

Centralized Admissions for Engineering Colleges in India

Surender Baswana ^{*} Partha Pratim Chakrabarti[†] Sharat Chandran[‡]
Yashodhan Kanoria[§] Utkarsh Patange[¶]

Abstract

We designed and implemented a new joint seat allocation process for undergraduate admissions to over 500 programs spread across 80 technical universities in India, including the prestigious Indian Institutes of Technology (IITs). Our process is based on the well known Deferred Acceptance algorithm, but complex affirmative action seat reservations led us to make a number of algorithmic innovations, including (i) a carefully constructed heuristic for incorporating non-nested common quotas that span multiple programs, (ii) a method to utilize unfilled reserved seats with no modifications to the core software, and (iii) a robust approach to reduce variability in the number of reserved category candidates admitted, while retaining fairness. Our new seat allocation process went live in 2015, and based on its success, including significant and provable reduction in vacancies, it has remained in successful use since, with continuing improvements.

Keywords: stable matching, college admission, deferred acceptance, affirmative action, algorithm, implementation, market design.

Introduction

Among the most select universities in the world, the prestigious Indian Institutes of Technology (IITs) are considered the Ivy League of India. The schools have an admission rate of less than

^{*}Indian Institute of Technology Kanpur

[†]Indian Institute of Technology Kharagpur

[‡]Indian Institute of Technology Bombay

[§]Columbia Business School, Email: ykanoria@columbia.edu

[¶]Columbia Business School

1 percent for the 1.2 million annual applicants who, in many cases, have spent a small fortune on specialized coaching to gain admission. Which made it all the more puzzling and frustrating that, until recently, about 6 percent of available seats at the Indian Institutes of Technology (IITs) were consistently unfilled. Over nearly five years, we have worked to correct this problem via innovative changes to the seat allocation process.

One key reason for seats remaining vacant was as follows. From the 1960s to 2014, the admissions to IITs were conducted under one umbrella. Only slightly less sought after than the IITs are the non-IIT Centrally Funded Technical Institutes (CFTIs). The admissions to the non-IIT CFTIs (henceforth referred to simply as “non-IITs”) were conducted under a separate umbrella, after completion of the IIT admissions. Each candidate was eligible to apply for a seat in each of the two sets of institutes, and several hundred candidates would indeed receive two offers, one at an IIT, and later, another one at a non-IIT. Each such candidate could use at most one of the seats, leaving a vacancy in the other seat; this would be noticed much later, in many cases after classes began. Such seats would either remain vacant or would be reallocated after classes began in an unregulated decentralized manner, leading to inefficiency in seat allocation in the form of unnecessary vacancies, and unfair allocation of seats. For example, a particular non-IIT seat could be offered to a candidate B, despite denying the same seat earlier to a candidate A with better rank, who had meanwhile taken some IIT seat and was no longer participating in the non-IIT process.

In 2015, we designed and implemented a new combined seat allocation process based on the Deferred Acceptance (DA) algorithm of Gale and Shapley (1962). The process brings all the over 80 CFTIs (IITs + non-IITs) under one umbrella for admissions, with approximately 34,000 available seats and over 1.3 million applicants. Each candidate submits a single preference list over all available programs, and receives no more than a single seat from the system, based on her submitted preferences and her rank in each relevant Merit List.

Despite the benefit in theory of a combined process in terms of allocating each candidate only one seat, merging the two seat allocation systems introduces several challenges. Key among these is that there is no longer a single ranking of candidates, and that the process must incorporate complex rules regarding multiple types of seat reservations for affirmative action (more than half of seats are reserved). In addition, we are not permitted, in anticipation of attrition, to speculatively admit more students than the capacity. Finally, despite complexities, the process is required to be completely transparent (unlike many other college admissions mechanisms worldwide). Our new joint seat allocation process that addresses all these challenges has now been running successfully for four years (2015 to 2018), and has provably reduced vacancies at the IITs by nearly three-fourths. We have continued to improve the process over the years, for example, to further reduce inefficiencies resulting from several thousand students surrendering seats that they had previously accepted.

Analytical and Algorithmic innovations. Our complex problem required a number of innovations that could be useful elsewhere. We highlight three novel algorithms we developed

(described in detail in the Algorithm Design section):

- **A practical heuristic for non-nested common quotas.** The Defense Services (DS) category reservation reserves a few seats at each IIT for eligible candidates; each institute has multiple programs and the reservation applies to the cumulative number of DS candidates across programs. Such a situation is typical in admissions settings: e.g., each institute may have multiple programs, and a limited number of scholarships it can give out across all programs. We provide a novel practical heuristic for incorporating such a “non-nested common quota” when the size of this quota is small relative to the total number of seats. (Theoretically, it is NP-complete even to determine if there is a stable matching when there is such a quota.) The idea is to *first run DA with temporary “phantom” spots for the non-nested common quota, and then to eliminate these phantom spots by running rejection chains one by one*. We explain why the heuristic should fail only rarely. We find no failures in practice and only one failure in 50 synthetically generated test cases. Failure is handled gracefully by creating an extra seat.
- **Dereservation with no software modification.** We were asked to make unfilled reserved seats available to all candidates, but without adding algorithmic complexity to minimize the risk of errors. We show how to implement such “dereservation” without making any change to a software that lacks this dereservation feature, by *iteratively rerunning the software with adjusted seat capacities in the input*. Our approach allows to seamlessly incorporate such process modifications. We argue that the number of software reruns needed is small; just 4 iterations suffice in practice.
- **Making the number of reserved category admits predictable.** In 2018, we were asked to design a transparent method for female reservations to ensure a target increased fraction of female admits in each program at the IITs and several non-IITs, but while ensuring that (i) the number of seats available to non-females remained nearly unchanged, and (ii) female candidates did not face a higher bar than non-females in any program (“fairness”). We provide an innovative algorithmic approach to achieve the demanded objectives. The idea is to *divide seats into reserved and unreserved seats (and not two sets of reserved seats, to retain fairness) and to consider a reserved category student first for a reserved seat and then for an unreserved seats (and not vice versa, to minimize variability)*. The algorithm we designed has superior properties to half a dozen algorithmic alternatives. Our careful design and prior simulation experiments had given us confidence to proceed with the female reservation and indeed we found that our expectations were closely met in practice in 2018: by design our process achieved transparency, fairness, and the target minimum number of female admits in each program. Moreover, the number of seats allotted to non-females was observed to be within 0.1% of the previous year at the IITs, far outperforming the next best algorithmic alternative.

It would be interesting to theoretically formalize our insights corresponding to the first and third bullet above, though such exercises are outside the scope of the current project.

Despite some of us having prior expertise in design of matching markets, many of our practical learnings on this project are hard earned. Throughout the paper, we highlight our possibly generalizable learnings about designing centralized admissions processes as “Design Insights”, accompanied by supporting analysis or facts as applicable. We would like to flag Design Insights 2-4 as being novel algorithmic insights corresponding to the list above, and Design Insights 1 and 7 as being novel process innovations. Design Insight 1 describes our novel Multi-Round seat allocation process that efficiently fills seats despite us not being allowed to speculatively admit more students than the capacity, and thousands of students rejecting allotted seats. Design Insight 7 describes our “Mock Seat Allocation” that allowed candidates to learn how to fill their preferences properly.

Provable impact. We analyze the impact of the new seat allocation process in the section “Impact of Joint Seat Allocation”. The introduction of a combined process in 2015 resulted in a reduction in vacancies when classes began: by over 50% at the IITs (on a baseline of 587 vacancies in 9,784 seats in 2014, see Table 2) and by nearly 8% at the non-IITs (on a baseline of 5,596 vacancies in about 21,285 seats in 2014, see Table 3). Further significant reductions in vacancies (by over 70% at the IITs) followed in 2016 when candidates who earlier accepted a seat but no longer wanted it were – as per our recommendation – permitted to Withdraw, allowing such seats to be assigned to other candidates before classes began. The reduction in vacancies at the IITs is shown in Figure 1.

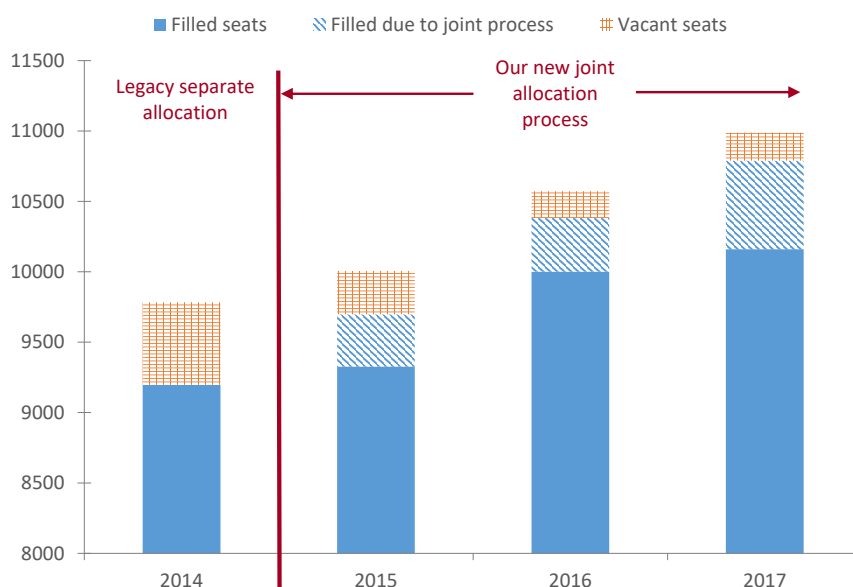


Figure 1: Vacancies in the IITs before and after the implementation of our joint seat allocation process in 2015. Seats that would not have been filled under the legacy process (rigorously estimated via a counterfactual experiment described in the Implementation section) are shown separately for 2015 onwards. The option to Withdraw after accepting a seat was introduced in 2016, leading to further reduction in vacancies. *Note:* The Y-axis begins at 8,000.

How do we know that our process was the cause of the reduction in vacancies? We rigorously quantify the benefits of the new process by conducting a careful counterfactual experiment described in the Implementation section. Based on the preferences filled by candidates in 2015, we simulate the disjoint allocation process as it used to happen until 2014. We first allot candidates to IITs, and then to non-IITs. Candidates who receive a better non-IIT seat vacate their IIT seat in the counterfactual. Figure 1 displays the counterfactual-based estimated reduction in IIT vacancies in each year, resulting from the new joint seat allocation process. For example, in 2017, the IITs had only 198 vacancies under our process but would have had 629 more vacancies under the legacy process. The new process was also able to give several candidates (3,672 candidates in 2017) more preferred programs than they would have got from the legacy process.

An additional benefit of the combined process was simplification of logistics for both colleges and students. Previously, IIT admissions would run until late July and admissions to non-IITs would happen only after that, often delaying the start of classes and continuing even after classes began. Now that the non-IIT admissions are conducted together with those of the IITs, the non-IITs are able to smoothly begin classes in late July.

Related work. By now there is a large number of matching markets that have been successfully centralized with a clearinghouse that collects preferences and determines matches. Most of these clearinghouses use versions of the Deferred Acceptance algorithm proposed by Gale and Shapley (1962), which produces a stable matching. In fact, stability has been found to be essential to the success of such clearinghouses (Kagel and Roth 2000). In our context, stability corresponds to a natural notion of fairness — a candidate should not be denied a program p if another candidate who was ranked lower by program p was offered a seat in the program.

One of the earliest documentations of this type of clearinghouse design was for the National Residency Matching Program (NRMP) (Roth and Peranson 1999). A number of cities in the United States use such a DA-based clearinghouse for admissions to public schools for instance New York (Abdulkadiroglu et al. 2005), Boston, Denver, Washington DC, New Orleans and Chicago. School admissions in Hungary (Biró 2008) (and some other countries) are similarly done using DA. Needless to say, there are hundreds of such marketplaces in addition to the ones mentioned. We remark here that our setting is different from the NRMP (and more similar to many school admission systems) in that the preferences of programs over candidates are determined entirely by exam scores, and so there is no strategic behavior on the part of the programs. On the other hand, indifferences in program rankings of students are rare by design and do not play a significant role in our setting (in contrast to many school admissions settings).

Following the seminal paper, by Gale and Shapley a vast theoretical literature has developed on the topic of stable matching. Instead of attempting to survey this literature, we point out a small subset of papers that are relevant to our project. Dubins and Freedman (1981) showed that candidate proposing DA cannot be manipulated by candidates, besides being candidate optimal, making it natural for us to appeal to candidate proposing DA. However, consistent with previous

empirical evidence starting with the NRMP, as well as theoretical work (most recently Ashlagi et al. 2015), we find that the set of stable matchings is small, in fact, we find that the candidate optimal stable match is identical to the program optimal stable match in all the datasets that we checked (so we would get the same allocation if we had instead used program proposing DA). Finally, a key issue we had to handle was complex business rules including a variety of quotas. Quotas that are nested preserve existence of stable matchings and allow a stable matching to be computed via a modification of DA. This helped with incorporating most of the key quotas. However, a problematic Defence Services (DS) quota was not nested, and in general this can lead to non-existence as well as computational difficulties Biró et al. (2010). We were able to construct a heuristic approach that almost always finds a stable matching despite this quota, but it made our algorithm much more complicated. We remark here that many of the serious issues we faced went beyond algorithm design, and involved understanding and engineering the details of the marketplace to make it work correctly (Roth 2002).

Outline of the paper. We provide some history and background regarding the CFTIs and their admissions processes in the Appendix. The Business Rules section follows immediately after the Introduction, followed in turn by the Algorithm Design, Implementation, and Impact of Joint Seat Allocation sections. In section “Reducing Vacancies Further” we propose process improvements to reduce vacancies further. We conclude with the Discussion section.

Business Rules

Due to the complexity of the Indian affirmative action program, and the goal of a completely fair and transparent process, the 16 (now 23) IITs had an intricate set of business rules for allocation of their 10,000 or so seats prior to 2014, based on Merit Lists (i.e., ranking) of candidates constructed from the nationwide Joint Entrance Examination (JEE) **Advanced** (see the appendix section “History and Background” for more information). Independently, the non-IITs had their own intricate business rules for allocation of over 20,000 seats in the non-IITs including the 30 (now 31) National Institutes of Technology (NITs) and the 12 (now 23) Indian Institutes of Information Technology (IIITs), based on a distinct set of Merit Lists (constructed using the JEE **Main** and high school graduation examination results). Our task of organizing Joint Seat Allocation in 2015 necessitated extensive coordination, starting with the business rules. Here is a summary of key aspects of the final Joint Seat Allocation Authority (JoSAA) business rules in 2015 and key changes since then (the business rules for the current year are available online, JoSAA 2018).

1. Merit Lists. Distinct Merit Lists are constructed for (i) admissions to the IITs using the JEE Advanced scores and (ii) admissions to the non-IITs using the JEE Main-based scores.
2. No overbooking. It is not permitted to admit more candidates than program capacity in anticipation of some offers being rejected.

3. Fairness. The seat allocation produced must satisfy the property that if a candidate is denied admission to a particular program, then no other candidate with an inferior rank as per the relevant Merit List should be admitted to that program. In other words, the allocation must be consistent with a *cutoff* rank for each program in the relevant Merit List. These cutoff ranks are publicly announced.

4. Reservation of seats for different affirmative action categories:

(i) By law, in each program, 15% of seats are reserved for the Scheduled Castes (SC), 7.5% for the Scheduled Tribes (ST), and 27% for Other Backward Castes (OBC). The remaining 50.5% of seats are in the Open category, and available for all.

(ii) 3% (now 5%) of seats in each category are reserved for Persons with Disability (PwD).

(iii) Two seats are preferentially allocated in every IIT under the Defence Services (DS) category. The DS category applies to candidates who are children of military personnel killed or disabled in service.

(iv) From 2018, 14% of admissions to IITs and NITs should be of female candidates, via creation of additional “supernumerary” seats if necessary, so that non-female candidates do not have to compete for a smaller number of seats. This number will gradually increase to 20% in the coming years. (At present, only 8 to 9% of IIT undergraduate students are female, reflecting (i) a gender skew at the top of the Merit Lists for admission, and (ii) only 1 in 3 female candidates who is offered an IIT seat accepts it, whereas 2 in 3 male candidates who are offered an IIT seat accept it. Once at the IITs, women do better academically than men on average.)

(v) 50% of seats at each NIT are reserved for candidates from the corresponding state.

(vi) A candidate should be considered for admission to a program through all the categories she is eligible in, in a particular order described in the business rules.

5. Dereservation of unfilled reserved seats. Unfilled OBC seats in a program must be “dere-served”, i.e., made available to Open candidates. SC/ST seats cannot be dereserved to the Open category. (However, unfilled SC-PwD seats must be made available to SC candidates, and similarly for other seats reserved for PwD candidates.)

6. Multiple rounds of allocation. Candidates fill and “lock” their preferences over programs at the start of the allocation process, after which the “first round” allocation is produced. In order to fill allotted seats that were Rejected (either actively, or via a no show by the candidate at the reporting center), additional fresh allocations and allocation “upgrades” are executed over multiple rounds. Candidates who accept a seat are provided the options “Float”, “Slide” and “Freeze”. *Float* indicates that the candidate wants to get upgraded as far up her preference list as possible. *Slide* indicates that the candidate wants to remain in the institute she has currently been allotted but to get her most desirable program available at that institute. *Freeze* indicates that she wants to remain assigned to the specific program she has been allotted, even if other options become available subsequently. Candidates are not permitted to change their preference

list midway through the process.

Candidates who accept a seat at an IIT are not permitted to apply to an IIT in future. (However, accepting a seat anywhere does not prevent a candidate from applying to a non-IIT in future.)

Our Float/Slide/Freeze mechanism may be useful in other Multi-Round allocation settings. We observed that the candidates valued this flexibility. For example, after the first round in 2017, 14.2% candidates chose to Freeze their allotted program, 8.8% chose Slide and 59.6% chose Float (the proportion of Freeze and Slide increases in later rounds). The rest (17.4%) decided to exit the system by rejecting their seat.

Design Insight 1 *In a Multi-Round seat allocation process, a candidate who is allotted a seat may be given the following options: “Float” (meaning to be open to upgrades in future rounds), “Slide” (be open to upgrades but only within the same institute), “Freeze” (ask to stick with the current allocation) or “Reject”.*

One additional difficult business rule was that if the category of a candidate changes due to misreporting by the candidate, the candidate must be penalized by giving a seat only from those that remained unfilled after the previous round.

The complexities of these rules necessitated a number of algorithmic and process-related innovations. We describe our algorithmic innovations in the next section, and then describe our process implementation in the Implementation section.

Algorithm Design

Early in the creation of a joint seat allocation process, the authorities suggested several times to keep the processes for the IITs and the non-IITs separate, but to require candidates who received two offers to reject one, and iterate. Even if implemented in the best possible way, such an approach is identical to program proposing DA, but with iterations occurring in the real world instead of on a computer. Our simulations showed that as many as six iterations would have been needed to obtain convergence. We ultimately convinced all concerned to instead collect integrated preferences from candidates and then run the algorithm on a computer.

Our software (see subsection “Software development and testing” in the Implementation section below for more) was created by the National Informatics Centre (NIC). NIC initially suggested that we should collect candidate preferences as a single list over all IIT and non-IIT programs and then run an algorithm similar to program proposing DA on the computer. We convinced them to use candidate proposing DA on the preferences instead, since it produces a candidate optimal allocation, and is incentive compatible for candidates. Incidentally, it turned out that the two versions of DA produced identical allocations on the preferences collected during 2015 to 2017. This is consistent with the finding in the literature that in typical matching markets there is an essentially unique stable matching, e.g., see Ashlagi et al. (2015).

The complex business rules necessitated a handcrafted variant of candidate proposing DA. Our full algorithm is specified in an 85-page document that includes pseudocode and also explanations and examples (Baswana et al. 2015). In the rest of this section we summarize how our algorithm incorporated, in turn, (i) affirmative action reservations, (ii) non-nested common quota reservations (for DS candidates), (iii) dereservation of unfilled seats, and (iv) the target of 14% female candidates in each program (in 2018), all while satisfying stringent practical requirements. While our solution for affirmative action reservations is straightforward, we constructed novel approaches to incorporate the other listed requirements. Since these types of requirements arise naturally in school and college admissions, our algorithmic design may help other market designers faced with similar problems choose the best practical solution.

Affirmative action reservations

Birth category affirmative action reservations (rule 4(i)-(ii) in the Business Rules section) are implemented using their nested structure by dividing each program into multiple “virtual” programs, one for each category. Their capacity is set to the number of reserved seats for that category in that program. The preference lists of candidates are modified to have virtual programs instead of academic programs. The sequence in which a candidate applies to virtual programs is based on business rules. For example, a candidate who appears in the Open, OBC, Open-PwD and OBC-PwD Merit Lists applies to virtual programs in the order $\text{Open} \rightarrow \text{Open-PwD} \rightarrow \text{OBC} \rightarrow \text{OBC-PwD}$.

Non-nested common quota reservations

One of the most difficult business rules to implement was the non-nested common quota for DS candidates (rule 4(iii)): at most two DS candidates are admitted in total across all programs in an institute. This quota is non-nested and as such a stable matching may not exist and checking whether a stable matching exists is computationally hard (Biró et al. 2010). We now present our practical solution to the challenge of incorporating a small non-nested common quota; such a quota may arise naturally in admissions settings, for example each institute may have multiple programs, and a limited number of scholarships it can give out across all programs. In principle, one could appeal to the integer programming method devised by Biró and McBride (2014) for this problem, that finds a stable outcome when it exists. However, such an approach was untenable in practice due to complexity, relative opaqueness, and the likelihood of an unreasonably large run time on our large problem.

Our heuristic algorithm. To implement the DS reservation, a new virtual DS program with capacity two is added *per institute* (not per program), e.g., IITK-DS for IIT Kanpur. (An equivalent way to view our setting is to think of one DS virtual program for each *program*, such that all DS virtual programs in a particular institute have common quota of two seats, and that individual DS virtual programs have no additional capacity constraints of their own. Under this view the DS

reservation is a non-nested common quota (Biró et al. 2010). However, for purposes of describing our algorithm, we define a single DS virtual program for the entire institute.) Only DS candidates are eligible for these virtual programs. Furthermore, the preference list of each DS candidate is modified as follows. If the preference list of candidate is $\langle p_1, p_2, p_3 \rangle$, then his preference list is first augmented as $\langle p_1, \text{Institute}(p_1)\text{-DS}, p_2, \text{Institute}(p_2)\text{-DS}, \dots \rangle$. Then p_1, p_2, \dots are each replaced by multiple virtual programs as described in the previous subsection. Note that any seat allotted based on the DS status of a candidate must be only from the Open category.

We start by running the DA algorithm to completion, while artificially allowing the institute DS virtual programs to admit up to two candidates, over and above the capacity of any of the other virtual program. Upon completion of DA, if each candidate allotted to a DS virtual program is given a seat in the respective Open category virtual program she had asked for, we may have artificially increased the capacity of some (Open category) programs by up to two seats per institute. To prevent this overage, we process candidates in DS virtual programs one by one as follows. Suppose candidate c allotted the IITB-DS virtual program asked for a seat in, say, the IITB-EE program. Let x be the candidate with the worst rank among those currently in the virtual program IITB-EE-Open. Then x is rejected by IITB-EE-Open. We run candidate proposing DA starting from the current allocation and with x applying to her next most preferred virtual program, triggering a *rejection chain*. Now there is a possibility of this rejection chain looping back to cause rejection of the candidate c who started it, in which case we fail to obtain a fair allocation. In any such situation, our algorithm rolls back the rejection chain and allocates that DS candidate a “supernumerary” seat (i.e., a seat in excess of program capacity). We provide an example of such a “failure” in the Appendix. Unsurprisingly, though, we did not observe a single failure in practice to date, and in our 50 synthetically generated test cases, there was only one instance of failure.

Why failures are rare. The example of failure in our handling of non-nested common quota described in the Appendix not only demonstrates a case of failure, but also throws light on why such failures are rare as long as the number of non-nested common quota (DS) seats and candidates is small (there are tens or fewer of each in our setting). In order for a failure to occur, one of the rejection chains initiated by a candidate occupying a seat in DS virtual program needs to displace a DS candidate from an Open virtual program, which means that at least one step of the rejection chain must involve an Open virtual program such that its lowest ranked Open candidate in fact has the DS tag. With fairly sizeable Open virtual programs, few DS candidates and relatively short rejection chains (as in our setting) this is already unlikely to happen. Further, if our heuristic smartly chooses the order in which to process the DS candidates, the rejection chain needs to loop back to the DS virtual program from which it started (most likely because the displaced DS candidate herself applies to that program), and this is also unlikely given the sizeable number of DS virtual programs (one per institute, with many institutes). In practice, we did not use smart (re)ordering of DS candidates, since we prioritized algorithmic simplicity over the small risk of creating an extra seat due to failure. Still, no extra seats were created in practice.

It would be an interesting theoretical exercise to formalize this intuition in future, along the lines of the analysis of the heuristic for accommodating couples in the NRMP by Kojima et al. (2013) and Ashlagi et al. (2014). The notion of influence tree (roughly, the programs and candidates that may be affected by the rejection chain of a DS candidate) in the latter paper may be especially useful.

Design Insight 2 *Non-nested common quotas that include a relatively small number of seats can be accommodated using our simple heuristic, while creating very few (or zero) extra seats. The idea of our heuristic is to first run DA with temporary extra spots for the non-nested common quota, and then to eliminate these extra spots by running rejection chains one by one.*

Dereservation of unfilled seats

Business rule 5 required unfilled OBC seats to be made available to Open category candidates. The approach we initially suggested involved construction of augmented Merit Lists making Open category candidates eligible for OBC seats but at a lower priority than all OBC candidates, and modification of virtual preference lists so that general candidates now apply for both the OPEN and the OBC virtual programs. We showed that running our algorithm on these modified inputs would produce the candidate optimal allocation satisfying the business rules. However, the authorities feared that this approach may cause issues with computing the closing rank correctly (see Design Insight 6), or have some other hidden problem. An authority running centralized college or school admissions is typically loathe to modify, add complexity to, or replace software that is tried and tested (seen also in Chilean college admissions and NYC public school admissions). Upon reflection, we realized that if we relaxed our goal of candidate optimality, and were willing to tolerate a slightly longer computation time, we could use the existing software as a black box, and yet incorporate dereservations.

Our algorithm. Our approach was remarkably simple: Run the core algorithm with no dereservations to completion. Move the vacant seat capacity in each OBC virtual program to the corresponding Open virtual program. Rerun the core algorithm. Iterate until convergence.

Properties of our algorithm. Each successive run of our core algorithm (which is essentially candidate proposing DA) will produce (weakly) fewer assignments to each OBC virtual program. This is true because adding more seat capacity to one or more virtual programs only makes candidates (weakly) better off (Roth and Sotomayor 1992), and so some candidates may get an upgrade *out* of an OBC virtual program, but no candidate gets an upgrade *to* an OBC virtual program that already had vacancies, given that the previous allocation was stable. As a result of this monotonicity, we can conclude that only a finite number of reruns is needed. In fact, each of our iterations resembles the iterations of Manjunath and Turhan (2016), who find fast convergence when there are two parallel school systems drawing upon the same set of candidates. In our setting, convergence is even faster, and only about 4 iterations were needed. Our intuition for

the observed rapid convergence is that accommodating several Open candidates in vacant OBC seats lowers the bar for Open category candidates, but this allows only a few OBC candidates to upgrade since most of the OBC candidates still get their allocation via OBC seats. As such, very few additional vacancies are generated in OBC seats.

We remark that our implementation of dereservation can be viewed as a hybrid of program proposing DA (in the outer loop of multiple runs) and candidate proposing DA (in the core algorithm), for the economy consisting of the virtual preferences and augmented Merit Lists defined in the first paragraph of this subsection. And so the allocation we arrive at is stable, but not necessarily candidate optimal in theory.

Design Insight 3 *Practitioners are loath to modify or replace existing software. Dereservation of unfilled seats, and conceptually similar problems like integration of two separate admission systems, can be practically implemented by treating the existing software(s) as a black box, and running it repeatedly while iteratively updating the input provided, until convergence.*

A target minimum fraction of females in each program

In 2018, we were asked to ensure that 14% of admitted candidates in each program in the IITs and the National Institutes of Technology (NITs) are female. (In 2017 and before that, gender played no role in admissions, and the fraction of females was only 8-9% overall in the IITs. The fraction was around 14% on average at the NITs, but lower in some programs and higher in others.) Further, we had to satisfy the following constraints:

1. The number of non-females admitted should not increase compared to 2017 since the institutes have resource constraints and cannot accommodate an increase in non-female and female candidates simultaneously.
2. The non-females should not be disadvantaged while admitting the target minimum fraction of females in a program. Therefore, the number of non-females admitted should not decrease significantly compared to 2017.
3. The program capacities have to be frozen prior to the Joint Seat Allocation since they are publicly announced, and moreover they should be determined in a simple, transparent, and fair way. The allocation produced should not violate the pre-announced capacities.
4. The allocative approach should ensure that the number of females is at least the larger of (i) the typical number in the past and (ii) 14% of the total capacity of the program, but not much more than this target.
5. For each program, the admission cutoff for female candidates should not be more stringent than that for non-female candidates, so that female candidates are not unfairly disadvantaged.

At first glance, the following simple algorithm may appear to meet all the above constraints: Divide each virtual program into two separate virtual programs for non-females and females respectively. In the seat allocation process, non-females compete for non-female virtual programs

and females compete for female virtual programs only. As per requirement 3, the capacities of these virtual programs may be fixed beforehand as follows — the capacity of the non-female virtual program is set equal to the number of non-females admitted to that virtual program in the previous year (thus satisfying requirements 1 and 2), whereas the capacity of the female virtual program is chosen so as to satisfy 4 under the assumption that the applicant pool will look like that of the previous year. Although this algorithm meets the first 4 constraints, it violates the 5th constraint, namely, some female candidates may be denied a seat under this algorithm despite non-females with inferior rank being admitted. Violation of 5th constraint would defeat the purpose of this exercise, since in pursuit of providing a guarantee of at least 14% seats to females, we would deny some female candidates of their right to compete for seats on the basis of merit. This serious problem in the algorithm was not merely a theoretical possibility; our simulations on 2017 preference data revealed this algorithm would deny seats to several deserving female candidates. We now present our algorithm that rectifies this serious problem and was used in 2018.

Our algorithm. The algorithm (and corresponding business rule) divides each virtual program p into two separate virtual programs as follows:

- Female(p): exclusively for females
- Gender-Neutral(p): admits candidates based on merit only

A Non-Female(p) virtual program is intentionally avoided, to satisfy constraint 5.

The key feature of our algorithm is that each female candidate interested in a program first competes for a seat in the relevant Female(p) virtual program(s), and only if she fails to get a seat does she compete for a seat in the Gender-Neutral(p) virtual programs, in contrast to Business Rule 4(vi) for other reservations. (In Boston school admissions, a similar “precedence order” design for Walk zone reservations ended up with an allocation almost identical to what would have happened with no reservation, because the number of reserved seats chosen was too small; see Dur et al. (2017). In our implementation, the fraction of seats allocated to females is guaranteed to increase from 9% to at least 14%, and the chosen precedence order will ensure it does not exceed 14% by too much.) As a result, if the cutoff for Female(p) ends up being less stringent than the one for Gender-Neutral(p), then females do not occupy any seat in Gender-Neutral(p) and hence the constraints 1 and 2 are satisfied exactly. On the other hand, if the cutoff for Female(p) is the more stringent one, then notice that the algorithm allows female candidates to compete for Gender-Neutral(p). This ensures fulfillment of constraint 5 and hence fairness to female candidates. One may be concerned that this outcome is unfair to non-female candidates since there may be female candidates evicting non-female candidates from their seats in p . However, in any scenario where even one female occupies a seat in Gender-Neutral(p), it can be observed that the allocation of all seats in p (including both Gender-Neutral(p) and Female(p)) is purely based on merit with no regard to gender. Therefore, neither females nor non-females are unfairly disadvantaged in our algorithm.

Note that the alternative of having females apply first to the Gender-Neutral virtual program

would lead to more variability in the fraction of females admitted, since now the *top* female candidates would compete for Gender-Neutral seats and an unpredictable number would gain admission via a Gender-Neutral virtual program (compared to very few with our approach, see below). Thus, the choice of precedence order affects not only the *number* of reserved category candidates admitted as noted by Dur et al. (2017), but crucially also the *unpredictability/variability* in the number of reserved category admits.

Design Insight 4 *If reserved category candidates are considered first for reserved seats and then for unreserved seats, this choice of precedence order can reduce the variability in the total number of reserved category candidates admitted.*

Table 1 summarizes the excellent performance of our design for female reservation in 2018. Note how only 10 female candidates at the IITs and 467 female candidates at the NITs took Gender-Neutral seats, thus ensuring that constraints 4 and 2 were closely met under our approach (constraints 1, 3 and 5 were already met *a priori* by design). In contrast, if female candidates had been first considered for Gender-Neutral virtual programs, we found that females would have captured 66 Gender-Neutral seats in the IITs and 847 in the NITs, unevenly and idiosyncratically distributed across programs. This would have caused less seats to be available for non-females (a violation of constraint 2), particularly in some programs.

Table 1: Summary of performance of our design for female reservation in 2018. Note the minimal reduction in seats for non-females.

	IITs	NITs
Female seats	1,852	2,947
Gender-Neutral seats	10,227	15,673
#Gender-Neutral seats taken by females	10	467
Average excess over target minimum #females	0.6%	15.8%
Average reduction in seats for non-females	0.1%	3.0%

Additional comments on algorithmic aspects

Needless to say, there were various additional complexities we had to handle. One noteworthy challenge was how to handle the several candidates who were found to have misreported their birth category. These candidates were then supposed to be allotted spots only from unfilled seats. We refer the interested reader to our full report (Baswana et al. 2015) for our careful algorithmic approach to handle such candidates.

In addition, it is fairly typical to have several candidates with identical ranks in our setting, and candidates with the same rank may apply to the same program. Our approach was to create additional seats to accommodate all candidates at the cutoff rank, corresponding to “L-stable

score-limit” solution in Biró and Kiselgof (2015) (though we learned of that paper only later). We employed the natural modification of DA to accommodate ties in this way.

We advocated for modification of the difficult DS and category change rules so as to simplify implementation. Our advice was eventually accepted: Since 2016, the DS seats have been allotted in a supernumerary manner, decoupling DS allotments from other allotments and greatly simplifying the process. In 2018, the category misreporting penalty has also been eliminated.

Implementation

“Market design involves a responsibility for detail, a need to deal with all of a market’s complications, not just its principal features.” Roth (2002)

The orders of a Delhi High Court judge under public interest court case W.P.(C) 2275/2010 catalyzed the launch of a joint seat allocation process for all CFTIs in 2015 (see the History and Background section in the Appendix).

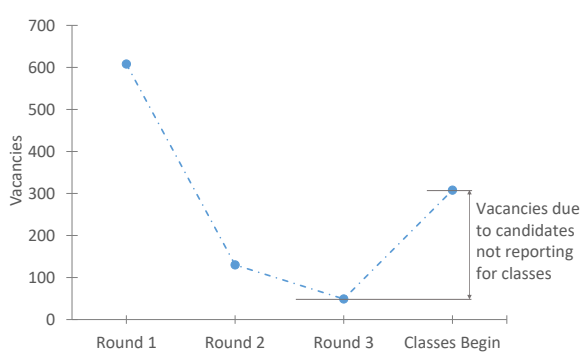
The separate processes under the IITs umbrella and the non-IITs umbrella each involved multiple (three or four) rounds of seat allocation conducted in rapid succession. The process began with candidates submitting preferences over programs, and then (a variant of) serial dictatorship was used to produce an allocation in the first round. Candidates allotted a seat were asked to pay fees and accept the allotted seat. Some candidates did not accept their seats, and these seats were then allotted again in the second round, and so on. Multiple rounds were especially important to fill seats since overbooking was not permitted (rule 2), and the yield was much less than 100% especially in the non-IITs. This Multi-Round structure had to be retained for joint seat allocation. But now the process in each round had to account for different Merit Lists for IIT programs and the non-IIT programs, necessitating that we construct a suitable Multi-Round adaptation of Deferred Acceptance.

Design Insight 5 *Multiple rounds of centralized allocation can serve to mitigate the number of vacancies without incurring the overage risk associated with admitting more candidates than the capacity of each program.*

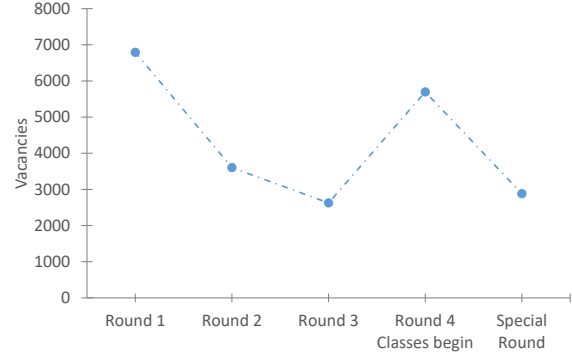
Figure 2 shows how multiple admission rounds enabled reduction in the number of vacancies using 2015 data (the scenario in 2016 and 2017 was complicated somewhat by the fact that candidates were permitted to Withdraw after accepting a seat, see the Impact of Joint Seat Allocation section).

Another important feature of the legacy processes that we retained was that the cutoff ranks for each program in each category were published in each round (rule 3).

Design Insight 6 *Conducting admissions based on strict program preferences derived from exam scores allows for the public announcement of closing ranks/cutoffs. Such public announcement provides (i) transparency, (ii) guidance to candidates regarding their chances of admission to each*



(a) Vacancies in IITs (10,006 total seats).



(b) Vacancies in non-IITs (24,068 total seats).

Figure 2: Vacancy progression after each admission round in 2015. IITs held 3 rounds, while non-IITs held a 4th round and a Special Round in addition. Since candidates were not allowed to Withdraw after accepting a seat, many vacancies were discovered after classes began, after which a Special Round was conducted in the non-IITs.

program (since the cutoff ranks from the previous year are available), and (iii) helps candidates to understand that they should report their true preferences without fear.

Regarding (ii), note that there are over 500 available programs. Typically, candidates have not been constrained on the number of preferences they can enter. Indeed, candidates submit extremely long preference lists of average length about 80 in practice, and constructing such detailed preferences is a Herculean task, so published cutoff ranks are very helpful in this regard.

As a result of (iii) (and because of explanatory material provided to candidates) we are not particularly concerned that candidates are misreporting their preferences, even though this has been observed in other centralized matching systems Hassidim et al. (2017). We have heard very few anecdotes about candidates being unsure whether they should report their true preferences.

Software development and testing

Figure 3 summarizes the timeline of the development of the new process. Two software implementations of our algorithm were developed. One was a database-based version developed by NIC, and the other was a main-memory version developed by us at IIT Kanpur. Since no dataset of integrated candidate preferences (over both IIT and non-IIT programs) was available for 2014 or earlier, synthetic datasets were prepared by software validation and testing teams from three IITs. The teams also prepared a set of validation modules to verify that the allocations generated satisfied all business rules. 50 test cases of varying sizes were tested and validated over two months. The IIT Kanpur software took 30-40 min to run on a dataset with 1 million candidates. It took 10 minutes per run (recall that multiple runs were used to implement dereservation of unfilled seats, see the subsection on Dereservation of unfilled seats in the Algorithm design section), and typically 3-4 runs were needed.

The preference filling started on July 1. Thereafter every day until the conclusion of the

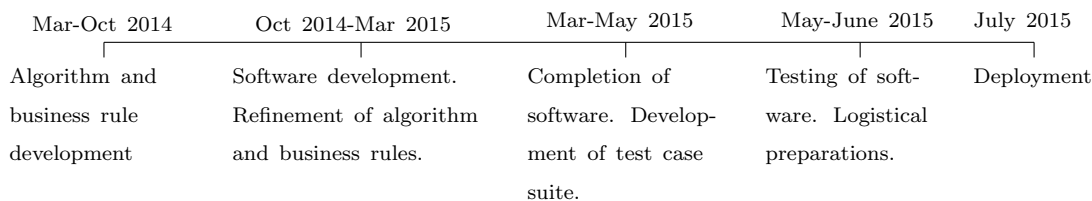


Figure 3: Timeline of the development of the new process.

process, two or three snapshots of the candidate preferences were taken and both softwares were run on that data and matched with each other. The matched allocations were then passed through validation modules prepared by the testing teams. This ensured that there were no surprises on allocation days (one for each round) when we had to publish the results.

In 2016, the software was modified to accommodate the changed business rules including the introduction of the Withdraw option. The running time of the software was also improved from 40 min to 3-4 min. One of the major problems in 2016 was the late addition of candidates and change in their marks. New candidates were added even after the 3rd round of allocation.

Allocation process timeline and details

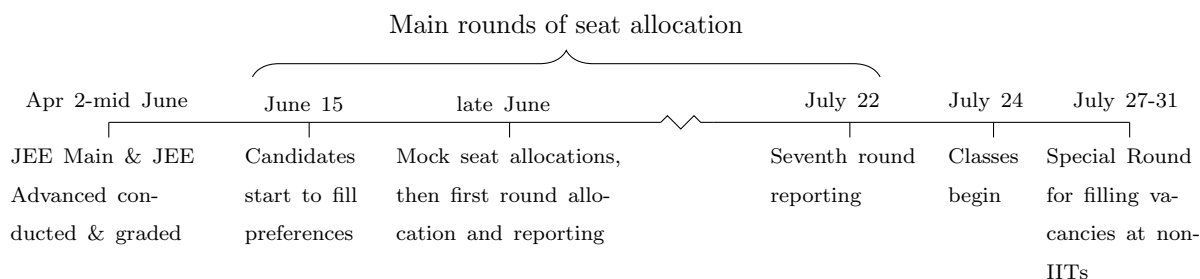


Figure 4: Timeline of the Multi-Round seat allocation process in 2017 (the timeline was similar in 2015 and 2016, except the Special Round occurred much later in 2015, and not at all in 2016).

Figure 4 shows the timeline of the process in 2017. Candidates filled in their preferences before the first round. A few days after the preference filling portal opened, two successive “Mock allocation” rounds were conducted online to make sure that candidates filled their preferences correctly and thus obtained their most desired program for which they cleared the cutoff. Analysis of the data reveals the benefits of the Mock rounds. In 2017, there were 2,063 candidates who did not get a seat in the first Mock allocation, but, upon suitably updating their preferences, were allotted a program in the actual first round. Furthermore, the Mock rounds (especially the second Mock round) were found to provide fairly accurate guidance in the sense that the closing ranks were close to those in the actual first round. The median errors in the estimated closing ranks (for

the Open category) was 13.8% for the IITs (18.4% for the non-IITs) in the first Mock round, and just 2.5% for the IITs (6.1% for the non-IITs) in the second Mock round.

Design Insight 7 *Conducting a Mock allocation based on tentative preferences and revealing the results to candidates helps them understand the purpose of the preference list they are submitting, and what allotment they may get as a result, and hence mitigates misreporting.*

Shortly after the Mock allocation rounds, candidates could “lock” their preferences, or let the system auto-lock. Candidates were not allowed to change their preferences after the deadline. (Allowing change of preferences in subsequent rounds would conflict with the requirement to have a single closing rank for each program in each category. It may also have produced other kinds of confusion in an environment with high stakes.)

After being allotted to a program, the candidates were asked to pay the fee and physically report to a reporting center for document verification and accepting the allotted seat. At the reporting center, each candidate had to choose an option between Freeze, Slide and Float (see the Business Rules section). If a candidate didn’t report, it was taken as a Reject. In 2016 an additional option “Withdraw” was added to the list. Withdraw was for candidates who accepted a seat in a past round, but did not want it anymore. Such candidates were given a refund of the seat acceptance fee, and asked to fill a short survey regarding their reasons for withdrawing, including where they were planning to study and whether they intended to write JEE again.

Impact of Joint Seat Allocation

In this section, we critically analyze the impact of the joint seat allocation during 2015, 2016 and 2017, relative to 2014 when the seat allocation was done separately for IITs and the non-IITs. We focus our analysis on seat vacancies.

Table 2 shows the vacancies in the IITs in each year during 2014-2017. In 2015 there was a significant reduction in the vacancies in IITs. This reduction can be attributed to combining the two seat allocation processes, primarily because in the prior system, the IIT admissions were completed first, and candidates who subsequently obtained a preferable allocation at a non-IIT surrendered their IIT seat, which then remained unfilled. We confirmed this benefit to the IITs by running a rigorous counterfactual experiment to estimate the number of vacancies that would have resulted from persisting with two separate allocation processes. We describe this experiment below in the next subsection. (We have omitted IIT Dhanbad from the list in Table 2. It had 7 vacancies in 2014, 33 in 2015, 37 in 2016 and 37 in 2017. The reason for the apparent increase is that this institute filled a large number of vacant seats locally via a “spot” round until 2014, but did not feel the need to fill the few vacant seats from 2015 onwards.)

In 2016 there was a further reduction in the vacancies in IITs, as a result of the introduction of a “Withdraw” option for candidates. In fact, the final vacancies reduced by over 70% (relative to 2014) in 2016 and 2017, because most of these vacated seats were successfully reallocated.

Table 3 shows the vacancies in the non-IITs during 2014-2017 after the main rounds. A plausible explanation for the reduction in vacancies in 2015 is that until 2014, some candidates would list programs under the non-IIT process that they did not truly prefer to the IIT allocation that they had already obtained a few days earlier, and later reject such non-IIT programs if allotted. The new process eliminates such waste. In 2016, after the introduction of the Withdraw option, there was a further reduction in the number of vacancies at the non-IITs (by 22% relative to 2014). However, in 2017, vacancies returned nearly to 2014 levels. We discuss possible reasons later in this section, and propose further changes to reduce vacancies in the next section.

Table 2: Vacancies when classes began at the IITs in 2014 (separate seat allocation), 2015 (joint seat allocation introduced), 2016 (Withdraw option introduced) and 2017. There are blanks for years in which that IIT did not exist. At the bottom, we make a before-after comparison based on only the IITs that existed in 2014. The IITs had a total of 9784 seats in 2014 across 17 IITs. This increased to 10988 seats in 2017 across 23 IITs.

IIT	2014	2015	2016	2017
BHU	197	72	55	51
Bombay	4	2	0	4
Bhubaneswar	21	18	12	12
Delhi	12	7	2	4
Gandhinagar	10	7	6	8
Guwahati	32	17	6	12
Hyderabad	15	13	0	2
Indore	2	6	4	13
Jodhpur	28	8	2	2
Kanpur	18	10	5	0
Kharagpur	97	34	22	22
Mandi	6	16	1	3
Madras	37	29	19	4
Roorkee	84	27	16	16
Patna	17	12	3	7
Ropar	7	8	6	3
Pallakad		11	7	3
Tirupati		11	4	7
Jammu			7	8
Dharwad			7	7
Goa			4	5
Bhilai			2	5
Total vacancies	587	308	190	198
Vacancies prevented by DA (see subsection on counterfactual experiment)	-	373	379	629
Vacancies in pre-2014 IITs	587	286	159	163
Reduction vs 2014		51%	73%	72%

Table 3: Vacancies when classes began across non-IITs in 2014, 2015, 2016 and 2017. The bottom set of numbers excludes institutes that were not a part of the system (or did not exist) in 2014 for a more fair comparison. There were 21,285 seats across non-IITs in 2014 (across 30 NITs, 12 IIITs, and 16 Other GFTIs). By 2017, there were 25,220 seats across non-IITs, with a big part of the increase in seats being due to the addition of 1 NIT, 11 IIITs and 4 Other GFTIs during this period.

	2014	2015	2016	2017
NITs	3208	3209	2613	3244
IIITs	578	709	666	1292
Other GFTIs	1710	1779	1622	1974
Total vacancies	5596	5697	4901	6510
pre-2014 NITs	3208	3111	2530	3112
pre-2014 IIITs	578	444	347	528
pre-2014 Other GFTIs	1710	1632	1502	1740
Total vacancies	5596	5141	4379	5380
Reduction vs 2014		8%	22%	4%

Joint Seat Allocation 2015. A total of 153k candidates filled in nearly 13 million preferred programs (so preference lists had an average length of 85 per candidate) for the academic programs offered by 87 institutes in the joint seat allocation in 2015 (The number of candidates who write the JEE Main is over 1 million each year but only about 150k of the best performing candidates qualify to fill in their preferences). The institutes were 19 IITs (including ISM Dhanbad, which was designated an IIT in 2016), 31 NITs, 18 IIITs and 18 Other-Government Funded Technical Institutes (Other-GFTIs). The seat allocation was carried out in 4 rounds from 1st July to 21st July.

Based on our advice, a systematic centralized Special Round was conducted by the non-IITs for the first time, to fill the nearly 5,697 vacant non-IIT seats after classes began. Previously, until 2014, after the final round of admission, the vacant seats in non-IIT institutes used to be filled by each institute locally by a spot round. (In some years, the spot round was centrally organized but candidates who already had seats could only get upgrades within the same institute. This led to unfairness and incentive issues in the overall process.) Unassigned candidates would physically visit as many institutes as possible (meaning just a couple) as required to participate in their spot rounds. This setup led to misallocation of seats: a seat could be given to a candidate with worse rank than another candidate because only the former could be present physically during the spot round of that institute. In the 2015 Special Round, unassigned candidates were required to pay fees to participate to reduce the number of frivolous applications. The best possible seat was assigned to each candidate in a fair and efficient manner. Candidates were allowed to submit new preferences for the Special Round. After the round, 2,148 candidates migrated to a different institute after joining. 1,492 stayed in the same institute but migrated to a different program.

5,354 fresh allotments were made, of which 2,683 candidates did not report leading to a final vacancy count of 2,883 at the non-IITs.

Joint Seat Allocation 2016. 92 institutes participated in the joint seat allocation in 2016. This includes 23 IITs, 31 NITs, 20 IIITs and 18 Other-Government Funded Technical Institutes (Other-GFTIs). The seat allocation was carried out in 6 rounds. The major change introduced in 2016 was that the candidates were allowed a Withdraw option: a candidate, who had previously accepted a seat allotted by JoSAA 2016, could Withdraw by reporting at a reporting center before the last round of seat allocation. This option provided candidates the ability to choose freely between her allotted CFTI seat and her outside (non-CFTI) options before the last round: a candidate who withdrew was refunded the seat acceptance fee and was allowed to appear for the JEE in the following year.

Design Insight 8 *A majority of candidates may be willingly share that they have decided not to take up their allotted seat if they are provided some monetary or other incentive.*

Withdrawal by 3,762 candidates allowed us to reallocate the surrendered seats in the later rounds of allocation, further reducing vacancies (but significant room for further improvement remained due to limited efficiency of filling vacant seats in the last rounds, see the next section).

No centralized Special Round was conducted by the non-IITs umbrella in 2016. It was later found that some of these institutes conducted “spot” rounds on their own to fill their vacant seats. We believe that a Special Round should have been conducted as in 2015.

Joint Seat Allocation 2017. 97 institutes participated in the joint seat allocation in 2017. This includes 23 IITs, 31 NITs, 23 IIITs and 20 Other-Government Funded Technical Institutes (Other-GFTIs). The seat allocation was carried out in 7 rounds. For the first time, the non-IITs did not give weightage to Board exam (high school graduation exam) marks in constructing their Merit Lists. Possibly this made more candidates want to appear for the JEE again the following year, leading to a large increase in the number of withdrawals to over 6k, and hence an increase in the final number of vacancies in non-IITs, almost to the same level as in 2014. Additionally, the increase in withdrawals could have been partly caused by more candidates listing and accepting programs they did not really want, because: (i) Candidates who did not receive any allocation during the Mock seat allocation were encouraged to list more programs. (ii) Awareness regarding the option to Withdraw later may have increased (in 2016 the option had just been introduced for the first time), leading candidates to list and accept programs more liberally. We propose some solutions to this issue in the next section.

Happily, a Special Round for non-IIT institutes was conducted centrally again in 2017 as in 2015. 5,352 of the 6,510 vacant seats were allotted in the Special Round, and 3,830 of these candidates actually joined classes, resulting in a final vacancy count of 2,680 at the non-IITs.

Counterfactual experiment to assess impact of Joint Seat Allocation

To conclusively establish that the reduction in vacancies was due to superiority of our DA-based joint seat allocation process over the legacy process, we simulate the legacy allotment process used until 2014 on data for each year since 2015. The resulting allocation is then compared with the allocation for that year under our DA-based process.

Until 2014, IITs conducted their allocation before non-IITs. Since IIT seat allocations were frozen by the time non-IIT allotments took place, many candidates abandoned their IIT seat in favor of some non-IIT seat. The abandoned IIT seat then remained vacant. To simulate the allocation that would have been produced under the legacy process, we proceed as follows. We first allot the IIT seats only, using the preferences submitted by the candidates over IIT programs (ignoring the non-IIT programs they had listed) and the IIT Merit Lists. Once the IIT seats are allotted, we then consider the submitted preferences of the candidates over non-IIT programs only, after removing their preference entries below their allotted IIT program (if any), and allot the non-IITs based on the non-IIT Merit Lists. (This simulation setup is based on the optimistic assumption that candidates would list only those programs in the old non-IITs process which they preferred to their IIT allocation. If candidates list additional programs as well, this would only serve to further increase the number of seats that would have been wasted under the legacy process.) The final allocation we obtain is compared with the first round allocation obtained under DA. One can easily show, mathematically, that every single candidate obtains a weakly better allocation under the new process than under the legacy process, and indeed we see this in the simulation results. Our simulation further proves that our DA-based joint process caused a large reduction in vacancies at the IITs — our process led to 373 prevented vacancies in IITs in 2015, 379 prevented in 2016 and as many as 629 prevented vacancies in 2017 (shown in Table 2 and Figure 1). Last but not least, we find that 1,890 candidates received a more preferred program due to the new process in 2015, 1,767 in 2016, and 3,672 in 2017.

Reducing vacancies further

As we have shown, our joint process has been very successful in reducing vacancies in the IITs, as per our mandate. Meanwhile, vacancies in the non-IITs have reduced only slightly, and concern us, though this was outside our mandate. The Withdraw option introduced in 2016 produced a modest further reduction in vacancies in the non-IITs, see Tables 2 and 3, by reducing the number of vacancies discovered only when classes begin. The reason that the further reduction was modest is revealed in the data — most of the seats from which candidates Withdraw are difficult to fill in late rounds. Over 70% of fresh allocations in late rounds are found to be rejected by candidates, and most of the withdrawals occurred in the penultimate round!

In 2017, there was an unexpected increase in the number of withdrawals. This number was 3762 in 2016 (of which 1114 said they plan to write JEE again), but went up to 6366 in 2017 (of which 1999 said they want to write JEE again). Over 4k of these withdrawals occurred in the penultimate round. Consistent with the pattern in 2016, most of these seats remained vacant at the end of Round 7 (the last round) due to rejection of the majority of fresh allocations. This leads us to the question: How can seats be effectively filled in the main rounds despite the significant number of Rejects and Withdraws (concentrated in a subset of the programs)? Data indicates that most of the vacancies at the end of the main rounds are avoidable in the sense that there are eligible candidates who want those seats.

We now argue for modification of candidate *incentives* to improve the efficiency of the seat allocation process. Currently, there is no bar on writing JEE Main again even if a candidate accepts a seat and does not report for classes (and does not Withdraw). In our view this leniency may be causing an unnecessarily large number of candidates to accept seats in programs they have no intention of joining. The problem is exacerbated by the fact that the non-IITs no longer use high school graduation exam marks to compute their Merit Lists, causing more candidates to consider the option of writing the JEE again. The policy of refunding the seat acceptance fee to candidates who Withdraw may be amplifying the problem further (now that the awareness about the Withdraw option is increasing). Note that the IITs do not allow candidates who accept a seat in an IIT program (and then do not Withdraw) to apply for an IIT seat in future, whereas there is no analogous rule for the non-IITs. We believe that this asymmetry between IIT and non-IIT programs should be removed. We advocate for the following approach:

1. Any candidate who is allotted a seat (whether in an IIT or a non-IIT) and then accepts should *not* be permitted to apply for admission to any CFTI in future. If they did not want the program they should not have listed it a few days earlier, and even if they made that mistake, they should not have accepted the seat.
2. The Withdraw option can be retained (primarily for candidates who just received better offers from outside the system and are no longer interested in the seat they were allotted), and incentivized, for instance with a speedy return of fees. But candidates who Withdraw should not be given permission to apply for admission to any CFTI in future, for the reason above. (Currently, a candidate who Withdraws from an IIT seat is allowed to reapply in future.)
3. Messaging should *not* encourage candidates to blindly list *more* programs. Instead, candidates should be encouraged to list programs they are truly interested in, and made clearly aware of the consequences of accepting a program they do not intend to join. (It will also hurt the candidate to list such a program in the first place, since when she is allotted the program, she will then have to Reject, in which case she will be eliminated from the seat allocation process entirely, with no opportunity to get a better allocation in a later round.)
4. Candidates should be allowed the flexibility to remove any program from their preference list at any time, except the program they are allotted to. Currently, a candidate's preference list remains locked for the entire process.

In the Appendix section “The challenge and the opportunity of overbooking”, we make further suggestions to (i) very conservatively admit some additional candidates in excess of program capacity based on somewhat predictable rejections of fresh allocations (the challenge is that many virtual programs are very small, and overage in specific categories can have political ramifications besides causing logistical issues), (ii) stop withdrawals two rounds before the last round to facilitate better utilization of the seats that get freed up, and (iii) try to ensure that only serious candidates participate in the last two rounds. These suggestions can be coupled with the ones listed above to maximize efficiency gains.

Design Insight 9 *The incentive properties (and related messaging) of the overall dynamic process may play a crucial role in determining how participants interact with it, and hence greatly impact its allocative efficiency. Monetary incentives can have an impact. Incentives perceived to affect a candidate’s career options may be yet more powerful.*

We advocated to include these changes (with the exception of overbooking) into the Business Rules for 2018, however only suggestion 3 was partly accepted. Our other suggestions were rejected citing that the proposed penalty for backing out was “too harsh” on candidates, etc. Consequently, we predicted several thousand vacancies when classes begin in 2018 as a result of the persistent improper incentive structure, and indeed, 6,133 seats remained vacant in the non-IITs at the end of the main rounds of admission, only slightly less than in 2017. We continue to advocate for our suggested changes to be made in future years. Some popular articles covering our recommendations have appeared recently (Chhopia 2018, Kurczy 2018).

Discussion

The theory and practice of one-shot seat allocation using the Deferred Acceptance algorithm is well developed. Nevertheless, we faced many challenges in bringing it to the high stakes setting of seat allocation for the most prestigious engineering colleges in India. Challenges included the complexity of business rules and other requirements such as a dynamic Multi-Round process to fill rejected seats. Since its implementation in 2015, our new joint process has provably reduced the vacancies at the IITs, which previously conducted their admissions independently of, and before, that of the non-IIT centrally funded institutes. The reduction in vacancies at the IITs was more than 50% in 2015 and has been in excess of 70% since 2016, relative to 2014. Though vacancies at the other non-IITs have reduced compared to 2014, a significant fraction of seats remain vacant at the end of the Multi-Round admissions process. We are advocating for multiple changes to address this issue: chiefly, a change in the incentives of candidates so that less seats remain unfilled due to being vacated at a late stage.

Our algorithmic innovations in this project include (i) a practical heuristic for incorporating a non-nested common quota, (ii) a method to “dereserve” seats with no modifications to the core

software, and (iii) a robust approach to reduce variability in the number of reserved category candidates admitted, while retaining fairness. Theoretically formalizing our insights regarding (i) and (iii) may be interesting for future work.

Overall, our experience developing, executing, adapting and improving this centralized seat allocation process has taught us many practical lessons, including those highlighted as “Design Insights” (supported by analysis and statistics) throughout the paper. We are optimistic that many practitioners faced with similar problems can benefit from our learnings, and that more countries will be inspired to collect the efficiency gains that come from centralizing admissions.

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Appendix

History and background

This appendix provides some background regarding the institutions and examinations involved.

Students in India must write a senior high school graduation exam (known as the “Board exam”); these are administered by the educational board to which their high school is affiliated. However, these exams necessarily account for the wide heterogeneity in the preparation of students across schools and geographies. Hence, scores in these exams are typically not considered appropriate for determining admissions to the country’s most prestigious engineering colleges.

History of admissions to the IITs. The first five IITs (Kharagpur, Bombay, Kanpur, Madras and Delhi) were founded during 1951-61, and almost immediately they created a countrywide Common Entrance Examination for admissions purposes. The examination was used to produce a single ranking called a “Merit List” of candidates (more precisely, one Merit List for each “category” of students, see the Business Rules section. Next, in a centralized process, candidates were then considered in the increasing order of rank (starting with the top ranker), and allotted their most desired program which was not already full based on the preferences over programs that they submitted after “counselling” at the closest IIT. (This mechanism is known as Serial dictatorship, e.g., see Abdulkadiroglu and Sönmez 1998) The name of the examination subsequently changed to Joint Entrance Exam (JEE), and the number of IITs has grown to 23. As the number of candidates grew, the IITs resorted to a two stage examination process, with the first “screening” stage used to select a subset of candidates who could then write a more detailed second stage exam.

History of admissions to the non-IIT CFTIs. Starting in 1959, Regional Engineering Colleges (RECs) were created in every major state to supplement the IITs. The admissions to these colleges was conducted in a decentralized manner and many of them conducted their own entrance exams, creating a logistical nightmare for high school students who were aspiring engineers. In 2002, the RECs were renamed National Institutes of Technology (NITs), and a single All India Engineering Entrance Examination (AIEEE) was created to centralize the examination and admissions process for the NITs, simplifying the logistics. (There are now 31 NITs.)

Several Indian Institutes of Information Technology (IIITs) were established starting in 1997; that exclusively offered programs allied to information technology. (There are now 23 IIITs.) Admissions to the IIITs as well as other engineering colleges funded by the central government were clubbed with admissions to the NITs.

Merger of Examinations. Subsequently the AIEEE was merged with the first stage of the two stage IIT exam process in 2012. The first examination is called the JEE Main. The JEE Main score of candidates is used as follows:

(i) The non-IIT CFTIs use the JEE Main scores to construct their Merit Lists and determine allocation of seats at the non-IIT CFTIs. Until 2016, the Merit Lists were created by a combining the JEE Main score of a candidate with her Board exam score. Since 2017 they are exclusively based on the JEE Main score.

(ii) The IITs use the JEE Main scores to determine a subset of about 150,000 candidates (as of 2015) who qualify to be permitted to write the second stage “JEE Advanced” examination.

The JEE Advanced examination is conducted subsequently by the IITs for their own admissions purposes, and typically consists of three separate exams for Physics, Chemistry and Math. Subject cutoffs are set for each of the three, and a Merit List of candidates who clear the cutoff is constructed based on their total score across the three subjects for purposes of admission to IIT programs. (Detailed tie-breaking rules ensure that ties in the Merit List play a negligible role, and similarly for the Merit Lists for the non-IITs umbrella.) The Board exam score of candidates is not used for ranking but purely to determine their eligibility based on a cutoff.

From 2012 to 2014, the seat allocation process for IITs (under IITs umbrella) remained separate from that for the non-IIT CFTIs (under non-IITs umbrella). The IITs conducted their admissions first, even before the Board exam scores had come in (since almost all successful candidates obtained the requisite Board exam score), and the non-IITs umbrella conducted its admissions process subsequently.

Merger of Seat Allocation processes. After the merger of examinations, an external nudge in the form of a public interest court case W.P.(C) 2275/2010 in the Delhi High Court (demanding coordination to reduce wastage of seats) caused the creation of a joint seat allocation process.

Following a false start in 2013, a common seat allocation process for all the CFTIs including the IITs was launched in 2015, run by the Joint Seat Allocation Authority (JoSAA). This joint seat allocation process is the subject of the current paper. JoSAA provided candidates with a single window for admission to any of the over 80 CFTIs.

Design Insight 10 *Centralization can greatly reduce the logistical burden of participation on both sides of the market (in addition to improving the allocative efficiency and reducing market congestion if the allocation is also done centrally).*

Related efforts elsewhere include the Common Application for applying to hundreds of colleges worldwide (in this case the allocation is not done centrally), centralized school admissions, and centralized labor markets (see Related Work in the Introduction). In the case of the CFTIs, centralization has occurred for the examination, application as well as the seat allocation processes.

In 2015 and 2016, the JoSAA seat allocation process was conducted after the JEE Main, JEE Advanced, and Board exam scores became available. Delays in announcement of Board marks was a major issue, indeed the joint process was very nearly called off the very first time in 2015

due to such delays. The IITs wanted to proceed with their allocation, whereas the other institutes were unable to rank candidates without Board marks being at hand.

Design Insight 11 *Aggregation of all relevant information and alignment of timelines of the concerned institutions can be bottleneck for centralized matching/allocation. If institutions construct their preferences based on the same information (and at the same time), this improves the chances of successful centralization.*

Since 2017 the non-IIT CFTIs chose to stop using Board exam scores for constructing their Merit Lists, eliminating this issue, consistent with the trend of logistics getting simpler over time.

Broader view of the Joint Entrance Examination (JEE). The JEE is generally viewed positively as a way to select high quality students. Some of the main concerns regarding JEE based admissions are: (i) Wealthy candidates have an advantage due to the need for expensive coaching. (ii) Candidates are “burned out” even before they start their studies at these institutes due to intense multi-year preparations, which may further cause them to “lose” years due to repeating the JEE after they have graduated from senior high school. (iii) Affirmative action reservation of seats is a contentious issue. We emphasize here that our mandate was restricted to designing an efficient and fair joint seat allocation mechanism for the 80+ institutions involved (the CFTIs), that respects a set of business rules, treating the JEE as a given.

Example of a rare failure in handling non-nested common quota reservation

In this appendix, we provide an example of a potential pathological situation in our heuristic algorithm to incorporate a non-nested common quota, forcing the creation of a supernumerary seat. Three IITs are involved in the example — Kanpur (IITK), Delhi (IITD) and Bombay (IITB).

Let Amar, Akbar, Chetan, and Dhanush be four DS candidates. At the end of the DA algorithm, Amar, Akbar, and Chetan get a DS seat in IITK-DS (wants Mechanical), IITD-DS (wants Metallurgy) and IITD-DS (wants Electrical) respectively; but Dhanush gets seat in IITB-Electrical-Open. Moreover, let Dhanush be the last ranked candidate getting a regular seat in IITB-Electrical-Open, and suppose there are already two (unnamed) DS candidates ranked above Dhanush who are occupying the two IITB-DS seats. Let Bharat, Krish, and Ekansh be the last ranked Open category candidates in IITK-Mechanical, IITD-Metallurgy, and IITD-Electrical respectively. The details of all these seven candidates along with the programs allocated by the DA algorithm is shown in Figure 5.

We now describe the processing of Amar, Akbar, and Chetan who got DS seats by our heuristic algorithm. These candidates have to be given Open seats. In order to accommodate Amar, we need to remove Bharat and this leads to Bharat getting some other less preferred program, and

Candidate	Open Rank	Program	Seat allocated	DS status	Preference list of Dhanush
Amar	1200	IITK-Mechanical	DS	Yes	IITB-Computer Sc
Akbar	400	IITD-Metallurgy	DS	Yes	IITB-Electrical
Chetan	500	IITD-Electrical	DS	Yes	IITD-Computer Sc
Dhanush	200	IITB-Electrical	GE	Yes	...

Candidate	Open Rank	Program	Seat allocated	DS status	Preference list of Ekansh
Bharat	600	IITK-Mechanical	GE	No	...
Krish	350	IITD-Metallurgy	GE	No	IITD-Electrical
Ekansh	150	IITD-Electrical	GE	No	IITB-Electrical
					...

Figure 5: Interim program allocation in the DA algorithm to 3 DS and 2 GE candidates.

possibly pushing another candidate out, and so on, in a rejection chain. In a similar manner, Akbar gets IITD-Metallurgy after removal of Krish and Krish gets some other less preferred program, in a rejection chain.

Next, we process Chetan. Since Chetan got seat IITD-Electrical through DS quota we need to remove Ekansh from IITD-Electrical. The next most preferred program for Ekansh is IITB-Electrical. Recall that Dhanush is the last ranked candidate getting IITB-Electrical-Open. Notice that though Dhanush is a DS candidate, he got Open seat as an Open candidate in IITB-Electrical. So Ekansh will remove Dhanush from IITB-Electrical-Open. Dhanush will be rejected from IITB-DS again (due to both spots being already occupied), will then apply for his next preferred program which is IITD-Computer Science, for which he does not clear the Open category cutoff, so he is rejected by the IITD-Computer Science virtual program and then applies to the IITD-DS virtual program. Now, there are already two DS candidates Akbar and Chetan who are occupying the two spots in IITD-DS. Since Dhanush has better rank than Akbar, and Akbar has better rank than Chetan, Dhanush will remove Chetan from DS virtual program of IITD, and we see that we have created a loop condition.

At this point we also realize that Ekansh should never have been rejected from IITD-Electrical since no DS candidate is taking a seat there, and Dhanush should not have been rejected from IITB-Electrical, but then why can't Chetan keep his spot in IITD-DS, and so on. We have run into trouble, and in fact, such examples may not have a stable matching at all. In this example, our algorithm gives Chetan a supernumerary seat in IITK-Electrical program.

We expect failures to be rare when the non-nested quota is small, because multiple improbable events as captured in this example must occur to produce a failure (see the discussion in subsection “Non-nested common quota reservation” of the Algorithm Design section). Indeed we have not observed any failures in practice to date.

The challenge and the opportunity of overbooking

The fraction of Rejects from among fresh allocations is quite large, especially in later rounds. One may think of using yield prediction (i.e., admitting more candidates than the capacity of the program) as a way to improve the efficiency of allocation and to have less vacancies at the end of all the main rounds. This is harder than it would seem, despite the fraction of rejected fresh allocations being 60% or more in later rounds. Consider Round 3 in 2015. There were 7342 candidates whose allocation changed including 3720 fresh allocations. There were 2666 Rejects, almost all (2651 of them) from among the fresh allocations, meaning that 71.6% of fresh allocations were rejected! It is tempting to think that one can use yield prediction to substantially take care of the issue of Rejects. Unfortunately, this does not work out as expected. Even for an individual program whose size may be 50 or 100 seats, a lot of virtual programs (split by category and further by Home State vs All India quota for NITs and many CFTIs) – with the exception of the OPEN and sometimes the OBC virtual programs – have a single digit number of seats, minimizing our ability to benefit from yield prediction when overage (i.e., admitting more candidates than the number of seats) is a problem. In fact, if we consider the subset of virtual programs with 10 or more rejected seats in that round, this accounts for only 740 of the 2666 Rejects. Thus, roughly, one could only hope to account for about 30% of the Rejects (roughly those that occur in virtual programs with 10 or more Rejects, since with 5 Rejects, Poisson(5) has a reasonable likelihood of being 0 and even more chance of being 1, so there is minimum benefit from yield prediction), without risking significant overage. 60 – 80% Rejects are present across categories, though the OPEN category has slightly higher fraction of Rejects closer to 80% and accounts for 1647 of the Rejects, meaning more than 60% of them. We do notice some patterns like if there are 4 or more fresh allocations then there is at least 1 Reject, if there are 6 or more fresh at least 2 Rejects, with 10 or more fresh at least 40% are Reject and with 20 or more at least 70% are rejected. Such observations may be the basis of refined business rules for conservative yield prediction to improve the efficiency of allocation. Use of an opaque/complex predictive model is not desirable due to lack of transparency and possibility of unfairness etc. In this context, it is imperative that the business rules must be transparently and completely specified, and be clearly fair to all concerned, in accordance with the prevailing laws. So far, this option has not been considered. However, it may be worth considering, based on the following reasoning: currently, about 70% of vacancies in a given round persist until the next round. So over two rounds, the number of vacancies is reduced to about 70% of 70% = half. Instead, suppose we use some conservative yield prediction as above, and it reduces the vacancies in the resulting allocation by about 30%. This means that now, 70% of $(100 - 30)\% = 50\%$ of vacancies in a given round persist until the next round. Over two rounds then, the number of vacancies is reduced to about 50% of 50% = 25%. The cost of doing this would be, in worst case, a handful of seats allocated in excess of capacity, maybe about 10 in total (though we would aim for 0 supernumerary based on

data from the previous year). In a system with over 30,000 seats, this would appear to be a very small aberration. On the other hand, the benefit may be substantial: In 2017, 4168 candidates withdrew in the reporting following the Round 6 allocation. After Round 6 there was only one more main round (Round 7), so 70% or more of these 4168 seats (i.e., over 3000 seats) remained unfilled at the end of all the main rounds. Note that Round 5 happened only two days before Round 6. One would expect that if withdrawal was permitted only until Round 5, then most of these 4168 candidates (say, 4000 of them) would have withdrawn by then, and then with two more main rounds of seat allocation with conservative yield prediction as above, only 25% or about 1000 of these vacancies would have remained at the end of the last main round (Round 7). Similarly, under such an approach, the vacancies resulting from rejected seats would also be more effectively dealt with.

Finally, we remark that steps should be taken to ensure that only serious candidates participate in the last rounds of admission after the Withdraw option is closed. For example, if a candidate wants to participate in such a round, she should be required to explicitly state that. Currently, all unallotted candidates are a part of future rounds by default. Making the candidate report physically, or pay the fee upfront could be a powerful tools to filter candidates who are not serious.