Recursive Calibration of Industrial Manipulators by Adaptive Filtering

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The calibration scheme of robot forward kinematics presented in this paper has a number of features. Firstly, robot kinematic errors are modeled in a recursive format and as such, the number of measurements that need to be taken for calibration can be determined by studying the rate of convergence of estimation error covariance. Secondly, a simplified adaptive filtering algorithm is used to deal with unknown measurement noise statistics and unknown robot motion repeatability characteristics in estimating the kinematic errors. Thirdly, a laser interferometry system is used to measure positions of a robot end-effector in world coordinates. The measurement system was implemented in experiments involving a three degree-of-freedom gantry robot. The adaptive filtering of the experimental data identified 0.5 to 1.5 percent errors in representative kinematic parameters of the given robot by taking into account measurement noise and robot repeatability.

1 Introduction

The open chain mechanism of robots determines that their end-effector's positioning accuracy relies on how well the kinematics is known. The goal of robot kinematic calibration is, therefore, to improve the accuracy of the kinematics. The inaccuracies in robot kinematics could be due to manufacturing deviations. They may also be attributed to slight changes or drifts, such as wear of parts, dimensional drifts and component replacement effects after a robot being in service for a period of time.

In general, robot calibration involves three steps: (*1*) deriving an error model to relate end-effector's positioning errors to robot kinematic errors, (*2*) measuring the end-effector's positioning errors in the world coordinates by an external system, and (*3*) using the measurement data to estimate the kinematic errors. The last step usually involves a numerical search procedure, since the kinematic errors are not an explicit function of the end-effector's positioning errors.

The most popular method to derive a kinematic error model is based on homogeneous transformations [1]. This technique has been used in [2, 3] to investigate robots with variations in their kinematic model. Numerical instability arises when two consecutive revolute joint axes are parallel. For this reason, Veitschegger and Wu [4] proposed to post multiply each transformation matrix by an additional rotation. These methods, like the Euler angle formulation used by Whitney et al. [5] and S-Model proposed by Stone et al. [6], are nonrecursive. Therefore, the adequate number of measurement data may be determined only on a trial and error basis.

The simplest approach to the estimation problem is to ignore the statistics of measurement noises, such as the least squares estimate [5, 7]. More accurate estimate can be expected, if probabilistic approaches are used, such as the minimum variance estimate employed by Wu and Lee [3], and Hayati and Mirmirani [8]. But the noise statistics is difficult to obtain.

In comparison with the methods reviewed above, the forward calibration scheme presented in this paper has the following advantages. Firstly, recursive formulation of robot calibration provides a framework to study the issues of the number of measurements that need to be taken as well as the effect of robot repeatability on estimation quality. Secondly, an adaptive filtering is used to deal with the unknown measurement noise statistics and robot repeatability characteristics in estimating kinematic errors. To take advantage of the special structure of the problem in question, an existing adaptive filtering scheme [9] was simplified for the calibration scheme. Thirdly, a measurement system capable of continuous path measurement of positional errors of robot end-effector was used in experiments involving a three degree-of-freedom gantry robot. The system is based on laser interferometry. Error models are first discussed. Simplification of the adaptive filtering is then shown. Finally, representative experimental results are presented and discussed.

2 Error Models

Define a 6 by 1 vector $\mathbf{p} = [p_x\, p_y\, p_z\, \phi_x\, \phi_y \phi_z]^T$, to describe the position/orientation of the end-effector in world coordinates. $\mathbf{p}$ could be given by

$$\mathbf{p}(j) = g(\mathbf{x}(j)) \tag{1}$$

where $\mathbf{x} = [\boldsymbol{\Theta}^T \mathbf{d}^T \mathbf{a}^T\, \mathbf{A}^T]^T$ is a $4N$ by 1 kinematic parameter vector and N is the degree-of-freedom of the robot. $\boldsymbol{\Theta} = [\theta_1, \ldots, \theta_N]^T$ represents joint angles, $\mathbf{d} = [d_1, \ldots, d_N]^T$ joint offsets, $\mathbf{a} = [a_1, \ldots, a_N]^T$ link lengths, and $\mathbf{A} = [\alpha_1, \ldots, \alpha_N]^T$ link twist angles. For a revolute joint, θ is the joint variable while for a prismatic joint, d is the joint variable.

Given $\mathbf{x}$, the known functional structure g predicts the end-effector position/orientation $\mathbf{p}$. Due to kinematic error $\Delta\mathbf{x}$, the actual end-effector position/orientation will differ from the predicted one, say, by $\Delta\mathbf{p}$

$$\mathbf{p} + \Delta\mathbf{p} = g(\mathbf{x} + \Delta\mathbf{x}) \tag{2}$$

where $\Delta\mathbf{x} = [\Delta\boldsymbol{\Theta}^T \Delta\mathbf{d}^T \Delta\mathbf{a}^T \Delta\mathbf{A}^T]^T$ contains the $4N$ kinematic errors generally. $\Delta\mathbf{p}$ is comprised of three translational error components, δp_x, δp_y and δp_z, and three orientational error components, $\delta\phi_x$, $\delta\phi_y$ and $\delta\phi_z$, all expressed in the world coordinate system. These errors are caused by the $4N$ kinematic errors.

The calibration is a process that uses the error between predicted and actual world coordinate measurements $\Delta\mathbf{p}$ to estimate more accurate kinematic parameters $\mathbf{x} + \Delta\mathbf{x}$. To enable a recursive estimation procedure, Eq. (2) may be perturbed with respect to nominal value $\mathbf{x}$, yielding a linear relationship $\mathbf{h}$ between errors $\Delta\mathbf{p}$ and $\Delta\mathbf{x}$ by neglecting higher order terms [2],

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$$\Delta \mathbf{p} = \mathbf{h}\Delta \mathbf{x} \tag{3}$$

where $\mathbf{h}$ is a 6 by $4N$ matrix, which depends on nominal kinematic parameters. It has been shown that the effect of the linearization on accuracy is negligible as the kinematic errors in question are usually small [10].

For simplicity without causing confusion, the incremental signs Δ are dropped, yielding

$$\mathbf{p} = \mathbf{h}\mathbf{x} \tag{4}$$

A minimum variance estimate of the kinematic error $\mathbf{x}$ based on m measurements of $\mathbf{p}$ taken by an external measurement system is given by [3]

$$\hat{\mathbf{x}} = (\Sigma_{\mathbf{x}}^{-1} + \mathbf{H}^T\mathbf{R}^{-1}\mathbf{H})^{-1}\mathbf{H}^T\mathbf{R}^{-1}\mathbf{Z} \tag{5}$$

where $\mathbf{R}$ and $\Sigma_{\mathbf{x}}$ are the covariance matrices of the measurements noise $\mathbf{v}$ and kinematic error $\mathbf{x}$, respectively. Both $\mathbf{x}$ and $\mathbf{v}$ are assumed to be Gaussian with zero mean and statistically independent. $\mathbf{H}$ and $\mathbf{Z}$ are equal to

$$\mathbf{H} = (\mathbf{h}^T(1), \ldots, \mathbf{h}^T(m))^T \tag{6}$$

$$\mathbf{Z} = (\mathbf{z}^T(1), \ldots, \mathbf{z}^T(m))^T \tag{7}$$

and

$$\mathbf{z}(j) = \mathbf{p}(j) + \mathbf{v}(j) = \mathbf{h}(j)\mathbf{x} + \mathbf{v}(j) \quad \text{for } j = 1, 2, \ldots, m \tag{8}$$

The estimation error covariance for Eq. (5) is given by [3]

$$\mathbf{M} = E[(\mathbf{x} - \hat{\mathbf{x}})(\mathbf{x} - \hat{\mathbf{x}})^T] = (\Sigma_{\mathbf{x}}^{-1} + \mathbf{H}^T\mathbf{R}^{-1}\mathbf{H})^{-1} \tag{9}$$

which relates the estimation error $\mathbf{M}$ to measurement noise $\mathbf{R}$ and the a priori kinematic error covariance $\Sigma_{\mathbf{x}}$. The drawback of the nonrecursive estimation, Eq. (5), is that it is usually impossible to determine a minimum set of measurement data that need to be taken in order to meet a pre-specified accuracy in estimation without trial and error.

A recursive estimation scheme is shown below. Define a $4N$ by 1 vector $\mathbf{w}$, that represents the effects of robot repeatability on estimation of kinematic error $\mathbf{x}$, together with Eq. (8) rewritten here, the recursive error model is:

$$\mathbf{x}(j+1) = \mathbf{x}(j) + \mathbf{w}(j) \tag{10}$$

$$\mathbf{z}(j) = \mathbf{h}(j)\mathbf{x}(j) + \mathbf{v}(j) \tag{8}$$

If it is assumed that $\mathbf{w}(j)$ and $\mathbf{v}(j)$ are independent Gaussian with zero mean and covariance matrices, $\mathbf{Q}$ and $\mathbf{R}$ ($\mathbf{Q} > 0$, $\mathbf{R} > 0$), that is

$$E\{\mathbf{w}(i)\} = 0; \quad E\{\mathbf{w}(i)\mathbf{w}^T(j)\} = \mathbf{Q}\delta_{ij} \tag{11}$$

$$E\{\mathbf{v}(i)\} = 0; \quad E\{\mathbf{v}(i)\mathbf{v}^T(j)\} = \mathbf{R}\delta_{ij} \tag{12}$$

$$E\{\mathbf{w}(i)\mathbf{v}^T(j)\} = 0; \quad \text{for all } i, j \tag{13}$$

and further assuming that the initial state $\mathbf{x}(0)$ is normally distributed with zero mean and covariance $\Sigma_{\mathbf{x}}(0)$, and $\mathbf{x}(0)$, $\mathbf{w}(j)$ and $\mathbf{v}(j)$ are mutually independent, the following well known solutions [11] can be obtained:

$$\hat{\mathbf{x}}(i+1/i) = \hat{\mathbf{x}}(i/i) \tag{14}$$

$$\hat{\mathbf{x}}(i/i) = \hat{\mathbf{x}}(i/i-1) + \mathbf{K}(i)[\mathbf{z}(i) - \mathbf{h}\hat{\mathbf{x}}(i/i-1)] \tag{15}$$

where

$$\mathbf{K}(i) = \mathbf{M}(i/i-1)\mathbf{h}^T(i)(\mathbf{h}(i)\mathbf{M}(i/i-1)\mathbf{h}^T(i) + \mathbf{R})^{-1} \tag{16}$$

$$\mathbf{M}(i/i-1)\mathbf{h}^T(i)(\mathbf{h}(i)\mathbf{M}(i/i-1)\mathbf{h}^T(i) + \mathbf{R})^{-1} \times \mathbf{h}^T(i)\mathbf{M}(i/i-1) - \mathbf{Q} = 0 \tag{17}$$

and $\mathbf{M}$ is the covariance matrix of the estimation error, namely

$$\mathbf{M}(i/i-1) = E\{[\mathbf{x}(i) - \hat{\mathbf{x}}(i/i-1)][\mathbf{x}(i) - \hat{\mathbf{x}}(i/i-1)]^T\} \tag{18}$$

Therefore, the adequate number of measurements can be determined by examining the rate of convergence of the estimation error covariance $\mathbf{M}$ after each iteration. One way to do that is to take the sum of diagonal elements of matrices $\mathbf{M}(i)$ and $\mathbf{M}(i-1)$ first and then check the difference between the sums against a present value C, that is

$$\left|\sum_{j=1}^{4N} (M_{jj}(i) - M_{jj}(i-1))\right| < C \tag{19}$$

where M_{jj}s are diagonal elements of the covariance matrix $\mathbf{M}$. Equation (19) only takes into account the variances of the estimation errors not the entire $\mathbf{M}$ matrix because the diagonal terms are usually more significant in comparison with the off diagonal terms. When Eq. (19) is satisfied, a calibration procedure may be terminated.

3 A Simplified Adaptive Filtering

The recursive error model, represented by Eqs. (8) and (10), requires an exact knowledge of robot repeatability covariance matrix $\mathbf{Q}$ and measurement noise covariance matrix $\mathbf{R}$, which are normally little known. Determination of $\mathbf{Q}$ and $\mathbf{R}$ analyti-

Nomenclature

$\mathbf{a}$ = the link length vector
$\mathbf{C}$ = the covariance matrix of $\mathbf{v}$
C = the error threshold
$\mathbf{d}$ = the joint offset vector
g = the mapping between $\mathbf{x}$ and $\mathbf{p}$
$\mathbf{h}$ = the linear relationship between errors $\Delta\mathbf{p}$ and $\Delta\mathbf{x}$
$\mathbf{H}$ = augmentation of $\mathbf{h}$ matrices
k_p = the number of measurement locations
k_r = the number of repeated measurements at one measurement location
$\mathbf{M}$ = the covariance matrix of the estimation error
m = the total number of measurements taken
N = the degree-of-freedom of the robot
$\mathbf{p}$ = the position/orientation error vector of the end-effector in world coordinates
$\mathbf{Q}$ = the covariance matrix of $\mathbf{w}$
$\mathbf{R}$ = the covariance matrix of $\mathbf{v}$
$\mathbf{v}$ = the noise vector
$\mathbf{w}$ = the error vector representing robot repeatability
$\mathbf{x}$ = the kinematic parameter error vector
$\mathbf{z}$ = the measurement with noise
$\mathbf{A}$ = the link twist angle vector
$\Delta\mathbf{p}$ = the end-effector position/orientation deviations expressed in the world coordinates (simplified as $\mathbf{p}$)
δp_x, δp_y and δp_z = translational components, and
$\delta\phi_x$, $\delta\phi_y$ and $\delta\phi_z$ = orientational components of $\Delta\mathbf{p}$, all expressed in the world coordinate system
$\Delta\mathbf{x}$ = kinematic error vector (simplified as $\mathbf{x}$)
υ = the innovation sequence
$\boldsymbol{\Theta}$ = the joint angle vector
$\Sigma_{\mathbf{x}}$ = the covariance matrix of $\mathbf{x}$

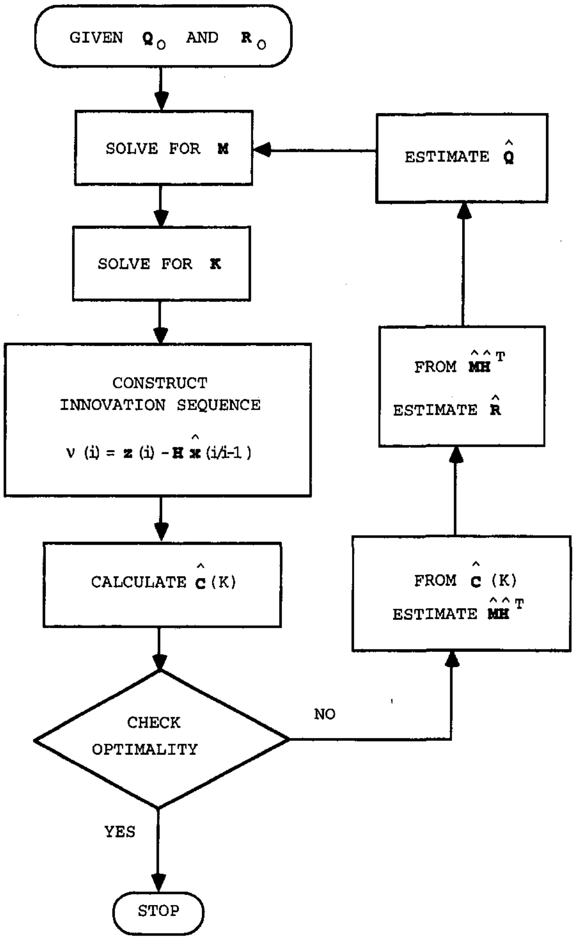

Fig. 1 Block diagram of simplified adaptive filtering

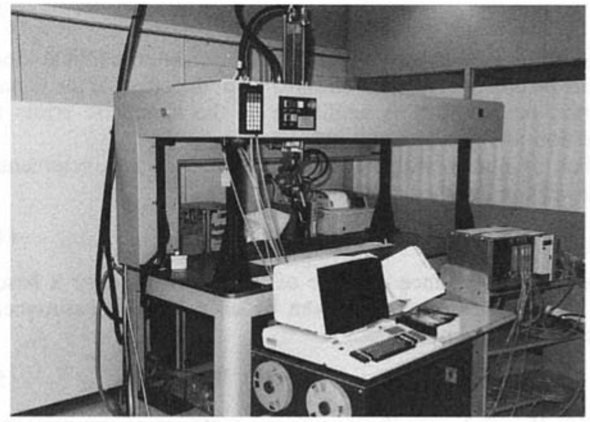

Fig. 3 Experiment setup

cally could be a difficult task. To properly model **Q**, one may have to consider machining tolerances of certain robot links, axis misalignment, encoder mounting, quantization noise, and many others. **R** depends on the accuracy and resolution of the end point sensors, machining tolerances of the calibration fixtures, the method by which the sensor data are processed.

Nishimura [12] has considered the effect of errors in **Q** and **R** on the performance of the estimation. Several other investigators [9, 13, 14] have proposed schemes, known as adaptive filtering, to identify **Q** and **R**. Shown below is a simplification of the adaptive filtering method proposed by Mehra [9]. Since the method is only suitable for time-invariant models, Eq. (5) is first converted to a time-invariant model below [15]. Let

$$m = k_p k_r \tag{20}$$

where m is the total number of measurements taken, k_p the number of measurement locations, and k_r the number of repeated measurements at one measurement location. The end-effector measurements, $\mathbf{z}(i)$, may be ordered in the following fashion with no loss of generality

$$\mathbf{z}_{k_p}(i) = \{\mathbf{z}^T[(i-1)k_p + 1], \ldots, \mathbf{z}^T[(i-1)k_p + k_p]\} \tag{21}$$

and

$$\mathbf{H}_{k_p} = (\mathbf{h}^T(1), \ldots, \mathbf{h}^T(k_p))^T \tag{22}$$

$$\mathbf{v}_{k_p} = (\mathbf{v}^T(1), \ldots, \mathbf{v}^T(k_p))^T \tag{23}$$

then Eq. (8) becomes a time-invariant version as follows

$$\mathbf{z}_{k_p}(i) = \mathbf{H}_{k_p}\mathbf{x}(i) + \mathbf{v}_{k_p}(i) \tag{24}$$

where $i = 1, 2, \ldots, k_r$. Although all **h**s depend on instantaneous robot configurations and therefore are time variant, $\mathbf{H}_{k_p}$ is time invariant when i advances from 1 to k_r, because it includes **h**s for all k_p locations of measurement. It can be seen that, with this formulation, the value of k_p has to be predetermined while that of k_r is determined by applying the criterion shown in Eq. (19) after each iteration.

The simplification was made possible because the coefficient matrices of $\mathbf{x}(i)$ and $\mathbf{w}(i)$ in Eq. (10) are identity matrices. The simplified version retains the property of asymptotically unbiased and consistent estimates of **Q** and **R** possessed by

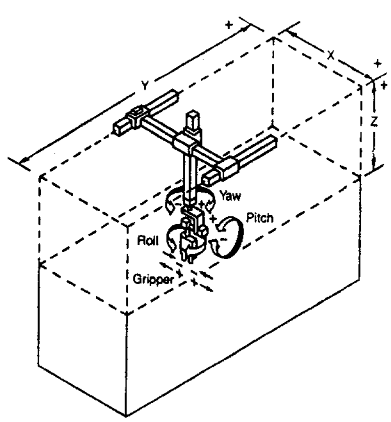

Fig. 2 IBM 7565 manipulator kinematic arrangement [16]

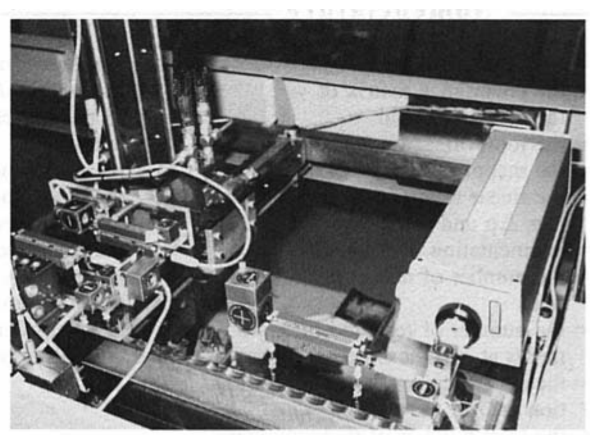

Fig. 4 Laser interferometry measurement system

Table 1 Summary of measurement results at end-effector displacement errors mm (1/1000 in.)

	x	y	z
Mean	-0.0390(-1.5336)	0.0074(0.2909)	0.0183(0.7208)
Peak-to-peak	0.1582(6.2276)	0.1905(7.5000)	0.1670(6.5719)
Std. Dev.	0.0258(1.0139)	0.0292(1.1478)	0.0294(1.1593)

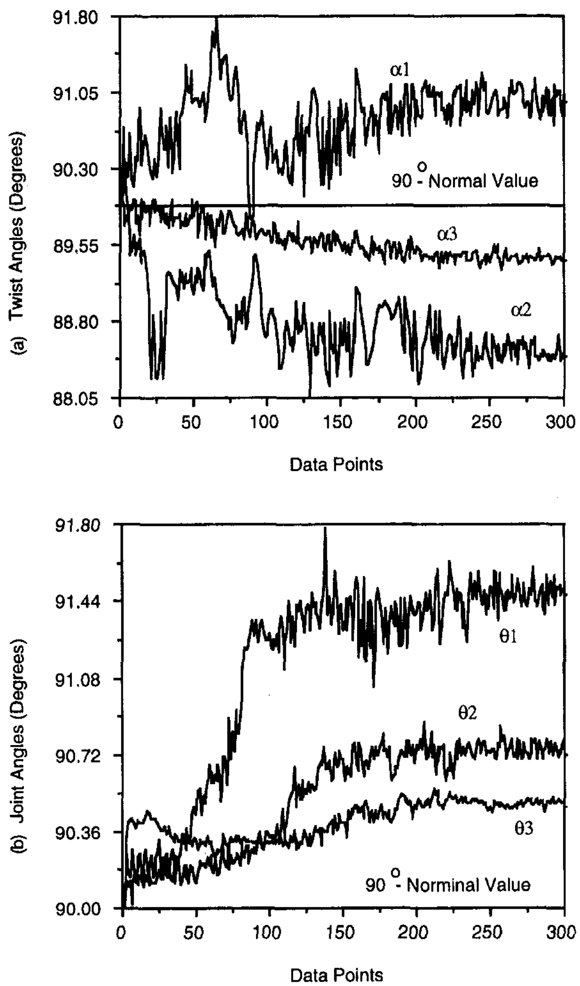

Fig. 5 Recursive calibration results of kinematic parameters $[\alpha_1\ \alpha_2\ \alpha_3\ \theta_1\ \theta_2\ \theta_3]^T$ (initial values are $[90°\ 90°\ 90°\ 90°\ 90°\ 90°]^T$)

the original method. Moreover, it estimates **Q** and **R** in an independent manner as opposed to the coupled estimation required in the original method.

From the innovation property of an optimal filter [9], the sequence

$$v(i) = \mathbf{z}(i) - \mathbf{H}\hat{\mathbf{x}}(i/i-1) \tag{25}$$

known as the innovation sequence, is a stationary Gaussian white noise sequence, that is,

$$\mathbf{C}(k) = \mathbf{HMH}^T + \mathbf{R} \quad k = 0 \tag{26}$$

$$\mathbf{C}(k) = 0 \quad k \neq 0 \tag{27}$$

where $\mathbf{C}(k)$ is defined as

$$\mathbf{C}(k) = E\{v(i)v^T(i-k)\} \tag{28}$$

It has been shown [9] that the necessary and sufficient condition for optimality of the filter is that the innovation sequence $v(i)$ be white. Therefore the non whiteness of the innovation sequence $v(i)$ is an indication of suboptimality where the true value of **Q** and **R** are unknown. It can be shown that in a suboptimal case the covariance of $v(i)$ corresponds to

$$\mathbf{C}(k) = \mathbf{HMH}^T + \mathbf{R} \quad k = 0 \tag{29}$$

$$\mathbf{C}(k) = \mathbf{H}(\mathbf{I} - \mathbf{KH})^{k-1}[\mathbf{MH}^T - \mathbf{KC}(0)] \quad k > 0 \tag{30}$$

where

$$\mathbf{M} = (\mathbf{I} - \mathbf{KH})\mathbf{M}(\mathbf{I} - \mathbf{KH})^T + \mathbf{KRK}^T + \mathbf{Q} \tag{31}$$

and **M** is defined by Eq. (18). Therefore, the simplified adaptive filtering can be summarized as follows.

(*1*) Calculate $\mathbf{C}(k)$ by using the ergodic property of a stationary random sequence

$$\hat{\mathbf{C}}(k) = (1/m) \sum_{i=1}^{m} v(i)v(i-k)^T \tag{32}$$

where m is the number of sample points and $k = 0, 1, 2, \ldots, 4N$.

(*2*) If the calculated $\mathbf{C}(k)s$ are not equal to zero for $k \neq 0$, that is, the filter is suboptimal, the next step will be to obtain better estimates of **Q** and **R**. An estimate of **Q** and **R** can be obtained by using Eqs. (26) and (30) as follows:

$$\hat{\mathbf{Q}} = \mathbf{K}(\hat{\mathbf{M}}\hat{\mathbf{H}}^T)^T + \hat{\mathbf{M}}\hat{\mathbf{H}}^T)\mathbf{K}^T - \mathrm{KH}(\hat{\mathbf{M}}\hat{\mathbf{H}}^T)\mathbf{K}^T - \mathbf{K}\hat{\mathbf{R}}\mathbf{K}^T \tag{33}$$

Table 2 Summary of calibration results of kinematic errors (degrees)

	α_1	α_2	α_3	θ_1	θ_2	θ_3
Nominal Value	90	90	90	90	90	90
Mean of the Last 100 Estimates	90.976	88.522	89.427	91.464	90.744	90.495
Terminal Estimate	90.959	88.447	89.399	91.474	90.724	90.474
Std. Dev.	0.1232	0.1389	0.0431	0.0757	0.0446	0.0203
% Error	1.084	-1.642	-0.637	1.627	0.827	0.550

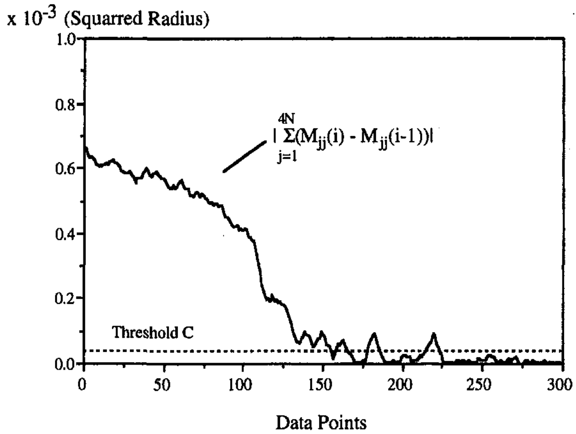

Fig. 6 Estimation errors of consecutive iterations against threshold C

$$\hat{\mathbf{R}} = \hat{\mathbf{C}}(0) - \mathbf{H}(\hat{\mathbf{M}}\hat{\mathbf{H}}^T) \tag{34}$$

The simplified adaptive filtering scheme is illustrated in Fig. 1.

4 Experimental Results and Discussions

The manipulator used for experiments was an IBM 7565 industrial manipulator, which is of a six-axis gantry configuration. The first three are prismatic joints, while the next three are revolute intersecting at the robot wrist (Fig. 2). The experiments were carried out by using the three prismatic joints, which determine the end-effector's position. A three axis laser interferometry measurement system was built using commercially available optical components as building blocks. The experiment set up is depicted in Fig. 3, and Fig. 4 shows the optical system.

4.1 Measurement. First, the robot was programmed to command x, y and z axes to move simultaneously at a constant speed of 12.7 mm/sec (0.5 in/sec). The commanded speed was chosen constant to be consistent with kinematic calibration. The laser interferometry system measured the actual displacement of the end-effector and 1,000 data points were collected for each of x, y and z axes at a sampling interval of 6 ms (166.7 Hz). The displacement errors, defined as the differences between the programmed values and the actual measurements, have the means, peak-to-peak values and standard deviations as summarized in Table 1.

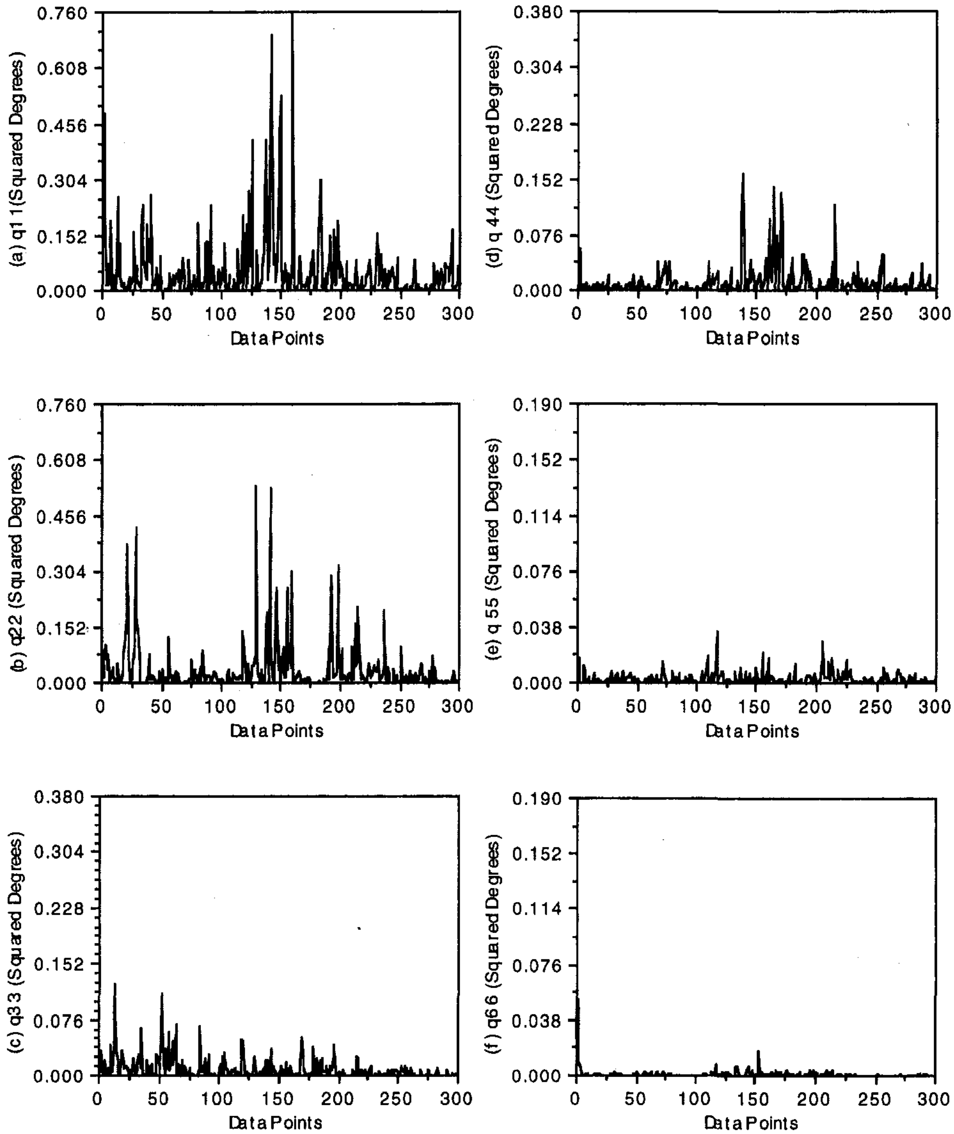

Fig. 7 Estimation results of robot repeatability characteristics (diagonal elements of matrix Q)

Table 3 Summary of estimation results of repeatability characteristics (squared degrees)

	q_{11}	q_{22}	q_{33}	q_{44}	q_{55}	q_{66}
Mean	0.0582	0.0355	0.0088	0.0118	0.0031	0.0010
Maximum	0.7552	0.5375	0.1239	0.1608	0.0346	0.0520

For certain accurate tasks, a robot is required to position its end-effector or trace a prescribed path in the order of thousandths of an inch accuracy. The laser system has a sufficient resolution to carry out measurements to that order of accuracy. It also has the continuous path measurement capability as opposed to point-to-point measurement methods. As seen from Table 1, Y axis exhibits the largest error in terms of peak-to-peak value because it has the longest travel span among the three axes (Fig. 2). To investigate the coupling effects between axes, motion combinations of various axes were programmed. For instance, x, y and z displacement errors were measured while only the x axis was commanded to move. It was found that there is little cross talking existing in this robot mainly because only three translational axes were set to motion. Subsequently, it seems reasonable to consider only the diagonal terms of the estimation error in Eq. (19).

4.2 Calibration of Kinematic Errors. For the experiments involving three prismatic joints, there are twelve kinematic parameters, that is, twist angles α_1, α_2 and α_3, link lengths a_1, a_2 and a_3, joint angles θ_1, θ_2 and θ_3 and joint offsets d_1, d_2 and d_3, where d_1, d_2 and d_3 are joint variables. The following assumptions were made for simplicity. The commanded joint variables d_1, d_2 and d_3 are assumed to be only offset by a constant lag because the translational drives are relatively accurate. The link lengths a_1, a_2 and a_3, whose nominal values are equal to zero since all x, y and z axes are prismatic, are also assumed to have no error because their effects on the end effector's accuracy is smaller than the angular parameters. Therefore, the kinematic error vector is reduced to $\mathbf{x} = [\Delta\alpha_1\ \Delta\alpha_2\ \Delta\alpha_3\ \Delta\theta_1\ \Delta\theta_2\ \Delta\theta_3]^T$ while the nominal values of α_1, α_2, α_3, θ_1, θ_2 and θ_3 are equal to 90 deg. as determined by the robot configuration. This simplification reduces the dimension of the repeatability covariance matrix $\mathbf{Q}$ to 6 by 6. Both $\mathbf{Q}$ and $\mathbf{R}$ are assumed to be diagonal matrices because little cross talking was observed in the foregoing measurements. The initial values of $\mathbf{x}$ and $\mathbf{Q}$ are taken as zero, while the initial values of diag($\mathbf{R}$) were taken as $(10.16 \times 10^{-6}\ 10.16 \times 10^{-6}\ 25.4 \times 10^{-6})$ mm or $(4 \times 10^{-7}\ 4 \times 10^{-7}\ 10^{-6})$ inches for each measurement. The kinematic errors were recursively estimated using the adaptive filtering and a representative result is plotted in Fig. 5. The result is also summarized in Table 2. Although 300 iterations are shown in Fig. 5, the estimates start to converge asymptotically after about 200 iterations. The number of iterations required depends on the value of the threshold C chosen in Eq. (19). Shown in Fig. 6 is an evolution of the number defined in Eq. (19) and the C value used.

4.3 Estimation of Repeatability Matrix. During the estimation, the innovation sequence, Eq. (25), was generated and its auto correlations were calculated using Eq. (32). Since the auto correlations of the innovation sequence were found not equal to zero for $k \neq 0$ [Eq. (27)], Eqs. (30), (32) and (33) were used to estimate the diagonal elements q_{ii} ($i = 1, 2, \ldots, 6$) of $\mathbf{Q}$. The results are plotted in Fig. 7. As seen, most q_{ii}s appear to approach a constant level after about 225 iterations, but this is not evident with q_{11} and q_{22}. This is due to the fact that robot repeatability is location-dependent. However, the estimation of $\mathbf{Q}$ provides a certain quantitative information on robot repeatability characteristics, which is summarized in Table 3.

5 Concluding Remarks

The recursive estimation allows one to determine an adequate number of measurements for achieving a prespecified calibration accuracy. The recursive formation in conjunction with the adaptive filtering identified 0.5 percent to 1.5 percent errors in kinematic parameters of a given robot. The adaptive filtering also provided quantitative information about robot repeatability characteristics.

Although robot accuracy is limited by robot repeatability, recursive determination of robot repeatability during the course of calibration offers better estimates of kinematic errors than other cases where repeatability characteristics are ignored or assumed.

The laser interferometry measurement system is precise and suitable for continuous path measurement. Although the system was built for a gantry robot experiment, using a microprocessor-controlled laser tracking system will extend the scheme to a wider range of applications.

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