Optimal Incentives for Teams

By Yeon-Koo Che and Seung-Weon Yoo*

Much of the existing theory of incentives describes a static relationship that lasts for just one transaction. This static assumption is not only unrealistic, but the resulting predictions appear to be at odds with many work organizations. The current paper introduces possible long-term interaction among agents, and studies how the design of explicit incentives and work organizations can exploit, and interact with, the implicit incentives generated by the repeated interaction of the agents. The optimal incentive scheme is shown to display observed features of the increasingly popular "teams," such as the use of low-powered, group incentives. (JEL D23, J33, J41, L23)

Teams have become increasingly popular in the U.S. workplace and elsewhere. Not only are teams adopted widely,¹ but many prominent firms boast of success stories associated with teams. Eastman Kodak Company reported a 70-percent decrease in emulation defect levels and considerable cost saving from their R&D Mul-timake Team;² AlliedSignal Aerospace reported an 11-percent increase in their aerospace revenue from a team-based approach in marketing;³ service firms such as Federal Express and IDS posted a productivity increase of up to 40 percent due to self-managed work teams (Dumain, 1994); Boeing used teams to cut the number of engineering hang-ups on its new 777 passenger jet by more than half (Dumain, 1994); quality improvement teams at Texas Instruments Malaysia are credited with saving $50 million in ten years by reducing their product-cycle time in half (Charles C. Manz and Henry P. Sims, Jr., 1993).⁴

While the specific characteristics of successful teams vary across cases, their common underlying philosophy is to foster cooperation among employees via several features of work practices. First, members of a self-managed team interact frequently and consistently over a long period ("long-term interaction").⁵ Second, members of teams are "empowered" to make

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¹ According to Paul Osterman’s (1995) survey based on the sample of 694, work teams are present in 54.5 percent of American establishments. Similarly, Brian Dumain (1994) estimated that about two-thirds of U.S. firms use work teams.

² See "Kodak Research and Development Receives Prestigious Award for Workforce Excellence" (Business Wire, October 14, 1999).

³ See "How We Brought Teamwork to Marketing" (The Wall Street Journal, August 26, 1996).

⁴ Other successful examples include Proctor & Gamble which treated teams as their trade secret, General Motor’s Saturn experiment, Gaines, Cummins Engine, Digital Equipment, Ford, Motorola, Tektronix, General Electric, Honeywell, LTV, Caterpillar, Monsanto, AT&T, and Xerox (Manz and Sims, 1993).

⁵ Not all team-based organizations have long life spans. Some problem-solving teams are temporary by design.
day-to-day decisions on project assignment and problem solving ("decentralized authority"). Third, instead of outside supervision, team members are encouraged to monitor and motivate each other ("peer monitoring"). Last, but not the least, rather than a competitive scheme such as relative performance evaluation, team members often work for cooperation-enhancing incentive schemes such as group incentives ("joint performance evaluation"). Recent empirical studies by Casey Ichniowski et al. (1997) and Brent Boning et al. (1998) based on steel finishing-line data find that adoption of work teams increases productivity, especially when they are combined with a set of "innovative" work practices mentioned above.

This heightened emphasis on the cooperative work practices marks a departure not only from the traditional work structure but also from the traditional economic theories. Up until recently, the economic theory of incentives largely focused on a short-term relationship among employees and advocated the use of incentive schemes that are sensitive to individual performance measures and induce competition among employees via tournaments or relative performance evaluation (see Oliver Hart and Bengt Holmstrom [1987] for a general overview). While these static models generated many useful findings and insights, their predictions are at odds with the fact that the actual incentive schemes adopted are low powered and that they seldom involve relative performance evaluation. It is often argued, for example, that relative performance evaluation discourages a cooperative work morale and encourages employees to adopt restrictive work norms and to penalize "rate busters" (see James N. Baron and David M. Kreps, 1999 p. 229). More recent work, such as Holmstrom and Milgrom (1990), Hal R. Varian (1990), Ram T. S. Ramakrishnan and Anjan V. Thakor (1991), Hideshi Itoh (1992, 1993), and Ines Macho-Stadler and J. David Perez-Castrillo (1993), does recognize the value of encouraging employees’ cooperation. In particular, Holmstrom and Milgrom (1990) and Itoh (1993) demonstrated that if employees coordinate their efforts and share risks in a Pareto-efficient fashion, then employers benefit from it by using a simple group-incentive scheme. Due to the static nature of these models, however, they left unexplained how cooperation among employees can be achieved.\(^7\)

The current paper explicitly addresses the enforcement issues associated with employees’ cooperation by introducing the possibility of their long-term, repeated interaction.\(^8\) Repeated worker interaction is realistic since many employment contracts have long-term or even open-ended life spans. Not surprisingly, introducing repeated interaction among employees enables us to explain their cooperation as self-enforcing behavior. But most importantly, it yields a host of new insights about the optimal design of work organizations that were absent in the extant literature. As we show below, repeated interaction can create implicit incentives by encouraging employees’ peer sanction. Furthermore, the design of the explicit incentives—through the design of compensation schemes and task assignment—influences the nature and scope of the employees’ dynamic interaction and hence the power of implicit incentives. By studying how the implicit incentives can be optimally induced by, and combined with, the explicit incentives, we develop a set of interrelated predictions on various aspects of organizational design.

Specifically, the current paper develops theoretical rationales for various team-oriented incentive features. First, we find that the compensation schemes that reward an employee when his coworkers perform well (what we call

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\(^7\) These papers simply assumed that the agents would achieve Pareto-efficient risk sharing and effort coordination. Not only are such risk-sharing agreements unobserved in practice but it is not clear how the agreements can be enforced. For example, if these agreements were enforced by an explicit side contract, then they would require agents’ effort decisions to be contractually specifiable. But the feasibility of such contracts would mean that the principal can directly contract on the efforts to solve the incentive problem completely, thus making agents’ cooperative arrangements superfluous. A similar enforcement issue exists with a literature on collusion (see, for instance, Jean Tirole, 1986; and Jean-Jacques Laffont and David Martimort, 1997).

\(^8\) In this sense, we view the current paper as complementary to the recent literature on worker cooperation mentioned above.
chian and Harold Demsetz (1972) and Holmstrom (1982), who focus on the effect that budget balancing has on the incentives of team members, and the work by R. Preston McAfee and John McMillan (1991), whose main concern is adverse selection in the multiple-agent environment.

The next section lays out basic features of the models. Sections II and III study the optimal compensation scheme and the optimal task assignment rule, respectively. Section IV concludes.

I. Preliminaries

A principal hires two identical agents, 1 and 2, to perform a project (or projects) in each period. All parties are risk neutral, except that the agents are subject to limited liability. We simply assume that the principal cannot impose negative wages on the agents. Limited liability of the agents may arise from workers’ having the freedom to quit but it may also arise from institutional constraints such as laws banning firms’ exacting payments from workers, workers’ liquidity constraints, or their extreme risk aversion for bearing loss. In each period, each agent makes a binary effort decision \( k \in \{0, 1\} \) at the cost of \( ke \), for \( e > 0 \); i.e., the agent either “shirks” \( k = 0 \) or “works” \( k = 1 \). The productive arrangement (or “team”) is an open-ended relationship, which is terminated at the end of each period with probability \( 1 - \delta \in (0, 1) \). The probability \( \delta \) can alternatively be interpreted as a common discount factor for the two agents. The former interpretation allows us to use \( \delta \) as a measure of the (expected) life span of the team relationship. Each agent’s reservation utility, which he can collect when the team is terminated or when he quits, is assumed to be zero.

Throughout, we assume that agents’ “work”

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9 The exception is Arya et al. (1997) who focus on group incentives via a two-period model.

10 This assumption yields the standard “efficiency wage” cost of incentive provision. By contrast, risk premium constitutes the incentive cost in the standard model of risk-averse agents.

11 Assuming that both the liability limit and the reservation utility are zero is consistent with the “quitting-constraint” interpretation of the former. The main results and the insight would continue to hold when the reservation utility is positive but is very close to zero.
is sufficiently valuable to the principal that she always prefers to induce the agents to “work.” Hence, the principal’s problem is simply to minimize the cost of motivating the agents. The incentives can be provided by the wages the principal offers at outset of the team arrangement, which satisfy several restrictions. First, the agents’ limited liability renders negative wages infeasible for the principal. Second, the wages are allowed to depend on verifiable signals that the principal receives in each period. In Section II, the principal receives a signal on individual performance, so the wage for each agent can, in general, depend on the realized signals on both agents, whereas in Section III the principal receives only a signal on joint performance. A more precise description of the signal structure will follow in each section. Third, the wage scheme is time invariant (or memoryless in the sense of Pierre-Andrê Chiappori et al., 1994) in that the wage scheme chosen initially applies to all subsequent periods. While this constitutes a restriction in the contract space, it can be justified as an equilibrium response by the principal when she cannot commit to a long-term contract, and it can even be justified as an optimal long-term contract if the agents’ efforts are sufficiently valuable. This assumption makes the comparison between the static and repeated settings transparent and allows us to focus on the strategic interaction among the agents.

Due to their close interaction, the agents observe each other’s effort decision in each period. As many authors have noted, such a mutual monitoring possibility is an important element of many team arrangements. As assumed, the principal can only observe a (imperfect) signal (or signals) about the agents’ effort decisions. Given mutual observability, the principal can conceivably require the agents to report their observations. Despite its theoretical interest, we limit our current scope of investigation by assuming that any communication between the principal and the agents is prohibitively costly. This abstraction is plausible in many environments where either the nature of the work/shirk cannot be easily described or is susceptible to manipulation.

As noted in the introduction, some recent articles demonstrated the value of cooperative arrangements such as teams, based on the assumption that the agents can directly side contract with each other. We do not allow such side-contracting abilities by the agents. In particular, the agents are not allowed to exchange side payments. Consequently, the agents can only interact through their effort decisions. Nonetheless, this limited interaction will be shown to generate a self-enforcing form of incentives. A strategy of each agent is a sequence of functions that map from any possible history into a probability distribution over the current effort choices. 13 A repeated setting of this kind can generate a host of (subgame-perfect) equilibria. In order to ensure a unique prediction, and more importantly to address the (negative) effect of the agents’ collusion, throughout the paper we focus on the subgame-perfect equilibrium that is most favorable to the agents in the sense that the chosen equilibrium yields the highest total payoff for the agents among all subgame-perfect equilibria. This restriction ensures that the agents have no incentive to “collude” against the selected subgame-perfect equilibrium, which is a natural concern when the agents have a close and repeated interaction. In what follows, we simply call such a subgame-perfect equilibrium a team equilibrium. An incentive scheme is said to be optimal if it induces the agents to “work” as a team equilibrium at the minimum cost.

II. Joint Performance Evaluation vs. Relative Performance Evaluation

An important issue in determining an employee’s compensation is whether it should be tied to the performance measures of his peers.

13 While we allow for correlated randomizations, their roles are very limited in our paper and, when they play any role, it is to strengthen our result. For instance, the team equilibrium concept that we introduce below requires an implemented outcome to be robust against some notion of collusion. Allowing correlated randomizations can only strengthen the burden of this robustness test and thus strengthen our notion of implementation.
and if so, how. Relative performance evaluation (hereafter RPE) penalizes an employee when his peers perform well whereas joint performance evaluation (hereafter JPE) rewards an employee when his peers perform well. Examples of JPE include divisional bonus, line incentives across “shifts” of workers (Ichniowski et al., 1997), and the group-based lending adopted in the Grameen bank (Varian, 1990). The existing literature has provided a rationale for RPE or its special form, tournaments, in a static setting. If the performance measures of workers have a common noise component, then an RPE-type scheme can be attractive since it insulates the workers from the risk of common shocks and thus generates a stronger incentive than other schemes, which allows the employer to lower the cost of providing any given level of incentive (see, for example, Lazear and Sherwin Rosen, 1981; Holmstrom, 1982; Jerry R. Green and Nancy L. Stokey, 1983; Barry J. Nalebuff and Joseph E. Stiglitz, 1983; Dilip Mookherjee, 1984; Holmstrom and Milgrom, 1990).

Several field studies, however, observed that RPE, besides being uncommon, is ineffective if workers interact closely with each other. For example, Carl E. Larson and Frank M. J. LaFasto (1989) document that work groups “pressure an individual into a slow pace and less output so as not to make other team members look bad” (p. 97). Since RPE entails extreme competition among team members, it does not provide an incentive for a team member to cooperate, such as to pick up the slack that occurs when another member falls behind. JPE may solve these problems. Susan A. Mohrman et al. (1995) suggest that performance evaluation practices should “minimize the competitive focus on individual performance, (and) tie together the fates of people who must work together” (pp. 233–34). Ichniowski et al. (1996) point out that “work groups may encourage both working harder and working smarter if their norms change from discouraging high performance, for example, punishing ‘rate busters,’ to rewarding high performance. These changes, in turn, are more likely if the group is rewarded for its collective success.” (p. 301). In this section, we provide a theoretical rationale for these observations by demonstrating how group incentives can be optimal when the agents interact over time.

To study optimal pay schemes for the agents, the principal is assumed to draw an individual signal for each agent’s effort. The individual signal, $x_i$, on agent $i$ is either good ($x_i = 1$) or bad ($x_i = 0$). Its probability distribution depends on the effort decision of that agent as well as a common environmental shock. The common shock is either favorable or unfavorable. The former event occurs with probability $\sigma \in (0, 1)$, in which case both agents receive the good signal, regardless of their individual effort decisions. If the common shock is unfavorable, which occurs with probability $1 - \sigma$, then the probability of getting a good signal depends on the individual effort of the agent: an agent receives a good signal with probability $q_1$ if he worked and with $q_0$ if he shirked, where $1 > q_1 > q_0 \geq 0$. In sum, if an agent picks $k \in \{0, 1\}$, he receives the good signal with probability $\sigma + (1 - \sigma)q_k$ and the bad signal with probability $(1 - \sigma)(1 - q_k)$. Notice that an agent’s effort decision does not affect the other agent’s probability of receiving a good signal. In this sense, the agents’ production technologies are independent.\(^{14}\)

Since the principal receives an individual signal on each agent, there are four possible combinations of signal realizations on which the wages for each agent can depend. Let $w_{ij}', w_{10}', w_{01}'$, denote the wage for agent $i$ if the principal receives signals $x_j \in \{0, 1\}$ and $x_j \in \{0, 1\}$ on agents $i$ and $j$, respectively. Let $w' = (w_{11}', w_{10}', w_{01}', w_{00}')$ denote a wage vector for agent $i$. Suppose that agents $i$ and $j$ choose efforts $k \in \{0, 1\}$ and $l \in \{0, 1\}$, respectively. Then, agent $i$’s expected wages under wage-scheme $w'$ is:

$$
\pi(k, l; w') = (\sigma + (1 - \sigma)q_k q_l)w_{11}' + (1 - \sigma)[q_k(1 - q_l)w_{10}' + (1 - q_k)q_l w_{01}'] + (1 - q_k)(1 - q_l)w_{00}']
$$

(Since the agents are symmetric, we will suppress superscripts unless necessary.)

\(^{14}\) Production externalities can clearly make joint performance evaluation an optimal method of internalizing them (see Itoh, 1991). We abstract from production externalities to focus on the peer sanction effect.
While the agents’ technologies may be independent, the agents’ payoffs can be interdependent if an agent’s wages are tied to his partner’s signal. It is useful to characterize a wage scheme based on how each agent’s wages are tied to his peer’s performance measure. For each agent, a wage scheme \( w \) exhibits JPE if \((w_{11}, w_{01}) > (w_{10}, w_{00})\). If \( w \) is JPE, then \( \pi(k, 1; w) > \pi(k, 0; w) \) for \( k \in \{0, 1\} \), so an agent’s work yields positive externalities to his partner. A wage scheme exhibits RPE if \((w_{11}, w_{01}) < (w_{10}, w_{00})\). In this case, \( \pi(k, 1; w) \leq \pi(k, 0; w) \) for \( k \in \{0, 1\} \), so an agent’s work generates a negative externality on his peer. Finally, a wage scheme exhibits “independent performance evaluation” (IPE hereafter) if \((w_{11}, w_{01}) = (w_{10}, w_{00})\). In this case, \( \pi(k, 1; w) = \pi(k, 0; w) \) for \( k \in \{0, 1\} \), so an agent’s work decision has no impact on his partner. As will be clear, the nature of payoff interdependence will influence how the incentive scheme shapes the strategic interaction among the agents and whether the agents will have an incentive to collude against “work.”

A. Static Benchmark

First, we consider the static benchmark in which the agents operate only for one period. A contract \( w \) induces both agents to work as a Nash equilibrium if and only if

\[
(IC_3) \quad \pi(1, 1; w) - e \geq \pi(0, 1; w),
\]

where the left-hand side represents an agent’s payoff when both agents work and the right-hand side represents his payoff when he unilaterally shirks. As mentioned earlier, the principal’s problem is to minimize her expected cost of inducing each agent to work. Consider the relaxed problem for the principal:

\[
[S] \quad \min_{w \geq 0} \pi(1, 1; w),
\]

subject to \((IC_3)\).

This is a relaxed program since \((IC_3)\) does not require the outcome to be a team equilibrium nor does it require the agents’ participation (although the latter holds trivially since \( w \geq 0 \), by limited liability.) The following proposition shows that the solution to \([S]\) displays a RPE scheme and induces both agents to work as a team equilibrium.

PROPOSITION 1: The optimal static wage scheme for each agent is an extreme form of RPE, \( w^* = (0, w_{10}^*, 0, 0) \), where

\[
w_{10}^* = \frac{e}{(1 - \sigma)(q_1 - q_0)(1 - q_1)}.
\]

PROOF:

It is straightforward to check that \( w^* \) solves \([S]\). We show here that the wage vector induces both agents to work as a team equilibrium. To show this, it suffices to show that, given the wage contract, the unique Nash equilibrium has both agents work. Given \( w^* \), \( \pi(k, l; w^*) \) is submodular in \((k, l)\):

\[
\pi(1, 1; w^*) + \pi(0, 0; w^*)
- [\pi(1, 0; w^*) + \pi(0, 1; w^*)]
= -(1 - \sigma)(q_1 - q_0)^2 w_{10}^* < 0.
\]

It follows that

\[
\pi(1, 0; w^*) - e > \pi(0, 0; w^*) + \pi(1, 1; w^*)
- e - \pi(0, 1; w^*) \geq \pi(0, 0; w^*),
\]

where the first inequality follows from the submodularity and the second follows from \((IC_3)\). This inequality implies that an agent strictly prefers to work if his coworker shirks with positive probability. It is then clear that the unique Nash equilibrium has both agents work, given \( w^* \).

This proposition replicates the well-known result from the literature on RPE and tournaments (see Holmstrom, 1982; Mookherjee, 1984). The intuition behind the result can be

15 As is conventionally the case, the inequality means weak inequality for each component and strict inequality for at least one component.

16 The optimality of the RPE scheme depends on the assumed specification of the common shock, and does not
more easily explained in the current framework. In the presence of the common shock assumed above, a good signal on an agent is a stronger indication of his effort ("work") when his partner receives a bad signal than a good signal. Thus, paying solely in the former event generates a stronger incentive for the agent, which lowers the cost of motivating him to work.

B. Repeated Setting

We now study the optimal incentive scheme in the repeated setting. In particular, we will explore the value of JPE in exploiting the agents’ mutual monitoring abilities. To analyze the agents’ strategic interaction, we study a repeated game played by the agents for a given wage scheme \((w^1, w^2)\). Let a history at time \(t = 1, 2, \ldots\) be a sequence of effort decisions made by the agents up to \(t - 1\). Then, a strategy profile is a sequence of functions that map from any possible history at each period into a probability distribution over effort choice profiles at that period. As mentioned, we are interested in a strategy that yields a repeated play of "work" as a team equilibrium outcome.

We first consider a necessary condition for the desired equilibrium to be subgame perfect. Fix an agent and his wage scheme \(w \geq 0\). By shirking, the agent can guarantee a payoff of at least \(\min\{\pi(0, 0; w), \pi(0, 1; w)\}\) in each period. Hence, this payoff constitutes a lower bound for the worst payoff sustainable in any subgame-perfect equilibrium. Therefore, in order to sustain a subgame-perfect equilibrium in which both agents work, we must have

\[
(I_C) \quad \pi(1, 1; w) - e \geq (1 - \delta)\pi(0, 1; w) + \delta \min\{\pi(0, 0; w), \pi(0, 1; w)\},
\]

where the left-hand side represents the average present-discounted expected payoff from working given that the other agent works, and the right-hand side represents the average present-discounted payoff from unilaterally "shirking" for one period and being subsequently punished by the worst possible equilibrium payoff. \(^{18}\) Note that \((I_C)\) is a necessary condition since the implied punishment may not be self-enforcing.

Inspecting \((I_C)\) reveals why JPE may be more effective than RPE in exploiting the strategic interaction among the agents in the repeated setting. Under JPE, \(\pi(0, 1; w) > \pi(0, 0; w)\). Thus, \((I_C)\) becomes

\[
(I_C) \quad \pi(1, 1; w) - e \\
\geq (1 - \delta)\pi(0, 1; w) + \delta \pi(0, 0; w).
\]

Since \(\pi(0, 1; w) > \pi(0, 0; w)\), the condition \((I_C)\) is weaker than the static incentive constraint \((I_S)\). That is, a shirking agent can be more severely punished in the repeated setting: he is not just punished by a lowered chance of getting the good signal (i.e., the static incentive), but also punished by the subsequent shirking of his partner, provided that the latter behavior is self-enforcing. As mentioned before, under JPE, an agent’s shirking punishes his peer by lowering the latter’s expected payoff. Hence, JPE may create implicit incentives unavailable in the static setting. Whether such punishment is self-enforcing for the former is the issue that will be addressed later.

By contrast, RPE cannot generate implicit incentives. To see this, consider any RPE scheme with \(\pi(0, 0; w) > \pi(0, 1; w)\). Then, \((I_C)\) collapses to

\[
(I_R) \quad \pi(1, 1; w) - e \\
\geq (1 - \delta)\pi(0, 1; w) + \delta \pi(0, 1; w) \\
\geq \pi(0, 1; w),
\]

which coincides with the static incentive constraint, \((I_S)\). This means that RPE will be at least as costly to implement in the repeated setting.

\(^{18}\) The average present-discounted payoff is obtained by multiplying a present-discounted payoff by \(1 - \delta\). Since such a rescaling makes the repeated-game outcome comparable to a stage-game outcome, we use this convention throughout the paper.

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\(^{17}\) That is, a history does not contain the agents’ past performance signals. This is without loss of generality since the effort choices are a sufficient statistic of the signals. See Michihiro Kandori (1992).

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\(^{18}\) We thank a referee for pointing this out. Our specification serves the purpose of replicating the standard result of the static models and thus highlights the result that will be obtained in the repeated setting.
setting as in the static setting. In this sense, RPE does not exploit the agents’ repeated interaction.

We now find the optimal wage scheme in the repeated setting. Consider first a relaxed problem:

\[ L \]

\[
\min_{w \geq 0} \pi(1, 1; w)
\]

subject to \((IC_L)\).

This is a relaxed program since \((IC_L)\) may not induce the agents to work as a subgame-perfect (let alone a team) equilibrium. The following lemma characterizes the solution to \([L]\).

**Lemma 1:** Define \(\delta(\sigma) = \frac{\sigma}{(1 - \sigma)q_1q_0}\). If \(\delta \in [\delta(\sigma), 1]\), then a JPE scheme, \(w^J = (w^J_{11}, 0, 0, 0)\) with

\[
w^J_{11} = \frac{e}{(1 - \sigma)(q_1^2 - (1 - \delta)q_0q_1 - \delta q_0^2)},
\]

solves \([L]\). If \(\delta \in [0, \delta(\sigma)]\), then the RPE scheme, \(w^R\), presented in Proposition 1, solves \([L]\).

**Proof:**

See the Appendix.

When \(w^J\) solves the relaxed program, \([L]\), \(w^J\) will be the optimal scheme if it implements (work, work)\(^\infty\) as a team equilibrium. We show that this is indeed the case. Since \(w^J\) is a JPE scheme, \(\pi(0, 1; w^J) > \pi(0, 0; w^J)\). Hence, \(\min\{\pi(0, 0; w^J), \pi(0, 1; w^J)\} = \pi(0, 0; w^J)\), so the worst possible equilibrium punishment is attained when the workers play (shirk, shirk)\(^\infty\) under \(w^J\). We first show that this latter outcome is self-enforcing given \(w^J\). Since \((IC_J)\) is binding at \(w^J\),

\[
(1) \quad \pi(1, 1; w^J) - e = (1 - \delta)\pi(0, 1; w^J) + \delta \pi(0, 0; w^J).
\]

Given \(\pi(0, 1; w^J) > \pi(0, 0; w^J)\), condition (1) implies that

\[
(2) \quad \pi(1, 1; w^J) - e < \pi(0, 1; w^J).
\]

Next, notice that \(\pi(k, l; w^J)\) is supermodular in \((k, l)\):

\[
(3) \quad \pi(1, 1; w^J) + \pi(0, 0; w^J) - \pi(1, 0; w^J) - \pi(0, 1; w^J) = (1 - \sigma)(q_1 - q_0)^2 w^J_{11} > 0.
\]

Combining (2) and (3) gives

\[
(4) \quad \pi(0, 0; w^J) > \pi(1, 0; w^J) - e.
\]

In other words, (shirk, shirk) is a stage-game Nash equilibrium, which implies that its repeated play forms a subgame-perfect equilibrium. Thus, the worst possible sustainable punishment in this case is self-enforcing under \(w^J\). Since \(w^J\) satisfies the constraint of \([L]\), repeated play of (work, work) is subgame perfect given the threat of such a punishment.

Further, the JPE wage scheme, \(w^J\), has an added virtue of being collusion-proof: under that scheme, (work, work)\(^\infty\) yields a higher joint payoff to the agents than all the other possible subgame-perfect equilibria. To see this, observe from (1) and (2) that

\[
(5) \quad \pi(1, 1; w^J) - e > \pi(0, 0; w^J).
\]

That is, both agents are strictly better off when they both work than when they both shirk. Also, the joint payoff is higher when both agents work than when one of them shirks, since

\[
2[\pi(1, 1; w^J) - e] > \pi(1, 1; w^J) - e + \pi(0, 0; w^J)
\]

\[
> \pi(1, 0; w^J) - e + \pi(0, 1; w^J),
\]

where the first inequality follows from (5) and the second follows from (3). This feature of the JPE scheme is very intuitive. Under JPE, each agent’s work confers positive externalities to his peer, and in the implemented equilibrium, these positive externalities are already realized. Therefore, the agents cannot benefit from colluding against that outcome. These two obser-
vations, along with Lemma 1, lead to the following conclusion.

**PROPOSITION 2:** The JPE scheme, \( w' \), implements (work, work)\( ^a \) as a team equilibrium. There exists \( \delta(\sigma) \leq \delta(\sigma) \) such that the JPE scheme, \( w' \), is optimal for \( \delta \in (\delta(\sigma), 1) \).

Since \( \delta(\sigma) \) is increasing in \( \sigma \), the JPE scheme is optimal for small values of \( \sigma \) and large values of \( \delta \). It is generally difficult to analyze the optimal scheme in the region where the RPE solves \([L] \). In contrast to the JPE scheme, RPE schemes are susceptible to collusion. To see this, consider the RPE scheme, \( w^S \), defined in Proposition 1. Since \((IC_S)\) is binding at \( w^S \),

\[
(6) \quad \pi(1, 1; w^S) - e = \pi(0, 1; w^S) < \pi(0, 0; w^S),
\]

where the inequality follows from \( w^S \) being RPE. It follows that the agents are jointly better off when they both shirk. Moreover, (shirk, shirk)\( ^a \) can be sustained as a subgame-perfect equilibrium for \( \delta \) close to one.\(^{19}\) Consequently, \( w^S \) cannot implement (work, work)\( ^a \) as a team equilibrium outcome in that case. This susceptibility to collusion is inherent in RPE since the agents can benefit from jointly slacking off. The possibility of collusion raises the cost of motivating the agents via RPE in the repeated setting. Figure 1, which presents the optimal scheme assuming that \( w_{01} = w_{00} = 0 \), illustrates this point. If there were no collusion (that is, if the equilibrium is selected in the best interest of the principal), then \( w' \) would be optimal to the left of the dotted line whereas \( w^S \)

would be optimal to the right. Once the agents are allowed to collude, however, the cost of using an RPE scheme rises while that of the JPE scheme remains unchanged. This expands the region in which the JPE scheme is optimal and even creates a region in which an IPE scheme (which is also unsustainable to collusion) is optimal.

Several other remarks are useful to make. First, the optimal joint performance evaluation \( w' \) may be implemented by a group signal if the latter aggregates the individual signals properly. For example, suppose that there is a binary group signal \( X = \min\{x_1, x_2\} \), where \( x_i \in \{0, 1\} \) is the individual signal for \( i \) in our model. Then, the optimal joint performance evaluation can be implemented if the principal gives the wage \( w'_{11} X \) to each agent when signal \( X \in \{0, 1\} \) is observed.

Second, the intensity of explicit incentives under the optimal wage scheme decreases as the wage contract shifts from the static optimal RPE to the long-term optimal JPE, and it further decreases as the expected life span of the team increases. One can measure the intensity of an incentive scheme by \( \pi(1, 1; w) - \pi(0, 1; w) \), the net-payoff reward for an agent when he works rather than shirks. Under the static optimal RPE, \( w^S, (IC_S) \) binds, so

\[
\pi(1, 1; w^S) - \pi(0, 1; w^S) = e.
\]

\(^{19}\) Inequality (6) implies that, for \( \delta \) close to one,

\[
\pi(0, 0; w^S) > (1 - \delta)[\pi(1, 0; w^S) - e] + \delta[\pi(1, 1; w^S) - e],
\]

so a strategy that punishes a deviator by a repeated play of (work, work) can implement (shirk, shirk)\( ^a \). [A repeated play of (work, work) is in turn self-enforcing since \( w^S \) satisfies \((IC_S)\).] Furthermore, one can show that a correlated randomization between (shirk, work) and (work, shirk) generates a higher joint payoff than (work, work) and is subgame perfect, regardless of the value of \( \delta \).
With the optimal JPE scheme, \( w^J \),

\[
\pi(1, 1; w^J) - \pi(0, 1; w^J) = -\frac{q_1 e}{q_1 + \delta q_0} < e.
\]

Hence, the incentive is lowered under the optimal JPE scheme. In fact, the power of explicit incentive falls discontinuously as an increase in \( \delta \) causes a shift from either RPE or IPE to JPE. Moreover, its intensity also falls even further as \( \delta \) goes up, i.e., as the employment relationship is expected to last longer. The intuition for this lower-powered incentive under JPE is twofold: First, the explicit incentive does not have to be too high powered to motivate an agent, given the presence of additional implicit incentives. Second, the explicit incentive must be sufficiently low powered for shirking to be a credible punishment strategy. This result complements a similar prediction by Holmstrom and Milgrom (1991) based on multitask activities.

Third, while we restricted the wage contract of the principal to be memoryless, the restriction can be justified as an equilibrium response when the principal cannot commit to a long-term contract. Suppose that \( \delta > \delta(\sigma) \) so that \( w^J \) is the optimal repeated contract. Given the agents' (equilibrium) strategy, it never pays the principal to deviate from \( w^J \) for a single period. Hence, by the single deviation principle, it is an equilibrium behavior for the principal to offer \( w^J \) every period. That is, there is no profitable deviation from the stationary wage rule to any other (possibly nonstationary) strategy on the part of the principal. It may appear at first glance that, if the principal observes the bad signal repeatedly for an agent, then she will infer that as an evidence of shirking and deviate to offer some punishment wage or even to fire that agent. Such a deviation does not occur, however, since, on the equilibrium path, the principal puts zero prior on the agents' shirking and never updates that belief even in the face of a seemingly overwhelming, but imperfect, evidence of shirking. In other words, the principal attributes such a series of bad signals to bad luck. Moreover, our stationary wage scheme can be justified as an optimal long-term contract even when the principal can commit to one, if the value of agents' efforts is sufficiently high.

Finally, note that the JPE scheme provides not only a motivation but also a built-in punishment device for peer sanction. Specifically, the workers can punish each other effectively through their effort decisions without having direct access to extra social sanctions such as social ostracism. In practice, these other forms of social sanctions may be present, which will reinforce the results obtained above (see Kandel and Lazear, 1992, for an approach along this line).

### III. Optimal Task Design and Assignment

The previous section considers a somewhat special case in which workers' production technologies are independent, i.e., team production provides no technological synergy. This assumption was made there to highlight the importance of group incentives in the team environment. In this section, we seek to find the technological conditions that would make team production (i.e., joint task assignment) an optimal job design. Several recent articles address this issue. Holmstrom and Milgrom (1991) suggest that the tasks must be assigned to separate agents based on the signal characteristics, which implies that it is never optimal to assign a single...
task to the joint care of multiple agents. Itoh (1992) shows that, if an agent’s cost of performing a task is convex in effort, assigning multiple agents to a single task can lower the total cost of performance and can be optimal. Thomas Hemmer (1995) shows that, if there is a direct synergy from performing two tasks, assigning two agents to these tasks jointly rather than assigning each agent to a separate task exploits the synergy better. We investigate the same issue but focus on the implication of agents’ repeated interaction.

A. Static Setting

A task can be performed by a single agent (called hereafter “individual production”), or by a team of two agents (called hereafter “team production”). This task can either “succeed” or “fail,” rendering $R > 0$ and $0$ (gross) payoffs to the principal, respectively.

In the case of individual production, a (chosen) agent makes an effort decision $k = 0, 1, 2$ at the cost of $ke$, $e > 0$. (The choice of the agent is not an issue, since the agents are symmetric.) The task succeeds with probability $q_k$ when the agent picks an effort level $k = 0, 1, 2$, where $1 > q_2 > q_1 > q_0 ≥ 0$. We assume that $q_2 + q_0 ≥ 2q_1$, which ensures that the agent will never choose one unit of effort. We assume that $R$ is sufficiently high so that it is optimal to induce the agent to choose two units of effort, which we denote as “work” in keeping with our convention.

In the case of team production, each agent works or shirks, now at the disutility of $e$ and $0$, respectively. The task again succeeds or fails, which is the only signal available for the principal. That is, unlike in the previous section, only a team signal is available now. (Alternatively, one can view this group signal as the particular aggregation of the individual signals studied in the previous section.) The wage scheme for each agent simply depends on whether the project succeeds or fails. The probability of success is denoted $p_{kl}$, where $k \in \{0, 1\}$ and $l \in \{0, 1\}$ represent agent 1 and 2’s effort decisions, respectively, and it satisfies $1 > p_{11} > p_{10} = p_{01} ≥ p_{00} ≥ 0$.

We assume that $p_{10} > q_0$, so the project is more likely to succeed when it is run by a team with at least one working agent than when it is performed by a single shirking agent. We also assume that $p_{kl}$ is (weakly) supermodular in $(k, l)$:

$$(SUP) \quad p_{11} + p_{00} ≥ 2p_{01}.$$ 

This supermodularity condition means that an agent’s work increases (weakly) his partner’s productivity gain from work. Again, we assume that $R$ is sufficiently high so that it is optimal to induce both agents to work. That is, the principal wishes to implement the same aggregate efforts in both regimes.

Before proceeding, it is useful to develop some terminologies for characterizing the technology of team production. If $p_{11} > q_2$, then the team production is more productive than the individual production, assuming that the agent(s) work in both cases. In this case, we say that the team has “synergy.” Next suppose $p_{00} < q_0$. In this case, each agent’s shirking has a negative externality on his partner’s shirking productivity, so the shirking behavior can be interpreted as “sabotage.” Finally, fixing an agent’s effort $k$, $\Delta_k = p_{kl} - p_{k0}$ measures the extent to which his productivity depends on the effort decision of his peer. In this sense, we can

25 This condition will later ensure that the agents’ mutual sanctioning through repeated shirking is self-enforcing for all $\delta > 0$, as in the previous section. In fact, a weaker condition requiring $p_{kl}$ to be not too submodular will be sufficient. Recall that $(SUP)$ would also hold if the group signal were generated by the particular aggregation of the individual signals studied in the previous section.

26 Here, the sabotage activity, represented by shirking, is assumed to be costless. In this sense, our interpretation of sabotage is a little different from that of Lazear (1989). Nonetheless, our version captures the essential feature that sabotage affects the peer’s payoff negatively. There are circumstances in which such a sabotage activity seems plausible. For example, one can imagine a team of workers working on different “shifts” or different locations in a production line, and the communication of information by a predecessor of his production conditions to a successor may be important for a smooth operation of the production process. In such a case, withholding information by the predecessor may hurt the productivity of the successor, so it can be interpreted as costless sabotage.

24 That the agents can expend, at most, one unit of effort is not important, since we focus on implementing an outcome in which each agent picks one unit of effort in team production. Also notice that the aggregate cost of the agents’ work remains the same as that of an agent choosing two units of effort in individual production. This latter feature rules out the effect that Itoh (1992) focuses on.
take \((\Delta_0, \Delta_1)\) as a measure of technological interdependence between the agents under team production. That is, the higher \((\Delta_0, \Delta_1)\), the more interdependent the technology is.

**Individual Production.**—Let \(w \geq 0\) denote a chosen agent’s bonus when the project succeeds. Obviously, it is optimal to set the (base) wages equal to zero when the project fails. Given this scheme, an agent will choose two units of effort rather than zero effort in this regime if and only if \(q_2w - 2e \geq q_0w\). The minimum bonus satisfying this condition is

\[
w^* = \frac{2e}{q_2 - q_0}.
\]

Given this bonus, the agent indeed chooses two units of effort (i.e., there is no incentive to choose just one unit). The associated incentive cost to the principal is

\[
q_2w^* = \frac{2q_2e}{q_2 - q_0},
\]

so the principal’s net expected payoff is

\[
q_2R - \frac{2q_2e}{q_2 - q_0}.
\]

**Team Production.**—A similar exercise is performed for team production. Conditional on the other agent working, an agent will work only if

\[
p_{11}w - e \geq p_{01}w.
\]

Given this condition, it is a team equilibrium for both agents to work.\(^{27}\) Therefore, the optimal incentive wage for each agent is:

\[
W^* = \frac{e}{p_{11} - p_{01}},
\]

which results in the total incentive cost to the principal of

\[
2p_{11}W^* = \frac{2p_{11}e}{p_{11} - p_{01}}.
\]

The principal’s net expected payoff is:

\[
p_{11}R - \frac{2p_{11}e}{p_{11} - p_{01}}.
\]

Comparing (8) and (10), it is clear that the synergy among the team members is necessary for team production to be optimal in the static setting.

**PROPOSITION 3:** The principal strictly prefers individual production to team production if the team has no synergy; i.e., \(p_{11} \leq q_2\).

**PROOF:**

Suppose that \(p_{11} \leq q_2\). Then,

\[
p_{11}\left[R - \frac{2e}{p_{11} - p_{01}}\right] < p_{11}\left[R - \frac{2e}{q_2 - q_0}\right] \leq q_2R - \frac{2q_2e}{q_2 - q_0},
\]

where the first inequality holds since \(p_{11} - p_{01} < q_2 - q_0\), and the last inequality holds since \(p_{11} \leq q_2\) and \(R - [2e/(q_2 - q_0)] > 0\).

The intuition behind the result is best understood in the special case where \(p_{11} = q_2\). In this case, a team productivity of the two workers each incurring \(e\) is precisely the same as the productivity of a single worker exerting \(2e\). Yet, the principal prefers individual production because the incentive cost is higher in team production. The reason for this is the presence of the fixed cost associated with providing incentives for each agent. With team production, a bonus used to motivate an agent’s effort simply confers positive externalities to his partner without generating the latter’s incentive for effort, whereas with individual production, the bonus used to motivate the first unit of effort for an agent will have a spillover effect on the second unit of his effort (see Holmstrom and Milgrom, 1991, for finding a similar effect with respect to a job design). Hence, to provide the same incentives in team production, the principal must award a larger total bonus to the agents than would be needed to motivate a single agent. This fact makes individual production more attractive than team production in the static setting.

\(^{27}\) Given this condition, it is a Nash equilibrium for both agents to work. Even though there may exist another equilibrium in which the agents choose (shirk, shirk), this latter pair yields a lower joint payoff to the agents since \(2p_{01}w \leq 2p_{01}w = 2(p_{11}w - e)\). Hence, (work, work) is a team equilibrium.
B. Repeated Setting

We show that a repeated interaction of agents makes team production more attractive. The result of individual production is the same as above in each stage and is characterized by (7) and (8). The case of team production is changed because of the strategic interaction among the agents. As before, consider the strategy of each agent: “start and keep playing ‘work’ unless an agent shirks in a previous stage, in which case both ‘shirk’ repeatedly thereafter.” Because of the (SUP) assumption, this penalty strategy generates the worst sustainable payoff for each agent.

Two conditions are necessary for this strategy to be subgame perfect. First, it must be self-enforcing for both agents to shirk repeatedly, which holds if it is a stage-game Nash equilibrium for each agent to shirk: $p_{00}w \geq p_{10}w - e$, or

$$w \leq \frac{e}{p_{10} - p_{00}}. \tag{11}$$

Second, each agent must have no incentive to shirk, when shirking is punished by the repeated shirking of the other agent: $p_{11}w - e \geq (1 - \delta)p_{01}w + \delta p_{00}w$, or

$$w \geq W^*(\delta) = \frac{e}{p_{11} - (1 - \delta)p_{01} - \delta p_{00}}. \tag{12}$$

Given the supermodularity assumption, (SUP), $W^*(\delta)$ satisfies (11). Hence, $W^*(\delta)$ is the lowest bonus that implements (work, work)$^w$ as a subgame-perfect equilibrium. Given supermodularity, $W^*(\delta)$ also implements (work, work)$^w$ as a team equilibrium (i.e., there is no incentive to collude against that outcome).²⁸ We thus conclude that $W^*(\delta)$ is the optimal level of incentive bonus. The resulting expected payoff to the principal is

$$p_{11}R = \frac{2p_{11}e}{p_{11} - (1 - \delta)p_{01} - \delta p_{00}}. \tag{13}$$

Comparing (13) with (10), we see that $p_{01}$ in the denominator of (10) is replaced by a smaller expression $(1 - \delta)p_{01} + \delta p_{00}$. In particular, the new element is the last term $p_{00}$, which is attributed to the dynamic penalty strategy of the agents. Consequently, it is less costly to motivate an agent in the repeated setting than in the static setting as long as $\delta > 0$ (i.e., $W^*(\delta) < W^* \delta$ for $\delta > 0$). Also, note that the denominator of the second term of (13) can be rewritten as $\Delta_1 + \delta \Delta_0$. Thus, the team production becomes more favorable if its technology is more interdependent [i.e., a higher $(\Delta_1, \Delta_0)$].

PROPOSITION 4: Team production with bonus $W^*(\delta)$ implements (work, work)$^w$ as a team equilibrium. If $p_{11} \geq q_2$ (i.e., the team has a synergy) and $p_{11} - (1 - \delta)p_{01} - \delta p_{00} \geq q_2 - q_0$, then the principal prefers team production to individual production in the repeated setting. Conversely, if $p_{11} < q_2$ and $p_{11} - (1 - \delta)p_{01} - \delta p_{00} < q_2 - q_0$, the principal prefers individual production to team production. Fixing $p_{11}$ or $p_{10}$, the value of team production is higher the more interdependent its technology is.

The proof is analogous to that of Proposition 3 and is omitted. Some insight can be gained by inspecting the conditions laid out in the proposition. Clearly, the condition for team production to be beneficial is weaker in the repeated setting. In particular, the term $p_{00}$, which played no role in the static analysis, now has an impact in the repeated setting. Suppose that $p_{11} = q_2$, so the team creates no synergy. Even in this case, if $p_{00} < q_0$ (i.e., each agent can sabotage his partner’s productivity) and if $\delta$ is sufficiently high, then

$$p_{11} - (1 - \delta)p_{01} - \delta p_{00} > q_2 - q_0,$$

so team production may be preferable to individual production. In this case, the sabotage possibility can help sustain a cooperative relationship among the agents, since it makes the agents’ mutual sanctioning power more credible and effective. Lazear (1989) shows the agents’ sabotage ability can make RPE ineffective. We have shown here that it can make JPE-type group incentives more effective. This finding is

²⁸ The arguments for this are the same as before and thus omitted.
also consistent with the view that “even if it is unnecessary on technological ground, a team-based job design is efficient whenever the firm can rely on internal monitoring and peer pressure” (Baron and Kreps, 1999 p. 324).

The value of technological interdependence can be seen by fixing $p_{10}$. As $p_{11}$ increases and/or $p_{00}$ decreases, the team technology becomes more interdependent. This latter change can occur when an organization adopts a technology that requires more coordination. Often, such a technology yields a higher return when coordination is successful but a more disastrous outcome when it fails. A case in point is the adoption of the just-in-time inventory system under which one employee’s shirking can slow down the entire production process and lower the performance of the other employees since there is no inventory buffer. Our theory suggests that such an increased technological interdependence can improve the incentives in the repeated setting.

IV. Concluding Remarks

This paper has shown that the optimal incentive scheme displays many observed features of team-oriented organizations when the relationship among agents has a long life span. Specifically, it has provided a rationale for group-based incentives and peer sanctioning. Furthermore, we predict that organizations with long life spans and mutual accountability are likely to be characterized by low-powered, group incentives whereas organizations with short life spans or no mutual accountability are characterized by high-powered, competitive incentive schemes such as relative performance evaluation. These findings not only provide a theoretical rationale for the use of “work teams” in many modern organizations, but it also explains why a set of “innovative” practices such as group incentive and long-term employment security are adopted as a cluster, as the recent human resource literature observes (see Ichniovski et al., 1996).

A caution must be exercised in interpreting our results, though. One theoretical problem is the multiplicity of (subgame-perfect) equilibria. While we resolved this issue in this paper by focusing on the equilibrium that is most favorable to the agents, the multiplicity leaves a range of different predictions supportable. For instance, the team equilibrium concept relies on the agents’ abilities to select the worst possible (subgame-perfect) punishment. In practice, such abilities may be limited if the agents can successfully renegotiate around such a punishment. While the equilibria identified in this paper are renegotiation proof according to some recent criterion, this issue is far from settled.\textsuperscript{29} Such limited punishment abilities diminish the effectiveness of the incentive scheme designed by the principal, which may explain a range of different experiences from team experiments that have been documented.

Although we have studied several aspects of organizational design in a stylized setting, our results generate insights that apply beyond the specific setting and organizational design issues that we have focused on. Our paper suggests that the employment practices that work well in short-term organizations may not work well in long-term organizations. In particular, the practices that promote competition or that separate the interests of the agents may not work well in the setting where the agents interact repeatedly. Rather, the latter setting requires the use of practices that interlock the fates of the agents, for such practices tend to increase the power of implicit incentives. Such practices range from adopting a technology, such as the just-in-time inventory system, which makes workers more dependent on each other, to encouraging formation of social ties among workers even beyond workplaces.\textsuperscript{30} Our results can also apply to models different from standard agency models. With some modifications, for instance, our repeated interaction model may explain the desirability of group-based

\textsuperscript{29} Repeated play of a stage-game Nash equilibrium considered in this paper satisfies the renegotiation-proofness notion developed by Dilip Abreu et al. (1993).

\textsuperscript{30} For instance, Thomas P. Rohlen (1975) cites practices to strengthen teamwork (adopted by Japanese firms) that involve overnight trips, monthly recreational activities, and office parties.
lending as well as the Japanese-style procurement partnerships.

APPENDIX

PROOF OF LEMMA 1:

Let \( W^L \) be the wage scheme that solves \([L]\). For \( i = J, R \), consider a program:

\[
[L_i] \quad \min_{\w = 0} \pi(1, 1; \w),
\]

subject to \((IC_i)\),

and let \( W^i \) be the solution to \([L_i]\). Now, observe that \((IC_L)\) holds if and only if either \((IC_R)\) or \((IC_J)\) holds. This can be seen by noting that \((IC_L)\) is weaker than \((IC_i)\), \( i = R, J \) and that if \( w \) satisfies \((IC_L)\) it must satisfy either \((IC_R)\) or \((IC_J)\). Consequently, \( \pi(1, 1; W^L) = \min\{\pi(1, 1; W^K), \pi(1, 1; W^J)\} \).

For this reason, we solve \([L_R]\) and \([L_J]\) separately. As noted in the text, since \((IC_R)\) is identical to \((IC_S)\), \([L_R]\) is identical to \([S]\). Thus, \( W^K = W^S \). Now, consider \([L_J]\). Its objective function is:

\[
\pi(1, 1; w) = (\sigma + (1 - \sigma)q_1^2)w_{11} + (1 - \sigma)q_1(1 - q_1)[w_{10} + w_{01}] + (1 - \sigma)(1 - q_1)w_{00},
\]

and \((IC_J)\) can be rewritten as:

\[
(II_J) \quad (q_1 + \delta_q_0)w_{11} + (1 - q_1 - \delta_q_0)w_{10} + (\delta - q_1 - \delta_q_0)w_{01} - (1 - q_1 + \delta(1 - q_0))w_{00} \geq \frac{e}{(1 - \sigma)(q_1 - q_0)}.
\]

The left-hand side of the constraint is decreasing in \( w_{00} \) while the objective function is increasing in \( w_{00} \). Therefore, it is optimal to set \( w_{00} = 0 \). Observe now that the coefficient of \( w_{10} \) is (weakly) greater than that of \( w_{01} \) in the left-hand side of \((IC_J)\) but that their coefficients are the same in the objective function. Suppose that \( w_{01} > 0 \). Then, lowering \( w_{01} \) and raising \( w_{10} \) simultaneously so that the left-hand side of \((IC_J')\) remains the same will reduce the value of the objective function. This latter observation suggests that it is optimal to set \( w_{01} = 0 \).

Since both the objective function and the constraint are linear in \( w \), only \( w_{11} \) or \( w_{10} \) is strictly positive. Suppose that \( w_{10} \) is strictly positive. In this case, \((IC_J')\) is more onerous than \((IC_2)\). Hence, in this case \( W^J > W^S \), so \( W^J \) can never solve \([L]\). Therefore, we focus on the case in which \( w_{11} \) is strictly positive. A straightforward argument shows that \( W^J = W^J \) in this case.

Therefore, it follows that \( \pi(1, 1; W^L) = \min(\pi(1, 1; W^J), \pi(1, 1; W^S)) \). Finally, observe that

\[
\pi(1, 1; W^J) - \pi(1, 1; W^S) = \frac{\sigma - (1 - \sigma)\delta q_1 q_0}{(1 - \sigma)(q_1^2 - (1 - \delta)q_1 q_0 - \delta q_0^2)} < 0
\]

if and only if \( \delta > \delta(\sigma) \). This proves Lemma 1.

REFERENCES


Baker, George; Gibbons, Robert and Murphy, Kevin J. “Relational Contracts and the Theory


