under group lending.

3 Repeated interaction with peer monitoring

In this section, we study a repeated version of the stage game studied in the previous section. Each agent can perform his project infinitely many times. Formally, time flows discretely with period $t = 0, 1, ...$ In each period, the stage game described in the previous section is played. The agents are long-lived and maximize the present discounted value of the long-term payoffs and they have a common discount factor $\delta \in (0, 1)$. We assume that the agents can observe the effort decisions made by other members in the past. This assumption is meant to capture the notion of "peer monitoring," which has been an important element of many group lending programs.

We consider a group loan of size $n \geq 1$. This restricted scope will entail no loss in some circumstances since group lending implements the first-best outcome. Any return left after repaying the loan is consumed in that period, so the revenue generated in a given period cannot be used to repay the loan in the next period.

We first consider a *restricted* game in which a group of $n \geq 1$ has an accepted group loan with per-capita interest charge, r^n, in each period, and ask what effort levels can be implemented in this (restricted) repeated game. Later, we will augment the restricted game to address the issue of contract offering. Studying the restricted game in isolation serves to highlight the effort incentives issues, independent of the underlying market structure or the relative bargaining power of the investor(s).

3.1 Analysis of the restricted game

Since the agents can observe other members' effort decisions, it is natural to consider a strategy conditional on these observations. Formally, let $h^t = (\mathbf{e}^1, ..., \mathbf{e}^t)$ denote the history at time t of effort decisions, $\mathbf{e}^t \in E^n$, $h^0 \equiv \emptyset$. Then, a strategy for agent i is a sequence of functions, $s_i \equiv (s_i^1, ...s_i^t, ...)$, where $s_i^t : h^{t-1} \to \mathcal{R}_+$ maps from the effort decisions up to period $t-1$ into an effort decision at time t. We are interested in a subgame perfect equilibrium in this restricted game. We say that *a group loan with* r^n *implements a common effort* $\hat{e}$ if there exists a subgame perfect equilibrium in which each agent chooses $\hat{e}$ every period.

Since the projects are completely independent across the agents, the only possible payoff interdependence can come from payment sharing. Since there is no payment sharing under individual lending (i.e., $n = 1$), the equilibrium outcome in the repeated setting is the same as that in the static game. By contrast, group lending can create an interesting payoff interdependence across the agents. Since an agent's low return increases the payment burden of the rest of the group members, an agent's shirking (i.e., lowering his effort) exerts a negative externality on the rest of the group. If the agents can thus observe the other members' effort decisions, they can employ a punishment strategy whereby a shirking agent is retaliated by a subsequent shirking by his peers.[12] When such a punishment strategy is self enforcing, group lending can alleviate the incentive problems facing the agents.

The joint liability feature of group lending implies that the worst sustainable (or minmax) payoff for a given agent is attained when his peers all choose the lowest effort. An effort level $\hat{e}$ is said to be *individually rational given* r if

$$\Pi^n(\hat{e}, \hat{e}; r) > \sup_{e \in \mathcal{R}_+} \Pi^n(e, 0; r) > 0.$$

The individual rationality condition says that an agent is better off when all group members choose $\hat{e}$ than when he plays his best response against his peers who choose the maximum shirking. Under group lending, shirking increases the payment burden of other members, so this condition is likely to hold as long as group lending involves a strong joint-liability feature.[13]

Lemma 1 *Fix any $n \geq 2$ and $r \geq 0$ and an individually rational $\hat{e}$. Then, there exists $\hat{\delta} \in (0, 1)$ such that, for all $\delta \geq \hat{\delta}$, a group loan with r^n implements $\hat{e}$.*

Our main results are obtained as applications of this lemma.

[12] Our model can be seen as capturing the notion that an agent is motivated by the effect that his shirking may have on the collective morale or work ethics of the other agents.

[13] Individual rationality would hold if the lenders were to adjust their rates in response to a shift to a minmax effort choice. Such adjustment does not occur, however, since lenders do not observe such a switch following deviation. While lenders observe imperfect signals of the change in effort decisions, it does not affect the lenders' posterior beliefs about the agents' effort choices since they concentrate entire prior beliefs on the equilibrium behavior. A similar phenomenon arises in Bagwell (1995).

Proposition 3 *There exists N such that, for any $n \geq N$, a group loan of size n with a competitive interest charge $r^n(e^*)$ implements the first-best effort e^*, for all $\delta \geq \hat{\delta}(n)$ for some $\hat{\delta}(n) \in (0,1)$.*

This result shows that group lending can solve the incentive problem completely if there are sufficiently many agents who are sufficiently patient. This result is intuitive since a group penalty becomes very strong when a large number of agents exert it. While the first-best result may not hold unless the group size is very large and the supply side of the market is competitive, one can show that group lending dominates individual lending, *regardless of the group size and of the market structure*, given the following assumption:

Condition B: $E[y|0] = 0$.

This assumption says that zero effort yields no returns, which seems plausible in many scenarios in which any positive return requires at least some minimal amount of effort.

Proposition 4 *Given Condition B, for any individual loan satisfying $(PC-R)$, $R \geq K$, there exists a group loan of arbitrary $n \geq 2$ that yields a weakly greater expected payment to the investor and implements a weakly higher effort if $\delta > \hat{\delta}(n)$ for some $\hat{\delta}(n) \in (0,1)$.*

While the proposition proves only weak dominance of group lending over individual lending, strict dominance holds in many cases, as illustrated by the following example.

Example 4: A binary return

Consider the following return profile.

e_i \ y_i	0	2
0	$\frac{2}{3}$	$\frac{1}{3}$
$\hat{e}$	$\frac{1}{3}$	$\frac{2}{3}$

The break-even (per capita) interest rates are $r^1(\hat{e}) = 3/2$ and $r^2(\hat{e}) = 5/4$ under individual and group lending, respectively, assuming that the agents work. Given these rates, an agent shirks in both formats if $\hat{e} > 1/6$, in the static setting. Given shirking, the project is not credit-worthy, so no lender will offer a loan in this case.

Group lending can work, however, in the repeated setting. If $1/6 < \hat{e} < 1/4$, then, for a sufficiently large δ, it is an equilibrium behavior for both agents to work under a group loan of $n = 2$, which makes the projects credit-worthy under group lending (but not under individual lending, which yields the same outcome as in the static setting). In that equilibrium, lenders charge the break-even rate of $r^2(\hat{e}) = 5/4$ and the agents work each period. Whenever a member deviates, the strategy triggers a punishment strategy which involves repeated shirking by both agents. (One can show that given $\hat{e} > 1/6$, it is a stage game Nash equilibrium for both agents to shirk, so its infinite repetition constitutes a subgame perfect equilibrium.) To be concrete, assume that $\hat{e} = 1/5$, then group lending will induce the agents to work if $\delta \geq 2/5$. In this circumstance, the projects can only be financed through group lending.

Remark 2 *An added virtue of the joint liability contract is that the maximal effort equilibrium tends to be also collusion proof in the sense that there exists no other equilibrium that would make all agents better off and some strictly better off. The reason is that, in a joint-liability contract, an agent's raising effort increases the payoff of the other agents, and such positive externalities are fully realized in the maximal effort equilibrium, leaving no scope for agents to exchange favors. Hence, there exists no scope for the agents to collusively select a Pareto superior equilibrium. While this point is difficult to prove in a general environment, it can be seen immediately for the case described by Proposition 3, in which the first-best outcome is implemented and the agents receive the entire surplus. The point can be seen also in the above example. When the group loan implements* $(\hat{e}, \hat{e})$ *in that example, the low effort pair,* $(0, 0)$*, can also be sustained as an equilibrium, but it yields a lower payoff to each agent — a point obvious by the fact that* $(0, 0)$ *is used there as a punishment to sustain* $(\hat{e}, \hat{e})$*. Less immediate, but one can also see that alternating between* $(0, \hat{e})$ *and* $(\hat{e}, 0)$ *yields a lower total payoff, so it can never Pareto-dominate the implemented pair, even when the former can be sustained as an equilibrium.* [14]

Remark 3 *Just as any other repeated games, our repeated game has many equilibria and is*

[14] See a more detailed analysis on this issue in Che and Yoo (2001).

susceptible to the possibility of renegotiation. That is, the agents may have incentives to renegotiate around a severe punishment, which undermines the sustainability of the implemented equilibrium. Authors have suggested different criteria for renegotiation proofness. According to the criterion suggested by Abreu, Pearce and Stachetti (1993), for instance, the repeated stage-game Nash equilibrium constitutes a renegotiation-proof punishment strategy. While our main theorem uses a severer punishment strategy than repeated Nash, our equilibrium appears to be also sustained in many circumstances by a repeated stage game Nash punishment, as can be seen in the above example.

3.2 Contract offering

We have so far considered a restricted game in which that a group of n faces a stationary contract. This stationarity feature of the contract can be justified as an equilibrium behavior in a richer game which involves either a single investor or multiple investors offering short-term contract(s) repeatedly. While constructing a precise contract form is cumbersome and is not the main focus of this paper, it is not difficult to sketch the arguments that illustrate this point.

Suppose that a group loan of $n \geq 2$ implements $\hat{e}$ with r^n in the restricted game. If the investor is making a substantial profit in that equilibrium of the restricted game, then that equilibrium is justified as part of an augmented game in which a single lender repeatedly offers a short-term contract. Specifically, suppose that the lender offers a contract satisfying (LL) and (M) in each period to any group of agents and that each agent of that group decides whether to accept that contract. If all agents of that group accept the contract, then that contract is in force for that period. Otherwise, no contract is in force for that period. Given this game form, suppose that each agent accepts any contract (satisfying (LL) and (M))[15] but that he

[15]Given the specification of the acceptance procedure, vetoing of any group contract can be sustained as an equilibrium in a stage game. This equilibrium, when repeated, can yield an even severer punishment strategy against any deviation on the part of the lender as well as on the part of the agents, than is considered in this paper. But this equilibrium is an artifact of an ad-hoc coordination failure and runs counter to the spirit of this paper (which refrains from introducing an artificial punishment device through group lending programs). Indeed, this coordination-failure equilibrium can be eliminated if we model the acceptance procedure as a sequential

behaves according to the equilibrium strategy of the restricted game *if and only if* the lender has previously offered the particular group loan of n with r^n. Any deviation from the lender is followed by the agents repeatedly choosing an one-period best-response effort against any subsequent contract the lender may offer. Clearly, this latter punishment behavior is subgame perfect. Given this behavior, if the lender ever deviates from the group lending contract, he will deviate to offer an individual loan (since the latter Pareto-dominates all other contracts satisfying (LL) and (M), by Proposition 1). Suppose that the group loan Pareto dominates the best individual contract in the restricted game, which will be the case, given Condition B, by Proposition 4. Then, the investor never deviates from the group loan, so long as it offers a stream of rents to the lender large enough to make deviation to the optimal individual loan unprofitable.

It is harder to sustain an equilibrium of the restricted game that offers low rents to the lender. The extreme case of zero equilibrium rents can be sustained as an equilibrium of an augmented game, however, in which two (or more) lenders competitively offer short-term contracts in each period. Much as before, suppose that the agents accept the assumed group loan in the restricted game (if it is offered) and behave according to its equilibrium strategy and that, if any other contract is offered (and accepted), they pick their short-term best responses given that contract. Suppose further that lenders all offer the suggested group loan repeatedly. This strategy profile forms a subgame perfect equilibrium, if the equilibrium of the augmented game implements an effort level weakly higher than that of the individual loan (which is satisfied in the circumstances described by Propositions 3 and 4). In the latter circumstances, a single-period deviation by a lender is not profitable, since, whenever a deviating lender offer a contract (satisfying the feasibility constraint) that would yield a strictly positive profit upon acceptance by some agents, all agents (at least weakly) prefer to choose the nondeviating lender.

---

process.

4 Conclusion

This paper has studied the incentive rationale for the use of group lending as a method of financing liquidity-constrained entrepreneurs. We have seen that, in a static setting, the individual loan is optimal among the monotonic return-sharing rules, individual or group, given a monotone likelihood ratio property. Hence, group loans can never be desirable in those circumstances. By contrast, the joint liability feature of group lending may turn out to have a desirable incentive benefit in the repeated setting. In particular, the associated free-riding problem provides the group members with credible means of exercising peer sanction. We find that, given a reasonable condition, group lending dominates individual lending, and that the former attain the first-best outcome if there are sufficiently many group members who are sufficiently patient. These results are obtained when the group members have no access to other means of social sanction, although the availability of other social sanctions will reinforce the appeal of group lending.

Our result suggests that interlocking the fates of the agents through a joint liability contract can be desirable in the repeated setting due to the dynamic punishment strategy that it makes available. This insight can shed some light on the issues outside the particular problem that we considered. For instance, it is a routine practice in Korea for a division of a *chaebol*, a Korean style conglomerate, to warrant loans obtained by the different divisions of the chaebol. Such a cross-warranting practice effectively makes the divisions jointly liable, just as in group lending. Following the logic of the current paper, then cross-warranting can promote peer monitoring among divisions, to the benefit of the lender. Internal capital markets can bring out similar peer monitoring incentives across divisions of a diversified company. Its presence often means a cross-subsidization of a losing division by a winning division, — the so-called “socialism within a firm”[16] — which interlocks the fates of the divisions much as in a joint liability contract. While such cross-subsidization can only cause free-riding by a weak division in the short run, it may motivate the divisions to peer monitor in the long run, more effectively than otherwise possible.

[16]See Meyer, Milgrom, and Roberts (1992), Rajan and Zingales (1996) and Scharfstein and Stein (1996). See also Shin and Stulz (1998) for empirical evidence of internal capital markets.

5 Appendix: Proofs

Proof of Proposition 2: We show that for any effort level e and interest r, a group loan of an arbitrary $n \geq 2$ can implement the same effort and the same expected payment, for some per-capita charge r^n.

This result holds trivially if $r \leq y^L$. In that case, individual lending implements the first-best effort (i.e., $e = e^*$) since the agent becomes a residual claimant. With a group loan of size n, setting $r^n = r$ will make each agent a residual claimant, thus implementing the first-best effort. The expected payment to the lender will be the same, given $r^n = r$. Therefore, assume $r \in (y^L, y^H)$. (If $r > y^H$, then the agent will pick the zero effort, which can be easily implemented by a group loan with $r^n = r$.)

Consider a group loan with per-capita charge, $r^n \in (y^L, y^H)$. Given such a charge, an agent pays y^L when his return is low. Let

$$P^n(y^H; r^n, e) := E_{\mathbf{y}_{-i}}[\min\{\rho(\mathbf{y}; r^n), y^H\}|e]$$

be the expected payment that a given agent makes when his return is high and all agents choose e. In particular, consider r^n that would yield the same expected payment to the lender as the individual loan with r, given the same effort e; i.e., r^n satisfies

$$\alpha(e)P^n(y^H; r_i^n, e) + (1 - \alpha(e))y^L = \alpha(e)r + (1 - \alpha(e))y^L,$$

or equivalently,

$$P^n(y^H; r_i^n, e) = r. \tag{4}$$

Such $r^n \in (y^L, y^H)$ exists, since $P^n(y^H; r^n, e) < r$ if $r^n \leq y^L$ and $P^n(y^H; r^n, e) > r$ if $r^n \geq y^H$ and since it is continuous in r^n. Observe now that an agent has no incentive to deviate from e when all other agents choose e: i.e., for any $e' \in \mathcal{R}_+$,

$$\begin{aligned}
& \alpha(e)[y^H - P^n(y^H; r^n, e)] - e \\
= \;& \alpha(e)[y^H - r] - e \\
\geq \;& \alpha(e')[y^H - r] - e' \\
= \;& \alpha(e')[y^H - P^n(y^H; r^n, e)] - e'
\end{aligned}$$

where the two equalities follow from (4), and the inequality follows from the fact that the individual loan with r implements e. Q.E.D.

Proof of Lemma 1: Consider two paths: (1) Compliance path: All agents repeatedly choose $\hat{e}$, and (2) Punishment path: All agents choose zero effort in the first period and then from the next period onwards repeatedly choose $\hat{e}$. We now construct a strategy: The players start with the compliance path unless a unilateral deviation occurs. The punishment path is triggered following a unilateral deviation. Any deviation from the punishment path triggers the punishment path anew.

We now prove that this strategy profile is subgame perfect. Invoking the single deviation principle, it suffices to show that there is no profitable single period deviation by any single agent from each path.

Consider first the punishment path. The (average) equilibrium payoff on the punishment phase is

$$\Pi_p^n := \delta \Pi^n(0, 0; r^n) + (1 - \delta)\Pi^n(\hat{e}, \hat{e}; r^n).$$

Since $\hat{e}$ is individually rational given r^n, there exists $\hat{\delta}_1 \in (0, 1)$ such that, for any $\delta \geq \hat{\delta}_1$, $\Pi^n(e, 0; r^n) < \Pi_p^n$ for all $e \in \mathcal{R}_+$. Hence, for such δ,

$$\Pi_p^n \geq \delta \Pi^n(e, 0; r^n) + (1 - \delta)\Pi_p^n, \quad \forall e \in \mathcal{R}_+.$$

Given this inequality, there is no profitable (single-period) deviation from the first period of the punishment path.

We now consider the compliance path (or equivalently, after the first period of the punishment path). Since $\hat{e}$ is individually rational,

$$\Pi^n(\hat{e}, \hat{e}; r^n) > \max_e \Pi^n(e, 0; r^n) \geq \Pi^n(0, 0; r^n).$$

Hence, there exists $\hat{\delta}_2 \in (0, 1)$ such that, for any $\delta \geq \hat{\delta}_2$,

$$\Pi^n(\hat{e}, \hat{e}; r^n) \geq \delta \Pi^n(e, \hat{e}; r_k^n) + (1 - \delta)\Pi_p^n \quad \forall e \in \mathcal{R}_+.$$

Therefore, there is no profitable deviation from the compliance path and from the punishment path from the second period onward.

Combining the arguments, if $\delta \geq \max\{\hat{\delta}_1, \hat{\delta}_2\}$, the strategy profile is subgame perfect. Q.E.D.

Proof of Proposition 3 It suffices to show that, e^* is individually rational under a group loan of size n with $r^n(e^*)$, for sufficiently large n. Define first $\hat{y}^n := \frac{\sum_{i=1}^n y_i}{n}$.

Fix $\epsilon \in (0, e^*)$. We then have

$$\begin{aligned}
& \lim_{n\to\infty} \{\Pi^n(e^*, e^*; r^n(e^*)) - \sup_{e\geq\epsilon} \Pi^n(e, 0; r^n(e^*))\} \\
= \; & \lim_{n\to\infty} \inf_{e\geq\epsilon} \{\Pi^n(e^*, e^*; r^n(e^*)) - \Pi^n(e, 0; r^n(e^*))\} \\
> \; & \lim_{n\to\infty} \inf_{e\geq\epsilon} \{\Pi^n(e^*, e^*; r^n(e^*)) - \Pi^n(e, e; r^n(e^*))\} \\
= \; & \lim_{n\to\infty} \inf_{e\geq\epsilon} \{E_{\hat{y}^n}[\max\{\hat{y}^n - r^n(e^*), 0\}|e^*] - e^* - \{E_{\hat{y}^n}[\max\{\hat{y}^n - r^n(e^*), 0\}|e] - e\}\} \\
= \; & E_y[y|e^*] - K - e^* - \sup_{e\geq\epsilon} \{E_y[\max\{y - K, 0\}|e] - e\} \\
\geq \; & E_y[y|e^*] - K - e^* - \sup_{e\geq\epsilon} \{E_y[y|e] - K - e\} \\
\geq \; & 0,
\end{aligned}$$

where the first inequality holds since $\Pi^n(e, 0; r^n(e^*)) < \Pi^n(e, e; r^n(e^*))$ for all $e > \epsilon$ (i.e., the joint liability feature), the second equality holds since, given a symmetric effort, each agent receives the equal share of what is left after repaying the debt, $\frac{1}{n}[\max\{\sum_{i=1}^n y_i - nr^n(e^*), 0\}] = \max\{\hat{y}^n - r^n(e^*), 0\}$, the third equality holds since $r^n(e^*) \to K$ as $n \to \infty$, and the last inequality follows since e^* is the first-best effort level.

Similarly,

$$\begin{aligned}
& \lim_{n\to\infty} \{\Pi^n(e^*, e^*; r^n(e^*)) - \sup_{e\geq\epsilon} \Pi^n(e, 0; r^n(e^*))\} \\
= \; & \lim_{n\to\infty} \inf_{e\in[0,\epsilon)} \{\Pi^n(e^*, e^*; r^n(e^*)) - \Pi^n(e, 0; r^n(e^*))\} \\
\geq \; & \lim_{n\to\infty} \inf_{e\in[0,\epsilon)} \{\Pi^n(e^*, e^*; r^n(e^*)) - \Pi^n(e, e; r^n(e^*))\} \\
= \; & \lim_{n\to\infty} \inf_{e\in[0,\epsilon)} \{E_{\hat{y}^n}[\max\{\hat{y}^n - r^n(e^*), 0\}|e^*] - e^* - \{E_{\hat{y}^n}[\max\{\hat{y}^n - r^n(e^*), 0\}|e] - e\}\} \\
= \; & E_y[y|e^*] - K - e^* - \sup_{e\in[0,\epsilon)} \{E_y[\max\{y - K, 0\}|e] - e\} \\
\geq \; & E_y[y|e^*] - K - e^* - \sup_{e\in[0,\epsilon)} \{E_y[y|e] - K - e\} \\
> \; & 0,
\end{aligned}$$

where the only differences are that the first inequality is now weak and the last inequality is now strict, which follows since the $E_y[y|e]$ is strictly concave in e.

Combining the arguments, we have

$$\lim_{n\to\infty}\{\Pi^n(e^*,e^*;r^n(e^*)) - \sup_{e\geq\epsilon}\Pi^n(e,0;r^n(e^*))\} > 0,$$

proving that e^* is individually rational for a large n. The result then follows directly from Lemma 1. Q.E.D.

Proof of Proposition 4. Suppose that an individual loan with r implements $\hat{e}$. Since the contract satisfies $(PC{-}R)$, $R \geq K$, we must have $r > 0$ and $\hat{e} > 0$ (or else the expected payment falls short of $K > 0$, given Condition B). It suffices to show that $\hat{e}$ is individually rational under a group loan of arbitrary $n \geq 2$ with r^n satisfying $E_{\hat{y}^n}[\min\{\hat{y}^n, r^n\}|\hat{e}] = E_y[\min\{y, r\}|\hat{e}]$.

First note that $nr^n > r$. To see this, suppose contrary to the claim that $nr^n \leq r$. Then, an agent, say i, pays at most nr^n. Specifically, when $y_i \geq nr^n$, he will no greater than r and strictly less than r with positive probability since other agents are contributing as well when they all choose $\hat{e}$. Likewise, when $y_i < nr^n$, the agent will pay no greater than y_1, and strictly less than y_i with positive probability, again because other agents are contributing with positive probability. In sum, each agent is paying strictly less under a group loan with r^n than he would under an individual loan with r. This fact contradicts the hypothesis that r^n satisfies $E_{\hat{y}^n}[\min\{\hat{y}^n, r^n\}|\hat{e}] = E_y[\min\{y, r\}|\hat{e}]$. We thus conclude that $nr^n > r$.

We now show that $\hat{e}$ is individually rational under a group loan with r^n. Let $\epsilon \in (0, \hat{e})$. Then,

$$\begin{aligned}\Pi^n(\hat{e},\hat{e};r^n) &= \Pi^1(\hat{e},\hat{e};r)\\ &\geq \sup_{e\geq\epsilon}\Pi^1(e,\hat{e};r)\\ &> \sup_{e\geq\epsilon}\Pi^n(e,0;r^n),\end{aligned}$$

where the equality holds since both formats yield the same expected payment to the investor, given $\hat{e}$, (thus yielding the same net surplus to each agent), the weak inequality holds since the individual loan with r implements $\hat{e}$, and the strict inequality holds since an agent in a group loan with r^n faces the same situation as in an individual loan with $nr^n > r$ if no other agents yield any returns (recall Condition B).

Similarly,

$$\begin{aligned}\Pi^n(\hat{e},\hat{e};r^n) &= \Pi^1(\hat{e},\hat{e};r)\\ &> \sup_{e\in[0,\epsilon]} \Pi^1(e,\hat{e};r)\\ &\geq \sup_{e\in[0,\epsilon]} \Pi^n(e,0;r^n),\end{aligned}$$

whose inequalities are explained as before except that the strict inequality now follows from the strict concavity of $\Pi^1(\cdot,\hat{e};r)$.

Combining the two arguments, we conclude that

$$\Pi^n(\hat{e},\hat{e};r^n) > \sup_{e\in\mathcal{R}_+} \Pi^n(e,0;r^n),$$

for any $n \geq 2$. Lemma 1 then proves that the group loan with r^n implements $\hat{e}$. The stated result holds since it is possible that a higher effort can be implemented under the group loan with the same r^n, which will mean a weakly higher expected payment to the investor. Q.E.D.

References

[1] Abreu, D., Pearce, D., and Stachetti, E., (1993), "Renegotiation and Symmetry in Repeated Games," *Journal of Economic Theory* 60, 217-240.

[2] Armendáriz de Aghion, B., (1999), "On the Design of a Credit Agreement with Peer Monitoring," *Journal of Development Economics*, 60, 79-104.

[3] Armendáriz de Aghion, B. and Gollier, C., (2000), "Peer Monitoring in an Adverse Selection Model," *Economic Journal*, 110, 632-643.

[4] Bagwell, K., (1995), "Commitment and Observability in Games," *Games and Economic Behavior*, 8, 271-80.

[5] Banerjee, A.V., Besley, T., and Guinnane, T.W., (1994), "Thy Neighbor's Keeper: the Design of a Credit Cooperative with Theory and a Test," *Quarterly Journal of Economics,* 491-515.

[6] Besley, T. and Coate, S., (1995), "Group Lending, Repayment Incentives and Social Collateral" *Journal of Development Economics,* 46, 1-18.

[7] Che, Y.-K., (2000), "Joint Liability and Peer Sanction under Group Lending," mimeo., University of Wisconsin.

[8] Che, Y.-K., and Yoo, S.-W., (2001), "Optimal Incentives for Teams" *American Economic Review, American Economic Review*, 91, 525-541.

[9] Conning, J., (1996), "Joint Liability, Peer Monitoring and the Creation of Social Collateral," IRIS Working Paper #165, University of Maryland.

[10] Ghatak, M., (2000), "Screening by the Company You Keep: Joint Liability Credit and the Peer Selection Effect," *Economic Journal*, 110, 601-631.

[11] Ghatak, M. and Guinnane, T.W., (1999), "The Economics of Lending with Joint Liability: A Review of Theory and Practice," *Journal of Development Economics,* 60, 195-228.

[12] Innes, R., (1990), "Limited Liability and Incentive Contracting with Ex-ante Action Choices," *Journal of Economic Theory*, 52, 45-67

[13] Martimort, D., (1999), "The Life Cycle of Regulatory Agencies: Dynamic Capture and Transaction Costs," *Review of Economic Studies*, 66, 929-947.

[14] Meyer, M., Milgrom, P., and Roberts, J., (1992), "Organization Prospects, Influence Costs, and Ownership Changes," *Journal of Economics and Management Strategy*, 1, 9-35.

[15] Rai, A. and Sjöström, T., (2000), "Is Grameen Lending Efficient?" mimeo., Penn State University.

[16] Rajan, R., and Zingales, L., (1996), "The Tyranny of the Inefficient: An Enquiry into the Adverse Consequences of Power Struggles," mimeo., University of Chicago.

[17] Sadoulet, L., (1998), "Non-Monotonic Matching in Group Lending: A Missing Insurance Market Story," mimeo., ECARE.

[18] Scharfstein, D.J., and Stein, J.C., (1996), "The Dark Side of Internal Capital Markets: Divisional Rent-Seeking and Inefficient Investment," mimeo., MIT.

[19] Shin, H.-H., and Stultz, R.M., (1998), "Are Internal Capital Markets Efficient?," *Quarterly Journal of Economics*, 113, 532-552.

[20] Spagnolo, G., (1998), "Social Relations and Cooperation in Organizations," *Journal of Economic Behavior and Organization*, 38, 1-25.

[21] Stiglitz, G.E., (1990), "Peer Monitoring and Credit Markets," *World Bank Economic Review,* 4, 351-366.

[22] Varian, H.R., (1990), "Monitoring Agents with Other Agents," *Journal of Institutional and Theoretical Economics*, 153-74.