Facilitating the search for partners on matching platforms: Restricting agent actions

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December 12, 2016

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Abstract

Two-sided matching platforms, such as those for labor, accommodation, dating, and taxi hailing, control many aspects of the search for partners. We consider a dynamic model of search by agents with costly discovery of match value and find that in many settings, the platform can mitigate wasteful competition in partner search via restricting what agents can see/do. For medium-sized screening costs (relative to idiosyncratic variation in utilities), the platform should prevent one side of the market from exercising choice (similar to Instant Book on Airbnb), whereas for large screening costs, the platform should centrally determine matches (similar to taxi hailing marketplaces). Restrictions can improve social welfare even when screening costs are small. In asymmetric markets where agents on one side have a tendency to be more selective (due to lower screening costs or greater market power), the platform should force the more selective side of the market to reach out first, by disallowing the less selective side from doing so. This allows the agents on the less selective side to exercise more choice in equilibrium.

When there is vertical differentiation between agents, the platform can further boost welfare by hiding quality information. In the limit of vanishing screening costs the boost in welfare (from each of the two interventions) remains significant, and a Pareto improvement in welfare is possible; the weakest agents can be helped without hurting other agents.

Keywords: matching market, market design, search frictions, stationary equilibrium, sharing economy, platform.

1 Introduction

During the last decade there has been rapid growth in the number of online platforms for matching in the contexts of dating, labor markets, accommodation, and taxi services, among others. These platforms allow agents to match with other agents for mutual benefit. In certain

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settings, the platform may know or may be able to elicit adequate information regarding the preferences of agents, and use this information to determine matches centrally. However, in many other settings, there is a significant heterogeneity in agents' preferences, which cannot be easily uncovered by the platform. In such situations, the platform must allow agents to take an active role in the search for a compatible partner. Therefore, one of the most important decisions made by these marketplaces relates to their "search environment" design, which determines the framework within which agents can acquire more information about other agents, and contact them to potentially form a match.

The "search environment" designs typically used in practice roughly fall into three categories: centralized, one-sided search, and two-sided search. In a *centralized matching* design, the marketplace essentially chooses matches and thus agents do not engage in active search for partners. Taxi services marketplaces like Uber/Lyft closely fit this category. In a *one-sided search* design, the marketplace allows only one side to search through available options and pick a suitable match. This match can then quickly be secured, as agents on the other side play a passive role and do not exercise choice. "Instant Book" on AirBnb, and "Quick Assign" on TaskRabbit are current examples that resemble this category. Finally, in a *"two-sided search"* design, agents on both sides of the market are able to screen potential partners, and a match results only upon approval by both sides of the market. In this setting, the platform may allow either side of the market to reach out (e.g. OkCupid, Upwork), or can further implement a *directional search* environment, in which one side of the market is required to reach out/apply/request, leaving the other side of the market to accept or reject the application (e.g. guests must "Request to Book" on Airbnb, the woman must send the first message on dating platform Bumble). Therefore, a key decision for the platform is which of these design types to choose. Furthermore, if a one-sided search is chosen, the platform must also decide which side can search. Similarly, when opting for a two sided search, the platform must still decide whether one or both sides can apply/reach out; if only one is allowed, the platform chooses which one.

The evidence strongly suggests that the choice of search design can be critical for the success of the platform. While the effort needed in the online world to acquire more information about a potential partner —e.g. scan the profile—, and reach out —send a message, or submit an application— is typically small, these costs can easily add up, and thus end up playing a significant role in the efficacy of a matching platform. In fact, the impact of search frictions has been documented in the context of AirBnB [15], Upwork [21] and TaskRabbit [11]. [11] further demonstrates the importance of search environment design; it finds a significant reduction of search costs and increase in the number of matches formed ("match efficiency") as a consequence of re-design of TaskRabbit in the spring of 2014.[1] The goal of this paper is to understand the impact that different design choices have on the overall welfare of market participants[2], and

[1] The importance of the search environment design has also been documented in a more traditional buyers-sellers setting in the context of E-Bay [12].

[2] We suppress issues of pricing and revenues here. This is partially motivated by the revenue model of several two-sided platforms that roughly charges commission at a fixed rate (e.g., Upwork, Airbnb, Uber), meaning that, if each user operates in a particular price range (e.g., $25-30/hour on Upwork), the platform does not have a strong incentive to "divert search" [18], since different matches yield similar revenues. There may still be some

use this to understand how the search environment and information disclosure policy can be designed to maximize welfare, as a function of market characteristics.

To that end, we introduce a simple model of dynamic two-sided matching without transfers, mediated by a platform. We initially consider ex-ante homogeneous agents on each side, and later allow two tiers of agents on one side. We call the two sides of the market men and women. A woman's value for matching with a particular man is the sum of two components: (1) a tier-specific component (quality) that depends only on the tier that the man belongs to, that all women agree on and which is common knowledge, and (2) an idiosyncratic component for the value that is random and independent across (ordered) agent pairs, and can only be discovered after a costly screening. The latter allows us to capture agent-pair level heterogeneity in preferences ("beauty lies in the eye of the beholder") that the platform cannot uncover a priori. A man's value for matching with a particular woman is analogously modeled. Agents on each side (in each tier) arrive at an exogenously given (and possibly unequal) rate. For a match (i, j) to form, one of the agents, say i, must propose (either after screening j, or without screening her), and the other agent j must accept the proposal (again either after screening i or without screening). If a match occurs, both agents leave the market. Agents unable to match over an extended period (exogenously) leave the market. (During their lifetime in the system, agents get several opportunities to propose, and they can choose either to actively exploit these or instead to wait for incoming proposals from agents on the other side.) We consider stationary equilibria [20], where agent best responses are utility maximizing solutions to the optimal stopping problem in the limiting steady state of the market. We study the welfare achieved under the equilibria that arise in this model, in which both sides are allowed to propose and quality information is readily available, which we refer to as the *no platform intervention* setting.

Next, we consider several types of market interventions. The platform can choose a search design different from two-sided search (see above), by preventing one or both sides of the market from screening, leading to a one-sided search or a centralized matching design respectively. Even if the platform chooses to retain a two-sided search design, it may block some agents from proposing (i.e., reaching out first). These interventions allow the platform to select among the multiple equilibria arising in the no intervention case and further, to create new equilibria which have higher welfare. We study the equilibrium welfare under these interventions, and derive design recommendations.

Main findings. We first study a market where agents on each side are ex ante homogeneous. In general, three equilibrium regimes arise: when screening costs are small, one side screens and proposes, and the other side screens and accepts/reject incoming proposals; with medium-sized screening costs, one side proposes without screening and the other side screens and accepts/rejects (this is equivalent to what occurs under a one-sided search design); finally, with

tension between maximizing revenues and maximizing user welfare, since user welfare incorporates not just the value from matching (which roughly grows with the number of matches and hence platform revenues), but also the cost of search (which does not directly affect platform revenues). However, we believe that this allows us to capture the main trade-off in the design.

large screening costs, both sides of the market propose and accept proposals without screening (this is equivalent to the result of centralized matching). We find that if the two sides of the market are symmetric —screening costs, valuation distributions, and arrival rates on the two sides of the market are identical—, then the platform cannot improve the average welfare by implementing one of the interventions we consider. However, we find that these interventions can significantly boost welfare in asymmetric settings.

In particular, we break the symmetry in two different ways. First, if agents on opposite sides of the market have different screening costs (but everything else is identical on the two sides), the platform is able to use interventions to *select* a high welfare equilibrium when screening costs are small/medium-sized. When screening costs are small, the platform should block the side with high screening costs from proposing, thus forcing the side with low screening costs to do so. This allows agents with high screening cost to be somewhat selective, improving their welfare at a small cost to the other side that now faces occasional rejection. For medium-sized screening costs, the platform can implement a one-sided search design in which the low screening cost side can choose. In particular, the previous design will cease to be optimal because the negative other-side externality of screening incoming proposals outweighs the benefit that the side with higher screening costs derives from screening.

The second asymmetric case we consider is that of an unbalanced market, modeled by having a faster arrival rate of agents on one side. (We refer to this side as the long side, and the other side as the short side.) In this setting, if the imbalance is not too small, the long side proposes in *all* equilibria. Moreover, proposals by agents on the long side are accepted only rarely (besides the risk of not matching at all), hence agents on the long side cannot affort to be too selective. The platform can significantly boost welfare by preventing the long side from proposing. This *creates* new equilibria in which the long side is able to also be somewhat selective when considering incoming proposals, since it does not have to fear rejection. This intervention provides a significant welfare boost to the long side at a small cost to the short side.

Finally, we allow the long side of the market (call them men) to have two quality levels – top men and bottom men. The utility of an agent on the short side (call them women) for matching with a man consists of the sum of a fixed quality term (which is larger for top men) and an idiosyncratic term, which is drawn independently across woman-man pairs. We assume that women arrive faster than top men, but slower than men overall. The platform knows the type of each man and, under no intervention, makes this information visible to the women. In sharp contrast with our results without vertical asymmetries, we find that there is an equilibrium with low welfare even as screening costs vanish. It is bottom men whose welfare is low, as a result two features of the equilibrium: bottom men propose *without screening* since most of their proposals are *ignored* by women waiting for a dream match (to a top man), and further, some of these women who are waiting for a dream match end up leaving unmatched, reducing the number of bottom men who match.

We identify a number of different interventions that the platform can employ to improve welfare. By preventing the men from proposing (as in the case with different arrival rates but no types), the platform is able to allow bottom men to be selective, providing a boost to welfare. By *hiding the types of men* from women in addition, the platform is able to further boost welfare by eliminating the issue of some women wastefully waiting to match with a top man, while ignoring bottom men. Importantly, in this setting, the platform can significantly boost the welfare of bottom men without significantly hurting any other type of agent; a Pareto improvement of welfare is possible in the limit of vanishing screening costs. One powerful alternative intervention we identify is that the platform can charge women a subscription fee for access to top men. This again produces the same welfare to each type of agent as the combination of interventions mentioned above, but in addition produces substantial revenues for the platform. Overall, in this setting, the platform is able to use simple interventions to once again *create* a high welfare equilibrium, this time with a Pareto improvement of welfare (in the small screening costs limit).

1.1 Related Literature

Our modeling approach draws upon the search frictions literature, which traditionally focused on macro level job growth and unemployment under search frictions, using relatively crude models of agent level behavior (e.g., see [28]). While traditional models of search typically assume that unmatched agents meet one another randomly (in direct proportion to their mass in the unmatched pool), some recent work has explored replacing this assumption by allowing for directed search where agents can target their applications (e.g [13]). However, these papers assume a common evaluation of the heterogeneous agents, and aim to understand phenomena such as an assortative matching arising in equilibrium. Our work differs from this literature in two important aspects. First, we incorporate both a common evaluation of agents through tiers, both also an idiosyncratic component. This allows us to capture elements of both random and directed search: agents can (sometimes) target a specific tier on the other side, but within-tier opportunities are presented randomly to them. In addition, we study not only the structure of the equilibria that arise, but also what interventions a matching platform can implement to improve the equilibrium welfare of agents, in other words, we are focused on the *impact of the matching technology on market performance.*

The literature on platforms (e.g. [29, 17]) has focused on platform-level effects of features like market thickness, and issues of attracting users and competition between platforms, and pricing, while using crude agent-level models. In particular, it has not modeled the search for partners, and hence does not lend itself to addressing questions regarding the role of search environment design.

The operations literature (including work on inventory management [30], revenue management [32], dynamic programming [7] and queueing [4]) has built a deep understanding of dynamics and decision making, but has traditionally focused on cases in which units on one side of the market (usually representing inventory/products/servers) do not have strategic considerations (e.g., inventory units can be purchased and sold, whereas agents may decide whether to participate and whom to match with). Recent papers explore how operational decisions can be used by a platform to improve its performance [3, 6, 16]. Perhaps the one most related to work is that by Allon et al [2]. They find that improving operational efficiency of a platform may reduce market efficiency due to negative externalities, similar in spirit to some of our findings albeit in a very different setting —while they consider a buyer-seller setting, where the seller sets a price and is indifferent as to whom he is serving, we consider a two-sided matching market, where agents on both sides have heterogeneous preferences over agents on the other side. In addition, in our analysis we use the notion of mean field equilibrium which has been effectively employed in the operations literature to study complex dynamic games involving many players [35, 22, 5].

Our work is loosely related to signaling in matching markets when there is a constraint on the number of signals (e.g., [10, 9]). However, we explicitly model search costs, rather than considering a budget on the number of signals/applications/options.

Finally, some of the interventions suggested in the paper require hiding information. Several papers [26, 23] find that hiding information about market participants can serve to prevent the market from unraveling or to reduce cherry-picking. [27] studies how to disclose information so as to optimally explore available options in the context of an online recommendation system. Our findings on the benefits of hiding information are similar in spirit; we find that the platform can induce agents to consider a larger set of potential partners by hiding tier information.

2 Model

We model a dynamic two-sided matching setting without transfers, mediated by a platform. Agents on each side of the market are ex-ante homogeneous. (Later, in Section 4, we show that our findings are robust to this assumption, by considering a model with vertical differentiation. For now, we introduce a very simple model in which to study the impact of platform design on the search for partners.) Each agent has an idiosyncratic valuation for every agent on the other side, represented by u_{ij}. We assume that the u_{ij}'s are independent identically distributed (i.i.d.) with distribution F for men over women, and i.i.d. with distribution G for women over men. Further, we assume that the expected value is positive under each of F and G. In order to obtain analytical results, we typically restrict ourselves to the case in which F and G are the uniform distribution over $[0, 1]$. We assume that only the distribution of the u_{ij}'s is common knowledge, and agent i can privately learn u_{ij} for any j on the other side by spending a screening cost; in particular, the u_{ij}'s are unknown to the platform.[3] Let this cost be c_m for men and c_w for women. With a small abuse of notation, we sometimes use c_i to denote the screening cost of agent i, i.e., if i is a man then $c_i = c_m$.

We consider dynamic arrival and departure of agents in a continuous time setting. Men arrive at rate λ_m and women arrive at a rate λ_w. Here, λ_m represents the "mass" of men that enter per unit time, and similarly for women; we employ a fluid limit model. When a match

[3]In Section 4, when we introduce vertical differentiation, we assume that agents have a quality component that, unless otherwise stated, is common knowledge. However, the idiosyncratic valuation can still only be learned by spending the cost.

forms, the concerned agents leave the market immediately; we describe the dynamics of search and match formation below. Unmatched agents die at a rate $\mu > 0$, common across all agents. While our results will be typically formulated in the limit $\mu \to 0$, the positive death rate ensures that the market reaches a steady state even if arrival rates are different for men and women.

Dynamics of search and matching. Each agent has a Poisson clock of rate 1. Each time the clock of an agent i ticks, the agent has the opportunity to costlessly request a potential match on the other side of the market. If a request is made and there are available agents on the other side, the platform chooses one of them j uniformly at random.[4] Agent i can decide whether to propose to j without screening, or to spend the cost c_i, learn her idiosyncratic valuation u_{ij} for j instantaneously, and then decide whether to propose to j. If i decides to propose, his proposal is conveyed to agent j. Agent j can choose whether to screen agent i at a cost c_j in order to learn u_{ji}. She then chooses between rejecting the proposal (in which case i and j remain unmatched), and accepting it, in which case a match is formed and the pair of agents leaves the market. All these events are assumed to occur instantaneously when the clock of an agent rings.

We call this the *baseline* or *no intervention* setting, where upon the ring of a clock an agent can decide whether to screen and propose, propose without screening, or do nothing, and similarly, upon an receiving an incoming proposal, an agent can decide to screen and then accept or reject, accept without screening, or reject without screening.

Potential interventions. We allow the platform to intervene by constraining agents' actions in specific ways:

- Shutting down screening: The platform can prevent agents on one or both sides of the market from screening, which we term a *one-sided search* or a *centralized matching* design respectively (see Section 1). When an agent is not allowed to screen, we assume that the agent will accept any proposed partner, without the need for a explicit proposal or acceptance/rejection.

- Directional search: When agents on both sides of the market are allowed to screen, this is a *two-sided search* design. Even here, the platform may constrain agents by preventing agents on one side of the market from proposing.

In addition, in Section 4 when we introduce a quality component on the men's side, we consider the possibility of the platform intervening by hiding information regarding the quality of potential partners.

We consider the equilibria that arise under no intervention, and compare them with equilibria under each of the considered platform designs.

[4]Here the platform ignores agents that i has seen before, and agents who have rejected i in the past. Note, however, that in the continuum limit we consider, the role of these considerations vanishes.

2.1 Equilibrium concept: Stationary equilibrium

Our equilibrium notion focuses on the stationary/steady-state behavior of the market and incorporates mean field informational assumptions on the part of the agents. The notion of mean field equilibrium has been effectively employed in the operations literature to study complex dynamical games involving many players [35, 22, 5, 8]. This equilibrium concept relaxes the informational requirements of agents, requiring them only to know the aggregate steady state description of the system, which makes it behaviorally appealing and tractable. The related concept of stationary equilibrium, introduced by Hopenhayn [20], studies game-theoretic equilibria that also correspond to dynamical steady state, again in a large market limit.[5] The equilibrium concept in the current work, which we also term *stationary equilibrium*, borrows from these related concepts.

Next we describe the equilibrium concept. A more detailed and formal description is deferred to Appendix A.

2.1.1 Agents' strategies

Recall that, in our setting, each agent's choice of strategy entails the following considerations: (i) If her clock rings should she ask for a potential match? If yes, then should she screen the potential match? (If she asks for a match and does not screen, then she proposes.) If she screens, then what threshold should she employ such that she proposes to the potential match if her valuation exceeds that threshold? (ii) If she receives a proposal, should she consider it? If yes, then should she screen? (If she does not screen then she accepts.) If she screens, then what threshold should she employ? Note that both the actions and threshold employed could, in principle, change over time. However, in our stationary setting, we assume agents play deterministic strategies that remain invariant over time as follows.

Definition 1 (Agents' strategies). *We consider* determistic, time-invariant *agent strategies* $s = (a, \theta)$ *defined by:*

1. *A deterministic set of actions a, which determines what an agent does when her clock rings, or an incoming proposal arrives. In more detail, each agent must choose:*

   (i) What to do with incoming proposals, namely to ignore them (I), or to accept without screening (A), or to screen and accept/reject (S+A/R).

   (ii) What to do when one's clock rings: The agent specifies whether to pass on the opportunity (N), or to propose without screening (P), or to screen and propose if the match value is above the threshold (S+P).

[5] Recent work [1] studies stationary equilibrium (incorporating mean field assumptions by players), finding conditions for it to exist and approximate Markov perfect equilibrium (in a finite market) well. However, our setting is different in that it involves entry and exit (among other differences), so those results do not directly apply. We conjecture but do not show that our stationary equilibria constitute approximate Markov perfect equilibria.

2. *A deterministic threshold θ used to screen participants: if her valuation for a participant is above θ, then the agent proposes/accepts the proposal. Otherwise, she skips the opportunity to propose/declines the incoming proposal.*

We justify our choice of strategy space as follows. First, note that our strategy is assumed to be time invariant. Recall that, under the mean field assumption (see below) each agent only observes the steady state distribution of the market, and their own history. Since agents are not distinguishable by others based on their history in the market, and no individual agent impacts the state of the rest of the market, there will always be a time invariant best response.[6] Second, though we assume that each individual agent chooses a deterministic strategy, we allow different agents to use different deterministic strategies. We expect that, for any candidate equilibrium that involves agents playing randomized strategies (that are best responses under the mean field assumption below), there is an equivalent equilibrium where each agent employs a deterministic strategy, with the appropriate distribution over determistic strategies on each side. Finally, we justify the assumption of threshold strategies: in steady state under the mean field assumption assumption below, agents are solving a Markov decision process problem that is memoryless (outside of the instants when an agent's clock rings or when the agent receives a potential option), so any best response where they screen will involve them proposing/accepting if and only if the match utility exceeds the continuation value. Hence, any best response must be a threshold strategy, employing the continuation value as the threshold.[7] Note how the threshold must be the same whether the agent is deciding whether to propose, or whether to accept. Further note that the continuation value must be the same for all agents on a particular side of the market in any equilibrium. Therefore, one can assume that all agents on a side of the market use the same threshold in all contexts regardless of their strategy. We call this property *threshold consistency.*

2.1.2 Mean field assumption

We now define the agent mean field assumption, when other agents are employing deterministic time invariant strategies satisfying threshold consistency, and where a fraction $f_m(s)$ of men employ strategy s, and similarly for women. Recall from Definition 1 that a strategy s consists of a threshold θ, along with a set of actions that indicate what to do with incoming proposals (three choices) and what to do when one's clock rings (three choices). That is, besides the choice of threshold, a strategy involves a choice between a finite set of options (in particular, nine different options). We denote this finite set by $\mathcal{S}$, and by $\mathcal{S}_\mathrm{p} \subset \mathcal{S}$ the set of strategies that consider potential options, and propose with or without screening. We will find it convenient to suppress the fact that θ is part of the strategy, since it is common across all agents on a side of the market when threshold consistency holds.

[6] When there are multiple best responses, agents may choose a best response such that their instantaneous strategy depends on their own history, but we expect that there will be an equivalent equilibrium such that agents act in a time invariant way as per the average (over agent histories) of time-dependent strategies in the original equilibrium.

[7] This fact has already been established, albeit in a different setting, by the traditional literature in search [25].

Assumption 1 (Agent mean field assumption)**.** *Fix the fractions $f_m(s)$ and $f_w(s)$, and the thresholds θ_m and θ_w. For all $s \in \mathcal{S}$, suppose that the steady state mass of men in the system using strategy s is $L_m(s)$ for some $L_m(s) \geq 0$, and similarly for women. (We assume that $F(\theta_m) < 1$, since this must be the case in any equilibrium that involves screening, and similarly for women.)*

Each woman i makes the following assumptions:

1. **Potential options (when they are always available):** *If $L_m(s_m) > 0$ for some $s_m \in S$, then the woman expects that each time her clock rings a potential option will be offered (if she asks for it), and the strategy of that man will be s_m with likelihood $L_m(s_m)/\big(\sum_{s\in\mathcal{S}} L_m(s)\big)$.*

2. **Potential options (when they are available only sometimes):** *On the other hand, if $L_m(s) = 0$, it must be that $L_m(s') = 0$ for all strategies that men are employing. Each woman expects that each time her clock rings and she asks for a potential option,*

$$\begin{aligned}&\Pr(\text{She is offered an option AND the strategy of that man is } s_m)\\ &\qquad = \frac{\xi f_m(s_m)}{1 - \mathbb{I}(s_m \text{ involves S+A/R})F(\theta_m)}\,,\end{aligned} \tag{1}$$

where ξ is the unique solution of

$$\begin{aligned}L_w(s_w) &= \frac{\lambda_w f_w(s_w)}{\xi\big(\mathbb{I}(s_w \text{ involves P}) - \mathbb{I}(s_w \text{ involves S+P})G(\theta_w)\big) + \mu} \quad \forall s_w \in \mathcal{S}_{\mathrm{P}}\,,\\ \mu \cdot \sum_{s_w\in\mathcal{S}} L_w(s_w) &= \lambda_w - \lambda_m\,,\end{aligned} \tag{2}$$

(which also gives the values of $L_w(s_w)$).

3. **Incoming proposals.** *Each woman will receive proposals from men following strategy s_m at rate $\rho_w(s_m)$, given by*

$$\rho_w(s_m) = \begin{cases} \infty & \text{if } \sum_{s_w} L_w(s_w) = 0\,,\\ \frac{L_m(s_m)\mathbb{I}(s_m \text{ involves P})(1-F(\theta_m)\mathbb{I}(s_m \text{ involves S+P}))}{\sum_{s_w\in\mathcal{S}} L_w(s_w)} & \text{otherwise.}\end{cases} \tag{3}$$

The quantifications in the statement above are arrived at by elementary but sometimes tedious accounting of arrivals, departures, proposals and matches. We provide a full explanation of these in Appendix A.1.

2.2 Stationary equilibrium

Having defined the strategies considered as well as the mean field assumption, we are now in a position to define our solution concept.

Definition 2 (Stationary equilibrium)**.** *Distributions of agent strategies* $(f_m(s))_{s\in\mathcal{S}}$ *and* $(f_w(s))_{s\in\mathcal{S}}$ *constitute a* stationary equilibrium (SE) *if there is a resulting steady-state such that for each* $s \in \mathcal{S}$ *such that* $f_m(s) > 0$*, it holds that* s *is a best response for a man under the agent mean field assumption, and similarly for women.*

Each of our results claiming a stationary equilibrium characterizes the corresponding steady state as part of the proof.

2.3 Evolutionary stability

We focus on the subset of stationary equilibria that are evolutionarily stable. We formalize the notion of evolutionary stability in Appendix A.2. Intuitively, the idea is that in a plausible equilibrium if the mix of strategies employed by agents changes slightly (this slightly changes the utility derived from different strategies), when agents begin to change their strategies in reaction to this change in the environment, this reaction should push the system back towards the equilibrium mix of strategies. To formalize this, we write a set of coupled differential equations that capture the evolution of the mass of agents in the system following different strategies. Any fixed point of this set of equations corresponds to a stationary equilibrium, whereas evolutionarily stable stationary equilibria correspond to *attractive* fixed points.

2.4 Preliminaries

Throughout the rest of the paper, we shall think about the agents' strategies as a function of the screening cost. Therefore, we conclude this section with the following simple lemma (proved in Appendix B), that yields the intuitively and analytically valuable notion of "effective" screening cost.

Lemma 1 (Effective screening cost)**.** *Consider the following two systems. In each case, the death rate is* μ*, and the value of an item to an agent is drawn i.i.d. from some distribution* F*. Any incoming option is screened (at some cost) revealing the true value of the option, and then accepted/requested if this value exceeds a threshold* θ*.*

- *System 1: "Potential options" arise according to a point process of rate* η*. Each potential option is screened at a cost* c*, to reveal its value, and requested if the value exceeds* θ*. The request is approved i.i.d. with probability* q*, in which case the agent obtains the item and leaves. If there is no request or the request is denied, the agent remains active.*
- *System 2: Options arise according to a point process of rate* ηq*. Each option is screened at a cost* c/q*, to reveal its value. The agent chooses to obtain the item if the value exceeds* θ*.*

Then, the two systems produce the same expected value.

Lemma 1 allows us to relate the threshold strategies at the equilibria as follows. For a given c, consider an equilibrium where (women screen and propose, men screen and accept/reject) with strategies $\theta_m(c)$ and $\theta_w(c)$ respectively. Let $\theta_w^*(c)$ be the threshold of women at an equilibrium when the proposing side is reverted. Then, we have that $\theta_w(c) = \theta_w^*\left(\frac{c}{1-\bar{F}(1-\theta_m(c))}\right)$.

3 Ex ante homogeneous agents on each side

As captured in our model in Section 2, we first study the case where on each side of the market all agents are ex ante homogeneous. We consider the limit of small death rate, namely $\mu \to 0$.

We begin by characterizing the equilibria in the fully symmetric case. For small or medium-sized search costs, one side takes the role of proposer and the other side waits for proposals. Both sides screen when the search cost is small, whereas only the side receiving proposals screens for intermediate search costs. When screening costs are large, both sides propose without screening and also accept incoming proposals without screening. Furthermore, the platform cannot increase the welfare through an intervention. (In the case of intermediate screening costs, the platform can obtain the same outcome via a one-sided search design, whereas for large screening costs, the platform can obtain the same outcome via a centralized matching design.) However, the fully symmetric case is an exception: we find that search design interventions can be very helpful when the two sides of the market are asymmetric, and this occurs even with ex ante homogeneous agents on each side of the market. We illustrate this by considering two types of asymmetries: agents on opposite sides face different screening costs (Section 3.1), and agents arrive at different rates (Section 3.2).

Simplest case: sides are symmetric. The simplest case under our model occurs with $\lambda_m = \lambda_w$, screening costs equal to c on both sides, and identical valuation distributions on both sides which we assume to be Uniform$(0, 1)$.[8] We find that, under no intervention, for every $c \in (0, 1/8)$ there exist only two equilibria which are the symmetric counterparts of each other when the roles of men and women are reversed. For $c \geq 1/8$, there is a unique equilibrium where both sides propose and accept without screening, earning expected utility of $1/2$ per agent.

For $c \in (0, 1/32)$, one side, say men, screen and propose, and women screen and accept/reject. Each side follows a threshold strategy, proposing/accepting if their valuation for the potential partner is large enough. The thresholds are $\theta_m(c) = 1-(2c)^{1/4}$ and $\theta_w(c) = 1-\sqrt{2c}$. The average utility earned by a man is $\theta_m(c)$ and for women it is $\theta_w(c)$. Women impose a negative externality on men when they reject an incoming proposal. For $c \in [1/32, 1/8)$, one side proposes without screening (achieving an expected utility of $1/2$), and the other screens and accepts/rejects with threshold $1 - \sqrt{2c}$ and identical expected utility.

It turns out that, in this setting, the platform cannot increase the welfare by implementing one of the interventions described in Section 2. However, the equilibria suggest the following search design: if $c < 1/32$, the platform should allow both sides to screen; if $1/32 \leq c < 1/8$, the platform can implement a one sided search where only one side screens; finally, if $c \geq 1/8$, the platform can opt for a centralized matching. We will see in the next sections that, as soon

[8] A distribution of Uniform $(a, a + 1)$ for $a \geq 0$ behaves identically with a translation of utilities.

as some asymmetry is introduced in the market, interventions can be useful to either select the highest welfare equilibria, or create equilibria with higher welfare.

3.1 Different screening costs markets

We now consider the case where sides might face different screening costs. For the sake of discussion, we assume that men's screening cost is greater than women's; without loss of generality, assume $c_m = \alpha c_w$ for some $\alpha \geq 1$. We assume also a balanced market, where both men and women arrive at the same rate. As usual, we are interested in the limiting description of the equilibria, considering $\mu \to 0$ for each fixed c_w. Note that as $\mu \to 0$, all men and women will leave the market matched. The difference between equilibria is then given by which side proposes and whether each side screens or not.

As men face a higher screening cost, all else being equal, they cannot afford to be as selective as women. As a consequence, we find that when screening costs are small the average welfare is higher if women propose, as then men have the chance to screen. However, for medium sized screening costs, the highest welfare equilibrium is one in which only the women screen. The following theorem characterizes the equilibria for different values of screening cost c_w (holding α fixed).

Theorem 1 (No intervention equilibria)**.** *Consider a market with $\lambda_m = \lambda_w = \lambda$ and $c_m = \alpha c_w$ for some $\alpha \geq 1$. For agents on both sides, their valuations for potential partners are drawn i.i.d. from a U(0,1) distribution. Fix α, and consider the limit $\mu \to 0$ for each fixed c_w. Then, the following are the stable stationary equilibria as a function of c_w:*

1. *(women screen + propose, men screen + accept/reject) with thresholds $\theta_w = 1-(2c_w/\alpha)^{1/4}$ and $\theta_m = 1 - \sqrt{2\alpha c_w}$. This is an equilibrium for $c_w \in \left(0, \min\left(\frac{1}{8\alpha}, \frac{\alpha}{32}\right)\right)$.*
2. *(women propose w.o. screening, men screen + accept/reject) with threshold $\theta_m = 1 - \sqrt{2\alpha c_w}$. Women get an expected utility of $1/2$. This is an equilibrium for $c_w \in \left[\frac{\alpha}{32}, \frac{1}{8\alpha}\right)$ if the interval is non-empty ($\alpha \leq 2$).*
3. *(women screen + propose, men accept w.o. screening) with threshold: $\theta_w = 1 - \sqrt{2c_w}$. This is an equilibrium for $c_w \in \left[\frac{1}{8\alpha}, \frac{1}{8}\right)$.*
4. *(men screen + propose, women screen + accept/reject) with thresholds: $\theta_m = 1-(2\alpha^2 c_w)^{1/4}$ and $\theta_w = 1 - \sqrt{2c_w}$. This is an equilibrium for $c_w \in \left(0, \frac{1}{32\alpha^2}\right)$.*
5. *(men propose w.o. screening, women screen + accept/reject) with threshold: $\theta_w = 1 - \sqrt{2c_w}$. Women get an expected utility of $1/2$. This is an equilibrium for $c_w \in \left[\frac{1}{32\alpha^2}, \frac{1}{8}\right)$.*
6. *Agents on both sides propose without screening whenever they get a chance, and accept all incoming proposals without screening. This happens when $c_w \geq \frac{1}{8}$.*

The proof of Theorem 1 can be found in Appendix C. Figure 1 illustrates the welfare at the different equilibria when $\alpha = 2$.

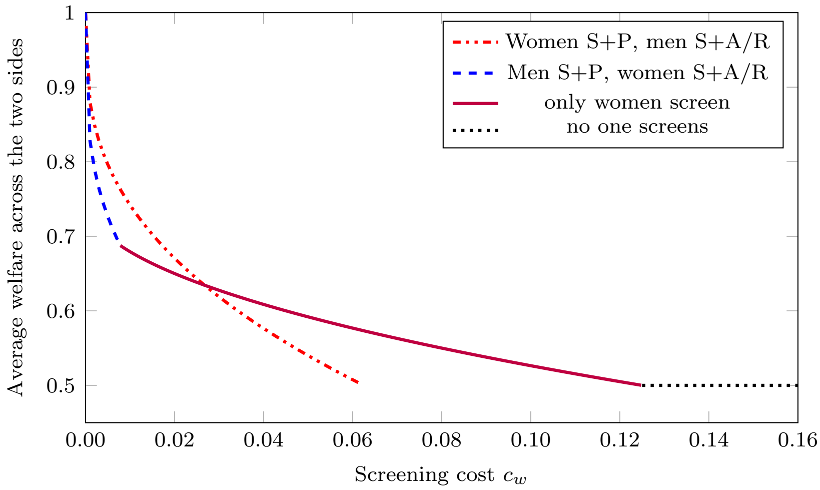

Figure 1: Welfare of equilibria with unequal screening costs $c_m = 2c_w$, same arrival rate on both sides, and i.i.d. $U(0,1)$ valuations on both sides of the market. In the legend, S+P=screen and propose, S+A/R=screen and then accept or reject.

First, note that any of these equilibria may arise without market intervention. When c is close to zero, the only equilibria are those in which one side proposes and both sides screen. When both these equilibria exist, the average welfare is higher when women play the role of proposers. To see why, compare the thresholds of men and women when they are the ones waiting incoming proposals: women use a higher threshold than men, as women's screening cost is smaller. Hence, when men are proposing, not only do they face a higher cost per opportunity screened, but also a smaller likelihood of their proposals being accepted. These two effects cause men's selectivity to decline rapidly with c_w (recall that c_m/c_w is fixed) when they are proposing, and for $c_w \geq 1/(32\alpha^2)$ men propose without screening.

For intermediate values of c_w, in particular for $c_w \geq 1/(32\alpha^2)$, typically[9] two equilibria coexist: (women screen + propose, men screen + accept/reject), and (men propose w.o. screening, women screen + accept/reject). For values of c_w just above $1/(32\alpha^2)$, the welfare is larger under the former equilibrium; men screen incoming proposals and this improves men's welfare (relative to not screening) more than it hurts women (recall the negative externality on proposers of selectivity by recipients). However, as c_w increases, the negative externality on women dominates and the equilibrium (men propose w.o. screening, women screen + accept/reject) has a larger average welfare.

Thus, the platform can significantly improve the average welfare by using an appropriate intervention to select a good equilibrium:

Corollary 1 (Selecting good equilibria via design)**.** *Consider again the setting of Theorem 1 and the equilibria described there.*

[9]Exactly two equilibria will exist if $\alpha \geq 2$. We focus on this case for the sake of the argument. If $\alpha \leq 2$, a third equilibrium exists: (women propose w.o. screening, men screen + accept/reject). However, the welfare of this equilibrium is dominated by that of (men propose w.o. screening, women screen + accept/reject), so the discussion on the main text remains valid.

- *For $c_w \leq c_*$, the welfare maximizing equilibrium is equilibrium 1. The platform can eliminate other equilibria by preventing men from proposing.*[10]
- *For $c_w \in [c_*, 1/8)$, equilibrium 5 maximizes welfare. The platform can implement one sided search where only women choose to obtain this welfare (and outcome) in the unique resulting equilibrium.*
- *For $c_w \geq 1/8$, equilibrium 6 maximizes welfare. The platform can implement centralized matching (agents do not choose) to obtain this welfare (and outcome).*

Here c_* is defined as

$$c_* = c_*(\alpha) = \frac{1}{32}\left(\sqrt{\left(\frac{1}{\sqrt[4]{\alpha}} - 1\right)^2 + 2} - \frac{1}{\sqrt[4]{\alpha}} + 1\right)^4 \tag{4}$$

Here, c_* is chosen such that equilibrium 1 and equilibrium 5 have identical welfare when[11] $c_w = c_*$. Note that implementing the suggested interventions does not decrease the average welfare. Furthermore, the improvement in average welfare can be substantial, for instance we get a 14.6% average welfare improvement when $\alpha = 2$ and $c_w = 1/16$.

3.2 Unbalanced markets

We now study the effect of unequal arrival rates on the two sides of the market. Assume, without loss of generality, that men are on the long side, i.e., they arrive faster than women. In this setting we expect women to be better off than men since they are overdemanded. We find that men compete for scarce women and hence cannot afford to be selective in equilibrium; even those who are lucky enough to match will not necessarily match with a woman they value very highly. However, the platform can significantly alleviate this issue by preventing men from proposing. In equilibrium, this allows men to be more selective, and boosts their welfare significantly at a small cost to women.

To formalize this, consider a market with $\lambda_w = 1$ and $\lambda_m = \lambda$ for $\lambda > 1$; that is, men arrive faster than women, or men are on the long side. For agents on both sides, their valuations for potential partners are drawn i.i.d. from a Uniform(0,1) distribution, and both sides face the same screening cost c. The next theorem characterizes the no-intervention equilibria as a function of the screening cost c.

Theorem 2 (No intervention equilibria)**.** *Consider a market with $\lambda_w = 1$ and $\lambda_m = \lambda$ for $\lambda > 1$. For agents on both sides, their valuations for potential partners are drawn i.i.d. from a Uniform(0,1) distribution. Both sides face the same search cost of $c > 0$. Then, the following is*

[10]Note here that the platform can slightly boost welfare by artificially increasing the value of c_m to $c_w^{2/3}/32^{1/3}$, if c_m is smaller than this. However, the boost obtained from this is very small.

[11]Recall that $\alpha \geq 1$. There is a unique solution to c_* that makes the welfare at both equilibria equal, since the difference between the welfares of equilibrium 5 and equilibrium 1 is strictly increasing in c_w, negative at $c_w = 1/(32\alpha^2)$ and positive at $c_w = \alpha/32$. This also means that equilibrium 5 maximizes welfare for $c_w > c_*$, and equilibrium 1 maximizes welfare for $c_w < c_*$.

the limiting description of a subset of stable stationary equilibria as a function of c (considering $\mu \to 0$ for each fixed c):

1. *(men screen + propose, women screen + accept/reject) with thresholds $\theta_m = \xi(\lambda, \sqrt{\frac{c}{2}})$ and $\theta_w = 1 - \sqrt{2c}$. This is an equilibrium for $c \in (0, 2\bar{c}^2)$.*

2. *(men propose w.o. screening, women screen + accept/reject) with threshold $\theta_w = 1 - \sqrt{2c}$. This is an equilibrium for*[12] *$c \in \left[2\underline{c}^2, \frac{1}{8}\right)$.*

3. *Agents on both sides propose without screening whenever they get a chance, and accept all incoming proposals without screening. This happens when $c \geq \frac{1}{8}$.*

Here

$$\xi(\lambda, c) = \frac{\lambda - \sqrt{\lambda^2 - 2\lambda(1 - 2c) + (1 - 2c)}}{2\lambda - 1} = \frac{\lambda - \sqrt{(\lambda - 1)^2 + 2c(2\lambda - 1)}}{2\lambda - 1} \tag{5}$$

(the function $\xi(\lambda, c)$ captures the equilibrium threshold for accepting a proposal used by the men when women are proposing),

$$\bar{c} = \frac{1}{4}\left(1 - \frac{\lambda}{1 + \sqrt{1 + (\lambda - 1)^2}}\right), \tag{6}$$

and

$$\underline{c} = \frac{1}{8\lambda^2}. \tag{7}$$

Furthermore, if $\lambda \geq 1.25$, there are no other stable equilibria.

The proof of Theorem 2 can be found in Appendix D. (We also state and prove Theorem 4, that captures additional equilibria in this setting for $\lambda < 1.25$.) We describe the main findings next. As a reference, Figure 2 illustrates the welfare under the different equilibria when $\lambda = 2$.

Under the equilibria described, the expected utility of women is greater than the expected utility of the men. This payoff inequality has two main causes. First, all women match (as $\mu \to 0$), while only a fraction $1/\lambda$ of men match. This is a consequence of the market primitives (recall that we take arrival rates to be exogenous), and hence cannot be fixed through intervention. Second, competition prevents men from being too selective in equilibrium. Note that for all but very small values of c (in particular, for $c \geq 2\bar{c}^2$), men propose without screening and get no better than a random woman (yielding expected match value 1/2) when they are lucky enough to match.

Further, when both sides are allowed to propose, there is no equilibrium where only women propose for $\lambda > 1.25$, thus causing men to have low welfare in all equilibria. The intuition is that it is always a best response for a man to reach out to a woman if he gets the opportunity to

[12]Depending on λ and c, women may or may not want to propose if they are given the chance. However, this does not play a significant role because all but a vanishing fraction of women will match before their clocks ring. In particular, as $\mu \to 0$, women would only propose if men do not screen the incoming proposals.

propose, as this would increase his chances of matching before dying. Given that men are thus active, women would rather wait for incoming proposals than screen and propose. However, when men propose they face large effective screening costs due to selectivity by women (see Lemma 1), and this hurts welfare in all equilibria.

Remark 1. *Note that there is a non-trivial interval of c's for which both equilibria 1 and 2 co-exist. The reason is that* selectivity by agents on the long side has a positive same-side externality*; when a man rejects a woman, this makes the woman available to match with other men. This externality leads to virtuous cycle in which selectivity by other men increases the availability of options for a man, and allows him to be more selective. As a result, both equilibria can co-exist at a particular value of c; the equilibrium where men screen results in higher welfare for men than the equilibrium in which men don't screen.*

The welfare of men can be significantly increased via intervention. We find that the platform can significantly boost the welfare by blocking men from proposing and thus forcing the women to propose. As a result of this intervention, men do not face rejection and can be reasonably selective. Selectivity by men again has a positive externality on other men which enhances the resulting welfare gains. Note that, in contrast with the case with different screening costs, this intervention *creates* new equilibria if the imbalance is not too small ($\lambda \geq 1.25$). These new equilibria are characterized next.

Theorem 3 (Intervention equilibria)**.** *Consider the market described in the statement of Theorem 2. If men are not allowed to propose, the following are all the stable equilibria as a function of c. (Again, this is the limiting description as $\mu \to 0$ for each fixed c.):*

1. *(women screen + propose, men screen + accept/reject) with thresholds $\theta_m = \xi(\lambda, c)$ and $\theta_w = 1 - \sqrt{2c/(1-\theta_m)}$. This is an equilibrium for $c \in (0, \min(\bar{c}, \hat{c}))$.*
2. *(women screen + propose, men accept w.o. screening) with threshold $\theta_w = 1 - \sqrt{2c}$. This is an equilibrium for $c \in [\underline{c}, \frac{1}{8})$.*
3. *(women propose w.o. screening, men screen + accept/reject) with threshold $\theta_m = \xi(\lambda, c)$. This is an equilibrium for $c \in [\hat{c}, \bar{c})$, and only exists if $\hat{c} < \bar{c}$ which occurs for $\lambda < 1.46$.*
4. *(women propose w.o. screening, men accept w.o. screening). This is an equilibrium for $c \geq 1/8$.*

where $\xi(\lambda, c)$ is as defined by Eq. (5)*, $\bar{c}$ and $\underline{c}$ are as defined by Eqs.* (6) *and* (7) *respectively, and*

$$\hat{c} = \frac{8\lambda - 7}{32(2\lambda - 1)}. \tag{8}$$

To illustrate the implications of Theorem 3, we consider the case of $\lambda > 1.46$. (The same insights apply when $\lambda < 1.46$, but the discussion becomes cumbersome as additional equilibria exist in that case.) Recall that when $\lambda > 1.46$, the only equilibria under no intervention are

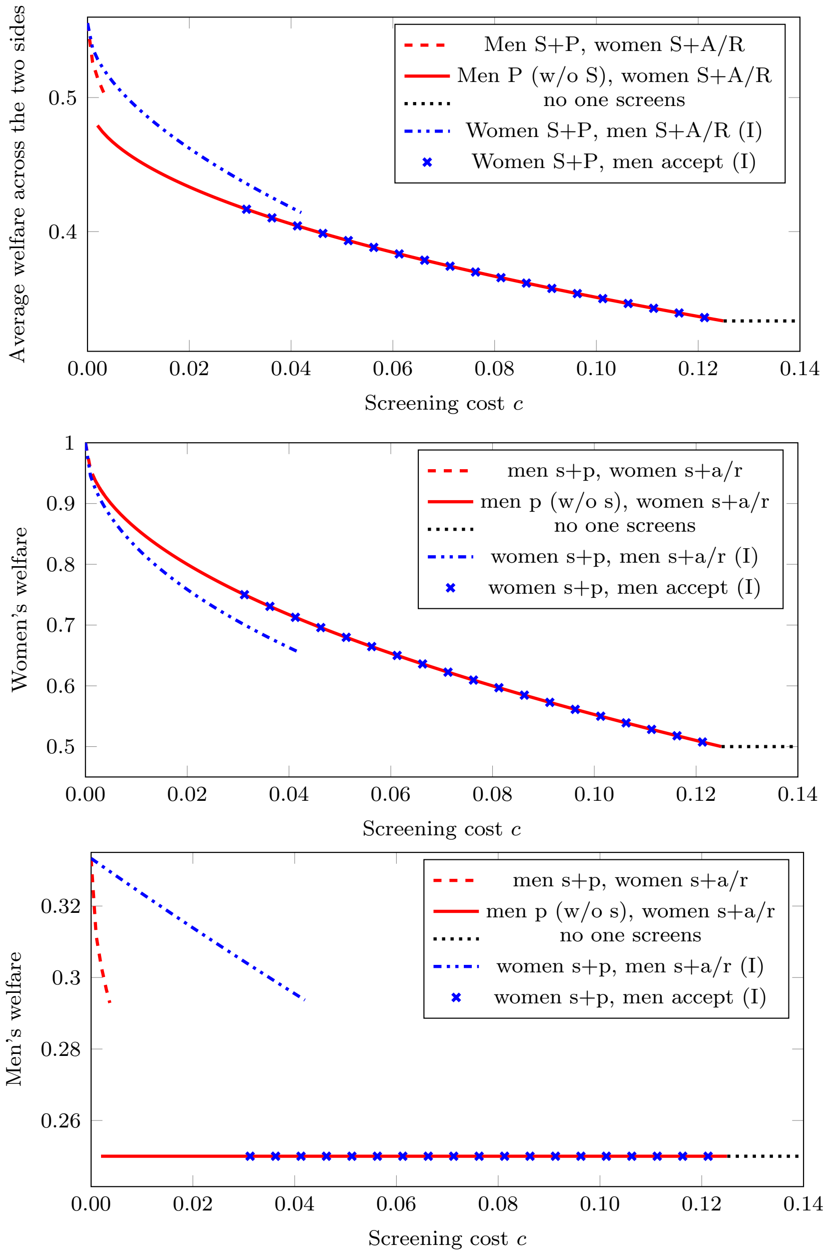

Figure 2: (*Top*) Equilibria with symmetric screening cost c, men arriving $\lambda = 2$ times as fast as women, and i.i.d. Uniform(0, 1) valuations on both sides of the market. Average agent welfare is shown, accounting for the fact that twice as many men pass through the system as women. In the legend, s+p = screen and propose, s+a/r = screen and then accept/reject, and (I) denotes that the equilibria is only obtained after the proposed intervention. (*Middle*) Women's welfare. (*Bottom*) Men's welfare.

those described in Theorem 2, whereas under the suggested intervention the equilibria are 1,2, and 4 in Theorem 3.

Our recommendation, based on Theorems 2 and 3, is as follows.

Corollary 2. *The platform can boost average welfare by preventing men from proposing if Theorem 3 equilibrium 1 exists when men are blocked from proposing, and the average welfare under this equilibrium is larger than that under the equilibria that can exist under no intervention (Theorem 2 equilibria 1 and 2 are the candidates).*

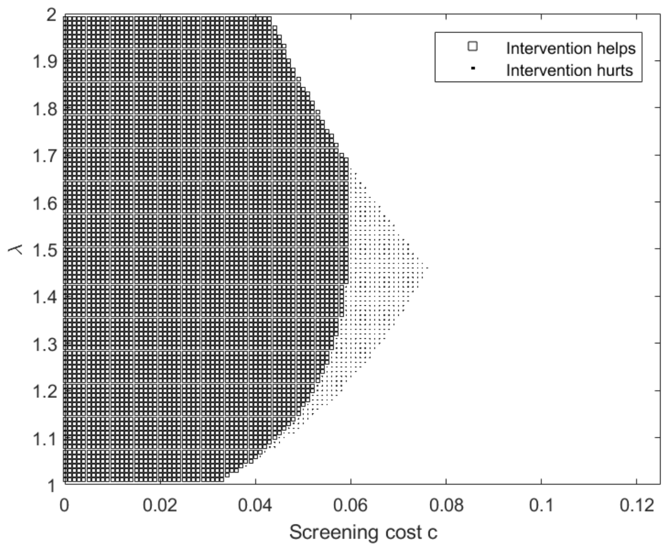

Figure 3: This figure shows the values of (λ, c) for which the platform should intervene by blocking men from proposing (see Corollary 2), shown via squares above: This is the region where Theorem 3 equilibrium 1 (women screen and propose, men screen and accept/reject) exists, and has welfare larger than all no intervention equilibria. Dots show the region where Theorem 3 equilibrium 1 exists but has welfare *smaller* than Theorem 2 equilibrium 2, and the blank area is where Theorem 3 equilibrium 1 does not exist. Recall that Theorem 3 equilibrium 1 exists for all $c < \hat{c}$ for $\lambda < 1.46$, and for all $c < \bar{c}$ for $\lambda > 1.46$.

Numerics based on this corollary reveal, see Figure 3, that the platform should block men from proposing for all c less than a threshold, where the threshold is equal to $\bar{c}$ for $\lambda > 1.67$ (including $\lambda = 2$, see Figure 2), and smaller than $\min(\hat{c}, \bar{c})$ for $\lambda < 1.67$.

Since women must propose in order to match, a man who receives a proposal has no fear of rejection —the decision of whether that match is formed is entirely up to him. Hence, he finds it worthwhile to screen, and accept only if match quality is reasonably good. This has a small negative externality on women; as they are the ones proposing, they face a higher effective screening cost (recall Lemma 1). The welfare loss incurred by women due to the intervention can be bounded as follows:

Remark 2. *For $\lambda = 2$, women's welfare decreases by less than 8%, across possible values of c.*

Furthermore, the fact that women are now proposing has two effects on men. First, as women are proposing, the effective screening cost of men decreases from $\sqrt{c/2}$ to c, and thus men are now more selective and screen. Second, this increased selectivity by men has a positive externality on other men, see Remark 1. Overall, this leads to a significant increase in the welfare of men:

Remark 3. *Men are always better off under the intervention. Men's maximum welfare increase occurs at $c = 2\bar{c}^2$; the increase can be up to 41%, and this welfare gain is decreasing in λ. For $\lambda = 2$, the welfare increase at $c = 2\bar{c}^2$ is 31%.*

Despite women being worse off, the fact that men's welfare increases after the intervention translates into the following average welfare increase:

Remark 4. *Following the recommendation in Corollary 2, the average welfare in the intervention is always (weakly) larger than under no intervention. When $\lambda = 2$, the welfare gain can be up to* 10%, *which is the gain achieved at $c = 2\bar{c}^2$.*

4 Vertical differentiation

We now augment the model from Section 3.2 to incorporate vertical differentiation on one side of the market. We see that the intuition from Section 3.2 generalizes: the weakest agents in the market propose in all equilibria when the platform does not intervene, which hurts their welfare as they are unable to be selective. However, many interesting new features emerge in the model with tiers, which makes the analysis more challenging. For instance, inefficiencies due to bad equilibria under no intervention persist as screening costs go to zero, in part because the weakest group of agents have most of their proposals completely *ignored* (as opposed to being rejected after screening, which also occurs), preventing them from being selective. Moreover, some agents who could find a suitable match may die unmatched while waiting for an ideal partner, which further harms the weakest group.

We consider the simplest case to study the impact of vertical differentiation. We augment our model to have one tier of women, and two potential tiers of men: top (high-quality) men, and bottom men. While quality is known to the platform (and to agents, unless otherwise stated), we retain an (additive) idiosyncratic component of utility which can only be discovered by spending a cost.

We first describe the augmented model, as well as the main changes to the equilibrium concepts defined in Section 2.1. We then characterize the equilibrium when no intervention is present. Next, we propose several interventions. Similarly to the case with no horizontal differentiation, we show that directing search by blocking outgoing proposals from one side of the market can result in significant welfare gains. In addition, we show that welfare gains can be further improved if the platform hides information regarding the agents' quality. We also discuss an intervention where the platform can learn and reveal the quality (top/bottom) of potential partners a particular agent (woman) is considering. Finally, we discuss how subscription fees for access to high quality potential matches (or for being presented matches at a faster rate) can help achieving an asymptotically socially optimum outcome when the search cost tends to zero.

Parts of this section are presented informally in this draft. Some interventions presented in this section (identifying women who will consider bottom men, hiding quality information and charging subscription fees) may become part of a separate paper (see footnote 20).

4.1 Augmented model: one tier of women, two tiers of men

We augment our model to incorporate vertical differentiation in the simplest possible setting. Women are ex ante homogeneous as before, but there are two *tiers* of men, "top" (t) men and "bottom" (b) men. A woman's utility for matching with a man of quality/tier τ is distributed as $a_\tau + \text{Uniform}(0,1)$, independently across pairs. We consider $a_{\text{t}} = a > 0$ and $a_{\text{b}} = 0$, and these values are common knowledge. A man's utility for matching with a woman is i.i.d. $\text{Uniform}(0,1)$. Women arrive at rate λ_w, top men arrive at rate λ_m^t and bottom men arrive at rate λ_m^b. We assume $\lambda_w > \lambda_m^t$ and $\lambda_w < \lambda_m^b + \lambda_m^t$.

Next, we augment our model of search as follows. When a woman receives a proposal, she knows, at no cost, whether it is from a top man or from a bottom man. When a woman has an opportunity to request a potential match, she can specify what tiers of men she is interested in, and in what order of priority. The platform will show her a potential option of her most preferred tier of man who is currently available, or do nothing if there are no men of the tier(s) desired by the woman currently available. Again, the woman knows the tier of the potential partner she has been presented with, and can accordingly decide whether she wants to screen, etc. The woman knows $a_{\text{t}} = a$ and $a_{\text{b}} = 0$ but a screening cost of c is incurred if she wants to learn her idiosyncratic term. Men, as usual, must spend c to learn the idiosyncratic term regardless of their tier.

As before, for every fixed c we will consider the limit $\mu \to 0$. After taking this limit, we will further take $c \to 0$, and find that many of our simple interventions can considerably improve welfare in this limit. This is in sharp contrast with the no vertical differentiation case, where the social welfare obtained by the equilibrium outcome in the no intervention case as $c \to 0$ cannot be improved by one of our simple interventions (see Section 3).

4.1.1 Extending the equilibrium concept and the mean field assumption

We now briefly comment on the extensions that we need make to the definitions in Section 2.1.

- **Strategy:** in the augmented model, the strategies of both bottom and top men are still as defined in Definition 1. However, the strategy of the women must be modified to account for the tiers. In particular, we allow women to have different strategies depending on the tier of the proposing agents. That is, women must now decide what to do with incoming proposals by agents from *each tier*: ignore the proposals, accept without screening, or screen and accept/reject (S+A/R).

In addition, to specify a woman's strategy for proposing, each woman has a (possibly incomplete) preference list over tiers of men. For every tier in the list, the women specifies whether to propose without screening (P), or to screen and propose if the match value is above the threshold (S+P).[13] When her clock rings, the platform will present the woman with a random man belonging to her top listed tier if there is a such a man in the system. If not, and the woman has not reached the end of her preference ranking over tiers, the platform will present her with a random man belonging to her second listed tier if there is a such a man in the system.

Finally, each women has a deterministic threshold θ_w used to screen participants.[14]

- **Mean field assumption:** We now informally extend Assumption 1 to the case with tiers. We will use the superscripts t and b to refer to top an bottom men respectively. Again, we fix the fractions $f_m^t(s)$, $f_m^b(s)$ and $f_w(s)$, and the thresholds θ_m^t, θ_m^b and θ_w. The limiting steady state will now consist of quantities $\{L_m^t(s), L_m^b(s), L_w(s)\}_{s\in\mathcal{S}}$, where $L_m^t(s), L_m^b(s), L_w(s)$ denote the mass of top men, bottom men, and women following each strategy under the steady-state distribution. Similar to case without tiers, each agent makes the following assumptions:

  1. *Potential options (when they are always available):* If $L_k^\tau(s_k) > 0$ for some side $k \in \{m, w\}$, a tier τ, and some $s_k \in S$, then an agent on the other side expects that each time his/her clock rings a potential option of tier τ will be offered if he/she asks for it, and the strategy of that agent will be s_k with likelihood $L_k^\tau(s_k)/\big(\sum_{s\in\mathcal{S}} L_k^\tau(s)\big)$. Note that this assumption is equivalent to the corresponding one for the case of no tiers.
  2. *Potential options (when they are available only sometimes):* On the other hand, if $L_k^\tau(s) = 0$, it must be that $L_k^\tau(s') = 0$ for all strategies that side k, tier τ agents are employing. (Under our assumption $\lambda_w < \lambda_m^b + \lambda_m^t$, this cannot simultaneously be the case for both tiers of men.)

     Each agent on the other side expects that each time his/her clock rings and he/she asks for a potential option from tier τ, an option will be provided and will be following strategy s with probability $\psi_k^\tau(s)$, that is uniquely determined. (We omit the general specification of ψ_k^τ in this draft.)
  3. *Incoming proposals.* Women will receive proposals from a tier τ, strategy s_m men at known rate $\rho_w^\tau(s_m)$. Similarly, τ-th tier men will receive proposals from women following s_w at known rate $\rho_m^\tau(s_w)$. (Again, we omit the specification of the ρ's in this draft.)

- **Stationary equilibrium and stability**: The definitions of stationary equilibrium (Definition 2) and evolutionary stability remain analogous to those for the case of no tiers.

[13] If a tier is not listed, we assume that the woman is not interested in that tier.

[14] Recall that, at equilibrium, θ_w will be equal for all women regardless of their strategy (this is what we defined as threshold consistency).

4.2 Equilibrium when no intervention is present

We now describe the equilibrium when no intervention is present. We find a low welfare equilibrium where bottom men propose without screening, and women screen and propose only to top men in all equilibria. Note that this is similar to the insights obtained in Section 3, in the sense that agents propose to those with more market power. However, here, we find that the equilibrium has poor welfare, in particular for bottom men, even in the limit $c \to 0$.

Fix $a \in (0, 1)$. Define $\delta_{\mathrm{t}} = \lambda_{\mathrm{t}} a/2 > 0$ and assume[15] $\lambda_w \in (\lambda_{\mathrm{t}} + \delta_{\mathrm{t}}, \lambda_{\mathrm{t}} + \lambda_{\mathrm{b}} + \delta_{\mathrm{t}})$. Then, there is a stable equilibrium[16] with the following description when we take $\mu \to 0$ and then $c \to 0$:

- Bottom men propose without screening.
- Top men do not propose, and screen and accept/reject incoming proposals using a threshold of $\theta_m^{\mathrm{t}} = 1 - \sqrt{2c}$.
- Women do not propose to bottom men. When the opportunity arises, women screen an available top man, and propose to him with the same threshold of[17] $\theta_w = 1 - \sqrt{2c}$. The women split into two types based on how they respond to proposals from bottom men.
  - **Seekers:** A fraction $\frac{\lambda_{\mathrm{t}}+\delta_{\mathrm{t}}}{\lambda_w}$ of women will ignore proposals from bottom men, and instead wait in the hope of matching with a top man.
  - **Settlers:** Other women will screen and accept/reject incoming proposals from bottom men with a threshold of θ_w, as a result they typically match with bottom men.

In steady state, there is a mass $\delta_{\mathrm{t}}/\mu = \Theta(1/\mu)$ of reachers in the market (to leading order), a mass

$$\frac{(\lambda_w - \lambda_{\mathrm{t}} - \delta_{\mathrm{t}})\delta_{\mathrm{t}}}{(\lambda_{\mathrm{b}} + \lambda_{\mathrm{t}} + \delta_{\mathrm{t}} - \lambda_w)\sqrt{2c}} = \Theta(1/\sqrt{c})$$

of settlers, and a mass $(\lambda_{\mathrm{b}} + \lambda_{\mathrm{t}} + \delta_{\mathrm{t}} - \lambda_w)/\mu = \Theta(1/\mu)$ of bottom men, whereas a top man is present in the market only a fraction

$$\frac{\mu\sqrt{2}}{a^2\sqrt{c}} = \Theta(\mu/\sqrt{c})$$

of the time. The mass of top men in the system is always 0.

- **"Top submarket":** Top men very quickly match (typically to seeker women) and leave, earning expected utility $\theta_m^{\mathrm{t}} \to 1$. A fraction $1/(1 + a/2) + o(1)$ of seeker women match with top men (earning a utility that is Uniform$(\delta_w, 1 + a)$ whereas the rest die without matching, consistent with their equilibrium utility of $\theta_w \to 1$.

[15] This is the "interesting" range for λ_w. If $\lambda_w < \lambda_{\mathrm{t}} + \delta_{\mathrm{t}}$, then women do not match with bottom men at all in equilibrium, and the interaction between top men and women is analogous to that captured in Section 3.2. On the other hand, if $\lambda_w > \lambda_{\mathrm{t}} + \delta_{\mathrm{t}} + \lambda_{\mathrm{b}}$, then all bottom men (and all top men) match in equilibrium and the situation again becomes similar to that in Section 3.2 with men being on the short side.

[16] We further conjecture that this is the only stable equilibrium.

[17] Note that, at an equilibrium, all women will be using the same threshold θ_w, which is also equal to their expected pay-off.

- **"Bottom submarket":** Settler women earn the same utility of θ_w by typically matching with a bottom man (whom they like). A fraction $(\lambda_w - \lambda_t - \delta_t)/\lambda_b$ of bottom men are lucky enough to match earning expected match utility 1/2 each (they like their partner no more than average), the rest die without matching. Thus, the overall expected utility of bottom men is $(\lambda_w - \lambda_t - \delta_t)/(2\lambda_b)$.

Note that the death-rate of top-seeking women is determined endogenously: In equilibrium, a woman must be indifferent between being seeker or a settler. For instance as a increases top men become more attractive, and the fraction of seeker women that die in equilibrium must increase to maintain this indifference.

We can relate the above results with the ones obtained in Section 3.2. The behavior of agents under equilibrium in the top submarket resembles that in Theorem 2 equilibrium 1, where the long side screens and proposes.[18] In the bottom submarket, the behavior resembles that in Theorem 2 equilibrium 2, where the long side proposed without screening. However, there is a major difference between the present case and an unbalanced market without vertical differentiation. Here, bottom men do not know who the settler women are when proposing. Further, since $\mu \to 0$ (and $\mu/\sqrt{c} \to 0$) most of the women present in the market are reachers who ignore proposals from bottom men; this makes it highly unattractive for bottom men to screen before proposing even when $c \to 0$, resulting in the welfare being low for bottom men even in this limit.

Furthermore, the inefficiencies we find appear related to phenomena observed in real online platforms (we do not pursue a detailed mapping between our model and reality at this point). [15] finds that searchers on Airbnb often leave the market although they could have found a suitable partner. A similar effect has been uncovered in the context of O-Desk [21]. Consider also some empirical findings from Tinder [34]: A third of men on Tinder report that they casually "like" most profiles, cf. the equilibrium behavior of bottom men. Women are much more selective, and 59% of women (as compared to just 9% of men) report that they like fewer than 10% of all profiles that they encounter, cf. the equilibrium behavior of seeker women. Less than 1% of likes by men result in match (a match occurs when two users like each other), whereas the corresponding number is over 10% for women, cf. our equilibrium finding that most proposals by bottom men are ignored.

4.3 Interventions

The equilibrium described in Section 4.2 suffers from two main problems, each of which cause a loss in welfare. The first issue is that it is that agents with low market power (in this case, bottom men) are unable to be selective. This is because it is very likely that their proposal will go to a seeker woman, who will ignore it. Hence, bottom men who are lucky enough to match only match with an average women. The second issue is that a fraction of the seeker women die without matching instead of matching with bottom men, which further decreases the welfare.

[18] The small modification is that here, a woman's utility for a man is uniformly distributed in $(a, a + 1)$.

Intervention	θ_m^{t}	θ_w	θ_m^{b}	Women die or not	Bottom men can choose or not
No intervention	1	1	$1/(2\lambda_\delta)$	Die!	Can't choose!
Men can't propose	1	1	$1/(2\lambda_\delta - 1)$	Die!	Can choose
Identify women who consider b-men	1	1	$1/(2\lambda_\delta - 1)$	Die!	Can choose
Hide quality of men and men can't propose	1	1	$1/(2\lambda - 1)$	Don't die	Can choose
Subscription a for access to t-men[19] *Platform earns rev at rate* $\lambda_{\mathrm{t}} a$	1	1	$1/(2\lambda - 1)$	Don't die	Can choose

Table 1: Welfare under different platform interventions in markets with vertical differentiation, parameterized by $\lambda_{\mathrm{t}}, \lambda_{\mathrm{b}}, \lambda_w$ and $a \in (0,1)$ where $\lambda_w \in (\lambda_{\mathrm{t}}(1 + a/2), \lambda_{\mathrm{t}} + \lambda_{\mathrm{b}})$, and we consider $\mu \to 0$ and then $c \to 0$. We use $\lambda_\delta = \lambda_{\mathrm{b}}/(\lambda_w - \lambda_{\mathrm{t}}(1 + a/2))$ and $\lambda = \lambda_{\mathrm{b}}/(\lambda_w - \lambda_{\mathrm{t}})$. Note that $\lambda_\delta > \lambda > 1$.

Next, we present a number of approaches that can be employed by the platform to mitigate or resolve these issues. (Here, we impose the further restriction $\lambda_w < \lambda_{\mathrm{t}} + \lambda_{\mathrm{b}}$ which strengthens our earlier requirement $\lambda_w < \lambda_{\mathrm{t}}(1 + a/2) + \lambda_{\mathrm{b}}$.) The resulting welfare under the different interventions is summarized in Table 4.3. We summarize the key ideas leading to our results in each setting next. In our analysis, we often refer to the quantity λ_δ which is formally defined as:

$$\lambda_\delta = \lambda_{\mathrm{b}}/(\lambda_w - \lambda_{\mathrm{t}}(1 + a/2)) \,. \tag{9}$$

Block men from proposing. Suppose that men are not allowed to propose, regardless of their tier. Then, there is a unique equilibrium when $\mu \to 0$ and then $c \to 0$. As in the case without intervention, we have two types of women in equilibrium: seekers and settlers. When a seeker's clock rings she requests a top man; if none is available, she just waits in the system. Settlers first request a top man and, if none is available, request a bottom man. All men screen and accept/reject. In particular, bottom men are also now able to be selective. Similarly to the no intervention case, two submarkets essentially arise at equilibrium: seekers match with top men or die without matching, while in the settlers match (primarily) with bottom men. There is a small reduction in the welfare of women relative to the no intervention case because each settler woman now faces the risk of rejection by a bottom man when she proposes. However, in the limit $c \to 0$, we still have $\theta_w \to 1$. We have $\theta_m^{\mathrm{t}} = 1 - \sqrt{2c} \to 1$ as before. The bottom men get utility $1/(2\lambda_\delta - 1)$, where λ_δ is as defined by Eq. (9), while the limiting values obtained by top men and women remain 1 in each case. The improvement in utility of bottom men over the no intervention utility of $1/(2\lambda_\delta)$ results from the fact that *bottom men are now able to be selective, because they receive proposals instead of having to make them.* Overall, as $c \to 0$, blocking men from proposing *helps bottom men without affecting other agents.*However, this intervention does

[19] Women who subscribe use a threshold of $1 + a - o(1)$ but pay a for their subscription, resulting in a net welfare of $1 - o(1)$.

not fix the fact that some women are dying unmatched.

Identifying women who will consider bottom men. [20]Recall that, under no intervention, one of main factors damaging welfare is the fact that bottom men cannot afford to screen. The reason is that most of their proposals go to the more numerous type of woman present in the market in steady state, namely, seeker women, who ignore such proposals (see Section 4.2). To alleviate this, suppose the platform is able to identify "settler" women who will consider proposals by bottom men. This information can then be revealed to bottom men, who can direct their search efforts towards such women exclusively. Whereas such a classification ("interested" or "not interested" in bottom men) may not be binary, here we optimistically assume that it is binary *and* that the platform can learn it, and study the resulting equilibrium.

Unsurprisingly, the welfare remains unchanged for top men and women as knowing this information is irrelevant for their strategy. However, it considerably helps bottom men whose proposals are no longer ignored, and can hence afford to screen before proposing. In fact, we find that the limiting utilities for all agents agree with those obtained under the intervention which blocks men from proposing (see Table 4.3). Again, the problem of some women dying without matching persists.

We remark that a platform can implement this intervention via its recommendation engine.

Hiding quality information. We now consider the intervention where the platform does not reveal to women whether a man is a top man or a bottom man. For simplicity, we consider the combination of this intervention together with the one where the platform blocks men from proposing and consider the equilibrium that results.[21]

In equilibrium, women screen and propose, while ignoring all incoming proposals, top men screen and accept/reject, and bottom men screen and accept/reject. Under this equilibrium, the limiting utilities are 1 each for top men and top women (as has been the case under all settings discussed so far), and $1/(2\lambda - 1)$ for bottom men where $\lambda = \lambda_{\mathrm{b}}/(\lambda_w - \lambda_{\mathrm{t}}) < \lambda_\delta$. Note that the limiting utility for bottom men exceeds that under the previously considered interventions (and that under no intervention). This is due to the fact that this equilibrium is good in both problem areas: bottom men are able to be selective, but also *almost all women match in equilibrium.*

We remark that the intervention of hiding information fits well with what many dating platforms do already: for instance, Tinder learns the attractiveness of a user's profile, and encodes this internally in a vertical "Elo" rating (that it uses to guide its recommendations), but does not reveal this rating to its users. We further remark that completely hiding information quality information may not always be the best approach. When there are multiple quality levels, the platform may maximize welfare by providing *partial* quality information, allowing users to prune the consideration set somewhat, but not to differentiate between those whom she

[20]The writeup from here until the end of this section is superseded by an ongoing project with Irene Lo.

[21]In the platform allows men to propose, then another equilibrium (or equilibria) arise where only men propose. This equilibrium (or equilibria) results in a lower limiting welfare for bottom men (as $c \to 0$) because they face rejection with likelihood $1 - \Theta(c)$, leading to an effective screening cost (see Lemma 1) of $\Theta(1)$.

can realistically "seek" and those whom she can settle for.[22] The platform may simulate this partial revelation of quality information by a combination of recommendation engine design and keeping quality information invisible.

Charging a subscription fee for access. We now consider a possibly more heavy-handed intervention, which involves charging agents a subscription fee for premium service.

In particular, suppose the platform was able to charge women a subscription fee to be able to see top men as potential options. A fee just below a turns out to yield remarkable welfare benefits. In equilibrium, a fraction approaching λ_t/λ_w of women (seekers) will be willing to pay the fee and will match with top men, earning match utility approaching $1 + a$, and hence net utility approaching 1. The other women will not pay the fee and will match with bottom men, also earning utility of 1. Almost no women die. In equilibrium, the limiting values of the agents are as high as those attained by the "hide quality information and men can't propose" intervention.[23] In addition, the platform earns a revenues at a rate of $a\lambda_t$. Thus, this intervention Pareto dominates the previously described interventions in terms of welfare.

We remark that the intervention considered here closely resembles the recently launched subscription service Upwork Pro, which provides access to high quality freelancers. We further remark that the same benefits can be achieved via a subscription service that charges women for receiving potential options at a faster rate (formally, for an increase in the speed at which their clock rings). This resembles the subscription service Tinder Plus that currently allows for unlimited "Likes" and five "Super Likes" each day, relative to about 100 "Likes" and one "Super Like" per day for free users.

5 Discussion

In this section we discuss some questions that arise in the context of our model and results.

One can consider the merits of our modeling assumption that agent departures without matching are *exogenous*. (Agent departures, in itself, is reasonable to assume since it occurs in most real platforms, and is necessary to obtain a steady state even when arrival rates are different on the two sides.) An alternative modelling approach may consider strategic departures (this could equivalently be interpreted as strategic market entry/participation): we could assume that agents have an outside option, and depart if their expected utility from participation under the same mean field assumptions is smaller than this outside option. It would appear that such a model may *not* capture the reality in many matching platforms, i.e., users do not appear to enter/exit strategically based on (aggregate) market state: for instance, multiple investigations have found that agents who face rejection are much less likely to match, even if suitable matches are available [15, 21]. Another possible piece of suggestive evidence is that the dating app Bumble, which requires women to send the first message, may actually benefit men and hurt

[22] We anticipate that formal results along these lines may be hard to obtain in the current setup.

[23] This is the case for both the possible equilibria in the bottom submarket: the one where bottom men propose, and the one where the settler women propose.

women slightly (see Section 3), and yet it has a more balanced set of users (about 50-50) relative to other platforms where the majority of users (60-70%) are male.

We allow the platform to prevent one or both sides of the market from proposing, and find that intervening in this manner (and a careful choice of *which side* is disallowed from proposing) can boost welfare in many situations by improving the efficiency of screening and match formation. It is natural to ask whether further improvements are possible by limiting (not necessarily to zero) the permitted rate of making proposals/seeing potential options on one or both sides of the market. Indeed such interventions are implemented in several real world matching platforms. We leave this question for future work, remarking that this relates with the literature on signaling in matching markets (see Section 1.1), and a recent paper on this issue in the context of competition between matching platforms [19].

In Section 3, we argue for certain interventions because they improve aggregate welfare in certain settings. The thought is that if the welfare of one set of agents improves substantially, at the cost of a small reduction in the welfare of another set of agents, this is desirable overall, especially in light of the fact that platforms often have tools as their disposal to transfer welfare roughly via "charging" one group of agents while subsidizing another group [14]. Further, user welfare is a primary objective for any platform in terms of attracting and retaining users, even if the ultimate goal is to maximize revenues. We suppress issues of platform revenue through most of this paper.

We did not compare against a planner's benchmark nor even on optimal mechanism designn, instead focusing on the best welfare a platform can achieve by choosing between a small set of *simple interventions*. We remark that a simple upper bound on aggregate welfare can be obtained by considering the best that a planner can do when screening costs are zero, and the planner can control the actions of all agents. One can check that:

- This benchmark is achieved in the limit of small screening costs when costs differ on the two sides (Section 3.1) under the high welfare equilibrium.
- This benchmark is *not* achieved in the limit of small screening costs in the case of unbalanced markets (Section 3.2), even when the long side is blocked from proposing. The reason is that the acceptance threshold on the long side does not converge to the upper bound of the support, simply because agents on the long side are at a risk of dying before they match.
- In the case with two tiers of men, and one tier of women considered in Section 4, among the interventions we consider, the benchmark welfare is only achieved when the platform charges women a subscription fee approaching the quality difference a between tiers in order to access top men.

In the interest of simplicity and tractability, we assumed idiosyncratic values are drawn i.i.d. uniformly between 0 and 1, and that a man m's value for a woman w is independent of w's value for m. We expect our insights to be reasonably robust to these assumptions. If m's value for w is strongly correlated with w's value for m, certain equilibrium features may be modified, though we expect that the welfare maximizing designs in the case of ex ante homogenous agents on each side will still be: (i) to have the side with lower screening cost go first/choose (cf. Section 3.1), and (ii) to have the short side go first.

We now briefly discuss the dynamic aspects of our model and results. One may ask if we can obtain similar insights in a simpler setup such as synchronous matching game [19] or a flow economy [24], instead of taking on the challenge of studying a dynamic steady state. It would appear that such alternate approaches would yield some of our insights but not others (and related to this, be somewhat unreliable in the intuition they provide). Consider the no intervention equilibrium in the setting with tiers. Here, bottom men suffer from most (all but a vanishing fraction) of their proposals being ignored (because most of the women in the system at any time are seekers hoping to match with top men), and as a result propose without screening, even when screening costs are very small, significantly hurting welfare. This phenomenon would be missed under a typical synchronous matching game approach or a flow economy model, which would miss that the preponderance of women who are present and searching ignore bottom men (cf. the inspection paradox [31]), even though the fraction who match with bottom men may be substantial. Another question one may ask is what happens if proposal responses are not immediate. We believe that this modification to the model will preserve or even increase the welfare gains from the interventions we propose in Section 3. The intuition is that our recommended interventions will replace proposals with low likelihood of being accepted with proposals with high likelihood of being accepted (in particular in unbalanced markets, most proposals by agents on the long side will be ignored if there is a delay in responding to proposals, causing an even lower likelihood of acceptance under no intervention).

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A Equilibrium concept, steady state, and mean field assumption

A.1 Justifying the statement of the mean field assumption (Assumption 1)

We justify each part of the assumption in turn.

1. This follows from the fact that the mass of men in the market playing each strategy s, $N_m(s)$, concentrates at $L_m(s)$, and the woman has interacted with only $O(1)$ of them before.

2. In this case, all men are considering incoming proposals. This implies that men are short-lived in the system, regardless of their strategy. In this case, there must be some strategy s_w for women such that $L_w(s_w) > 0$ and such that under s_w a woman considers a potential option when offered one (and hence proposes with positive probability).[24] Here, whether a man is making use of the opportunity to propose or not is irrelevant (since the mass of men in the system concentrates at zero, only a negligible mass of men have opportunities to propose). Proposals by men can hence be disregarded.

   If all men are following a single strategy s_m, then women assume that the likelihood of a potential option being available when their clock rings is[25]

$$\frac{\lambda_m}{\big(1 - \mathbb{I}(s_m \text{ involves S+A/R})F(\theta_m)\big)\sum_{s_w \in \mathcal{S}_\text{p}} L_w(s_w)\big(1 - G(\theta_w)\mathbb{I}(s_w \text{ involves S+P})\big)}, \tag{10}$$

   i.i.d. across clock rings. That is, if all men employ the same strategy, the numerator is equal to the arrival rate of men. The denominator must be equal to the rate at which proposals are issued and accepted, which can be obtained by multiplying the probability that a proposal is accepted (first term) [26], with the rate at which proposals are issued (second term)[27]. Note that the cumulative rate of men leaving due to death is negligible since $L_m(s_m) = 0$.

   Eqs. (1) and (2) in the main text generalize the above to men mixing according to $(f_m(s_m))_{s_m \in \mathcal{S}}$. Here ξ can be interpreted as the likelihood (when the clock of a woman rings) of the woman receiving a potential option who will accept if she proposes. Eq. (2) simply captures that in steady state, the arrival rate of women following each strategy must equal their departure rate, and further, that women must die at a rate of $\lambda_w - \lambda_m$,

[24] Since the mass of men following s in the system is 0, it follows that all but a negligible mass of men leave due to match formation, and moreover, men form matches due to incoming proposals. Hence, there *must* be some such strategy s_w that involves proposing under which women spend $\Theta(1)$ time in the system (since they must propose and thus must wait until their clock ticks), which means that $L_w(s_w) > 0$. Also, it must be $\sum_{s' \in \mathcal{S}} L_m(s') = 0$, to ensure that a man receives proposals at an $\omega(1)$ rate.

[25] In the following equation we use the notation introduced in Definition 1

[26] Recall that only women are proposing in this setting. Therefore, a proposal will be always be accepted if s_m does not involve screening (i.e., s_m involves A). However, if s_m involves screening + accepting/rejecting, each proposal is rejected with probability $F(\theta_m)$.

[27] Here, women with strategy $s_w \in S_p$ issue proposals at a cumulative rate $L_w(s_w)$ if s_w does not involve screening, and at cumulative rate $L_w(s_w)(1 - G(\theta_w))$ if it involves screening. Recall that this follows from the fact that the clock of all women tick at rate 1, regardless of their strategy.

since they match with men at a rate λ_m. One can check that in the special case that all men are following the same strategy we recover Eq. (10).

3. If $\sum_{s_w} L_w(s_w) = 0$, it is easy to deduce as before that women receive proposals at an individual rate that goes to ∞ with R.

   If $\sum_{s_w} L_w(s_w) > 0$, the individual rate $\rho_w(s_m)$ must be consistent with $L_m(s_m)$ for $s_m \in \mathcal{S}$, which leads to Eq. (3). The reasoning is that if $L_m(s_m) > 0$ and strategy s_m involves proposing, then each woman expects to receive proposals from men following strategy s_m at an individual rate of

$$\begin{aligned} \frac{L_m(s_m)(1-F(\theta_m)\mathbb{I}(s_m \text{ involves S+P}))}{\sum_{s_w} L_w(s_w)} \qquad & \text{if } \sum_{s_w} L_w(s_w) > 0\,, \\ \infty \qquad & \text{if } \sum_{s_w} L_w(s_w) = 0. \end{aligned}$$

A.2 Evolutionarily stable equilibrium

Let

$$N_m(s) = \text{Mass of men in the system following strategy } s. \tag{11}$$

Define $N_w(s)$ similarly. Further, let $\bar{N}_m = (N_m(s))_{s\in\mathcal{S}}$ and $N_m = \sum_{s\in\mathcal{S}} N_m(s)$, and similarly for women. Let $\bar{N} = (\bar{N}_m, \bar{N}_w)$. This implies a rate $\rho_m(s; \bar{N})$ at which matches involving men following strategy s are formed, and similarly for women. (These rates are easy to characterize in the case $N_m > 0$ and $N_w > 0$. We explicitly characterize these rates for the complementary case below.) When a new man enters, he considers the continuation value $V_m(s; \bar{N})$ that would result from using strategy s assuming $\bar{N}$ will remain unchanged over time. (Again this is easy to characterize in the case $N_m > 0$ and $N_w > 0$, and we explicitly consider the complementary case below.) The man chooses strategy $s^*_m(\bar{N}) = \arg\max_{s\in\mathcal{S}} V_m(s; \bar{N})$. (When there are ties, we will allow them to be broken arbitrarily, including possible mixing. This will ensure that all stationary equilibria will be captured as fixed points of the differential equations below.) We think of the thresholds θ_m, θ_w as being fixed.[28] Agents do not change their strategy during their lifetime. This leads to the following coupled ODEs capturing system evolution in the continuum limit

$$\begin{aligned} \frac{\mathrm{d}N_m(s)}{\mathrm{d}t} &= \mathbb{I}\big(s = s^*_m(\bar{N})\big)(\lambda_m + \xi N_m) - N_m(s)\big(\mu + \xi\big) - \rho_m(s; \bar{N}) \qquad \forall s \in \mathcal{S}\,, \\ \frac{\mathrm{d}N_w(s)}{\mathrm{d}t} &= \mathbb{I}\big(s = s^*_w(\bar{N})\big)(\lambda_w + \xi N_w) - N_m(s)\big(\mu + \xi\big) - \rho_w(s; \bar{N}) \qquad \forall s \in \mathcal{S} \end{aligned} \tag{12}$$

We now characterize the matching rates and continuation values in the hard case of interest.

[28] These thresholds are chosen to match the continuation value at the equilibrium/fixed point. We expect that holding these thresholds fixed generally should not impact whether an equilibrium classifies as stable or not, since the utility loss due to error in the choice of threshold should grow only quadratically with distance from the equilibrium, whereas the difference between utilities of different strategies in $\mathcal{S}$ should grow linearly with the distance from the equilibrium.

Suppose $N_m = 0$. (The case $N_w = 0$ is analogous.) Define

$$\begin{aligned} &\eta_w(s_w, s_m) \\ &= \big(\mathbb{I}(s_w \text{ involves P}) - G(\theta_w)\mathbb{I}(s_w \text{ involves S+P})\big)\big(\mathbb{I}(s_m \text{ involves A}) - F(\theta_m)\mathbb{I}(s_m \text{ involves S+A/R})\big)\,, \end{aligned} \tag{13}$$

i.e., the fraction of strategy s_m options shown to women following s_w that result in matches. Then we have that the likelihood of an incoming man almost immediately matching with a woman following s_w is proportional to $N_w(s_w)\eta_w(s_w, s_m^*(\bar{N}))$, leading to

$$\rho_w(s; \bar{N}) = N_w(s_w)\eta_w(s, s_m^*(\bar{N}))\min\left(\frac{\lambda_m}{\sum_{s_w\in\mathcal{S}} N_w(s_w)\eta_w(s_w, s_m^*(\bar{N}))}, 1\right)\,,$$

and $V_w(s; \bar{N})$ is the utility for a woman which results from being offered a potential match (following strategy $s_m^*(\bar{N})$) each time the woman's clock rings with likelihood equal to the last term $\min(\,\cdot\,, 1)$ above. For the men, $V_m(s; \bar{N})$ is the payoff from receiving proposals at rate ∞ with the proposer strategy being s_w with likelihood proportional to $N_w(s_w)\big(\mathbb{I}(s_w \text{ involves P}) - G(\theta_w)\mathbb{I}(s_w \text{ involves S+P})\big)$, and (relevant only if s ignores incoming proposals) always being offered a potential match, the strategy of the potential match being s_w with likelihood proportional to $N_w(s_w)$. The rate of matching is given by

$$\rho_m(s_m^*(\bar{N}); \bar{N}) = \min\left(\lambda_m, \sum_{s_w\in\mathcal{S}} N_w(s_w)\eta_w(s_w, s_m^*(\bar{N}))\right)\,.$$

(The rates of matching for $s_m \neq s_m^*(\bar{N})$ are irrelevant.) When the min is the second term, we see that $\frac{\mathrm{d}N_m}{\mathrm{d}t} > 0$ leading to $N_m > 0$ in future.

As mentioned above, all stationary equilibria correspond to fixed points of our dynamical system (12). We focus on the subset of stationary equilibria that are plausible from an evolutionary/dynamical standpoint.

Definition 3. *Each stationary equilibrium corresponds to a fixed point of the dynamical system* (12) *when the threshold θ_m (and θ_w) is equal to the continuation value of the best response for men (women) at the fixed point, and conversely. A stationary equilibrium is* evolutionarily stable *if the corresponding fixed point is attractive. We refer to this simply as a* stable equilibrium.

An attractive/stable fixed point of a dynamical system is a point such that if the state starts sufficiently close to the fixed point, it remains close to the fixed point and converges to it.

A.3 Dynamics when agents on each side follow a single strategy

We now analyze the system dynamics when all agents on the same side use the same strategy. We will find that the corresponding dynamical system always has a unique steady state/fixed point L, that is always stable. When the fixed strategies employed are the unique best responses on each side of the market to L, they are also best responses in a neighborhood of L, hence the system dynamics precisely matches the dynamics under best responses (Eq. (12)) in a neighborhood of L, implying that L corresponds to a stable equilibrium. As we will argue later, in all the settings consider in Section 3, in each stable equilibrium, all agents on the same side of the market do, in fact, use the same strategy. In other words, there is no mixed stable equilibrium.

Suppose all men employ strategy s_m and all women employ strategy s_w. Let η_w and η_m be defined as

$$\eta_w = \big(\mathbb{I}(s_w \text{ involves P}) - G(\theta_w)\mathbb{I}(s_w \text{ involves S+P})\big)\big(\mathbb{I}(s_m \text{ involves A}) - F(\theta_m)\mathbb{I}(s_m \text{ involves S+A/R})\big)$$

i.e., the fraction of options shown to women that result in matches, and similarly

$$\eta_m = \big(\mathbb{I}(s_m \text{ involves P}) - F(\theta_m)\mathbb{I}(s_m \text{ involves S+P})\big)\big(\mathbb{I}(s_w \text{ involves A}) - G(\theta_w)\mathbb{I}(s_w \text{ involves S+A/R})\big)\,. \tag{14}$$

Note that the expressions in Eq. (14) are special cases of the expressions defined in Eq. (13). We will show convergence to a limiting mass of men and women (i.e. L_m and L_w respectively[29]) and calculate the limits assuming that:

$$\lambda_m \neq \tfrac{\eta_w \lambda_w}{\mu+\eta_w} \tag{15}$$

$$\lambda_w \neq \tfrac{\eta_m \lambda_m}{\mu+\eta_m} \tag{16}$$

If $\eta_w = 0$ because the women do not propose under s_w, then condition (15) holds automatically. Suppose $\eta_w > 0$. We will find that the limiting values of L_m and L_w resulting from $\lambda_m \to \left(\frac{\eta_w\lambda_w}{\mu+\eta_w}\right)_+$ and $\lambda_m \to \left(\frac{\eta_w\lambda_w}{\mu+\eta_w}\right)_-$, holding everything else fixed, are identical. Though we omit the details, a coupling argument can be used to establish that this pair of values matches the L_m and L_w that arise from $\lambda_m = \left(\frac{\eta_w\lambda_w}{\mu+\eta_w}\right)$.

Note that the mass of men in the system in steady state is bounded above by λ_m/μ, since agents die at rate $\mu > 0$ (even if they don't leave by matching), and similarly for women. Also, note that the only way agents can have a vanishing expected lifetime in the system is if they receive incoming proposals at a rate of ∞. All other agents have a positive expected lifetime in the system. We will argue that:

(i) All agents have a positive expected lifetime in the system if the left-hand side is more than the right in both conditions (15) and (16), and

(ii) If the left-hand side is smaller in (15), then men will have a vanishing lifetime in the system. Similarly, if the left-hand side is smaller in (16), then women will have a vanishing lifetime in the system.

To that end, suppose that the left-hand side is greater than the right-hand side in both conditions (15) and (16). Note that, even if a woman is given a new potential option each time her clock rings, her likelihood of matching before she dies is only $\eta_w/(\mu+\eta_w)$. Hence, the maximum rate at which men match due to proposals by women is $R\lambda_w\eta_w/(\mu+\eta_w)$. If the

[29]Here, we abused notation and suppressed the dependence on the strategy, as it is the same for all agents on the same side.

left-hand side is larger in condition (15), then a positive fraction of men do not match as a result of proposals by women; thus, the expected time a man spends in the system is positive. Analogously, if the left-hand side is larger in condition (15), then the expected time a woman spends in the system is positive. Therefore, all agents have a positive expected lifetime in the system if the left-hand side is more than the right in both conditions (15) and (16).

Next, suppose the left-hand side is smaller than the right-hand side in condition (15). Then, we know that $\lambda_w > \lambda_m$, so a simple argument can be used to show that at any time, the mass of women in the system is positive (since women die at rate at least $(\lambda_w - \lambda_m)$ for all $t \geq t_0$, for some t_0). Also, women do not get a potential option with positive probability when their clock rings in steady state (else they would form matches faster than the rate of arrival of men, which is impossible). Therefore, there are no men in the system with positive probability, in particular, the limiting mass of men is 0. Moreover, the mass of men will stay at that level —if it starts to build up, then women will have a new potential option each time their clock rings, and will form matches at rate $\lambda_w \eta_w/(\mu+\eta_w) > \lambda_m$, reducing the mass of men. Hence, in steady state, the mass of men remains 0. Therefore, if the left-hand side is smaller than the right-hand side in condition (15), men will have a vanishing lifetime in the system. Note that the analogous argument can be applied in the case in which the left-hand side is smaller than the right-hand side in condition (16).

Limiting steady state when left-hand side is smaller than the right-hand side in condition (15)**.** In fact, we can precisely characterize the steady state. Since the mass of men is 0, the rate at which men die is 0, meaning that men form matches at λ_m (moreover, these matches occur at a steady pace). Hence, women match with men at a rate λ_m, meaning that women die at a rate of $\lambda_w - \lambda_m$, hence the mass of women in the system is $(\lambda_w - \lambda_m)/\mu$, and in fact the mass of women remains steady near this value since matches occur in a steady fashion. It follows that the mass of women concentrates around the limiting value of $L_w = \frac{\lambda_w - \lambda_m}{\mu}$, whereas the mass of men in the system is 0. Note that $\lambda_m < L_w \eta_w$, consistent with the mass of men remaining 0. (Interpreting our continuum model as the limit of a model with finite arrival rates, in the limit, the absolute number of men follows a birth-death process, where the number increases by 1 at rate proportional to λ_m and the number decreases by 1 at rate proportional to $L_w \eta_w$ (which is positive). In steady-state the birth death process is at value $k \in \mathbb{N} \cup \{0\}$ a fraction $\big(1 - \frac{\lambda_m}{L_w \eta_w}\big)\big(\frac{\lambda_m}{L_w \eta_w}\big)^k$ of the time. The fraction of the time the birth-death process has at least one man in the system is $\frac{\lambda_m}{L_w \eta_w}$, hence, the rate at which a woman can match in the limit is $\eta_w \frac{\lambda_m}{L_w \eta_w} = \frac{\lambda_m}{L_w}$, leading to a chance $\frac{\lambda_m}{L_w} / \big(\frac{\lambda_m}{L_w} + \mu\big) = \frac{\lambda_m}{\lambda_m + \mu L_w} = \frac{\lambda_m}{\lambda_w}$ of matching before the woman dies, as we expect.)

Note that as $\lambda_m \to \big(\frac{\eta_w \lambda_w}{\mu+\eta_w}\big)_-$ we have $L_m = 0$ (in fact, this holds everywhere in this case) and $L_w \to \frac{\lambda_w}{\mu+\eta_w}$.

Limiting steady state when the left-hand side is larger in both conditions (15) **and** (16)**.** Let N_m be the mass of men in the system at time t. (Recall that all men are using the same strategy s_m.) Define N_w similarly. The limiting dynamical system when the left-hand side is larger in both conditions (15) and (16), is given by (refer to the definitions of the η's in Eq. (14))

$$\frac{\mathrm{d}N_m}{\mathrm{d}t} = A\bar{N} + b\,, \text{ for}$$

$$\bar{N} = \begin{bmatrix} N_w \\ N_m \end{bmatrix},\ b = \begin{bmatrix} \lambda_w \\ \lambda_m \end{bmatrix},\ A = \begin{bmatrix} -\mu - \eta_w & -\eta_m \\ -\eta_w & -\mu - \eta_m \end{bmatrix}$$

This is a pair of coupled linear differential equations in N_m and N_w, and note that is a special case of those defined in Eq. (12). Matches resulting from options shown to women form at rate $N_w\eta_w$ and matches resulting from options shown to men form at rate $N_m\eta_m$. In addition, individual agents die at rate μ, leading to the form of the equations.

The eigenvalues of A are $-\mu$ and $-\mu - \eta_w - \eta_m$. Since the eigenvalues are negative, we deduce [33] that

$$L = \begin{bmatrix} \frac{\lambda_w(\mu+\eta_m)-\lambda_m\eta_m}{\mu(\mu+\eta_w+\eta_m)} \\ \frac{\lambda_m(\mu+\eta_w)-\lambda_w\eta_w}{\mu(\mu+\eta_w+\eta_m)} \end{bmatrix},$$

which solves $A\bar{N} + b = 0$, is a stable fixed point of the dynamical system with a global basin of attraction. Hence, the dynamical system converges globally to L.

Proposition 1. *When agents on each side follow a single strategy the steady-state of the system* $L = [L_w, L_m]$ *can be characterized as follows:*

1. *When the left-hand side is smaller than the right-hand side in condition* (15)*, the system converges to a steady state of*

$$L = \begin{bmatrix} \frac{\lambda_w-\lambda_m}{\mu} \\ 0 \end{bmatrix}. \tag{17}$$

2. *When the left-hand side is larger in both conditions* (15) *and* (16)*, the system converges to a steady state of*

$$L = \begin{bmatrix} \frac{\lambda_w(\mu+\eta_m)-\lambda_m\eta_m}{\mu(\mu+\eta_w+\eta_m)} \\ \frac{\lambda_m(\mu+\eta_w)-\lambda_w\eta_w}{\mu(\mu+\eta_w+\eta_m)} \end{bmatrix}.$$

Note that as $\lambda_m \to \left(\frac{\eta_w\lambda_w}{\mu+\eta_w}\right)_+$ we have $L_m \to 0$ and $L_w \to \frac{\lambda_w}{\mu+\eta_w}$. These limiting values of L_m and L_w match the limiting values that arise when $\lambda_m \to \left(\frac{\eta_w\lambda_w}{\mu+\eta_w}\right)_-$.

B Proof of Lemma 1

Proof of Lemma 1. Consider the first system. Let q' be the likelihood that a value exceeds θ. The probability that a potential option is both requested and approved is qq'. Hence:

- The expected screening cost spent per obtained item is $c/(qq')$.
- The likelihood of obtaining an item before death is the likelihood that a Poisson clock of rate $\eta qq'$ rings before the death Poisson clock of rate μ.

It is easy to check that the two parts of this description each apply also to the second system, since the probability of a value exceeding θ is again q'. Finally, the expected value of an obtained item is just $\mathbb{E}_{X\sim F}[X|X > \theta]$ in each system. Combining, we obtain the claim. □

C Appendix to Section 3.1

Sketch of proof of Theorem 1. First, note that in all equilibria listed in the statement of Theorem 1, agents on each side of the market are using a unique strategy. Therefore, as argued in Section A.3, the steady state will exist in each case, and can be characterized. In particular, using Proposition 1, we have $L_m = L_w = \frac{\lambda}{\mu+\eta_w+\eta_m} = \Theta(1)$, when $\mu \to 0$.

We next argue that each of these is an equilibrium in the proposed regime.

1. (women screen + propose, men screen + accept/reject). Note that, if women are proposing, for men is optimal to use the threshold $\theta_m = 1 - \sqrt{2\alpha c_w}$.[30] Furthermore, they do not have an incentive to propose themselves. Therefore, by using Lemma 1, we can think of women facing an effective screening cost of $c_{\text{eff}} = c_w/\sqrt{2\alpha c_w} = \sqrt{c_w/(2\alpha)}$, and hence their threshold will be $\theta_w(c)$. In addition, note that men don't want to screen beyond $c_w = \frac{1}{8\alpha}$; at this point, they would rather accept without screening. Also, women don't want to screen beyond $c_w = \frac{\alpha}{32}$; at this point, it also becomes profitable for them to stop screening.

2. (women propose w.o. screening, men screen + accept/reject). Note that this equilibrium occurs if women stop screening in the above equilibrium before men, that is, if $\frac{1}{8\alpha} \geq \frac{\alpha}{32}$. This is the only equilibrium whose existence depends on the value of α, and it occurs for some c_w's if $\alpha < 2$. Note that, again, if women are proposing, for men is optimal to use the threshold $\theta_m = 1 - \sqrt{2\alpha c_w}$ as men are not affected by whether a women screens or not.

3. (women screen + propose, men accept w.o. screening). As argued in equilibrium (1), if women are proposing, men will give up on screening only for $c_w \geq \frac{1}{8\alpha}$. Therefore, this will be an equilibrium for $c_w \in \left[\frac{1}{8\alpha}, \frac{1}{8}\right)$.

4. (men screen + propose, women screen + accept/reject). Here, if men are proposing, it is optimal for a women to screen and accept/reject with threshold $\theta_w = 1 - \sqrt{2c}$. Therefore, men face an effective screening cost of $c_{\text{eff}} = \alpha c_w/\sqrt{2c_w} = \sqrt{\alpha^2 c_w/2}$ (see Lemma 1), which gives us θ_m. Note that, in this setting, women will never stop screening before men, as they have a lower screening cost plus they are not facing rejection (i.e. they are not proposing). Therefore, this will be an equilibrium as long as men continue to screen, which happens if $c_w < \frac{1}{32\alpha^2}$.

5. (men propose w.o. screening, women screen + accept/reject). This equilibrium occurs when men no longer want to screen, that is, when $c \geq \frac{1}{32\alpha^2}$. In addition, women will have an incentive to screen as long as $c_w \leq \frac{1}{8}$, which defines the range for which this is an equilibrium.

[30] This follows from the fact that θ_m is equal to the continuation value of the men. As $\mu \to 0$ the fraction of men who does not match vanishes, we have that in the limit $\mu \to 0$ the continuation value satisfies $\theta_m = -\alpha c_m + (1-\theta_m)\frac{1+\theta_m}{2} + \theta_m$. Solving for θ_m yields the desired expression.

6. Agents on both sides propose without screening whenever they get a chance, and accept all incoming proposals without screening when $c_w \geq 1/8$. By our previous arguments, it can easily be seen that agents will have an incentive to deviate and screen if $c_w < 1/8$.

To conclude the proof, we note that there cannot be any mixed equilibria that is evolutionary stable. If one side, say women, mixes between proposing and not, then at least a fraction of men must be proposing; otherwise, women who are not proposing will never get match and thus proposing is a profitable deviation. However, once the number agents on the other side who are proposing is slightly perturbed, that will affect the number of women who want to propose, and thus this cannot be stable. Therefore, in this setting, we have that either all agents on one side propose, or none agent does. The difference can then be in whether they screen or not; however, at an equilibrium, all agents on the same side will have the same continuation value and thus they must use the same θ if they screen, or none of them must screen.[31] Finally, the difference between the strategies of the agents on the same side can also be on how they handle incoming proposals. However, as we argued before, one side takes the role of proposer and the other one just receives proposals. For the proposers, the decision as to what to do with incoming proposals does not play a role, so we can ignore differences in this. On the other hand, those receiving proposals must either accept without screening or screen and accept/reject; ignoring proposals can never be an equilibrium strategy. However, as we argued before, all agents must have the same utility, and thus must follow the same strategy. □

Proof of Corollary 1. c_* is defined to make the welfares of equilibrium 1 and equilibrium 5 equal when $c_w = c_*$. We note the following:

1. Equilibrium 1 has higher welfare than equilibrium 4, when both exist.

2. The left hand side of Eq. (4) is at most 1/2 when $c = 1/(32\alpha^2)$ and is at least 1/2 when $c = \alpha/32$.

Both are basic calculus. It is easy to see that the welfare of equilibrium 5 is higher than that of equilibrium 3, when both these equilibria exist. Together with footnote 11 this is a proof of the theorem □

[31] Here, we assume that an agent only screens if $\theta > 0$.

D Appendix to Section 3.2

We now prove the results which are stated in Section 3.2. Note that in all equilibria listed in the statements of Theorems 2 and 3, agents on each side of the market are using a unique strategy. Therefore, as argued in Section A.3, the steady state exists in each case, and can be characterized using Proposition 1. In particular, when men are proposing, the steady state is given by $L = \left[0, \frac{\lambda(R-1)}{\mu}\right]$, and the fraction of time that there is at least one women in the system is $\frac{\mu}{(R-1)\eta_m}$ where η_m is as defined in Eq. (14). On the other hand, when women are proposing, the steady state is given by $L = \left[\frac{\lambda}{\mu+\eta_w}, \frac{\lambda(R\mu+(R-1)\eta_w)}{\mu(\mu+\eta_w)}\right]$ where η_w is as defined in Eq. (14).

In the rest of this section, we do not refer to the steady state explicitly.

Semantic definition of $\bar{c}$. Consider a setting where women screen and propose, and men are not permitted to propose. When a proposal arrives, a man must decide between screening or accepting it without screening. We define $\bar{c}$ to be the largest screening cost (as $\mu \to 0$) such that there exists a symmetric equilibrium between men where they screen and accept/reject based on a threshold of $\theta_m = \xi(\lambda, c)$ as given by Eq. (5). Next we show that Eq. (18) holds.

Now, the expected value of a man who uses strategy θ_m is identical to θ_m, since the process of arrival of proposals/death as seen by a man is memoryless, by our mean field assumption. Let V be the expected value from participation, just after a man m has received a proposal. Let p be the likelihood that a man receives a proposal before he dies.

Lemma 2. *In the limit $\mu \to 0$, we have*

$$p = \frac{1}{\theta_m + \lambda(1-\theta_m)} = \frac{1}{1+(\lambda-1)(1-\theta_m)}\,.$$

Proof. Women make $R/(1-\theta_m)$ proposals per unit time (as $\mu \to 0$, since a vanishing mass of women die without being matched. In comparison, a mass of λR men arrive per unit time. Hence, the expected number of proposals received by a man (who uses strategy θ_m) during his lifetime is

$$n_m = 1/(\lambda(1-\theta_m))\,. \tag{18}$$

Let p be the likelihood that a man receives a proposal from a woman before he dies. Checking for consistency when men screen with threshold θ_m, we obtain

$$n_m = p(1+\theta_m n_m)\,. \tag{19}$$

Combining Eqs. (18) and (19), we obtain

$$p = \frac{1}{\theta_m + \lambda(1-\theta_m)} = \frac{1}{1+(\lambda-1)(1-\theta_m)}\,. \tag{20}$$

(Notice that identical quantities appear in the analysis of the case where men propose and women screen and accept. Now, n_m is defined as the average number of opportunities that a man receives to propose to a woman who will accept him, during his lifetime, if he adopts strategy θ_m. And p is the likelihood of receiving such an opportunity before he dies.) □

Remark 5. *Lemma 2 also applies to the likelihood p that a man will get an opportunity to propose to a woman who will accept him, in the case where men propose and women screen and accept.*

Considering the possible cases —either a man receives a proposal before he dies, or he does not—, we obtain

$$\theta_m = pV + (1-p) \cdot 0 = pV \,. \tag{21}$$

Note that if a man simply accepts an incoming proposal, his expected value is 1/2. Hence, if the man is indifferent between accepting without screening, and using strategy θ_m, we have $V = 1/2$. Using this together with Lemma 2 in Eq. (21), and making the dependence on c explicit, we obtain $1/2 = \theta_m(\bar{c})/p(\bar{c}) = \theta_m(\bar{c})\,(1 + (\lambda - 1)(1 - \theta_m(\bar{c})))$. Solving for $\theta_m(\bar{c})$ we obtain that $\theta_m(\bar{c}) = \frac{\lambda - \sqrt{(\lambda-1)^2+1}}{2(\lambda-1)}$. Using the expression for $\xi(\cdot, c)$ in Eq. (5) we can solve for $\bar{c}$ to obtain Eq. (6).

Semantic definition of $\underline{c}$. We define $\underline{c}$ to be the smallest value of c (as $\mu \to 0$), such that, if the women are proposing (and men are not permitted to propose), there is a symmetric equilibrium between men where they accept incoming proposals without screening. Suppose other men are not screening (i.e., $\theta_m = 0$). Using Lemma 2, we know that the likelihood that a man will receive a proposal before he dies, is $p = 1/\lambda$. Note that the value obtained by accepting without screening is $V' = p/2 = 1/(2\lambda)$. Now, suppose a man receives a proposal. By accepting without screening, he can earn $V = 1/2$. This is a best response if and only if the man cannot do better by screening the current proposal, accepting with a threshold of $1/(2\lambda)$ (this threshold is exceeded with likelihood $1 - 1/(2\lambda)$, and the expected value of the match, conditioned on the threshold being exceeded, is $(1/2)(1+1/(2\lambda)))$, and if the value is below the threshold, accepting the next proposal, if any, without screening (this follows from the idea of a "rollout" in dynamic programming [7]). The value obtained from the latter strategy is

$$-c + (1/2)(1 + 1/(2\lambda))(1 - 1/(2\lambda)) + V'/(2\lambda) \,.$$

Comparing with $V = 1/2$ and using $V' = 1/(2\lambda)$, we find that the deviation does not increase welfare if and only if

$$c \geq 1/(8\lambda^2) \,, \tag{22}$$

leading to Eq. (7).

Semantic definition of $\hat{c}$. Again consider the setting where women screen and propose and men screen and accept/reject. Men are not permitted to propose. We define $\hat{c}$ to be the screening cost (as $\mu \to 0$) at which women are indifferent between screening, and proposing without screening, assuming men are screening with threshold $\theta_m(c)$. (Women do not have any externality on each other, being on the short side.)

Note that if men are screening, then the effective cost for women is equal to $c_{\text{eff}} = c/(1 - \theta_m(c))$, using Lemma 1. Now the value and threshold for women when they screen before proposing is $\theta_w(c) = 1 - \sqrt{2c_{\text{eff}}}$. The value when women don't screen is 1/2. It follows that $1 - \sqrt{2c_{\text{eff}}} = 1/2$ for $c = \hat{c}$, since women are indifferent between screening and not screening for $c = \hat{c}$. We deduce that $\theta_m(\hat{c}) - 1 + 8\hat{c} = 0$, which yields Eq. (8).

Proof of Theorem 2. We first establish that the following is a subset of equilibria, as a function of c, taking $\mu \to 0$:

- (men screen + propose, women screen + accept/reject) with thresholds: $\theta_m = \theta_m(\sqrt{\frac{c}{2}})$ and $\theta_w = 1 - \sqrt{2c}$. This is an equilibrium for $c \in (0, 2\bar{c}^2)$.
- (men propose w.o. screening, women screen + accept/reject) with threshold: $\theta_w = 1 - \sqrt{2c}$. This is an equilibrium for $c \in \left(2\underline{c}^2, \frac{1}{8}\right)$.
- Agents on both sides propose without screening whenever they get a chance, and accept all incoming proposals without screening. This happens when $c \geq \frac{1}{8}$.

Suppose the men are proposing. Then, it is clear that as $\mu \to 0$, the value of the women is upper-bounded by $\max(1/2, 1 - \sqrt{2c})$ which is the value women can get if they are guaranteed not to die. We show that if women wait for incoming proposals, then the value they obtain approaches the upper bound as $\mu \to 0$. If $c \leq 1/8$, the women screen and accept/reject, employing a threshold of $1 - \sqrt{2c}$, and producing a utility which tends to $1 - \sqrt{2c} \geq 1/2$ for women, showing that this is a best response for women. If $c \geq 1/8$, women accept without screening. In this case, they will also propose if they are given the chance as, by symmetry, men will not screen either. It remains to characterize the symmetric equilibria between men in response to this behavior of women. For $c \geq 1/8$, it is clearly a best response for men to propose without screening, thus establishing the third bullet. Consider $c < 1/8$. The effective screening cost faced by men is $c_{\text{eff}} = c/\sqrt{2c} = \sqrt{c/2}$, see Lemma 1. Suppose other men are not screening (i.e., $\theta_m = 0$). Using Lemma 2, we know that the likelihood that a man will receive an opportunity to propose to a woman who will accept him before he dies, is $p = 1/\lambda$. Using Lemma 1, it suffices to analyze an alternate situation where a man is receiving instead of making proposals, but screening costs are c_{eff} and the likelihood of getting a proposal before he dies is p. Consider this alternate situation, simultaneously for all men. Then the condition for existence of a symmetric equilibrium where men accept without screening is

$$c_{\text{eff}} \geq \underline{c} \quad \Rightarrow \quad c \geq 2\underline{c}^2.$$

Thus, we have established the second bullet.

For the first bullet, suppose there is a symmetric equilibrium between men where they screen, with a threshold of θ_m. If men have incentive to screen, then clearly so do women, since the women are not facing any possibility of rejection (with proposals being incoming) and the women have unlimited opportunities to match, being on the short side of the market. Hence, we know that women are screening, and using a threshold of $1 - \sqrt{2c}$. Thus, men again face an effective screening cost of $c_{\text{eff}} = c/\sqrt{2c} = \sqrt{c/2}$. Using Lemma 2, we have $p = 1/(1+(\lambda-1)(1-\theta_w))$. We again consider the alternate situation suggested by Lemma 1, with proposals guaranteed to be accepted, screening cost c_{eff}, and probability p of getting an opportunity before death. Then the value obtained a by a man if he screens and uses the optimal threshold is[32] $\xi(c_{\text{eff}})$, cf. Eq. (5). In comparison, the value obtained by taking the first proposal opportunity without screening is $p(1/2) = p/2$. The best response condition is thus $\xi(c_{\text{eff}}) \geq p/2$, which yields $c_{\text{eff}} \leq \bar{c}$ by definition of $\bar{c}$. Plugging in $c_{\text{eff}} = \sqrt{c/2}$, we obtain $c \leq 2\bar{c}^2$, yielding the first bullet.

Finally, we argue that there are no other stable equilibria if $\lambda \geq 1.25$. We rule out the possibilities one by one. Suppose both sides mix between proposing and don't proposing. Then if a few more men start proposing, this will make less women propose, since proposing becomes relatively less attractive for women. In turn, this will make more men propose and so on. Therefore, an equilibrium where both sides mix between proposing and not cannot be stable.

Suppose one side mixes between proposing and don't proposing, but the other side does not propose. This is ruled out because all agents on the first side will want to propose. In addition, suppose one side mixes between proposing and don't proposing, but the other side proposes. Suppose the men (long side) are mixing. Compared to the case where all men are not proposing, some men proposing makes things worse for the other men. So the stable situation can only be that the long side is either all proposing or none are proposing. In addition, we can rule out the case in which women mixing, because if all men are proposing, then women don't want to propose. Therefore, we must have that either all men propose or all men don't propose, and similarly for women.

Furthermore, for $c < 1/8$, it also can't be that both sides propose. (For $c \geq 1/8$, both sides proposing and accepting without screening will be the unique equilibrium.) Hence, it must be that one side proposes and the other side does not. When $\lambda > 1.25$, we argued that all men will want to propose as the unique best response, and thus women will never propose. In addition, we can rule out that men will mix between screening and not screening in any stable equilibrium. Therefore, men's best response will be to propose if $\lambda \geq 1.25$. □

Proof of Theorem 3. We want to prove that if men are not allowed to propose, the following equilibria exist as a function of c, taking $\mu \to 0$:

- (women screen + propose, men screen + accept/reject) with thresholds: $\theta_m = \xi(\lambda, c)$ and $\theta_w = 1 - \sqrt{2c/(1-\theta_m)}$. This is an equilibrium for $c \in (0, \min(\bar{c}, \hat{c})]$.

- (women screen + propose, men accept) with threshold: $\theta_w = 1 - \sqrt{2c}$. This is an equilibrium for $c \in [\frac{1}{8\lambda^2}, \frac{1}{8}]$.

[32] Here we suppress dependence on λ.

- (women propose w.o. screening, men screen + accept/reject) with threshold: $\theta_m = \xi(\lambda, c)$. This is an equilibrium for $c \in [\hat{c}, \bar{c}]$, and only exists if $\hat{c} < \bar{c}$ (might not exist at all).

The first bullet follows from the fact that it is a best response for men to screen and accept/reject with threshold $\xi(\lambda, c)$ for $c \leq \bar{c}$, provided other men are doing the same; and it is a best response for women to screen and propose if $c \leq \hat{c}$.

The second bullet follows from the definition of $\underline{c}$ (hence accepting without screening is an equilibrium among men), and the fact that when the men are not screening and $c \leq 1/8$, it is a best response for the women to screen and propose, with a threshold of $1 - \sqrt{2c}$.

It is easy to see that if men are screening with a threshold of $\xi(\lambda, c)$, it is a best response for women to propose without screening if $c > \hat{c}$. (Women do not exert any externality on each other, hence fixing the way men respond to proposals, exactly one of the two equilibria exist between women.) Combining with the definition of $\bar{c}$ (implying that men are playing a best response), we deduce the third bullet. □

Proof of Corollary 2. Theorem 3 equilibrium 1 exists for all $c < \min(\bar{c}, \hat{c})$.

- It may coexist with Theorem 3 equilibrium 2 (but not with the other two equilibria in Theorem 3). If this is the case, Theorem 2 equilibrium 2 exists for the same market under no intervention, and has welfare identical to Theorem 3 equilibrium 2.
- For the same market under no intervention, the possible equilibria are Theorem 2 equilibria 1 and 2. One or both of them may exist for the market under consideration.

It follows that if the stated condition holds, then preventing men from proposing can only increase (or leave unchanged) average welfare in equilibrium, relative to the case of no intervention. □

Theorem 4. *Consider the market defined in the statement of Theorem 2. Then, in addition to those defined in Theorem 2, the following equilibria might also exist. (Here, the equilibria are characterized by their limiting description as a function of c, considering $\mu \to 0$ for each fixed c):*

1. *(women screen + propose, men screen + accept/reject) with thresholds: $\theta_m = \theta_m(c)$ and $\theta_w = 1 - \sqrt{2c/(1-\theta_m)}$. This is an equilibrium for $c \in (\bar{c}_2, \min(\bar{c}, \hat{c}, \bar{c}_4))$. Furthermore, this equilibrium exists if and only if $\lambda \leq 1.25$.*
2. *(women propose w.o. screening, men screen + accept/reject) with threshold: $\theta_m = \xi(\lambda, c)$. This is an equilibrium for $c \in [\max(\hat{c}, \bar{c}_3), \min(\bar{c}, \bar{c}_4)]$. Furthermore, this equilibrium exists if and only if $\lambda \leq 1.25$.*

where $\xi(\cdot, \cdot)$ is as defined by Eq. (5), $\bar{c}$ and $\hat{c}$ are as defined by Eqs. (6) and (8) respectively, and

$$\bar{c}_2 = 4(\lambda - 1)^3 , \tag{23}$$

$$\bar{c}_3 = \frac{2(\lambda-1)^2}{(4\lambda-3)^2}\,. \tag{24}$$

$$\bar{c}_4 = \frac{(3-2\lambda)}{8}\,. \tag{25}$$

Furthermore, (women screen + propose, men accept) cannot be equilibrium unless there is a system intervention.

Proof of Theorem 4. To prove when (women screen + propose, men screen + accept/reject) is an equilibrium, recall that in Theorem 3 we showed that, when men are not allowed to propose, (women screen + propose, men screen + accept/reject) with thresholds $\theta_m = \theta_m(c)$ and $\theta_w = 1-\sqrt{2c/(1-\theta_m)}$ is an equilibrium for $c \in (0, \min(\bar{c}, \hat{c})]$. For this to be an equilibrium without any intervention, we must make sure that, given a chance to propose, a men would prefer to ignore it.

To that end, suppose that all men and women follow the strategies described above, and a single man deviates from this strategy by proposing if he gets the chance to do so. It is easy to see that a woman who receives this proposal will screen it with the same threshold θ_w as this maximizes her value. Using Lemma 1, a man will face an effective cost of $c/(1-\theta_w) = \sqrt{\frac{c}{2}(1-\theta_m)}$ to screen that opportunity and decide whether to propose. Given this cost, if he decides to screen it, he will still do so with threshold θ_m. Then, he will only take the opportunity to screen and propose if:

$$-\sqrt{\frac{c}{2}(1-\theta_m)} + (1-\theta_m)\frac{1+\theta_m}{2} + \theta_m\theta_m \geq \theta_m.$$

The first term is the effective screening cost, the second term is the expected value if he likes the woman times the probability of liking her, the third term is the continuation value times the probability of not liking the proposed woman; this should exceed the continuation value obtained by doing nothing (θ_m). Rearranging the terms, we obtain that the deviation is profitable only if $\theta_m \leq 1-(2c)^{1/3}$. Therefore, for this to be an equilibrium we need to have $c \geq \bar{c}_2 = 4(\lambda-1)^3$ (and $c \leq 1/2$).

However, there is also the possibility that a man would want to propose without screening. In this case, his proposal will be accepted with probability $1-\theta_w$, and if accepted he gets an expected utility of $1/2$. Hence, this will be a profitable deviation if

$$\frac{1}{2}(1-\theta_w) + \theta_w\theta_m \geq \theta_m,$$

or equivalently, $\theta_m \leq 1/2$, which occurs only if $c \geq \bar{c}_4 = (3-2\lambda)/8$. Note that the interval $(\bar{c}_2, \min(\bar{c}, \hat{c}, \bar{c}_4))$ will be non-empty only if $\lambda \leq 1.25$. Furthermore, $\hat{c} = \min(\bar{c}, \hat{c}, \bar{c}_4)$ when $\lambda \in [1, 1.25]$, which completes the proof of the first claim.

To prove the second equilibria, again note that in Theorem 3 we showed that, when men are not allowed to propose, (women propose w.o. screening, men screen + accept/reject) with threshold $\theta_m = \theta_m(c)$ is an equilibrium for $c \in [\hat{c}, \bar{c}]$. As before, for this to be an equilibrium in a no-intervention setting, it must be the case that a man does not want to propose if he gets the chance. To that end, suppose that all men and women follow the strategies described above, and a single man deviates from this strategy by proposing if he gets the chance to do so. It is easy to see that a woman who receives this proposal will now screen it with a threshold equal to 1/2. Hence, the man will now face an effective cost of $2c$, if he wishes to screen such an opportunity. For him to choose not to screen and propose, and rather wait for a proposal, it must be that $\theta_m \geq 1 - \sqrt{4c}$, which happens only if $c \geq \bar{c}_3 = \frac{2(\lambda-1)^2}{(4\lambda-3)^2}$. As before, we must also consider the possibility that a man would rather propose without screening, which happens if $c \geq \bar{c}_4$. Therefore, (women propose w.o. screening, men screen + accept/reject) will be an equilibrium only if $c \in [\max(\hat{c}, \bar{c}_3), \min(\bar{c}, \bar{c}_4)]$. Noting that $\bar{c} \geq \bar{c}_4$ completes the proof.

Finally, note that (women screen + propose, men accept) cannot be equilibrium unless there is a system intervention. To see why, note that in the case a men who gets an opportunity to propose, will do so. □