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Assigning Resources to Budget-Constrained Agents

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This article studies different methods of assigning a good to budget-constrained agents. Schemes that assign the good randomly and allow resale may outperform the competitive market in terms of Utilitarian efficiency. The socially optimal mechanism involves random assignment at a discount—an in-kind subsidy—and a cash incentive to discourage low-valuation individuals from claiming the good.

Key words: Budget-constrained agents, Random rationing, Resale, Cash subsidy

JEL Codes: D02, D45, D61, D82, H42

When ye are passed over Jordan into the land of Canaan ... ye shall divide the land by lot for an inheritance among your families ... every man's inheritance shall be in the place where his lot falleth ... (Num. 33:51–54)

1. INTRODUCTION

Suppose that a government wishes to distribute a resource such as public land or radio spectrum. A natural option is to sell it at the market-clearing price, via a standard auction, but other methods have a long history, as the passage above indicates. Land was assigned on a first-come first-served basis during the 1889 *Oklahoma Land Rush*. In many countries, new housing units may be subject to binding price caps, with the units assigned by lottery.[1] Various priority rules are used as well. For example, children are accepted into public schools based on where they live, which school their siblings attend, or the outcome of a lottery. Priority rules also determine which transplant patients get organs, based on factors such as the patient's age and the severity of the condition.

According to the Coase theorem, the exact method of assigning a good does not affect the efficiency of the ultimate allocation since individuals will negotiate until they exhaust all gains

1. In Singapore, most citizens live in units sold by the government at below-market prices. Some 82 percent of Singapore's citizens live in "public housing flats," and about 95 percent of those residents own their units. (See "Building Homes, Shaping Communities," at http://www.mnd.gov.sg/, accessed on 13 November, 2008.) The price cap is as low as half of the price on the resale market (Tu and Wong 2002). The same fraction is given by Green *et al.* (1994) for Korea.

from trade.[2] In particular, if individuals are risk-neutral (i.e., have quasilinear preferences) and have unlimited wealth, the final allocation is exactly the same no matter the initial allocation, and it is Utilitarian efficient.

When individuals are budget constrained, however, these last conclusions do not follow. Then, resale may not correct an initial misallocation since nonrecipients of the good with higher valuations may be unable to buy from initial recipients with lower valuations. The final allocation may therefore not be Utilitarian efficient. More important, the initial assignment of the good matters. In particular, the ability to correct any misallocation depends on the purchasing power of those who fail to receive the good.

We study various methods of assigning an indivisible good to agents who may be budget constrained. The good could be a productive asset such as a license to operate a business (e.g., an airport takeoff-and-landing slot or a taxi medallion) or to exploit resources (e.g., a hunting or fishing license), in which case the valuation reflects the monetary payoff that the asset will generate. Or, it could be a consumption good such as housing or health care, in which case the valuation reflects the utility from consuming it. Individuals' budget constraints may stem from low lifetime earnings, but more generally stem from limited access to the capital market. When the good is a productive asset that generates a monetary return, an agent may borrow against the return, using the good as collateral. But the returns may be private information, prospective investors may not have good estimates of the returns, and the collateral could lose value due to inadequate care. Likewise, for human capital assets such as education and the right to immigrate, it is difficult to predict whether someone will develop marketable skills, and it is difficult to collateralize such skills. These adverse selection and moral hazard problems may limit agents' ability to finance purchases. In other words, budget constraints may result from imperfect capital markets.

Suppose that a mass $S\in(0,1)$ of a good is to be assigned to a unit mass of agents who differ in their initial wealths (budgets) and valuations of the good (which are distributed independently of each other). We compare three popular assignment methods: (1) competitive market (CM), wherein the good is sold at a market-clearing price; (2) random assignment without resale (RwoR), wherein the good is assigned randomly at a below-market price and the recipients are not allowed to resell the good; and (3) random assignment with resale (RwR), wherein the good is assigned randomly at a below-market price and then the recipients are allowed to resell the good on a competitive resale market. Some or all of these methods have been employed in contexts such as assignment of public land, housing, spectrum rights, immigration rights, public education, and military recruitment.

We show that while the competitive market method dominates random assignment without resale, it is strictly dominated by random assignment with resale in terms of Utilitarian efficiency of the final allocation. The reason is that a market assigns goods to those individuals who have high wealth as well as high valuations. In other words, the initial assignment is biased toward the wealthy. By contrast, random rationing (at a below-market price) has less of a bias. In particular, agents with low wealth who would not get the good in the competitive market may now receive it. When resale is permitted, the low-valuation recipients resell while the low-wealth high-valuation recipients do not. As a consequence, more high-valuation agents consume the good than in the competitive market. By the same logic, the lower the assignment price is, the more efficient the final allocation is under RwR, and the welfare gap between the two mechanisms is amplified if wealth inequality grows in a certain way.

2. The Coase Theorem is invoked frequently when new assignment schemes are proposed. One example concerns the Federal Communications Commission (FCC) spectrum license auctions. Opponents of the FCC's favored design argued that the design would not affect the ultimate allocation so revenue maximization should be the only goal. See the discussion in Milgrom (2004).

These three mechanisms are ex post individually rational, incentive compatible, and do not incur a budget deficit. Most important of all, they are quite simple, which makes them practically implementable, perhaps explaining their common use. At the same time, a question arises as to the optimal mechanism for assigning the good. In particular, RwR involves an in-kind subsidy (the good is offered initially at a below-market price) and it invites speculators—those participating solely to profit from resale—which reduces the amount of the good accruing to the low-wealth high-valuation agents. Questions arise as to whether efficiency can be improved via cash subsidies (perhaps in lieu of the in-kind subsidy) and whether participation by low-valuation agents can be more effectively controlled.

In order to investigate these questions, we study the optimal mechanism for assigning the good within a class of mechanisms that are ex post individually rational, incentive compatible, and do not incur a budget deficit. We provide a partial characterization of the optimal mechanism for the general model, showing that it involves cash transfers and random assignment to a positive measure of agents. Thus, the optimal mechanism retains the key features of RwR; i.e., a cash subsidy and an in-kind subsidy.

The intuition behind the role of the subsidies is made clear by studying a simple 2×2 type model with binary types for the budget and valuation, for which we obtain a complete characterization of the optimal mechanism. The optimal mechanism makes the good affordable to high-valuation but cash-poor agents. Since this makes it attractive for the low-valuation agents to mimic, a cash payment must be offered to induce them not to claim the good. The cash payment must in turn be financed by charging a high price to the high-valuation high-wealth agents; for this to be incentive compatible, the optimal mechanism "degrades" the contract for the target group by offering them a random assignment. We also demonstrate that when the budget constraint binds sufficiently, the optimal mechanism can be implemented via *random assignment with regulated resale and cash subsidy*. Unlike in RwR, the resale market is taxed so as to make speculation unprofitable for low-valuation agents, who then accept a cash subsidy in return for not participating. (The cash subsidy is financed through the resale tax.) Finally, we extend the 2×2 type model by adding (verifiable) signals that are correlated with wealth levels, and show that the optimal mechanism involves *need-based assignment* that favors those who are more likely to be poor.

Our findings yield useful insights that are policy relevant. First, despite the widespread use of nonmarket methods such as assigning goods at below-market prices, their *efficiency* properties are not well appreciated. The superior performance here provides a rationale for their use when budget constraints are important. This observation is not a criticism of the fundamental merits of markets, however, since nonmarket schemes succeed in conjunction with a resale market.

Second, our findings provide a new basis for subsidizing the poor. Need-based schemes are common in college admissions, subsidized housing programs, and license auctions. While these programs are often motivated by redistributive goals, our results suggest an efficiency rationale.

Third, our findings shed light on the *method* of subsidization as well. Our analysis finds that when an individual's subsidy-worthiness (e.g., her wealth) is not observable, an in-kind subsidy will be part of an optimal policy as it is less susceptible to mimicking by those outside the target group.[3] The optimal mechanism includes a cash subsidy as well, but its role differs

3. Currie and Gahvari (2008) and references therein have also shown that in-kind subsidies are effective at preventing the wealthy from mimicking the poor. For instance, Gahvari and Mattos (2007) find that an in-kind subsidy can be used to raise the utility of the poor, while keeping the wealthy from mimicking. Their setting is different from

from the one envisioned by conventional wisdom (i.e., that subsidizing an individual with cash is less distortionary than subsidizing her in-kind). In our model, the cash subsidy is used to discourage low-valuation agents from claiming the good. While our analysis is efficiency-based, we conjecture that these features would continue to apply when the planner's objective is just to benefit the poor.

Fourth, while resale performs a beneficial role, there may be benefits from regulating the resale market. We show that a tax on the resale market can be used as a revenue source for cash subsidies, which strengthen incentive compatibility.

The remainder of the article is organized as follows. Section 2 lays out the model and studies the performance of three common assignment methods. We study the optimal mechanism in Section 3. Further implications of our model are explored in Section 4 and 5 while related work is described in Section 6. Concluding remarks are in Section 7.

2. ALTERNATIVE METHODS OF ASSIGNING RESOURCES

2.1. *Preliminaries*

A planner wishes to assign a mass $S \in (0,1)$ of an indivisible good to a unit mass of agents. Each agent consumes at most one unit of the good in addition to a divisible numeraire called "money." Each agent has two attributes: her endowment of money or *wealth*, w; and her *valuation* of the good, v. She is privately informed of her *type*, (w,v). The attributes w and v are distributed independently over $[0,1]^2$, according to the cumulative distribution functions, $G(w)$ and $F(v)$, respectively, each of which has nonzero density in the support. Independence is assumed largely for analytical ease; the results are robust to introducing (even large) correlations between w and v. Further, independence helps to isolate the role that each attribute plays.[4]

A type-(w,v) agent gets utility $vx+w-t$ if she consumes the good with probability $x \in [0,1]$ and pays $t \leq w$. The agent cannot spend more than w. If $w < v$, we say she is *wealth constrained* as she is unable to pay as much as she is willing to pay. As noted above, the limited ability to pay may stem from capital market imperfections.[5]

The welfare criterion we use is a Utilitarian welfare function. Given quasi-linear preferences, the total value realized is an equivalent criterion.[6] This involves no restriction when the value of the good is a monetary return, as with productive assets or human capital: if one allocation dominates another in terms of total value, there is a way for the realized value to be redistributed from the former allocation to Pareto dominate the latter. The only reason that final allocations are inefficient in the competitive market, for example, is the lack of a means for such redistribution to occur; i.e., missing capital markets.

ours, as consumers all have the same strongly quasiconcave utility function. In addition, the consumers have the choice of either the in-kind subsidy from the government or a higher-quality alternative from the private market. The welfare criterion is also different as Gahvari and Mattos focus on achieving the Pareto-efficient frontier, whereas our planner provides subsidies in order to maximize Utilitarian welfare.

4. If the poor are more likely than the wealthy to have high valuations, schemes that benefit the poor could be desirable simply because low wealth serves as a proxy for a high valuation. Assuming independence avoids this confounding of effects.

5. For instance, the good could be an asset with a random return. If the return is not verifiable, the owner may be able to abscond with the proceeds. Then, lenders will be unwilling to invest in the project, causing market failure. Even if the cash flow cannot be hidden, if the project requires noncontractible effort by the borrower to be successful, the financing contract would involve capital rationing, thus causing capital market imperfections.

6. The wealth distribution has no effect because the marginal utility of wealth is constant and equal for all agents.

If the valuations of the good are nonmonetary in nature, Pareto efficiency does not imply Utilitarian efficiency, but the latter is still a compelling criterion from an ex ante perspective. As argued by Vickrey (1945), each agent will rank alternative allocations using Utilitarian welfare, prior to realizing her preferences, knowing only that she "has an equal chance of landing in the shoes of each member of the society."[7] Other criteria would give the same ranking.

Given the scarcity of the good, the entire supply will be assigned in any reasonable mechanism, including the ones we consider. Hence, the realized total value can be equivalently represented by the realized *average per-unit value*. The first-best allocation can be represented in this way. Let $v^* > 0$ denote the critical valuation such that $1-F(v^*)=S$. When all S individuals with valuations above v^* consume the good, total value is maximized. The average per-unit value realized in the first-best allocation is therefore given by

$$V^* := \frac{\int_0^1 \int_{v^*}^1 v dF(v) dG(w)}{S} = \frac{\int_{v^*}^1 v dF(v)}{1-F(v^*)} = \phi(v^*),$$

where

$$\phi(z) := \frac{\int_z^1 v dF(v)}{1-F(z)}$$

is the expectation of an agent's valuation, conditional on its exceeding z. (Note that $\phi(\cdot)$ is strictly increasing, a fact we will use later.) The typical allocation in our model, including that of the competitive market, will not attain V^*.

We now study the performance of three assignment methods: (1) *Competitive market (CM)*; (2) *random assignment without resale (RwoR)*; and (3) *random assignment with resale (RwR)*. A competitive market operates according to the standard textbook description, and can be implemented by (a continuous version of) a uniform-price multi-unit auction. At each price, each agent indicates whether she demands a unit, and the price adjusts to clear the market. The competitive market outcome can also be replicated when lobbyists offer nonrefundable bids ("burn resources") to a government official who makes inferences about their merits (based on the bids) and then assigns the good.[8] Under random assignment, the planner offers the good at a below-market price, and excess demand is rationed uniform randomly. The recipients of the good are allowed to resell it in (3), but not in (2).

These three methods do not require the planner to observe the agents' types, so they are incentive compatible, and they do not require the planner to infuse money into the system. Most of all, these methods are simple to implement. Not surprisingly, they are observed widely in the markets for a variety of goods and services. Some prominent examples follow:

- *Fugitive Property, Entitlements, and Government Resources:* fugitive property—a good or resource whose ownership is not yet established—can be assigned to the individual who claims it first (*the rule of first possession*). This method corresponds to RwR. The Korean housing market gives another example of this method. New construction in Korea is subject to below-market price caps, with excess demand rationed randomly. And the recipients are allowed to resell (Kim 2002). Many countries assign transferable fishing

7. Vickrey (1960) reprised this argument and described a potential immigrant, unsure of her standing, deciding between two communities. Harsanyi (1953, 1955) had a similar thought experiment, but he allowed for heterogeneous preferences. In Harsanyi's *Impartial Observer Theorem*, an observer forms a social ranking of alternatives by imagining that she has an equal chance of being any individual in society. For further discussion, see Mongin (2001).

8. See the discussion of Esteban and Ray (2006) in Section 6.

rights at nominal fees (Shotton 2001). At the same time, many government entitlements are not transferable. Immigration visas are often assigned by lottery, and they are not transferable. Radio spectrum was once assigned by lottery, but resale was allowed. A shift to auctions marked a change in regime from RwR to CM in many countries. The market is also used for other resources such as the rights to harvest timber and to drill for oil.

- *Education:* in many locales, school-aged children are assigned to public schools in the vicinity of their residence. Although public schooling involves little or no tuition, the demand for schools is reflected in housing prices, so the housing prices in good school districts are higher than those in bad school districts, all else equal.[9] This case corresponds to the market regime; the housing market assigns both schooling and housing. Seats in certain public schools are assigned by lottery, and admission is not transferable. This corresponds to random assignment without resale. The final regime would arise if a lottery were used to award *transferable* vouchers that confer attendance rights.
- *Military Recruitment:* an all-volunteer army corresponds to the competitive market as tax-financed salaries and benefits are used to attract the enlistees. A draft lottery is effectively an RwoR scheme. A draft lottery with tradable deferments represents RwR. This is essentially what occurred during the US Civil War when conscripts avoided service in the Union Army by paying nondraftees to take their places.

We now study each of the three mechanisms in the remainder of this section.

2.2. *Competitive Market (CM)*

We study the equilibrium price at which the market clears. Formally, the equilibrium price is the value of p at which demand equals supply (or it equals zero and supply exceeds demand). In our model, the planner makes the entire supply, S, available for sale at any nonnegative price. The demand at price $p \geq 0$ is given by the measure of agents willing and able to pay p:

$$D(p) := [1-G(p)][1-F(p)].$$

Note that the demand is continuous and strictly decreasing in p for any $p \in (0,1)$, and satisfies $D(0)=1>S$ and $D(1)=0<S$. Hence, there is a unique market-clearing price $p^e>0$ such that

$$D(p^e) = [1-G(p^e)][1-F(p^e)] = S. \tag{1}$$

Since $1-F(v^*)=S$, we have $[1-G(v^*)][1-F(v^*)]<S$, so $p^e<v^*$. This means that the equilibrium allocation does not maximize welfare.[10]

In Figure 1, the welfare-maximizing allocation would give the good to all agents in region $A+B$ whereas the market assigns it to those in $B+C$. In that sense, the market favors high-wealth low-valuation agents (region C) over low-wealth high-valuation agents (region A).

The competitive market yields an average per-unit value of

$$V_{CM} := \frac{\int_{p^e}^1 \int_{p^e}^1 v dF(v) dG(w)}{S} = \frac{[1-G(p^e)] \int_{p^e}^1 v dF(v)}{[1-G(p^e)][1-F(p^e)]} = \phi(p^e) < \phi(v^*) = V^*.$$

The second equality holds since $[1-G(p^e)][1-F(p^e)]=S$, by (1), and the inequality holds since $p^e<v^*$ and ϕ is a strictly increasing function.

9. See Black (1999) for the effect that parental valuation of public education has on housing prices.

10. If no agents were wealth constrained (i.e., $w \geq v$ for all agents), the competitive market equilibrium would maximize welfare.

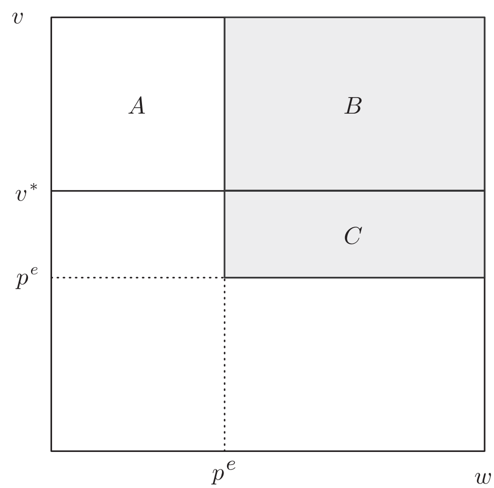

FIGURE 1

Benchmark allocations

Two points are worth making. First, the inefficiency of the competitive market is attributable to the wealth constraints. If the agents were not wealth constrained, the first-best outcome would arise. Second, the inefficiency will not be mitigated by opening another market. Suppose that a resale market opens. If the agents do not anticipate that it will open, the resale market will not trigger further sales since no mutually beneficial trades remain. (Individuals who purchased the good have $v \geq p^e$ so they would only sell at prices exceeding p^e, but there would be no additional demand at such prices.) Now suppose that agents do anticipate that a resale market will open, and suppose further that the resale market is *active*. Then, the prices would need to be equal in the two markets; otherwise, individuals would have an incentive to switch from one market to the other. Either way, the ultimate allocation is the same as above.

2.3. *Random Assignment without Resale (RwoR)*

We now analyze the simplest nonmarket assignment scheme: the planner offers the good at $\overline{p} \in [0, p^e)$ and those who demand the good at $\overline{p}$ have an equal probability of receiving it. This scheme is particularly easy to implement in that no knowledge of individuals' preferences or wealth is required. We assume that each agent can participate in the assignment scheme only once, and resale is not permitted. Agents can be kept from participating multiple times by requiring that all participants register.[11]

Since resale is not permitted, only agents whose wealth and valuations both exceed $\overline{p}$ will participate and attempt to acquire the good. Given uniform random rationing, each participant receives the good with probability

$$\frac{S}{[1-F(\overline{p})][1-G(\overline{p})]}.$$

Then, the average per-unit value realized equals $\phi(\overline{p})$. Since $\overline{p} < p^e$, we have $\phi(\overline{p}) < \phi(p^e)$, meaning that the allocation is inferior to the competitive market allocation.

11. This issue is particularly important when resale is possible, in which case it pays to purchase multiple units to resell. The impact of that is similar to the impact of adding pure speculators, which we study later.

A shift from CM to RwoR alters the set of recipients in two ways. First, it allows some agents with $(w,v)\in[\overline{p},p^e)\times[p^e,1]$ to receive the good. Redistribution to agents with low wealth is welfare-neutral, all else equal, because of the independence of wealth and valuations. Second, the shift to uniform assignment allows those with valuations $v\in[\overline{p},p^e)$ to receive the good with positive probability. This latter effect lowers welfare. As the price cap is lowered, RwoR performs even worse since more low-valuation buyers get the good.

2.4. *Random Assignment with Resale (RwR)*

We now assume the good is assigned at the below-market price $\overline{p}<p^e$ and those who participate are rationed uniform randomly (receiving at most one unit per participant). Unlike RwoR, recipients of the good are permitted to resell the good. The resale market operates in the same way as the competitive market discussed earlier. In particular, the equilibrium resale price is the value of r at which demand equals supply (or else it is zero and there must be excess supply).

We begin our analysis with the observation that the equilibrium resale price, $r_{\overline{p}}$, exceeds the cap $\overline{p}$. This is because any agent with a budget $w\geq r_{\overline{p}}$ and valuation $v>r_{\overline{p}}$ must obtain the good with probability one in equilibrium, and the measure of these agents exceeds S if $r_{\overline{p}}\leq\overline{p}$ since $\overline{p}<p^e$. Given this, any agent who receives the good can pocket $r_{\overline{p}}-\overline{p}>0$ by reselling. Hence, anyone who is able to pay $\overline{p}$ will participate. Since a measure $1-G(\overline{p})$ of agents will participate in the assignment, each participant receives the good with probability:

$$\sigma(\overline{p}):=\frac{S}{1-G(\overline{p})}.$$

We now study the determination of the equilibrium resale price. To this end, fix any resale price $r>\overline{p}$. Resale demand at that price comprises the agents who did not receive the good initially but who are willing and able to pay r:

$$RD(r):=[1-F(r)][1-G(r)](1-\sigma(\overline{p})). \tag{2}$$

Now consider resale supply. If a recipient of the good keeps it, she will receive utility $v+w-\overline{p}$, whereas reselling gives her $r+w-\overline{p}$. It is thus optimal to resell if and only if $v<r$. It follows that the resale supply at r equals the quantity initially assigned to agents with $v<r$. Since the probability of being in the latter group is simply $F(r)$, the resale supply at r is given by

$$RS(r):=SF(r). \tag{3}$$

Equating resale demand and supply yields:

$$[1-F(r)][1-G(r)](1-\sigma(\overline{p}))=SF(r). \tag{4}$$

The left-hand side of (4) is continuous and strictly decreasing in r, for $r\geq\overline{p}$, and its right-hand side is continuous and strictly increasing in r. At $r=\overline{p}$, the left-hand side exceeds the right-hand side,[12] and vice versa at $r=1$. Hence, there exists a unique $r_{\overline{p}}>\overline{p}$ that satisfies (4).

The final allocation of RwR thus has two different groups of agents receiving the good. The agents with $w\geq r_{\overline{p}}$ and $v>r_{\overline{p}}$ (in region B' in Figure 2) obtain the good with probability one, since they purchase the good on the resale market if they are not assigned the good initially. Those agents with $w\in[\overline{p},r_{\overline{p}})$ and $v>r_{\overline{p}}$ (in region A' of figure 2) obtain the good with probability $\sigma(\overline{p})$.

12. The left-hand side of (4) simplifies to $[1-F(\overline{p})][1-G(\overline{p})]-[1-F(\overline{p})]S$, which exceeds the right-hand side, since $[1-F(\overline{p})][1-G(\overline{p})]=D(\overline{p})>D(p^e)=S$.

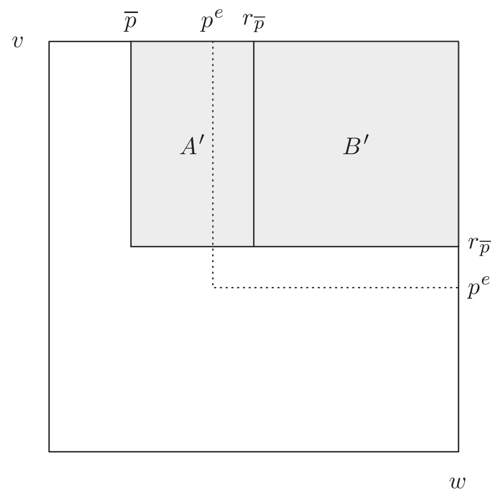

FIGURE 2
Random assignment with transferability

RwR yields an average per-unit value of

$$V_{RR}(\overline{p}) := \frac{\sigma(\overline{p}) \int_{\overline{p}}^{r_{\overline{p}}} \int_{r_{\overline{p}}}^{1} vf(v)dvg(w)dw + \int_{r_{\overline{p}}}^{1} \int_{r_{\overline{p}}}^{1} vf(v)dvg(w)dw}{S}$$

$$= \left(\frac{\sigma(\overline{p})[G(r_{\overline{p}}) - G(\overline{p})][1 - F(r_{\overline{p}})] + [1 - G(r_{\overline{p}})][1 - F(r_{\overline{p}})]}{S} \right) \left(\frac{\int_{r_{\overline{p}}}^{1} vf(v)dv}{1 - F(r_{\overline{p}})} \right)$$

$$= \phi(r_{\overline{p}}),$$

where the last equality follows from the fact that the numerator of the first term in the second line is the measure of those who end up with the good, which equals S.

Since a competitive market produces an average per-unit value of $\phi(p^e)$, to show that RwR outperforms CM, it suffices to show that $r_{\overline{p}} > p^e$. Suppose, to the contrary, that the resale price were $r \leq p^e$. By (1), $D(r) \geq S$, so the agents who are willing and able to pay r (region B') would exhaust S by themselves. In addition, some agents in region A' would receive the good and would not resell it. Since there will be excess demand on the resale market when $r \leq p^e$, the equilibrium resale price must exceed p^e. A consequence is that $V_{RR}(\overline{p}) = \phi(r_{\overline{p}}) > \phi(p^e) = V_{CM}$, and we conclude that random assignment with resale yields strictly higher welfare than either random assignment without resale or the competitive market.[13]

Random assignment allows the poor to receive the good with positive probability. Since the poor lack the ability to buy the good on the market, shifting the assignment toward them enhances welfare if resale is possible. The high-valuation poor will keep the good whereas low-valuation agents will resell to high-valuation agents.[14]

13. High-valuation agents with $w < \overline{p}$ do not get the good so $r_{\overline{p}} < v^*$, which means that welfare is not maximized.

14. Note that both regimes entail competitive markets in the end. Essentially, RwR preendows the good to the agents before the opening of a competitive market. Hence, when the realized values are not transferable, the first welfare theorem suggests that the outcomes of both regimes are Pareto efficient; one simply attains a higher Utilitarian welfare

A similar logic applies to reductions in the price cap. As $\overline{p}$ falls, more of the good accrues to the poor. The ultimate allocation improves since $r_{\overline{p}}$ rises as $\overline{p}$ falls. The results are now summarized.

Proposition 1. *Random assignment with resale yields higher welfare than the competitive market, and welfare rises as the price cap falls. Random assignment without resale yields lower welfare than the competitive market, and welfare falls as the price cap, $\overline{p}$, falls.*

Remark 1. *Although independence of wealth and valuations simplified the comparison, it is not crucial. The results are robust to either positive or negative correlation between the two. The first statement of Proposition 1 holds as long as there is a sufficient measure of agents with wealth just below the competitive price and sufficiently high valuations. The second statement of Proposition 1 holds as long as there are enough agents with high wealth and low valuations.*

Given the intuition behind the proposition, it is also not difficult to see the effects of changing wealth inequality. There exists a kind of increasing inequality that worsens the outcome of CM and improves that of RwR with $\overline{p}=0$.

Corollary 1. *Let G be the initial wealth distribution and fix $\overline{p}=0$. Now let p^e denote the equilibrium price under CM, let $r_{\overline{p}}$ denote the equilibrium resale price under RwR, and fix $\hat{w}\in(p^e,r_{\overline{p}})$. If the distribution of wealth shifts to $\hat{G}$, where $\hat{G}(w)<G(w)$ for $w\in(\hat{w},1)$ and $\hat{G}(w)>G(w)$ for $w\in(0,\hat{w})$, then the equilibrium prices under CM and RwR, respectively, are $\hat{p}^e$ and $\hat{r}_{\overline{p}}$ that satisfy $\hat{p}^e<p^e$ and $\hat{r}_{\overline{p}}>r_{\overline{p}}$. Consequently, the shift lowers welfare under CM and raises it under RwR.*

Proof We have $[1-F(p^e)][1-\hat{G}(p^e)]<[1-F(p^e)][1-G(p^e)]=S$, which implies that $\hat{p}^e<p^e$, by the single-crossing property of net demand. Likewise, we have

$$[1-F(r_{\overline{p}})][1-\hat{G}(r_{\overline{p}})](1-S)>[1-F(r_{\overline{p}})][1-G(r_{\overline{p}})](1-S)=SF(r_{\overline{p}}),$$

where the strict inequality follows from $r_{\overline{p}}\in(\hat{w},1)$ and $p^e<\hat{w}$, and the equality follows from $r_{\overline{p}}$ being the equilibrium price in CM under G. The inequalities, along with the single-crossing property of net (resale) demand, mean that $\hat{r}_{\overline{p}}>r_{\overline{p}}$. The last statement then follows from the monotonicity of ϕ. ∥

3. OPTIMAL MECHANISM

The previous section studied the implications of agents' wealth constraints for three common allocation mechanisms. A natural question is: what mechanism is optimal in this environment? RwR involves in-kind subsidies (the good is offered initially at a below-market price). An immediate question is whether one can do just as well using cash subsidies instead of in-kind subsidies, along with the competitive market. RwR also attracts pure speculators—low-valuation agents who participate solely to profit from resale—and their participation undermines efficiency by reducing the probability that the good accrues to the low-wealth high-valuation agents. Another question, then, is whether participation by low-valuation agents can be more effectively controlled.

than the other. If the values of the good are transferable, however, then neither outcome is Pareto efficient, since there is a Pareto-improving redistribution of values. This result is not inconsistent with the first welfare theorem; it is explained by a missing capital market.

To investigate these issues, we study an optimal mechanism in the class of mechanisms that are individually rational, incentive compatible, and budget balanced. Our analysis of an optimal mechanism shows that it retains the central features of RwR as it provides an in-kind subsidy to high-valuation cash-poor agents and a cash incentive to low-valuation agents.

3.1. *Formulation of the problem and partial characterization*

We first formulate the mechanism-design problem in a general setting that includes our original model as well as the 2×2 type model that is the focus of Section 3.2. Let W and V be the supports of budgets and valuations, respectively. By the revelation principle, we can restrict attention to a direct mechanism, $\Gamma=(x,t): W\times V \rightarrow [0,1]\times\mathbb{R}$, which maps from an agent's reported type, $(w',v')\in W\times V$, into the probability, $x(w',v')$, that the agent obtains the good; and the expected payment, $t(w',v')$, that the agent must make. Note that this payment can be negative; i.e., the mechanism may involve a transfer from the planner. We impose several conditions on feasible mechanisms, some of which are standard. For instance, we require that the amount of the good assigned cannot exceed the supply:

$$\mathbb{E}[x(w,v)]\leq S, \tag{S}$$

and that each agent must be willing to participate:

$$vx(w,v)-t(w,v)\geq 0, \forall w,v. \tag{IR}$$

While we seek to consider a general mechanism, the primary aim is to yield insights that have some relevance for real-world institutions. This motivates the following restrictions. First, we assume that the agent has limited ability to sustain a loss; in particular, the agent cannot be forced to make a positive payment when she does not receive the good. This means that $t(w,v)\leq 0$ whenever $x(w,v)=0$. We are effectively assuming that an agent makes a conditional payment $p(w,v):=t(w,v)/x(w,v)$ if $x(w,v)>0$. In particular, the budget constraint for a type-(w,v) agent takes the form:

$$t(w,v)\leq wx(w,v), \forall w,v. \tag{BC}$$

This rules out lotteries with positive entry fees. (While such mechanisms are of intellectual interest, we are not aware of any examples with substantial entry fees.[15]) The assumption is justified if the agents are sufficiently loss-averse at a zero payoff. Based on Che and Gale (2000) (which allows for charging entry fees for lotteries), though, we conjecture that the optimal mechanism involves qualitatively similar features as the one we find. The crucial difference is that our framework has realistic implications, as will be shown by the implementation results.

Second, even though it is conceivable that the planner could force agents to demonstrate their "wealth" (e.g., by requiring a bond) to mitigate incentive compatibility, such a mechanism is unrealistic in practice since one's wealth may not be easily verifiable.[16] This means that an

15. Lotteries with upfront fees are susceptible to manipulation; the sponsor may convey the prize to a confederate. Further, they are illegal in the USA. Actual lotteries may be disguised as in essay contests, for instance, but these practices are not common.

16. We believe that this assumption is also without loss since any such mechanism can only discriminate against agents with lower wealth, and this discrimination tends not to be optimal given independence of wealth and valuations. In fact, we prove it for the 2×2 model by showing (in the proof of Proposition 3) that the optimal mechanism involves no such discrimination when (IC) is relaxed to allow for such discrimination. See also Che and Gale (2000) for an argument making the same point for the general model, albeit in a slightly different context.

agent may choose any contract that requires a conditional payment less than her wealth. Incentive compatibility therefore means:

$$vx(w,v)-t(w,v)\geq vx(w',v')-t(w',v'), \forall(w',v') \text{ such that } t(w',v')\leq x(w',v')w. \tag{IC}$$

Third, the planner should not incur a deficit:

$$\mathbb{E}[t(w,v)]\geq 0. \tag{BB}$$

If the planner can subsidize without limit, the first-best can be attained easily. But, the result will not be very useful since there is a social cost of raising public funds. Some mechanisms may yield a strictly positive budget surplus; the surplus can then be redistributed (say equally) without violating incentive compatibility (since agents are atomless). For this reason, we shall continue to focus on Utilitarian efficiency; i.e., maximizing the aggregate value of the good realized.

Summarizing, the planner solves

$$\max_{x,t}\mathbb{E}[vx(w,v)] \qquad [P]$$

subject to

$$(S),(IR),(BC),(IC), \text{ and } (BB).$$

Let us first consider our original model with $W=V=[0,1]$, and absolutely continuous distributions G and F for these variables, with densities that are bounded. We provide the following (partial) characterization of the optimal mechanism.

Proposition 2. *The optimal mechanism for the general model involves cash transfers to a positive measure of agents and provides random assignment to a positive measure of agents; i.e., it induces a positive measure to obtain the good with interior probabilities (strictly between zero and one).*

The proposition shows that the optimal mechanism contains the two main features of RwR discussed earlier: a cash subsidy and random assignment. (Recall that RwR confers a cash subsidy on low-valuation agents in the form of resale profit.) These two features will emerge explicitly, and the intuition behind them will be made clear, in the next section.

3.2. *Optimal mechanism in the* 2×2 *type model*

The multidimensional types and the budget constraints make complete characterization difficult for our model with continuous types. We therefore solve a "2×2" version of the model in which an agent's wealth and valuation each take one of two values. Specifically, we assume that each agent draws a wealth from $W:=\{w_L,w_H\}$ and a valuation from $V:=\{v_L,v_H\}$, where $0\leq w_L<w_H$ and $0\leq v_L<v_H$. In order for the problem to be interesting, we also assume that $w_H\geq v_H>w_L$. Then, the high-wealth type is never constrained whereas the low-wealth type may be. We assume that a mass $g(w)f(v)>0$ of agents has wealth $w\in W$ and valuation $v\in V$, where $g(w_L)+g(w_H)=1$ and $f(v_L)+f(v_H)=1$. As before, we assume that the supply of the good is limited to $S<1$. While simpler, this model captures the salient features of the general model.

In this environment, the first-best allocation is defined as an assignment $x(\cdot,\cdot)$ such that (S) is binding and, whenever $x(w,v_L)>0$ for some $w\in W$, we must have $x(w',v_H)=1$ for all $w'\in W$. For ease of exposition, let $g_i:=g(w_i)$, $f_j:=f(v_j)$, $x_{ij}:=x(w_i,v_j)$, and $t_{ij}:=t(w_i,v_j)$ for $i,j\in\{L,H\}$. Then, $\Gamma=\{(x_{ij},t_{ij})\}_{i,j\in\{L,H\}}$ denotes a mechanism. We now give a characterization of the optimal mechanism.

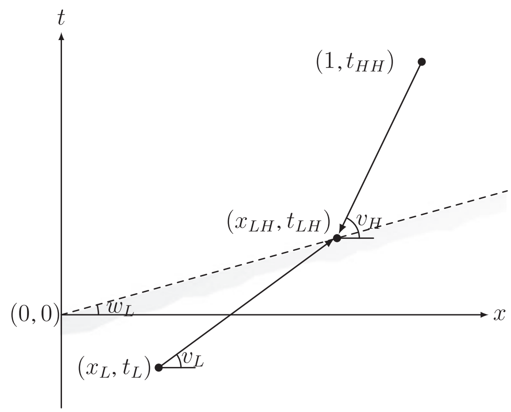

FIGURE 3
The second-best menu

Proposition 3. *The optimal solution to [P] attains the first-best*[17] *if and only if either*

$$S \le g_H f_H \text{ or } w_L \ge \min\left\{(1-S)v_L, f_L v_L - g_H f_H v_H \left(\frac{f_H - S}{S - g_H f_H}\right)\right\}. \tag{5}$$

When (5) fails, the optimal (second-best) mechanism involves a menu of three contracts, $\{(x_L, t_L), (x_{LH}, t_{LH}), (1, t_{HH})\}$, *where* $x_L = \frac{S - f_H(g_H + g_L x_{LH})}{f_L}$, $x_{LH} = \frac{g_H f_H (v_H - v_L) + S v_L}{g_H f_H (v_H - v_L) + v_L - w_L}$, $t_L = v_L x_L - (v_L x_{LH} - t_{LH})$, $t_{LH} = x_{LH} w_L$, *and* $t_{HH} = v_H - (v_H x_{LH} - t_{LH})$.

This proposition indicates that if agents' budget constraints are sufficiently severe, then the optimal mechanism involves a menu of three contracts. The first contract involves sale of the good at the "full price," $(x,t) = (1, t_{HH})$, and the remaining contracts involve two types of subsidies: (x_{LH}, t_{LH}) offers the good with an interior probability at a discount per-unit price $(t_{LH}/x_{LH} < t_{HH})$, and is chosen by low-wealth high-valuation agents; and (x_L, t_L) offers the good with an even lower probability, possibly zero, at a negative price (i.e., a cash subsidy), and is chosen only by low-valuation agents.

Further intuition can be gained from Figure 3, which graphs the optimal mechanism as a nonlinear tariff in (x,t) space, when the first-best is infeasible.

The three bold-faced points represent the contracts offered in the optimal mechanism. Their linear envelop forms a nonlinear tariff, which implements the optimal outcome. The figure illustrates how the agents' constraints affect the nature of the socially optimal mechanism. When no agents are wealth constrained, the socially optimal tariff is linear, with a slope equal to

17. When (5) holds, the first-best outcome is implemented as follows:

- If $S \le g_H f_H$ or $w_L \ge v_L$, then an optimal (first-best) mechanism involves a menu, $\{(0,0),(1,t^*)\}$, where $t^* \in (w_L, v_H]$ if $S \le g_H f_H$, $t^* = w_L$ if $S \in (g_H f_H, f_H]$, and $t^* = v_L$ if $S \in [f_H, 1)$.
- If $S \ge f_H$ and $w_L < v_L$, then an optimal (first-best) mechanism involves a menu, $\{(x_L, t_L), (1, t_H)\}$, where $x_L = \frac{S - f_H}{f_L}$, $t_H = w_L$, and $t_L = x_L v_L - (v_L - t_H)$.
- If $S \in (g_H f_H, f_H)$ and $w_L < v_L$, then an optimal (first-best) mechanism involves a menu, $\{(0, t_L), (x_{LH}, t_{LH}), (1, t_{HH})\}$, where $x_{LH} = \frac{S - g_H f_H}{g_L f_H}$, $t_{LH} = x_{LH} w_L$, $t_L = v_L x_L - (v_L x_{LH} - t_{LH})$, and $t_{HH} = v_H - (v_H x_{LH} - t_{LH})$.

the (per-unit) value of the marginal consumer (i.e., the market-clearing price). Such a mechanism selects agents based solely on their valuations: those above the marginal valuation consume the good with probability one and those below consume nothing. No agents are induced to consume a random amount.

Budget constraints introduce three new features into the optimal mechanism. First, low-wealth high-valuation agents are induced to consume a random amount. Second, a cash subsidy is given to the low-valuation agents. Third, the tariff is convex.

These features can be explained via Figure 3. Note first that the conditional payment associated with a given point (i.e., contract) is represented by the slope (t/x) of the ray from the origin to that point. Hence, a low-wealth agent cannot afford a contract above the shaded area (whose boundary is the dashed ray with slope equal to w_L).[18] For a low-wealth high-valuation agent to get the good with positive probability, the implicit price must be sufficiently low. This strains incentive compatibility as the low-valuation agents may find the contract for the low-wealth high-valuation agents attractive. Since $v_L > w_L$, the former cannot be dissuaded from consuming the good unless they are offered a cash subsidy, which explains the second feature. This is depicted by their contract being below the horizontal axis. To balance the budget, the cash subsidy is financed by charging the high-wealth high-valuation agents more than w_L. But, for them to pay more than the low-wealth high-valuation agents do, the latter agents' consumption must be distorted downward, which explains their less-than-certain consumption of the good—the first feature. Finally, the incentive constraints for the high-wealth high-valuation agents and for the low-valuation agents must be binding with respect to the contract for the low-wealth high-valuation agents (this is depicted by the two arrows pointing to the middle contract).[19] This means that the latter's contract must be on their indifference curves, with slopes v_H and v_L, respectively. This explains the third feature: convexity of the nonlinear tariff.

3.3. *Implementation via regulated resale and cash subsidy*

Here we discuss how the optimal mechanisms identified in Proposition 3 can be implemented via common assignment schemes. Those schemes include two that we already considered: CM and RwR. There are cases, however, in which these schemes are not sufficient, so they need to be modified to implement the optimal mechanism. For that purpose, we introduce two new schemes:

- *Competitive Market with Cash Subsidy (CMC)*: the planner offers a cash subsidy C to every agent who participates and assigns the good via a competitive market (e.g., standard multiunit auction).[20]
- *Random Assignment with Regulated Resale and Cash Subsidy (RwRRC)*: the planner initially assigns the good randomly (with uniform probability) at a below-market price

18. If charging entry fees for lotteries were allowed (i.e., if (BD) were relaxed so that $t(v,w) \le w$, then the feasible set for a type (w,v) would be the area below the horizontal line $t=w$. We note that the qualitative features of the optimal menu remain the same given a sufficiently binding constraint.

19. This pattern of binding incentive constraints is indicative of the difficulty encountered when doing mechanism design analysis in the current setting. In the standard one-dimensional problem, often the single-crossing property means that the constraints are binding adjacently in a monotone fashion. In the current setting (with two-dimensional types and budget constraints), the direction of binding incentive constraints (arrows in Figure 3) does not have an obvious order.

20. We note that any allocation that can be achieved under CMC can also be achieved using RwR with a cash subsidy. To be concrete, let p be the equilibrium price under CMC. The following scheme achieves the same allocation as under CMC: randomly assign the good at a price equal to $p-C$, and give a cash subsidy equal to C to those who were not assigned the good. Then, the high-valuation agents who were not assigned the good buy it on the resale market (with the cash subsidy). The ultimate assignment is the same as under CMC, and the budget balances for the same reason as under CMC.

$\overline{p}$, awarding at most one unit to each participant, and offers a cash subsidy C to those who do not participate in the assignment. The resale market opens next, but it is regulated, with a per-unit sales tax of τ.

The results are as follows.

Proposition 4. *(i) If $S \leq g_H f_H$ or $w_L \geq v_L$, CM implements the first-best outcome.*
(ii) If $S \in (g_H f_H, g_H f_H/(1-g_L f_H)]$, then the optimal mechanism (the first-best) is implemented by RwR with initial price $\overline{p} \leq w_L$.
(iii) If $S \geq f_H$ and $w_L \in [(1-S)v_L, v_L]$, then the optimal mechanism (the first-best) is implemented by CMC with a cash subsidy $C = v_L - w_L$.
(iv) In the remaining cases, including the case in which (5) fails (so the first-best is not implementable), the optimal mechanism is implemented by RwRRC with $C = v_L x_L - t_L$, $\overline{p} = t_{LH} = w_L$ and $\tau = v_H - v_L$.

The first three cases are relatively easy to explain. The first case (i) is indeed obvious. To explain (ii), suppose that RwR is employed with $\overline{p} \leq w_L$. Then, all agents will participate and each gets the good with probability S, so a fraction S of high-valuation agents obtain the good and consume it. A measure $(1-S)g_H f_H$ of high-valuation high-wealth agents are not initially assigned, so they demand the good on the resale market. A measure Sf_L of low-valuation agents are assigned and attempt to resell. Given the condition, the former is no less than the latter, meaning that only high-valuation agents will end up with the good. Hence, RwR implements the first-best.

To understand (iii), suppose that the planner offers a cash subsidy of $C = v_L - w_L$, and assigns the good via a competitive market. Then, every agent, including the low-wealth types, is willing and able to spend at least v_L for the good. At the price $p^e = v_L$, all of the high-valuation agents and a measure $S - f_H$ of low-valuation agents will demand the good, clearing the market.[21] The total subsidy required, $v_L - w_L$, can then be financed from the sales proceeds, $p^e S = v_L S$, given the condition.

Case (iv) is most interesting since it deals with the situation where the agents' budget constraints are severe enough to preclude the first-best. The proof for this case is provided in the Appendix. Here we simply explain the idea of how RwRRC can improve the outcome. The primary challenge now is to maximize the consumption of those with a high valuation but low wealth. This is difficult because of the incentive problem; the low-valuation agents are tempted to mimic the target group. The problem is addressed in RwR solely by allowing resale of the assigned good. Resale profit induces the successful low-valuation agents to give up the good. The optimal mechanism here employs two additional instruments to address the incentive problem. First, it offers a direct cash incentive to discourage low-valuation agents from participating in the initial assignment, which increases the probability that the high-valuation low-wealth agents obtain the good. The bigger the cash subsidy is, the stronger is their incentive not to participate, so the higher is the probability that the low-wealth high-valuation agents obtain the good. And the cash subsidy can be financed in particular by those high-wealth high-valuation agents. The sales tax is used as a means to collect extra revenue from this group. These two instruments enable the current optimal mechanism to improve upon the RwR mechanism.

As mentioned in the introduction, the fact that the optimal mechanism contains both in-kind and cash subsidies is interesting. This has a useful implication for the optimal method of

21. In fact, this is the only market-clearing price. If $p < v_L$, then there will be excess demand. If $p > v_L$, then only high-valuation and high-wealth agents will demand; since $S \geq f_H > g_H f_H$, we will have excess supply.

subsidizing the poor. An in-kind subsidy may be an important aspect of subsidy design when the subsidy-worthiness of an agent is not observable to the policy maker. Cash subsidies may also be employed, but they may play a role different from the one envisioned by the conventional wisdom. In particular, the cash subsidy may be offered not to benefit the target subsidy group but to keep the subsidy unworthy from claiming the scarce benefits.

4. NEED-BASED ASSIGNMENT

The preceding analysis has assumed that each agent's wealth and valuation are unobservable to the planner. In reality, however, information about agents' characteristics is often available and used in the assignment of resources. For instance, the awarding of *need-based* scholarships provides an example of assignment based on wealth.[22] It is obvious that schemes based on signals of valuations would be desirable from a welfare standpoint. In this section, we show the less-obvious property that need-based assignment schemes have the same effect of improving welfare, using the mechanism design framework from the previous section.

We simplify the 2×2 model in Section 3 by assuming that each of four wealth-valuation types has an equal mass, 1/4. We then introduce a binary signal $s\in\{\ell,h\}$ that is observable and verifiable by the mechanism designer. The signal s is assumed to be positively correlated with the wealth type. Specifically, for $\rho\in(1/2,1)$, the mass of each type (w_i,v_j) is given by the following table:

	v_L	v_H
w_L	$\frac{1}{4}\rho$	$\frac{1}{4}\rho$
w_H	$\frac{1}{4}(1-\rho)$	$\frac{1}{4}(1-\rho)$

$s=\ell$

	v_L	v_H
w_L	$\frac{1}{4}(1-\rho)$	$\frac{1}{4}(1-\rho)$
w_H	$\frac{1}{4}\rho$	$\frac{1}{4}\rho$

$s=h$

Note that the masses of the four agent types sum to 1/2 for each signal, and the low (high) signal is more likely when the agent has low (high) wealth. We will refer to the agents with signal s as *group s*. We will sometimes use n_{ij}^s to denote the mass of the type (w_i,v_j) in group s; e.g., $n_{HL}^h=\frac{\rho}{4}$.

The mechanism design problem is reformulated in a natural way. The direct mechanism specifies the assignment and payment rules as functions of an agent's wealth signal in addition to her report on her type.[23] We let x_{ij}^s and t_{ij}^s denote the probability of assignment and payment for each type (w_i,v_j) in group s. As before, we let $x_L^s:=x_{iL}^s$ when $x_{HL}^s=x_{LL}^s$.

It is useful to think of the planner facing two separate subprograms $[P^s]$ indexed by $s=\ell,h$. Each subprogram is similar to the problem in Section 3; in particular, constraints (*BC*), (*IR*), and (*IC*) can be required for each subprogram separately.[24] The subprograms cannot be solved in isolation, however, since (*BB*) need not hold for each group separately, as one group can cross-subsidize the other, and the amount of the good assigned to each group is endogenously determined as part of the optimal design. Letting $S^s:=\sum_{i,j\in\{L,H\}}n_{ij}^s x_{ij}^s$ denote the amount of the

22. Likewise, many government transfer programs in the USA are means-tested. See Currie and Gahvari (2008).

23. In the mechanism design literature, Riordan and Sappington (1988) and Faure-Grimaud *et al.* (2003), among others, study a similar setup in which an informative signal is available to the principal. There are a couple of crucial differences, however. First, in this article and Faure-Grimaud *et al.* (2003), the signal is publicly available at the time the principal offers a contract, whereas in Riordan and Sappington (1988), the signal is unobserved at the time of contracting, which leads to the full-extraction first-best result of Cremer and McLean (1988). More important is that the signal pertains only to one component of the agent's two-dimensional private information in the current model, whereas it pertains to the agent's single-dimensional private information in both Riordan and Sappington (1988) and Faure-Grimaud *et al.* (2003).

24. For instance, there is no issue of an agent in one group mimicking those in the other.

good assigned to group s, the constraint (S) changes to $S^{\ell}+S^{h}\leq S$. Lastly, we rewrite (BB) as

$$B:=\sum_{\substack{i,j\in\{L,H\}\\ s\in\{\ell,h\}}} n_{ij}^{s}t_{ij}^{s}=B^{\ell}+B^{h}\geq 0, \tag{6}$$

where $B^{s}:=\sum_{i,j\in\{L,H\}}n_{ij}^{s}t_{ij}^{s}$ is the net payment collected from group s. Cross-subsidization between the two groups is allowed since it is possible to have $B^{s}>0>B^{s'}$ and $B^{s}+B^{s'}\geq 0$. Under these constraints, our task is to find the mechanism that maximizes the sum of surpluses for the two groups. As before, the first-best is always implementable if $w_L\geq v_L$ or $S\leq 1/4$,[25] so we focus on the remaining cases.

Before proceeding, we demonstrate why need-based assignment is desirable by considering a special case in which the agents' wealth levels are perfectly observable (i.e., $\rho=1$). In this case, the planner can achieve the first-best with RwR that favors the low-wealth group $s=\ell$ in the initial assignment. Specifically, the planner assigns as much of the good as possible to the low-wealth group at the price $\overline{p}^{\ell}\leq w_L$. If $S\leq 1/2$, then the entire supply will be allocated (uniform-randomly) to the members of this group. If $S>1/2$, then every agent of the low-wealth group gets the good. The remainder is then allocated uniform-randomly to the members of the high-wealth group $s=h$ at the price $\overline{p}^{h}=v_L>w_L$.

It is easy to see that this need-based RwR implements the first-best. Suppose, first, that $S\leq 1/2$. Then, the resale market clears at the price v_H (in the unique equilibrium). Given $r=v_H$, a low-wealth recipient of the good keeps it if her valuation is v_H but otherwise sells it to a high-valuation agent in the high-wealth group. Since the mass of high-valuation agents exceeds the supply, the market clears at the price v_H. If $S>1/2$, the first-best is attained at the resale price of v_L. Given the resale price, each high-valuation agent keeps the good if she receives it, and demands it on the resale market if she did not receive it initially. Meanwhile, a low-valuation recipient of the good is indifferent to reselling it. The resale market clears at the price v_L since the supply exceeds the demand from high-valuation agents. Need-based assignment works in both cases by excluding the high-wealth low-valuation agents who would otherwise mimic the low-wealth high-valuation agents. The following proposition establishes the optimality of need-based assignment for the general case.

Proposition 5. *Suppose that $S\in(1/4,1)$ and $w_L<v_L$. Then,*

(i) the first-best outcome is attainable if and only if $w_L\geq\hat{w}_L(\rho)$ for some $\hat{w}_L(\rho)\in[0,(1-S)v_L]$. Moreover, it is implemented by a mechanism with $x_{HH}^{\ell}=x_{HH}^{h}=1$ and $x_{LH}^{\ell}=\min\{1,\frac{S-(1/4)}{(\rho/4)}\}\geq x_{LH}^{h}$.

Suppose, in addition, that $w_L<\hat{w}_L(\rho)$ (so the first-best is not attainable) and $v_H\geq\frac{(2-\rho)v_L}{(1-\rho)}-\frac{w_L}{(1-\rho)}$.[26] Then,

(ii) the optimal (second-best) mechanism has $x_{HH}^{s}=1,\forall s$, and either $1=x_{LH}^{\ell}>x_{LH}^{h}\geq 0$ or $1>x_{LH}^{\ell}>x_{LH}^{h}=0$;

(iii) each agent type in group ℓ is better off than the corresponding type in group h.

25. Note that 1/4 is the mass of high-wealth high-valuation types across the two groups.
26. The latter inequality is a technical condition that guarantees $x_{LH}^{s}\geq x_{HL}^{s},\forall s=\ell,h$, which facilitates our analysis.

Statement (i) suggests that the first-best outcome, if attainable, can be implemented via need-based assignment: the first-best mechanism favors the low-wealth high-valuation agents in group ℓ over the same type of agents in group h. Statement (ii) establishes the optimality of favoring group ℓ in the second-best mechanism in terms of both the probability of assignment and payoff.

This result is explained by the differing incentive costs of favoring cash-poor high-valuation agents across the two groups. Consider assigning an extra unit to the low-wealth high-valuation agents in either group. To make this incentive compatible, additional information rents must accrue to all other types in that group. [27] Since the mass of all other types is relatively smaller in group ℓ, smaller information rents are needed to assign the extra unit to low-wealth high-valuation agents in group ℓ than to the same types in group h. This alleviates the budget-balancing constraint (BB) and thus raises total surplus. Naturally, need-based assignment makes the agents in group ℓ better off compared with those in group h, as shown in (iii).

5. SPECULATION AND NON-MARKET ASSIGNMENT

The preceding analysis assumes that the planner can control recipients' ability to resell the good. This assumption is not without loss of generality. Even though resale can improve welfare ex post, it could have an adverse effect ex ante. The ability to resell may invite (low-valuation) agents to participate solely to profit from resale when the good is offered for sale at a below-market price, as in our model.[28] This kind of speculation is harmful for welfare since it reduces the probability that the good is assigned to those with high valuations but low wealth, thereby undermining efficiency.

The effect of speculation can be seen most clearly when the mass of potential speculators is large. Consider our continuous-types model. (The same result holds for our 2×2 model.) Suppose that, in addition to the unit mass of agents, there is a mass m of agents with zero valuation and wealth distributed according to density g. These agents can be interpreted as "outsiders" who have no intrinsic demand for the good, but can respond to any speculative opportunity. We envision a potentially large number of these agents. For a given economy indexed by m, we consider an arbitrary feasible mechanism, (x_m, t_m), that is in the class studied in Section 3, except that we now assume that agents who receive the good can resell it in a (unregulated) competitive market that operates whenever there is a price at which a positive measure of demand and supply exist. In other words, we assume that the resale process is beyond the control of the planner.

Proposition 6. *With unrestricted resale, in the limit as $m \to \infty$, the optimal mechanism converges to the one obtained under a competitive market.*

This result is reminiscent of the Coase theorem in that the initial assignment does not matter much; however, the ultimate allocation is not efficient here.[29] A significant presence of pure speculators thwarts any attempt to assign the good to the agents with low wealth and high valuations (the target group), since a cash incentive large enough to keep them from participating

27. This can be seen from the fact that all other types than (w_L, v_H) are indifferent between their own contracts and (x_{LH}, t_{LH}).

28. When the FCC used a lottery to assign cellular telephone licenses, it received nearly 400,000 applications. The application fee was zero until 1987, and only \$230 in 1993 (Kwerel and Williams 1993). The use of lotteries to assign housing in Korea engendered so much speculation that it was blamed for volatility in housing prices (Malpezzi and Wachter 2005).

29. Jehiel and Moldovanu (1999) also find that different ownerships result in the same but inefficient allocation; but the main source of inefficiency in their model is externalities that one's ownership imposes on the other agents.

will entail a budget deficit, whereas allowing the pure speculators to participate will simply undermine any real attempt to distribute the good to the wealth-constrained agents with high valuations. The above result also underscores the robustness of competitive markets against speculation, which may explain their prevalence and persistence.

6. RELATED LITERATURE

The current article follows the well-known theme that wealth constraints may impact allocative efficiency. Che and Gale (1998, 2006) study standard auctions in which bidders differ in their valuations and wealth. They showed that standard auctions differ in terms of allocative efficiency and the seller's expected revenue.[30] Che and Gale (2000) solved for the profit-maximizing mechanism for selling to a single buyer with private information about her valuation and wealth. They found that the optimal mechanism may contain a menu of lotteries indexed by the probability of sale and the corresponding fee.[31] Random assignment offers a similar benefit in the current study. Pai and Vohra (2011) derive the revenue-maximizing mechanism for selling a single unit to multiple buyers. Agents draw budgets and valuations independently from the same discrete distribution. Pai and Vohra's optimal mechanism involves an in-kind subsidy (i.e., random assignment with a price discount) but no cash subsidy. This is driven by their objective of revenue maximization. A cash subsidy will likely be part of the optimal mechanism in their setup if the designer maximizes welfare subject to a revenue target.

Fernandez and Gali (1999) study the assignment of workers to inputs when workers differ in their ability and wealth, and inputs differ in their productivity. In particular, they compare a competitive market to a tournament. In the tournament, each worker is assigned to an input based on a signal that depends on the worker's ability and her investment in the signal. When capital markets are perfect, there is positive assortative matching in equilibrium in both the competitive market and the tournament. With imperfect capital markets, borrowing constraints prevent some low-wealth high-ability workers from matching with high-productivity inputs. Then, tournaments outperform the market in terms of allocative efficiency, although the total surplus ranking could go either way because of the resource cost of signaling. Although their model involves matching, market assignment in Fernandez and Gali corresponds to the competitive market in our model. Their tournament can be interpreted as merit-based assignment since it is based on a signal on v.

Esteban and Ray (2006) consider a government that awards licenses to agents who differ in their productivity and wealth. The government's objective is to maximize allocative efficiency. Agents are able to signal their productivity via lobbying, and licenses are awarded based on that lobbying. Wealthier sectors find it less costly to lobby, which jams the productivity signal. The resulting allocation again corresponds to the market regime here. Esteban and Ray focus on how allocative efficiency changes with the underlying wealth distribution. Increasing inequality means that more of the high-productivity agents find themselves financially constrained when lobbying. Consequently, allocative efficiency and total welfare may both fall. Similarly, increasing inequality lowers welfare in the competitive market here but, interestingly, raises welfare under rationing with resale, making it even more attractive.

Our results also yield an interesting implication of their setup: if the government is unconcerned about welfare, lobbying will not arise, which means that licenses will be assigned randomly. If resale of licenses is allowed, the latter assignment method will then dominate the allocation generated by a government that is responsive to productivity signals.

30. See Kotowski (2010) for a further characterization of first-price auctions in that setting.

31. Making the good "divisible" by selling in probability units still does not maximize welfare here.

A related literature rationalizes market intervention based on welfare criteria different from those used here. For instance, Weitzman (1977) took as a benchmark the allocation of goods that would prevail if all agents had the mean income. He then showed that an equal allocation of goods may be closer to the benchmark than the market allocation is. Sah (1987) compared different regimes from the perspective of the members of the poorest group. Our article is also related to the literature on redistribution via in-kind subsidies.[32] Unlike our assignment problem, this literature is primarily concerned with wealth redistribution, with the goal of shifting from a point on the Pareto frontier (laissez-faire) to another point that is more favorable to the poor (those with a smaller endowment of the numeraire good). If the target group is observable, then the solution would simply be to tax the rich and subsidize the poor (using the numeraire). Difficulty in identifying the target group calls for a more complex mechanism to induce "self-targeting." Similar to our in-kind subsidy, this literature suggests degrading the quality of the subsidized good in order to discourage individuals outside the target group from claiming it (Besley and Coate 1991); and combining it with cash payments/transfers in order to make self-targeting more effective (Gahvari and Mattos 2007).

Some recent papers deal with mechanism design with financially constrained agents. Condorelli (2011) considers the problem of assigning a good to a finite number of agents, each of whom has private information about her valuation (v) and cost of spending a dollar ($1+r$), and he analyzes the problem of maximizing the sum of realized valuations. In his model, the only payoff-relevant information is $v/(1+r)$, so the model reduces to a problem with one-dimensional private information. Despite the similarity, his model does not exhibit nonlinearity in the payoff function caused by budget constraints,[33] so a cash subsidy or resale would play no role in his analysis. Richter (2011) analyzes a problem similar to ours with a continuum of agents who differ in their valuations and wealth, but his agents have linear preferences for *unlimited* quantities of the good. This difference gives rise to the optimality of linear pricing with a uniform cash subsidy to *all* agents.

7. CONCLUDING REMARKS

This article has studied methods of assigning resources to agents who are wealth constrained, from a Utilitarian efficiency perspective. We find that simple nonmarket schemes such as uniform random assignment may outperform market assignment, if the recipients are allowed to resell. We have also studied the optimal mechanism and showed that it may be implemented by a scheme that employs random assignment, regulated resale, and cash subsidies. The results here could apply to the assignment of resources and entitlements ranging from rights to exploit natural resources to exemptions from civic duties such as military service or jury duty.[34]

Many goods are assigned using nonmarket assignment schemes. In addition to providing justification for these schemes, the results here have implications for how existing schemes can be improved. In particular, the introduction of transferability may offer benefits to programs that do not currently permit it.[35] The USA assigns 50,000 permanent resident visas each year

32. Contributions include Besley and Coate (1991), Blackorby and Donaldson (1988), Gahvari and Mattos (2007), and Nichols and Zeckhauser (1982).

33. Our model can be seen to involve convexity of the spending function. In our model, when an agent with budget w spends π, she incurs a cost $C(w,\pi)=\pi$ if $\pi \leq w$ but $C(w,\pi)=\infty$ if $\pi > w$.

34. The results also apply to government-led industrialization processes in many developing economies as well. The Korean industrialization process was marked by licensing policies that targeted industries and firms for export quotas, trade protection, and other privileges (Amsden 1989). During the period dubbed the "licence raj," the Indian government controlled large areas of economic activity through the awarding of rights and "permissions" (Esteban and Ray 2006).

35. The resale rules can be adapted to control speculation and to accommodate other institutional constraints.

by lottery.[36] Becker (1987), Chiswick (1982), and Simon (1989) all discuss the sale of visas to qualified applicants, stressing the efficiency benefits (e.g., the immigrants who will pay the most will be the most productive). An alternative to straight sales is to retain the lottery system but permit the recipients themselves to sell their visas to other qualified applicants. Our results suggest that this change could yield an improvement relative to the current system and the straight sales proposal. In one sense, there are sales of visas already. Canada and the USA have allowed entrepreneurs to immigrate if they will start a business (e.g., the USA "immigrant investor program"). Resale of visas is not permitted, however.

A lottery could also be used to assign transferable educational vouchers. With the transfer process regulated to discourage speculation, such a system may select students more efficiently than would a system of local attendance zones or random assignment of nontransferable vouchers. In the same vein, one could imagine a military draft with tradable deferments.[37] Other objectives or institutional details may loom large in these cases, but the results here argue for consideration of nonmarket assignment schemes and transferability.

APPENDIX: PROOFS

Proof of Proposition 2. To prove the first statement,[38] it suffices to establish that the budget-balancing constraint is binding in the optimal mechanism. If the constraint is binding, a positive measure of agents must be receiving monetary transfers, or else (almost) all agents make a zero payment, in which case the only feasible assignment is random assignment, which the competitive market obviously dominates. Let (x,t) be the optimal mechanism. It is useful to invoke the taxation principle and consider the tariff scheme that implements the optimal mechanism. Let $X_0 := \{x \in [0,1] | \exists (w,v) \text{ s.t. } x(w,v) = x\}$ be the set of x values chosen in the optimal mechanism and, for $x \in X_0$, let $\bar{\tau}(x) = \{t(w,v) | \exists (w,v) \text{ s.t. } x = x(w,v)\}$ be the associated payment. (Note that $\bar{\tau}(x)$ must be a singleton since no two payments can be charged for the same x.) The optimal mechanism can be represented as $\{\bar{\tau}(x)\}_{x \in X_0}$. Also, $\bar{\tau}(\cdot)$ must be nondecreasing; otherwise, there exists $x < x'$ in X_0 such that $\bar{\tau}(x) > \bar{\tau}(x')$, in which case x will never be chosen by any type.

Let X be the closure of X_0, and let $\tau : X \to \mathbb{R}$ be the continuous extension of $\bar{\tau}$ from X_0 to X.[39] It can be seen that a mechanism $\{\tau(x)\}_{x \in X}$ implements $\{\bar{\tau}(x)\}_{x \in X_0}$.[40] Note that τ is continuous on X.[41]

We first make a few observations. Let $\underline{x} := \min X$. We must have $\tau(\underline{x}) \leq 0$. If $\tau(\underline{x}) > 0$, then $\inf_{x' \in X} \frac{\tau(x')}{x'} > 0$, and all agents with $w < \inf_{x' \in X} \frac{\tau(x')}{x'}$ must be choosing $(0,0)$, so $\underline{x} = 0 = \tau(\underline{x})$, a

36. Eligibility for the *Diversity Immigrant Visa Program* is restricted to individuals from countries with low rates of immigration to the USA. See http://www.travel.state.gov/pdf/dv07FinalBulletin.pdf, accessed 8 July, 2006.

37. Tobin (1970) noted that the same conclusion can be reached on the basis of egalitarian concerns. He first pointed out that the all-volunteer army was "just a more civilized and less obvious way of ... allocating military service to those eligible young men who put the least monetary value on their safety and on alternative uses of their time." He added that the difference between the two schemes is who pays—general taxpayers or the individuals who wish to avoid military service.

38. The proofs of some technical steps are provided in the Supplementary Appendix, to which we will refer readers as those steps are needed.

39. That is, if $x \in X_0$, then $\tau(x) := \bar{\tau}(x)$; and if $x \in X \setminus X_0$, then there exists $\{x^n\}_{n=1}^{\infty}$, $x^n \in X_0$, converging to x, such that $\tau(x) := \lim_{n \to \infty} \bar{\tau}(x^n)$. The existence of such a sequence is guaranteed by the fact that $\bar{\tau}(X)$ lies within a compact set $[-M, 1]$ for some $M > 0$.

40. If a positive measure of agents would strictly gain from deviating to some $x \in X \setminus X_0$, then they will gain strictly from deviating to some $x'' \in X_0$ that is sufficiently close to x.

41. If this is not the case, τ must jump up at some point $x \in X_0$, but in this case, no point x' slightly above x would ever be chosen by any type, so x' can never be an element of X_0.

contradiction. Also, $\frac{\tau(x)}{x}$ must be weakly increasing in x since no agent type would choose any $x \in X$ (or any $x' \in X$ close to x) if there is some $\tilde{x} \in X$ with $\tilde{x} > x$ and $\tau(\tilde{x})/\tilde{x} < \tau(x)/x$. Lastly, the constraint (B) must be binding; i.e., $\mathbb{E}[x(w,v)] = S$, since otherwise the remaining supply could be uniformly distributed among all agent types for free, which would increase the total surplus.

To prove that the budget-balancing constraint binds at the optimal solution, we suppose that it does not, and then draw a contradiction by modifying the mechanism so that it satisfies all of the constraints but generates higher welfare than τ does. Before doing so, we need to define some notation. First, for any $z > 0$,

$$v_z := \min_{x \in X} \frac{\tau(x) - \tau(\underline{x}) + z}{x - \underline{x}},$$

where, by a slight abuse of notation, the value of the objective is taken to be its right-hand limit at $x = \underline{x}$. The minimum is well defined since the objective function is continuous and X is a compact set.[42] Let $x_z := \arg\min_{x \in X} (\tau(x) - \tau(\underline{x}) + z)/(x - \underline{x})$ (or any selection from the set of minimizers). It is straightforward to see that v_z is continuously increasing in z and x_z is weakly increasing in z.[43] Second, for any $\sigma \in (0, \varepsilon]$, define a mechanism $\overline{\tau}^\sigma$ as follows: $\overline{\tau}^\sigma(x) := \tau(x)$ for all $x \in X \setminus \{\underline{x}\}$ and $\overline{\tau}^\sigma(\underline{x}) = \tau(\underline{x}) - \sigma$. Clearly, no agent types consume more under $\overline{\tau}^\sigma$ than under τ. There is a positive mass of agents who consume strictly less: all agent types with $w \geq v_\sigma$ and $v \in (v_0, v_\sigma)$ would choose $x > \underline{x}$ under τ, whereas they switch to $\underline{x}$ under $\overline{\tau}^\sigma$. Thus, $\overline{\tau}^\sigma$ would generate excess supply.

Fix a sufficiently small $\varepsilon > 0$. The argument consists of potentially two steps, depending on whether a positive mass of agents choose from the interval $(\underline{x}, x_\varepsilon)$ under τ. If the condition is satisfied, we say that τ satisfies Property (M).

If τ satisfies (M), then we start with Step 1. If τ does not satisfy (M), then we skip Step 1 and start with Step 2.

Step 1: For $s \in [v_\varepsilon, 1]$, modify τ to define τ^s as follows: For all $x \in X$ with $x > x_\varepsilon$, $\tau^s(x) := \tau(x)$; for all $x \in (\underline{x}, x_\varepsilon)$, $\tau^s(x) := \min\{\tau(x), \tau(\underline{x}) - \varepsilon + s(x - \underline{x})\}$. Let $\{x^s(w,v)\}_{(w,v) \in [0,1]^2}$ denote the associated incentive compatible choices. Note that this mechanism introduces some new contracts on a straight line, $(x, \tau(\underline{x}) - \varepsilon + s(x - \underline{x}))$, if they cost less than the original contracts, $(x, \tau(x))$. Clearly, if $s = 1$, then no agent type with $v < 1$ would choose the new contracts, except for $(\underline{x}, \tau(\underline{x}) - \varepsilon)$. This means that $\tau^1(\cdot)$ will induce the same consumption behavior as $\overline{\tau}^\varepsilon$ defined above (with $\sigma = \varepsilon$), thus generating excess supply; i.e., $\mathbb{E}[x^1(w,v)] < S$.

In the Supplementary Appendix, we establish the following facts: (i) for any $s \in [v_\varepsilon, 1]$, all agent types with $v < (>)\, s$ consume weakly less (more) under τ^s than under τ, and $x^s(w,v)$ weakly increases as s decreases (Supplementary Lemma S.1); (ii) if ε is small enough, then no mechanism τ^s for any $s \in [v_\varepsilon, 1]$ runs a budget deficit (Supplementary Lemma S.2); (iii) total consumption, $\mathbb{E}[x^s(w,v)]$, is continuously increasing as s decreases (Supplementary Lemma S.3).

Now choose ε sufficiently small that (ii) holds true. We first consider the case where $\mathbb{E}[x^1(w,v)] < S \leq \mathbb{E}[x^{v_\varepsilon}(w,v)]$. Then, by (iii), there exists some $\hat{s} \in [v_\varepsilon, 1)$ such that $\mathbb{E}[x^{\hat{s}}(w,v)] = S$, which leads to a contradiction since, according to (i), $\tau^{\hat{s}}(w,v)$ generates higher welfare than τ does. In case $\mathbb{E}[x^{v_\varepsilon}(w,v)] < S$, we move on to Step 2.

42. X is compact since it is a closed subset of the interval $[0,1]$.

43. To see the latter fact, suppose, to the contrary, that $x_{z'} < x_z$ for some z, z' with $z' > z$. Then, there arises a contradiction since

$$\frac{\tau(x_{z'}) - \tau(\underline{x}) + z'}{x_{z'} - \underline{x}} = \frac{\tau(x_{z'}) - \tau(\underline{x}) + z}{x_{z'} - \underline{x}} + \frac{z' - z}{x_{z'} - \underline{x}} > \frac{\tau(x_z) - \tau(\underline{x}) + z}{x_z - \underline{x}} + \frac{z' - z}{x_z - \underline{x}} = \frac{\tau(x_z) - \tau(\underline{x}) + z'}{x_z - \underline{x}},$$

where the inequality follows from the definition of x_z and the assumption that $x_{z'} < x_z$.

Step 2: For each $\delta \in [0,\varepsilon]$, we construct another mechanism, τ_δ, depending on whether τ satisfies Property (M). In case it does, let $\tau_\delta(x) := \tau(\underline{x}) - \varepsilon + v_{\varepsilon-\delta}(x-\underline{x})$ for any $x \in [\underline{x}, x_{\varepsilon-\delta})$ and $\tau_\delta(x) := \tau(x) - \delta$ for any $x \in X \cap [x_{\varepsilon-\delta}, 1]$. In case it fails, let $\tau_\delta(\underline{x}) := \tau(x) - \varepsilon$ and $\tau_\delta(x) := \tau(x) - \delta$ for any $x \in X \cap (\underline{x}, 1]$. In the subsequent proof, we focus on the case in which Property (M) holds. The argument for the other case is similar and omitted. Let $\{x_\delta(w,v)\}_{(w,v)\in[0,1]^2}$ denote the incentive compatible choices associated with τ_δ.

First, we prove the following claim:

Claim A1. *For any $\delta \in [0,\varepsilon]$, all agents with $v < v_{\varepsilon-\delta}$ consume weakly less, all agents with $v > v_{\varepsilon-\delta}$ weakly more, and a positive measure of them strictly more under τ_δ than under τ.*

Proof First, all agents with $v < v_{\varepsilon-\delta}$ choose $\underline{x}$ under τ_δ, thus reducing their consumption at least weakly.

Now consider agents with $v > v_{\varepsilon-\delta}$. All such agents strictly prefer $x_{\varepsilon-\delta}$ to any $x < x_{\varepsilon-\delta}$, and (given the way τ_δ is constructed) they all prefer a larger x to a smaller x in $[\underline{x}, x_{\varepsilon-\delta}]$. Since $\tau_\delta(\cdot) < \tau(\cdot)$, any agent with $v > v_{\varepsilon-\delta}$ who chose $x \in [\underline{x}, x_{\varepsilon-\delta})$ under τ will definitely choose $x' \geq x$ under τ_δ. Now consider any agent $v > v_{\varepsilon-\delta}$ who chose $x \geq x_{\varepsilon-\delta}$ under τ. Such an agent can afford x and at least weakly prefers it to any $x'' < x$ under τ_δ. This is trivially the case if $x'' < x_{\varepsilon-\delta}$ or the agent has $w \geq \tau(x'')/x''$. It is also true even if $w < \tau(x'')/x''$ since in that case

$$vx - \tau(x) = x\left[v - \frac{\tau(x)}{x}\right] \geq x''\left[v - \frac{\tau(x)}{x}\right] \geq x''[v-w] > vx'' - \tau(x'').$$

We have thus shown that all agents with $v > v_{\varepsilon-\delta}$ consume at least weakly more under τ_δ than under τ.

To see that a positive mass of agents consumes strictly more, consider the agent types with $v > v_{\varepsilon-\delta}$ and $w \in [\tau_\delta(x_{\varepsilon-\delta})/(x_{\varepsilon-\delta}), \tau(x_{\varepsilon-\delta})/(x_{\varepsilon-\delta}))$. Under τ, any such agent consumes less than $x_{\varepsilon-\delta}$ (due to an insufficient budget) but will surely choose some $x \geq x_{\varepsilon-\delta}$ under τ_δ, thereby increasing her consumption. ||

Next, in the Supplementary Appendix, we establish the following facts: (iv) if $\varepsilon > 0$ is small enough, then no mechanism τ_δ for any $\delta \in [0,\varepsilon]$ runs a budget deficit (Supplementary Lemma S.4); (v) $\mathbb{E}[x_\delta(w,v)]$ is continuously increasing with δ (Supplementary Lemma S.6). Now note that $\tau_0 = \tau^{v_\varepsilon}$ so τ_0 generates excess supply by assumption; i.e. $\mathbb{E}[x_0(w,v)] = \mathbb{E}[x^{v_\varepsilon}(w,v)] < S$. Also, since $v_{\varepsilon-\varepsilon} = v_0$, Claim A1 implies that all agents consume weakly more under τ_ϵ than under τ so $\mathbb{E}[x_\varepsilon(w,v)] \geq S$. By (v), one can find some $\hat{\delta} \in (0,\varepsilon]$ with $\mathbb{E}[x_{\hat{\delta}}(w,v)] = S = \mathbb{E}[x(w,v)]$. This is a contradiction since Claim A1 implies that $\tau_{\hat{\delta}}$ generates greater welfare than τ does.

To prove the second part of the Proposition, suppose that a measure zero of agents receives the good with an interior probability. Then, the optimal mechanism effectively consists of a menu of two contracts, $\{(0,t_0),(1,t_1)\}$. Given this, it is obvious that $t_1 > t_0$, or else only the second contract will be chosen, which one can easily show to be suboptimal. Optimality requires that $t_1 < 1$ and $t_0 < 0$ (this follows from the above result that the budget-balancing constraint binds). Under this mechanism, agents with $v \geq t_1 - t_0$ and wealth $w \geq t_1$ obtain the good (one unit apiece) so the total assignment is

$$[1 - F(t_1 - t_0)][1 - G(t_1)] := \tilde{D}(t_1), \tag{A.1}$$

and the revenue it generates is at most

$$t_1 \tilde{D}(t_1) + t_0[1 - \tilde{D}(t_1)]. \tag{A.2}$$

The average per-unit surplus realized is $\phi(t_1 - t_0)$. We shall argue that an alternative mechanism assigns the same mass as (A.1), generates weakly higher revenue than (A.2) and yields an average per-unit surplus strictly greater than $\phi(t_1 - t_0)$.

Consider a mechanism that offers a menu of three contracts, $\{(0, t_0), (\delta, t_0 + \delta(t - t_0)), (1, t)\}$, where $\delta \in (0,1)$ and t are chosen so that

$$\tilde{D}(t) + \delta[1 - F(t - t_0)]G(t) = \tilde{D}(t_1) \tag{A.3}$$

and $t_0 + \delta(t - t_0) < 0$. Note that the left-hand side of (A.3) represents the total assignment under the new mechanism; the first term again accounts for those who choose $(1, t)$ and the second term accounts for those who choose the middle contract, $(\delta, t_0 + \delta(t - t_0))$ (these agents have $v \geq t - t_0$ so they prefer a higher probability of obtaining the good but cannot afford the last contract since $w \in [0, t)$). It follows that the average per-unit surplus under the new mechanism is $\phi(t - t_0)$. This mechanism thus generates a budget surplus of

$$t\tilde{D}(t) + (t_0 + \delta(t - t_0))[1 - F(t - t_0)]G(t) + t_0 F(t - t_0). \tag{A.4}$$

Totally differentiating (A.3) and evaluating it at $(\delta, t) = (0, t_1)$, we get

$$(d\delta)[1 - F(t_1 - t_0)]G(t_1) + (dt)\tilde{D}'(t_1) = 0, \tag{A.5}$$

where we used the fact that $t_0 < 0$. Equation (A.5) suggests that $dt/d\delta$ is well defined (by the implicit function theorem) and is strictly positive since $\tilde{D}'(t_1) < 0$. Taking the derivative of (A.4) at $(\delta, t) = (0, t_1)$ along (A.3) yields:

$$\begin{aligned} &(t_1 - t_0)[1 - F(t_1 - t_0)]G(t_1) + \left.\frac{dt}{d\delta}\right|_{(A.3)} \left(\tilde{D}(t_1) + (t_1 - t_0)\tilde{D}'(t_1)\right) \\ &= (t_1 - t_0)\left[[1 - F(t_1 - t_0)]G(t_1) + \left.\frac{dt}{d\delta}\right|_{(A.3)} \tilde{D}'(t_1)\right] + \left.\frac{dt}{d\delta}\right|_{(A.3)} \tilde{D}(t_1) \\ &= \left.\frac{dt}{d\delta}\right|_{(A.3)} \tilde{D}(t_1) \\ &> 0, \end{aligned}$$

where the second equality follows from (A.5) and the strict inequality holds since $\frac{dt}{d\delta}|_{(A.3)} > 0$ and $\tilde{D}(t_1) > 0$.

The results so far imply that the alternative mechanism with δ sufficiently small assigns the same mass of the good and yields higher revenue than the original mechanism. It is easy to see that the mechanism improves (Utilitarian) efficiency since $\frac{dt}{d\delta}|_{(A.3)}\phi'(t_1 - t_0) > 0$. We have thus proven that the original mechanism could not have been optimal, as was to be shown. ||

Proof of Proposition 3. We begin with the following simple observation.

Lemma A1. *The first-best outcome is attainable if either $S \leq g_H f_H$ or $w_L \geq v_L$.*

Proof If $S \leq g_H f_H$, then selling the good at price $p = v_H$ achieves the first-best allocation. Next, if $w_L \geq v_L$ and $S \in (g_H f_H, f_H]$, then the first-best can be achieved by assigning the good to all high-valuation types with probability S/f_H and charging them $\min\{v_H, w_L\}$ upon receipt. If $w_L \geq v_L$

and $S \in (f_H, 1]$, then the first-best can be achieved by assigning the good to high-valuation types with probability 1 and low-valuation types with probability $\frac{S-f_H}{1-f_H}$, while charging them all v_L upon receipt. ‖

Given Lemma A1, we assume from now on that $S > g_H f_H$ and $v_L > w_L$. We consider a relaxed program $[P']$, which is the same as $[P]$ except that (IC) is replaced by a weaker requirement:

$$vx(w,v) - t(w,v) \geq vx(w',v') - t(w',v') \text{ for all } (w,v) \text{ and } (w',v') \text{ such that } w' \leq w. \qquad (IC')$$

The condition (IC') is weaker than (IC) since the former only requires that incentive compatibility hold with respect to contracts offered to agents with equal or lower wealth. The set of mechanisms satisfying (IC') is closed (unlike those satisfying (IC)). Hence, the set $\mathcal{M}$ of mechanisms satisfying $(S), (IR), (BC), (IC')$, and (BB) is also closed. One can easily see that $\mathcal{M}$ is also bounded, so it is compact. Since the objective function of $[P']$ is linear (and thus continuous), the set $\mathcal{M}^* \subset \mathcal{M}$ of solutions to $[P']$ is nonempty. It can also be easily seen that $\mathcal{M}^*$ is closed. Hence, the following result is obtained.

Lemma A2. *$\mathcal{M}^*$ is non-empty and compact.*

We next establish a series of lemmas characterizing the properties of an optimal mechanism in $\mathcal{M}^*$. To this end, we often use b_{ij} to denote a contract (x_{ij}, t_{ij}) for the type (w_i, v_j). We first show that the total budget is balanced and the entire supply is allocated at any optimal mechanism whenever it does not implement the first-best.

Lemma A3. *Suppose that the solution to $[P']$ does not achieve the first-best outcome. Then, both (BB) and (S) must be binding at any $\Gamma \in \mathcal{M}^*$.*

Proof Suppose, to the contrary, that there is an optimal mechanism $\Gamma = \{b_{ij}\}_{i,j\in\{L,H\}} \in \mathcal{M}^*$ at which (BB) does not bind. Now consider a relaxed program in which (BB) is absent. Its solution, $\Gamma^F = \{b_{ij}^F\}_{i,j\in\{L,H\}}$, must implement the first-best outcome. (Each agent can be provided a subsidy of $M \geq v_H$, and the good can be sold in a competitive market.) Next, consider an alternative mechanism, $\Gamma^\lambda = \{b_{ij}^\lambda\}_{i,j\in\{L,H\}}$, where $b_{ij}^\lambda = \lambda b_{ij} + (1-\lambda) b_{ij}^F$ for $\lambda \in [0,1]$. By hypothesis, the optimal mechanism does not implement the first-best, for any $\lambda \in [0,1)$, so Γ^λ must yield a higher aggregate surplus than Γ does. Since $\mathcal{M}$ is convex, Γ^λ satisfies (S), (IR), (BC), and (IC') for any $\lambda \in [0,1]$. Further, since (BB) is not binding at Γ, Γ^λ satisfies (BB), for $\lambda < 1$ sufficiently close to one. In sum, Γ^λ, with $\lambda < 1$ but sufficiently close to one, is feasible and yields a higher surplus than Γ, which contradicts the optimality of Γ. That (S) is binding at the optimum can be shown analogously, so we omit the proof. ‖

By Lemma A3, we assume that (BB) and (S) are binding throughout. We now further characterize the optimal mechanism.

Lemma A4. *Any optimal mechanism $\Gamma \in \mathcal{M}^*$ must have $x_{LH} \geq \max\{x_{HL}, x_{LL}\}$.*

Proof By definition, a mechanism implementing the first-best outcome must satisfy the inequality. Hence, assume that the first-best is not implementable and suppose, contrary to the premise, that $x_{LH} < \max\{x_{HL}, x_{LL}\}$ at some optimal mechanism $\Gamma \in \mathcal{M}^*$. Note that the usual incentive compatibility argument implies $x_{LH} \geq x_{LL}$ and $x_{HH} \geq x_{HL}$, which means $x_{LL} \leq x_{LH} < x_{HL} \leq x_{HH}$. We first establish the following claim:

Claim A2. *At any $\Gamma \in \mathcal{M}^*$, a type-(w_H, v_H) agent must strictly prefer b_{HH} to both b_{LH} and b_{LL}. Also, if $x_{LH} > x_{LL}$, then $t_{LL} < w_L x_{LL}$ and a type-(w_L, v_H) agent must strictly prefer b_{LH} to b_{LL}.*

Proof Condition (IC') for type (w_H, v_L) implies that $v_L x_{Lj} - t_{Lj} \leq v_L x_{HL} - t_{HL}, \forall j$. Given $x_{HL} > x_{LH} \geq x_{LL}$, this, together with (IC') for type (w_H, v_H), implies that

$$v_H x_{Lj} - t_{Lj} < v_H x_{HL} - t_{HL} \leq v_H x_{HH} - t_{HH}, \forall j.$$

Hence, a type-(w_H, v_H) strictly prefers b_{HH} to both b_{LH} and b_{LL}, which proves the first statement of the claim.

To prove the second statement, note that (IC') for type (w_L, v_L) implies $v_L x_{LL} - t_{LL} \geq v_L x_{LH} - t_{LH}$. This in turn implies

$$t_{LL} - w_L x_{LL} \leq t_{LH} - v_L(x_{LH} - x_{LL}) - w_L x_{LL} \leq w_L(x_{LH} - x_{LL}) - v_L(x_{LH} - x_{LL}) < 0,$$

where the second inequality follows from (BC) and the strict inequality follows from $w_L < v_L$. We have thus proven $t_{LL} < w_L x_{LL}$. To prove the last part, suppose, to the contrary, that a type-(w_L, v_H) agent is indifferent between b_{LH} and b_{LL} in Γ. Then, a type-(w_L, v_L) agent would strictly prefer b_{LL} to b_{LH}. Hence, we can raise t_{LL} slightly, holding everything else the same, and still satisfy all of the constraints of $[P']$. This will leave the aggregate surplus unchanged but increase the total revenue relative to Γ. This contradicts Lemma A3, according to which any optimal mechanism in $\mathcal{M}^*$ must balance the budget. ||

Claim A3. *If $x_{HL} > x_{LH}$ at $\Gamma \in \mathcal{M}^*$, then we must have $x_{LH} = x_{LL}$.*

Proof Suppose, to the contrary, that $x_{LH} > x_{LL}$. (This is the only case to check as the reverse inequality is inconsistent with incentive compatibility.) We then have $x_{LL} < x_{LH} < x_{HL} \leq x_{HH}$ and thus $t_{LL} < w_L x_{LL}$ by Claim A2. For a contradiction, we construct another mechanism, $\tilde{\Gamma} = \{\tilde{b}_{ij}\}_{i,j \in \{L,H\}}$, where $\tilde{b}_{LH} = b_{LH}$:

$$\tilde{b}_{HH} = (x_{HH}, t_{HH} + \delta),\ \tilde{b}_{HL} = (x_{HL} - \varepsilon, t_{HL} - \varepsilon v_L), \text{ and } \tilde{b}_{LL} = (x_{LL} + \varepsilon', t_{LL} + \varepsilon' v_L),$$

where $\varepsilon, \varepsilon', \delta > 0$ are chosen to be small enough to satisfy: (i) $\varepsilon g_H = \varepsilon' g_L$; (ii) $\tilde{t}_{LL} < w_L \tilde{x}_{LL}$; (iii) (w_H, v_H) prefers $\tilde{b}_{HH}$ to both $\tilde{b}_{LH}$ and $\tilde{b}_{LL}$; and (iv) (w_L, v_H) prefers $\tilde{b}_{LH}$ to $\tilde{b}_{LL}$. Note that such $\varepsilon, \varepsilon', \delta > 0$ exist by Claim A2.

We first show that $\tilde{\Gamma}$ satisfies (IC'). First, (iv) ensures that (w_L, v_H) does not mimic (w_L, v_L). The latter does not mimic the former either since (w_L, v_L) is indifferent between $\tilde{b}_{LL}$ and b_{LL} and prefers b_{LL} to b_{LH}. Next, a type-(w_H, v_L) agent does not deviate to $\tilde{b}_{LH}$, since she is indifferent between $\tilde{b}_{HL}$ and b_{HL}, and weakly prefers b_{HL} to $\tilde{b}_{LH} = b_{LH}$; she will also not deviate to $\tilde{b}_{HH}$ since $\tilde{b}_{HH}$ is worse than b_{HH} (by (i)), and she does not deviate to $\tilde{b}_{LL}$ since she is indifferent between $\tilde{b}_{LL}$ and b_{LL}, and she (weakly) prefers $\tilde{b}_{LH} = b_{LH}$ to b_{LL}. Given (iii), it now remains to see that a type-(w_H, v_H) agent has no incentive to deviate to $\tilde{b}_{HL}$. Since Γ satisfies (IC') for (w_H, v_H), $v_H x_{HH} - t_{HH} \geq v_H x_{HL} - t_{HL} > v_H \tilde{x}_{HL} - \tilde{t}_{HL}$, where the strict inequality follows from the fact that $v_L x_{iL} - t_{iL} = v_L \tilde{x}_{iL} - \tilde{t}_{iL}$ and $\tilde{x}_{HL} < x_{HL}$. So, for $\delta > 0$ sufficiently small, a type-(w_H, v_H) agent prefers $\tilde{b}_{HH}$ to $\tilde{b}_{HL}$.

Now observe that $\tilde{\Gamma}$ generates the same surplus that Γ does, and they assign the same quantity of the good since $\sum_{i,j \in \{L,H\}} g_i f_j \tilde{x}_{ij} = \sum_{i,j \in \{L,H\}} g_i f_j x_{ij}$. Yet, $\tilde{\Gamma}$ generates higher revenue than Γ since $\sum_{i,j \in \{L,H\}} g_i f_j \tilde{t}_{ij} = g_H f_H \delta + \sum_{i,j \in \{L,H\}} g_i f_j t_{ij}$, so we have a contradiction. ||

We are now ready to prove Lemma A4. It follows from Claim A3 that if $\Gamma \in \mathcal{M}^*$ has $x_{HL} > x_{LH}$, then $x_{LL} = x_{LH} < x_{HL} \leq x_{HH}$. For any such assignment probabilities, we can choose payments $t_{LL} = t_{LH} = 0$, $t_{HL} = v_L(x_{HL} - x_{LH}) > 0$, and $t_{HH} = t_{HL} + v_H(x_{HH} - x_{HL}) > 0$ to satisfy (IC').[44] This mechanism generates a budget surplus, which contradicts Lemma A3. We conclude that $x_{LH} \geq x_{HL}$, which in turn implies $x_{LH} \geq \max\{x_{HL}, x_{LL}\}$, given $x_{HL} \geq x_{LL}$ by (IC'). ‖

Let $\mathcal{M}_m \subset \mathcal{M}^*$ be the set of mechanisms that generate the highest total revenue among optimal mechanisms in $\mathcal{M}^*$. $\mathcal{M}_m$ is nonempty since $\mathcal{M}^*$ is compact by Lemma A2. Without loss of generality, we focus on these maximal-revenue optimal mechanisms.

Lemma A5. *Suppose that a mechanism in $\mathcal{M}_m$ satisfies $x_{LH} \geq \max\{x_{HL}, x_{LL}\}$. Then, there exists a mechanism in $\mathcal{M}_m$ in which $b_{HL} = b_{LL} = (x_L, t_L)$ and $x_L \leq x_{LH}$. Any such mechanism in $\mathcal{M}_m$ must satisfy*

$$t_{LH} = w_L x_{LH},\; t_{HH} = v_H x_{HH} - (v_H x_{LH} - t_{LH}),\; t_L = v_L x_L - (v_L x_{LH} - t_{LH}). \tag{A.6}$$

Proof Suppose that a mechanism $\Gamma \in \mathcal{M}_m$ has $b_{LL} \neq b_{HL}$. Then, we can construct an alternative mechanism Γ' in which $b_L = (x_L, t_L) = g_L b_{LL} + g_H b_{HL}$, and all other contracts remain the same as in Γ. Note that the low-valuation agents are indifferent among b_{HL}, b_{LL}, and b_L. And it is straightforward, and thus omitted, to check (IC'). Since Γ obviously yields the same total revenue and same total surplus as Γ, Γ' must also belong to $\mathcal{M}_m$. By hypothesis, we have $x_{LH} \geq \max\{x_{HL}, x_{LL}\} \geq x_L$.

Fix any $\Gamma \in \mathcal{M}_m$ with $b_{HL} = b_{LL} = (x_L, t_L)$. We prove that Γ must satisfy (A.6). To this end, note first that, given $x_{HH} \geq x_{LH} \geq x_L$, payments in (A.6) satisfy (IC'), (IR), and (BC).[45] To see that Γ must satisfy (A.6), note first that, given $x_{LH} \geq x_L$, a type-(w_H, v_H) agent would prefer b_{HH} to b_L whenever she prefers b_{HH} to b_{LH}. If the latter preference is strict, though, we can raise t_{HH} slightly, all else equal, so Γ could not be revenue-maximal. Hence, a type-(w_H, v_H) agent must be indifferent between b_{HH} and b_{LH}. This leads to the second equation of (A.6). By the same logic, a low-valuation agent must be indifferent between b_L and b_{LH}, which yields the third equality in (A.6). To prove the first equality, observe first that (BC) requires $t_{LH} \leq w_L x_{LH}$. Suppose $t_{LH} < w_L x_{LH}$. Then, we can raise t_{LH} slightly and raise t_{HH} and t_L so as to satisfy (A.6). This maintains (IC'), (IR), and (BC). Since total revenue rises in the process, Γ cannot be in $\mathcal{M}_m$. We conclude that $t_{LH} = w_L x_{LH}$. ‖

By Lemma A5, there exists an optimal mechanism $\Gamma_m = (x, t) \in \mathcal{M}_m$ with $b_{HL} = b_{LL} = (x_L, t_L)$.

Lemma A6. *Suppose $S \geq g_H f_H$. Then, Γ_m has $x_{HH} = 1$.*

Proof Suppose, to the contrary, that $x_{HH} < 1$ in Γ_m. By Lemma A4, $x_{LH} \geq x_{HL}$. Hence, Lemma A5 applies, so the payments are given by (A.6). We can then use (A.6) and $x_{HH} =$

44. Note that with these payments, (w_H, v_L) is indifferent between b_{HL} and b_{LH} while (w_H, v_H) is indifferent between b_{HH} and b_{HL}.

45. To see the latter, observe $t_{LH} = w_L x_{LH}$ and $t_L = v_L x_L - (v_L x_{LH} - t_{LH}) = v_L x_L - (v_L - w_L) x_{LH} \leq w_L x_L$, where the inequality follows from $(v_L - w_L) x_L \leq (v_L - w_L) x_{LH}$ and our assumption $v_L > w_L$.

$(S-g_Lf_Hx_{LH}-f_Lx_L)/(g_Hf_H)$ to show that the total revenue must equal

$$B=\sum_{i,j\in\{L,H\}} g_if_jt_{ij}$$
$$=-x_{LH}\big[(1+g_Lf_H)(v_H-w_L)+f_L(v_L-w_L)\big]-x_L\big[v_H-f_Lv_L\big]+v_HS. \tag{A.7}$$

Note that the expressions within both pairs of square brackets are positive. Given $x_{HH}<1$, either x_{LH} or x_L must be strictly positive, or else we will have a contradiction to the fact that $S\geq g_Hf_H$. We can then increase the revenue by lowering either x_{LH} or x_L, and increasing x_{HH} at the same time (while the payments are adjusted to satisfy equation (A.6)). Since this change keeps total surplus the same as in Γ_m, it contradicts the assertion that $\Gamma_m\in\mathcal{M}_m$. ‖

We are now ready to characterize an optimal mechanism; i.e., verify the statements of Proposition 3. In light of Lemma A1, we again focus on the case of $S>g_Hf_H$ and $v_L>w_L$.

We begin by showing that equation (5) is necessary and sufficient for implementing the first-best. By definition, the first-best assignment means that $x_{LH}\geq\max\{x_{HL},x_{LL}\}$. Hence, Lemma A5 applies.

Suppose that $S\geq f_H$. Then, the first-best assignment has $x_{HH}=x_{LH}=1$ and $x_L=(S-f_H)/f_L\geq 0$. The corresponding payments equation (A.6) then entails a budget surplus (i.e., $B\geq 0$) if and only if $w_L\geq(1-S)v_L$. This latter condition coincides with equation (5) if $S\geq f_H$.[46]

Now suppose that $S\in(g_Hf_H,f_H)$. Then, by Lemma A5, the first-best, if implementable, must be implemented by a mechanism that has $x_{HH}=1$ and $x_{LH}=(S-g_Hf_H)/(g_Lf_H)$. The corresponding payments equations (A.6) then entail a budget surplus if and only if $w_L\geq f_Lv_L-g_Hf_Hv_H\,((f_H-S)/(S-g_Hf_H))$, which is equivalent to equation (5) if $S\leq f_H$.

In sum, we conclude that equation (5) is necessary and sufficient for implementing the first-best. Suppose equation (5) fails. Then, since $S\geq g_Hf_H$ has been assumed, Lemma A6 implies that $x_{HH}=1$. By Lemma A4, we also have $x_{LH}\geq\max\{x_{HL},x_{LL}\}$, and this in turn implies, via Lemma A5, that $x_{HL}=x_{LL}=x_L$ (without loss), and the corresponding payments are given by equation (A.6). Invoking the fact that the entire supply S is allocated at the optimum (see Lemma A3), we obtain

$$1=x_{HH}=\frac{S-g_Lf_Hx_{LH}-f_Lx_L}{g_Hf_H}. \tag{A.8}$$

Finally, since $B=0$ at the optimum (by Lemma A3), equation (A.7) yields

$$0=-x_{LH}\big[(1+g_Lf_H)(v_H-w_L)+f_L(v_L-w_L)\big]-x_L\big[v_H-f_Lv_L\big]+v_HS. \tag{A.9}$$

Solving equations (A.8) and (A.9) simultaneously (together with equation (A.6)) provides the characterization stated in Proposition 3. The mechanism described in Proposition 3 also satisfies (IC). Hence, the mechanism solves [P] as well. ‖

Proof of Proposition 4. We first argue that in equilibrium all high-valuation agents must participate in the program. If a positive mass of low-valuation agents participate, the fact that low-valuation agents find it optimal to participate means that the high-valuation agents find it

46. It can be checked that

$$(1-S)v_L\leq(\geq)f_Lv_L-g_Hf_Hv_H\left(\frac{f_H-S}{S-g_Hf_H}\right)\text{ if } S\geq(\leq)f_H.$$

strictly optimal to participate. Now suppose that no low-valuation agents participate. Then, high-valuation agents must obtain the good with probability $\min\{\frac{S}{f_H},1\}>x_{LH}$.[47] Now recall that the low-valuation agents' incentive compatibility constraint is binding:

$$v_L x_L - t_L = v_L x_{LH} - t_{LH}, \tag{A.10}$$

which implies that

$$v_H\left(\min\left\{\frac{S}{f_H},1\right\}\right)-t_{LH}>v_H x_{LH}-t_{LH}>v_L x_{LH}-t_{LH}=v_L x_L-t_L=C,$$

proving that all high-valuation agents must strictly prefer to participate in the initial assignment.

We next argue that a mass of exactly $\hat{Z}=S/x_{LH}-f_H$ of low-valuation agents must participate in equilibrium. To see this, suppose that a mass $Z<\hat{Z}$ participate. Then, a low-valuation agent who participates will obtain the good with probability $S/(f_H+Z)>S/(f_H+\hat{Z})=x_{LH}$. Hence, (A.10) implies that

$$v_L\frac{S}{f_H+Z}-t_{LH}>v_L x_{LH}-t_{LH}=v_L x_L-t_L=C,$$

so all low-valuation agents must participate, contradicting $Z<\hat{Z}\leq 1$.

Next, suppose that a mass $Z>\hat{Z}$ of low-valuation agents participate. Given the tax rate $\tau=v_H-v_L$, no agent can earn more than v_L by selling the good on the resale market, so any participating agent must earn at most

$$v_L\frac{S}{f_H+Z}-t_{LH}<v_L x_{LH}-t_{LH}=v_L x_L-t_L=C,$$

47. This follows from the fact that (S) is binding in the optimal mechanism, which means that $g_H f_H+g_L f_H x_{LH}=S$, or $g_H+g_L x_{LH}=\frac{S}{f_H}$. The inequality then follows since $g_H+g_L=1$ and $x_{LH}<1$.

so no low-valuation agents participate, contradicting $Z > \hat{Z} \geq 0$. These arguments also prove that it is an equilibrium for a mass $\hat{Z}$ of low-valuation agents to participate in the initial assignment.

In any such equilibrium, a mass $\hat{Z}x_{LH}$ of the good is held by low-valuation agents. At the same time, a mass $g_H f_H(1-x_{LH})$ of high-valuation agent is not assigned the good in the initial assignment. Note that the former (weakly) exceeds the latter since

$$\hat{Z}x_{LH} = S - f_H x_{LH} = g_H f_H + g_L f_H x_{LH} + f_L x_L - f_H x_{LH} = g_H f_H(1-x_{LH}) + f_L x_L, \tag{A.11}$$

where the second equality follows from the fact that (S) is binding in the optimal mechanism (i.e., $S = g_H f_H + g_L f_H x_{LH} + f_L x_L$). The low-valuation agents who received the good are indifferent to selling it at the price v_H, which will yield after-tax revenue of v_L. Likewise, the high-valuation agents who have not received the good are indifferent to buying at the price v_H, and they are able to do so if they have high wealth. Hence, there exists an equilibrium in the resale market in which a mass $g_H f_H(1-x_{LH})$ of the good held by low-valuation agents is sold to high-valuation agents who did not receive the good in the initial assignment.

In the chosen equilibrium, the same assignment is implemented as in the optimal mechanism; namely, the high-wealth high-valuation agents receive the good with probability one, and the low-wealth high-valuation agents receive the good with probability x_{LH}. The two types enjoy the same payoff, just as in the optimal mechanism where the high-wealth high-valuation agents' incentive constraint is binding. Next, the low-valuation agents end up, collectively, with a mass

$$\hat{Z}x_{LH} - g_H f_H(1-x_{LH}) = f_L x_L.$$

of the good, by (A.11), and they each enjoy the payoff $C = v_L x_L - t_L$, just as in the optimal mechanism.

Remark A1. *Note that the implementation described above is not unique since any trading volume less than or equal to $g_H f_H(1-x_{LH})$ can be supported in the resale market. Yet, the second-best can be virtually uniquely implemented in the following sense. Suppose that the planner imposes a tax $\tau(\varepsilon) = \tau - \varepsilon$. For sufficiently small $\varepsilon > 0$, there exists a mechanism with cash subsidy $C(\varepsilon) = v_L(x_{LH}(\varepsilon)) - t_{LH}$ that implements uniquely the outcome in which a mass $\hat{Z}(\varepsilon) = \frac{S}{x_{LH}(\varepsilon)} - f_H$ of low-valuation agents participate in the program and sell at the price $v_L + \tau(\varepsilon)$ (since they are the long side of the market). One can show that there exists $x_{LH}(\varepsilon) = x_{LH} - O(\varepsilon)$ such that the mechanism is budget-balanced in equilibrium. Clearly, the implemented outcome converges to the optimal one as $\varepsilon \to 0$.* ||

Proof of Proposition 5. Following the proof of Proposition 3, we consider the relaxed problem with (IC) replaced by (IC'). Let $\mathcal{M}^*$ denote the set of optimal mechanisms. Analogously to Lemma A2, this set is well defined. Lemma A3 extends similarly: (BB) and (S) bind at any $\Gamma \in \mathcal{M}^*$. We shall prove that as with Proposition 3, our optimal mechanism satisfies $x^s_{HH} \geq x^s_{LH} \geq x^s_{HL} = x^s_{LL}$ for each $s = \ell, h$ with the payments given by equation (A.6). Such a mechanism satisfies (IC) and thus solves the original problem as well.

As in the proof of Proposition 3, we focus on the set $\mathcal{M}_m \subset \mathcal{M}^*$ of mechanisms that generate the highest total revenue among optimal mechanisms, $\mathcal{M}^*$, which is well defined. Given a mechanism Γ, let $\Gamma^s = \{(x^s_{ij}, t^s_{ij})\}_{i,j \in \{L,H\}}$ for each group $s = \ell, h$.

Lemma A7. *Any mechanism $\Gamma \in \mathcal{M}_m$ must satisfy $S^s \geq h^s_{HH}$ and $x^s_{HH} = 1$ for all $s = \ell, h$.*

Proof Suppose, to the contrary, that $S^h < h^h_{HH} = \frac{\rho}{4}$. (The argument for group ℓ is exactly the same.) Clearly, the revenue and surplus for group h are maximized by assigning the entire amount S^h to the type (w_H, v_H) and charging them v_H per unit. Since $S^\ell = S - S^h > \frac{1-\rho}{4} = h^\ell_{HH}$, some low-wealth or low-valuation agents in group ℓ must be receiving the good. Now reassign some of the good away from those types in group ℓ toward the type-(w_H, v_H) agents in group h, and charge the latter types v_H per additional unit. This requires some payments in Γ^ℓ_m to be reduced so as to maintain (IC'), but the reduction in each payment is strictly less than v_H, so revenue and total surplus both increase as a result of the reassignment. We therefore have a contradiction.

We next show that $x^s_{HH} = 1$ for each $s = \ell, h$. To this end, fix $s = \ell, h$ and consider two cases: $x^s_{LH} \geq x^s_{HL}$ and $x^s_{LH} < x^s_{HL}$. In the former case, the same argument as in Lemma A6 proves that $x^s_{HH} = 1$. Now consider $x^s_{LH} < x^s_{HL}$ and suppose, to the contrary, that $x^s_{HH} < 1$. To obtain a contradiction, we construct an alternative mechanism, $\tilde{\Gamma}$, which has the same contracts as Γ except that

$$\tilde{b}^s_{HH} = (x^s_{HH} + \varepsilon, t^s_{HH} + \varepsilon v_H) \text{ and } \tilde{b}^s_{HL} = \left(x^s_{HL} - \frac{n_{HH}\varepsilon}{n_{HL}}, t^s_{HL} - \frac{n_{HH}\varepsilon}{n_{HL}} v_L \right),$$

where $\varepsilon > 0$ is sufficiently small. It is straightforward to check (so we omit the details) that $\tilde{\Gamma}$ satisfies (IC'). Now observe that $\tilde{\Gamma}$ satisfies $\sum_{s,i,j} n^s_{ij} \tilde{x}^s_{ij} = \sum_{s,i,j} n^s_{ij} x^s_{ij}$ and $\sum_{s,i,j} n^s_{ij} \tilde{t}^s_{ij} = \sum_{s,i,j} n^s_{ij} t^s_{ij} + \varepsilon n_{HH}(v_H - v_L) > \sum_{s,i,j} n^s_{ij} t^s_{ij}$ but generates a higher total surplus than Γ. We now have the desired contradiction. ‖

We now show that the first-best is attained if and only if

$$w_L \geq \hat{w}_L(\rho) := \begin{cases} \frac{v_L}{2} - \frac{(1-4S+4\rho S)}{2(4S-1)} & \text{if } S \in (\frac{1}{4}, \frac{1+\rho}{4}) \\ \frac{v_L}{2} - \frac{\rho(1-2S)}{2(2S-\rho)} & \text{if } S \in [\frac{1+\rho}{4}, \frac{1}{2}) \\ (1-S)v_L & \text{if } S \in [\frac{1}{2}, 1) \end{cases}.$$

To this end, suppose that $\Gamma \in \mathcal{M}_m$ attains the first-best, and let Γ^s, $s = \ell, h$, be the associated submechanism for group s. By definition, Γ satisfies $x^s_{LH} \geq \max\{x^s_{HL}, x^s_{LL}\}$. Hence, Lemmas A5 and A7 apply for Γ^s for each $s = \ell, h$. Hence, there exists a mechanism $\Gamma \in \mathcal{M}_m$ where $b^s_{HL} = b^s_{LL} = (x^s_L, t^s_L)$ with $x^s_L \leq x^s_{LH}$ and the payments are given by equation (A.6) with $x^s_{HH} = 1$, for each $s = \ell, h$.

Suppose, first, that $S \geq 1/2$. Then, all high-valuation types must receive the good with probability 1 at the first-best, which requires $S^s \geq 1/4$ for each s. Thus, for each s, substituting into (A.6) gives

$$x^s_{HH} = x^s_{HL} = 1 \text{ and } x^s_L = \frac{S^s - n^s_{HH} - n^s_{LH}}{n^s_{HL} + n^s_{LL}}. \tag{A.12}$$

Given our type distribution, rearrangement of terms yields $B^s = \frac{1}{2}(w_L - (1-2S^s)v_L)$. Using $S = S^\ell + S^h$, the total net budget surplus then equals $B = B^\ell + B^h = w_L - (1-S)v_L$. This last expression is nonnegative if $w_L \geq (1-S)v_L = \hat{w}_L(\rho)$, as desired.

We next consider the case of $S < 1/2$. In this case, only high-valuation types must receive the good in the first-best, which requires $S^s \leq 1/4$ for each s. Thus, for each s, substituting into (A.6) gives

$$x^s_{HH} = 1, x^s_{LH} = \frac{S^s - n^s_{HH}}{n^s_{LH}}, \text{ and } x^s_L = 0, \tag{A.13}$$

and calculate B^ℓ and B^h to obtain

$$B^\ell = \frac{(1-\rho)(1-4S^\ell)}{4\rho} v_H + \left(w_L - \frac{1}{2}v_L\right)\frac{(4S^\ell-(1-\rho))}{2\rho} \tag{A.14}$$

$$B^h = \frac{\rho(1-4S^h)}{4(1-\rho)} v_H + \left(w_L - \frac{1}{2}v_L\right)\frac{(4S^h-\rho)}{2(1-\rho)}. \tag{A.15}$$

Next, substitute $S^\ell = S - S^h$ into (A.14), differentiate $B = B^\ell + B^h$ with respect to S^h, and rearrange terms to obtain

$$\frac{dB}{dS^h} = \frac{2\rho-1}{\rho(1-\rho)}(2w_L - v_L - v_H) < 0$$

since $v_H > v_L > w_L$ and $\rho > 1/2$. The negative derivative implies that the total revenue is maximized by making S^h (S^ℓ) as small (large) as possible. Thus, if $S \in (\frac{1}{4}, \frac{1+\rho}{4})$, then $S^h = n^h_{HH} = \frac{\rho}{4}$ and $S^\ell = S - S^h$, and if $S \in [\frac{1+\rho}{4}, \frac{1}{2})$, then $S^\ell = n^\ell_{HH} + n^\ell_{LH} = \frac{1}{4}$ and $S^h = S - S^\ell$. Substituting these into (A.15) and (A.14) and requiring $B = B^\ell + B^h \geq 0$ yields $w_L \geq \hat{w}_L(\rho)$. Also, one can substitute these values of S^h and S^ℓ into (A.12) and (A.13) to obtain the first-best assignment of the good for each type. This completes the proof of (i).

We now turn to (ii) and (iii). We first establish the following lemma, the proof of which is provided in the Supplementary Appendix.

Lemma A8. *Suppose that $w_L < \hat{w}_L(\rho)$ and $v_H \geq \frac{(2-\rho)v_L}{(1-\rho)} - \frac{w_L}{1-\rho}$. Then, any $\Gamma \in \mathcal{M}_m$ must satisfy $x^s_{LH} \geq \max\{x^s_{HL}, x^s_{LL}\}, \forall s = \ell, h$.*

Thanks to this lemma, we can apply Lemmas A5 and A7 to obtain a mechanism, $\Gamma \in \mathcal{M}_m$, where $b^s_{HL} = b^s_{LL} = (x^s_L, t^s_L)$, with $x^s_L \leq x^s_{LH}$, and the payments are given by (A.6), with $x^s_{HH} = 1$.

Substituting (A.6) and rearranging terms gives the total revenue from Γ:

$$B^\ell = \frac{(1-\rho)}{4} v_H(1-x^\ell_{LH}) + \frac{1}{4}v_L(x^\ell_L - x^\ell_{LH}) + \frac{1}{2}w_L x^\ell_{LH} \tag{A.16}$$

$$B^h = \frac{\rho}{4} v_H(1-x^h_{LH}) + \frac{1}{4}v_L(x^h_L - x^h_{LH}) + \frac{1}{2}w_L x^h_{LH}. \tag{A.17}$$

Using this and substituting $\frac{1}{4}(x^h_L + x^\ell_L) = S - \frac{1}{4} - \frac{(1-\rho)}{4}x^h_{LH} - \frac{\rho}{4}x^\ell_{LH}$, one can calculate

$$B = B^\ell + B^h = \frac{1}{4}v_H + (S - \frac{1}{4})v_L - \frac{1}{4}C^\ell x^\ell_{LH} - \frac{1}{4}C^h x^h_{LH},$$

where $C^\ell := (1-\rho)v_H + (\rho+1)v_L - 2w_L$ and $C^h := \rho v_H + (2-\rho)v_L - 2w_L$. This leads us to write the binding (BB) as

$$C^\ell x^\ell_{LH} + C^h x^h_{LH} = v_H + (4S-1)v_L. \tag{A.18}$$

Note that $C^h - C^\ell = (2\rho-1)(v_H - v_L) > 0$ and $C^\ell > (1-\rho)v_L + (\rho+1)v_L - 2w_L = 2(v_L - w_L) > 0$.

Now the total surplus can be written as

$$\begin{aligned}
&\sum_{i,s} v_H n_{iH}^s x_{iH}^s + \sum_{i,s} v_L n_{iL}^s x_{iL}^s \\
=& v_H\Big[\frac{1}{4} + \frac{(1-\rho)}{4} x_{LH}^h + \frac{\rho}{4} x_{LH}^\ell\Big] + v_L\Big[S_a - \frac{1}{4} - \frac{(1-\rho)}{4} x_{LH}^h - \frac{\rho}{4} x_{LH}^\ell\Big] \\
=& \frac{1}{4} v_H + (S - \frac{1}{4}) v_L + \frac{1}{4}(v_H - v_L)\big[(1-\rho)x_{LH}^h + \rho x_{LH}^\ell\big]. \qquad \text{(A.19)}
\end{aligned}$$

Thus, given $C^h > C^\ell$ and $\rho > (1-\rho)$, it is clear that (A.19) is maximized by making x_{LH}^ℓ as large as possible and then choosing x_{LH}^h to satisfy (A.18). Since $x_{LH}^\ell = x_{LH}^h = 1$ is impossible when the first-best is unimplementable, we must have ether $1 = x_{LH}^\ell > x_{LH}^h \geq 0$ or $1 > x_{LH}^\ell > x_{LH}^h = 0$, proving (ii).

To prove (iii), recall that all agent types in group s are indifferent between their own contract and the type (w_L, v_H)'s contract, $(x_{LH}^s, x_{LH}^s w_L)$. Thus, we can write the payoff to a type (w_i, v_j) as $(v_j - w_L)x_{LH}^s$. Since $x_{LH}^\ell > x_{LH}^h$ (from (ii)) and $v_L > w_L$, statement (iii) follows; i.e., each type in group ℓ is better off than the corresponding type in group h. ‖

Proof of Proposition 6. We argue that the total surplus from an optimal mechanism approaches the level that is realized under the competitive market. We demonstrate this by showing that as $m \to \infty$, any feasible assignment $x_m(w,v)$ prior to resale must converge to 0 for almost every (w,v) such that $w < p^e$, where p^e is the equilibrium price in the competitive market (defined in Section 2). Suppose that this is not the case. Then, there exists a positive-measure set, $A = \{(w,v)|w < p^e\}$, of agents such that $\limsup_{m\to\infty} \inf_{(w,v)\in A} x(w,v) = \varepsilon$ for some $\varepsilon > 0$. Take a sequence of $m \to \infty$ such that the limit of $\varepsilon_m := \inf_{(w,v)\in A} x_m(w,v)$ is ε. As $m \to \infty$ along that sequence, there is a positive measure of agents in $(w,v) \in [p^e, 1]^2$ who do not obtain the good in the initial assignment. These agents demand the good in the resale market at any price p^e or less. Hence, the equilibrium resale price r_∞ cannot be strictly less than p^e, or else all agents with $(w,v) > (r,r)$ will obtain the good for sure (either through initial assignment or from a resale purchase) and the measure of these agents exceeds the supply, S. It follows that $r_m \to r_\infty \in [p^e, 1)$. But then all agents with zero valuation and wealth no less than p^e earn surplus of at least $r\varepsilon_m$. Since the measure of these agents is $m(1-G(p^e))$, the total surplus that these agents earn must be at least $r\varepsilon_m m(1-G(p^e))$ which converges to ∞ as m increases along the sequence. As argued above, this is not feasible since budget-balancing and individual rationality mean that the aggregate surplus that these agents can enjoy is bounded above by $\phi(v^*)S$. This contradiction means that, in the limit, almost no agent with $w < p^e$ can receive the good in any feasible mechanism. The highest welfare that can be realized subject to this constraint is $\phi(p^e)S$, and this welfare level is implementable by the planner when the good is initially sold via a competitive market. ‖

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SUPPLEMENTARY DATA

Supplementary data are available at *Review of Economic Studies* online.

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