Process Synthesis of Laser Forming by Genetic Algorithm

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Abstract

Significant progress has been made in analyzing and predicting laser forming (LF) processes of sheet metal. Process synthesis in LF, concerned with determination of laser heat condition given a target shape to form, is crucial to make LF practical. This paper reports an effort towards this end using genetic algorithms (GAs). The problem is formulated for a class of 2D target shapes. The effects of control parameters on the GA synthesis process are investigated. The effects of the type of fitness function on achieving multiple objectives are also investigated. The synthesis process is validated through several cases under diverse conditions including one involves close to thirty decision variables.

1. Introduction

In metal forming industry, the primary goals to be met by process design may vary depending upon the need of manufacturers and customers, but forming a sound product in terms of less defects, good dimensional accuracy, and mechanical properties and enhancing production economy tend to receive unprecedented considerations. Efforts have been made to develop efficient methodologies for process optimal design. Some of them have been based on heuristic methods such as genetic algorithms (GAs) because the complexity involved in these processes does not lend themselves to methods such as gradient-based search methods. For instance, Roy, et al. (1996) applied GAs to optimal design of process variables in multi-pass wire drawing.

In laser forming process, sheet metal is progressively bent under a laser heating condition. The sheet metal can be formed into variety of shapes by tailoring the heating condition. Much research has been done to analyze the deformation given a heating condition. For laser forming to become a practical process, however, the issue of process synthesis needs to be addressed, that is, designing the heating condition for a given desired shape. It is impossible to use the gradient-based methods to obtain the solution from the differentiation of an objective function because the design variables (such as laser power, scanning velocity and heat path in particular) cannot be explicitly expressed as a function of the deformation. Shimizu (1997) applied GAs in laser forming of a dome shaped object to determine a heat condition assuming the heat path is known. He used discrete value to represent the heat conditions and a linear, 8-node FEM program to evaluate the fitness function. However, the result is not flexible and natural. It also experienced difficulty when an experimental validation of the result was attempted.

Genetic algorithms mimic the natural evolution process by which more superior creatures evolve while inferior ones fade out from the population as generations go on. GA based optimization techniques have been successfully implemented for a wide range of problems (Goldberg, 1989). GAs have been proved to be a robust, simple to implement method, which can handle a large set of parameters. The disadvantage of GAs includes their computational time, and the semi-empirical nature of the algorithm parameter selection procedure.

In this paper, a genetic algorithm based approach is presented for process synthesis applicable to laser forming of a class of two-dimensional shapes. The synthesis scheme developed in this study has the advantage of treating the number of scans and large set of laser heating conditions as decision variables during a given design cycle. This approach is applied to several cases. The effects of fitness function and control parameters of GAs (population size, crossover rate and mutation rate) on the convergence of the design process are also investigated.

2. Necessity of a Heuristic Approach

Given laser power P, scanning speed V, and beam diameter D; as well as geometric attributes of sheet metal ξ, laser scanning pattern ψ, and material properties ζ, the temperature distribution in the sheet metal being irradiated by a scanning laser beam could be analytically expressed as

$$\Delta T = f(x, y, z, t, P, V, D, \xi, \psi, \zeta) \quad (1)$$

An example is the well-known solution to the moving heating source problem

$$\Delta T = \frac{Q}{2\pi\lambda S_0}\exp\left(\frac{Vx}{2a}\right)K_0\left(\frac{Vr}{2a}\right) \quad (2)$$

where Q, λ, S_0, and a are heat input per unit time, thermal conductivity, sheet metal thickness, and thermal diffusivity; K_0 is 2^{nd} kind Bessel Function of zero order; r is the radial distance from heat source center; and x is the distance from heating source center along scanning direction.

The explicit expression of elastic and plastic deformation as a result of the temperature distribution

$$U_{x,y,z} = g(\Delta T, x, y, z, t, \xi, \zeta) \quad (3)$$

is, however, unobtainable in general. For one thing, this is because ΔT could contain a form of Bessel function (Eq. 2). As a result, $U_{x,y,z}$ is usually obtained numerically using finite element modeling or finite difference modeling (Magee, et al., 1998; Cheng and Yao, 2001).

As a result, analytically or even numerically solving the inverse problem, that is, determining the heat conditions in terms of P, V, D, and ψ given desired deformation is obviously even less feasible. This is why a heuristic approach such as GAs provides a viable alternative.

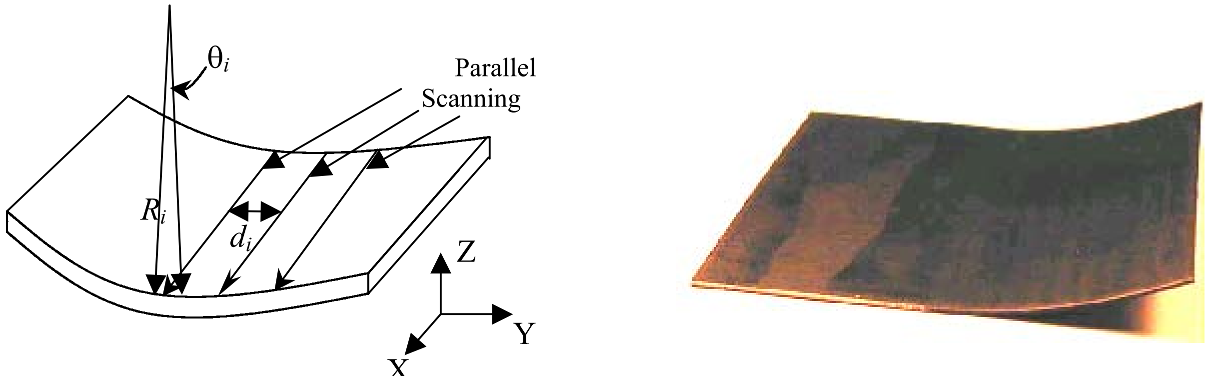
3. Problem Description

In this study, a two-dimensional class of target shapes is used to investigate the feasibility of using GAs for process synthesis. It is chosen to allow relative simplicity without loss of generality. As shown in Fig. 1, the class of target shapes assumes a general 2D profile in the Y-Z plane while the profile remains unchanged in the X direction. It is further assumed that the target shape can be formed by a laser-scanning pattern ξ consisting of straight-lines parallel to each other and to the X-axis.

The decision variables, therefore, are N: number of parallel scan lines; P_i (i=1 to N): laser power at the i^{th} scan; V_i (i=1 to N): laser-scanning velocity at the i^{th} scan; d_i (i=1 to N): distance between the i^{th} and $(i+1)^{th}$ scan lines or the sheet edge. Laser beam size D is assumed constant. This implies that the 2D profile will be either concave or convex. The value of D and sheet thickness S_0 used in this paper promote the temperature gradient mechanism and thus only make concave possible. The decision variables for the 2D laser-forming problem defined above can, therefore, be represented in a matrix. There are $3\times N$ independent decision variables.

$$E = \begin{bmatrix} P_1 & \dots & P_N \\ V_1 & \dots & V_N \\ d_1 & \dots & d_N \end{bmatrix} \quad (4)$$

The range of the decision variables are: P: [200, 900] W, V: [20, 90] mm/s, and $\sum d_i$ (i=1 to N) = sheet width/2. Only half of a sheet is considered assuming symmetry.



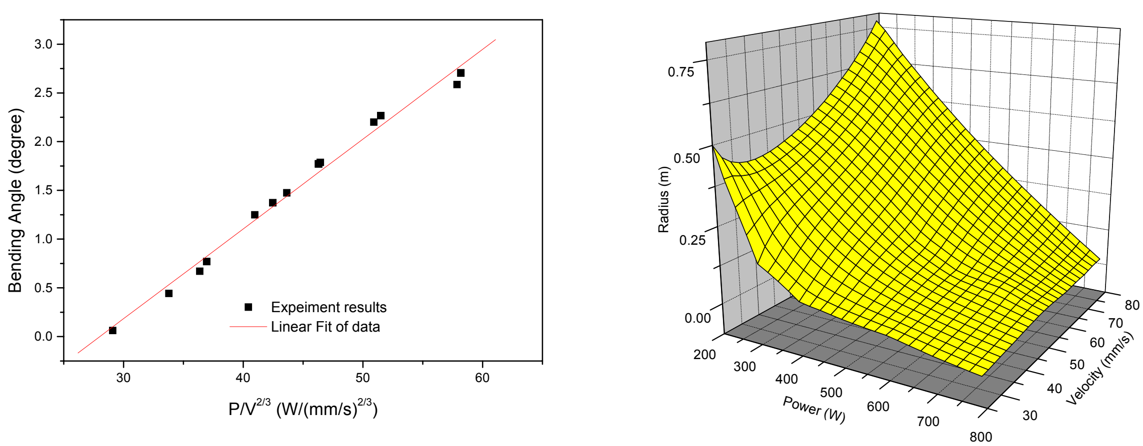
(a) (b)

Figure 1. (a) A class of 2D shapes and parallel scanning pattern; and (b) sheet metal (mild steel) after being laser formed using the scanning pattern in (a).

Let R_i and θ_i denote the radius of curvature and bending angle as the result of the i^{th} scan, respectively (Fig. 1). It is also assumed that $d_i \geq R_i\theta_i$, that is, adjacent scan lines don't overlap such that R_i and θ_i are determined solely by the i^{th} scan. The length of the straight portion between arc $R_i\theta_i$ and adjacent arc $R_{i+1}\theta_{i+1}$ is donated as $h_i = d_i - R_i\theta_i$. After the i^{th} scan, coordinates (Y_i, Z_i) of the partially formed 2D profile can be expressed as

$$Y_i = Y_{i-1} + y_i \cos\left(\sum_{k=1}^{i-1}\theta_k\right) - z_i \sin\left(\sum_{k=1}^{i-1}\theta_k\right), Z_i = Z_{i-1} + y_i \sin\left(\sum_{k=1}^{i-1}\theta_k\right) - z_i \cos\left(\sum_{k=1}^{i-1}\theta_k\right) \quad (5)$$

where $y_i = R_i \sin(\theta)$, and $z_i = R_i(1-\cos(\theta))$ for the arc section $R_i\theta_I$, and $\theta \in (0, \theta_i)$, and $y_i = R_i \sin(\theta_i) + h\cos(\theta_i)$, and $z_i = R_i(1-\cos(\theta_i)) + h\sin(\theta_i)$ for the straight segment h_i and $h \in (0, d_i - R_i\theta_i)$.



(a) (b)

Figure 2. (a) Bending angle and (b) radius of curvature vs. power/velocity, based on experimental data of 1010 steel coupons of 80 by 80 by 0.89 *mm* with laser beam size of 4 *mm*.

The above equations will be used to express the formed shapes in the fitness function, which is part of the GAs to be explained in the Section 5.2. Additionally, in the process of applying the GAs, it is necessary to determine θ_i and R_i for candidate design variables P_i and V_i. While this could be achieved via the well-established, full-blown FEM modeling, a more feasible approach is to use empirical relationships based on readily available experimental data since the number of such determination is large as GAs evaluate many candidate values of the design variables. Shown in Fig. 2 (a) and (b) are bending angle and radius of curvature vs. power and velocity, respectively, based on experimental data of 1010 steel coupons of 80 by 80 by 0.89 *mm* with beam size of 4 *mm*, the same condition to be used in the paper. As seen from Fig. 2 (a), bending angle correlates linearly with $P/V^{2/3}$. This result may somewhat differ from others (Masubuchi 1992; and Vollertsen, 1993) but it conveniently serves the purpose of this paper since the conditions are the same as the ones being considered in this paper.

4. Genetic Algorithms

GAs are a class of stochastic search methods that mimic the metaphor of natural biological evolution. GAs operate on a population of potential solutions applying the principle of survival of the fittest to produce better and better approximations to a solution, just as in natural adaptation. The basis steps of implementing GAs can be summarized as follows (Fig. 3). First, generate a random population of *n* chromosomes; secondly, evaluate the fitness of each chromosome in the population and create a new population by repeating following steps until the new population is complete: a) select parent chromosomes from a population according to their fitness (the better fitness, the bigger chance to be selected; b) with a crossover probability cross over the parents to form a new offspring; c) with a mutation probability mutate the new offspring at each locus (position in chromosome); d) place new offspring in a new population; e) use newly generated population for a further run of algorithm; f) if the end condition is satisfied, stop, and return the best solution in current population.

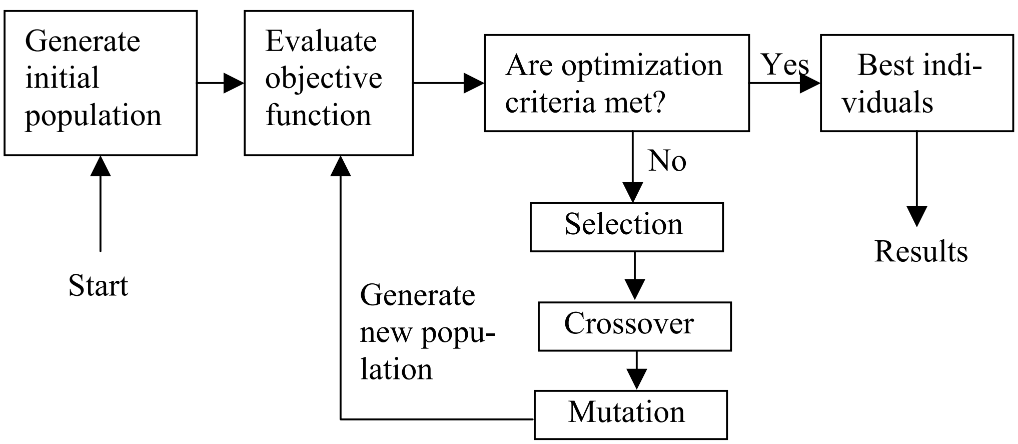


Figure 3. The basis steps of implementation of GAs.

How to choose the control parameters (such as population size, crossover rate, mutation rate, representation of decision parameters and others) is important to the performance of GAs. There have been studies investigating the interactions among different GA parameters for successful application of GAs (Goldberg, et al., 1992a) and control maps for operator probabilities (Goldberg, et al., 1992b; Thierens and Goldberg, 1993). More importantly, their interactions are largely dependent on the function being optimized.

Population size is the first parameter to choose. Generally, a highly undulating cost surface should have a larger population than a smooth cost surface. A small population size causes the genetic algorithm to quickly converge to a local optimum, while a large population size takes too long to find and assemble the building blocks to the optimum solution. The most effective population size is dependent on the problem being solved, the representation used, and the operator manipulating the parameters (Syswerda, 1991). Mühlenbein (1992) modeled the GAs based on Markov chain analysis. He calculated the transition probability of moving to the optimal string from any string and then estimates the expected transition time. He showed that this transition probability reduces with population size, giving rise to the concept of a minimum population size below which GAs are not expected to work. Goldberg, et al., (1992a) showed that a population size (N_p) is necessary to trigger correct building block processing based on correct schema processing:

$$N_p = 2ck\sigma_0^2 / f_0^2 \tag{6}$$

where f_0 and σ_0 are mean and variance of the fitness values respectively; k is the number of competing schema; and the factor c varies with error rate α as $\alpha = \exp(-c/2)/\sqrt{2\pi c}$.

Crossover operator is a constructive process, which can combine good partial solutions together to form the optimal solution. Crossover rate (C) determines the number of chromosomes that enter the mating pool. High crossover rates introduce many new chromosomes into the population. If crossover rate is too high, good building blocks don't have the opportunity to accumulate within a single chromosome. A low crossover rate, on the other hand, doesn't do much exploring of the cost surface. By mutation operator, offspring variables are mutated by small random values. Bäck (1993) reported that for unimodal functions a mutation rate of $1/N_p$ was the best choice. This value may still valid when fitness function is multimodal. Generally, with small mutation rate, the GAs is easy to converge to local optima, while large mutation rate destroy the already-found building blocks. In practice, these control parameters are often determined semi-empirically on a case-by-case basis in order to minimize the number of evaluations required, which is the product of population size and the generations that take to converge.

5. Results and Discussions

As explained in Section 3, the sheet dimension is 80×80×0.89 *mm*, and the laser beam diameter is 4 *mm*. Only half of the sheet is scanned (Fig. 1 (b)). The class of 2D target shapes is characterized by a profile on the *Y-Z* plane and the profile remains unchanged in the *X* direction (Fig. 1 (a)). The 2D target profiles investigated in the paper include a circular profile of constant radius of 0.1 *m* and a parabolic profile $z = 8y^2$. The scanning pattern is straight-lines and parallel to each other and parallel to the *X*-axis.

5.1 Effects of GA parameters

To investigate the effects of selection of GA parameters, the following case is considered. The target shape is the circular profile mentioned above. Due to its constant radius of curvature, $P_i=P$ and $d_i = d$ = half of the sheet width/N. The scanning velocity is set as an invariable constant, $V_i = V = 50$ *mm/s*, and P and N and the two design variables to determine.

First, the crossover rate, C, and mutation rate, M, are chosen as 0.8 and 0.05, respectively, while the population size varies from 5 to 500. Figure 4 (a) shows the development of the fitness with generation for several population sizes. The result shows that the number of generations to achieve convergence decrease with increasing the population size. The minimum population size below which GAs are not expected to work (Eq. 6) is determined as 17. The mean and variance of the fitness values $\overline{f_0} = 0.26$, and $\overline{\sigma_0} = 0.59$ were obtained by sampling 1000 chromosomes randomly generated. k=1, and c=1.67 (with 90% confidence) were used. Fig. 4 (c) shows that GAs with about N=*20* work the best. The larger the population size, fewer generations it takes to converge but more evaluations to perform within each generation.

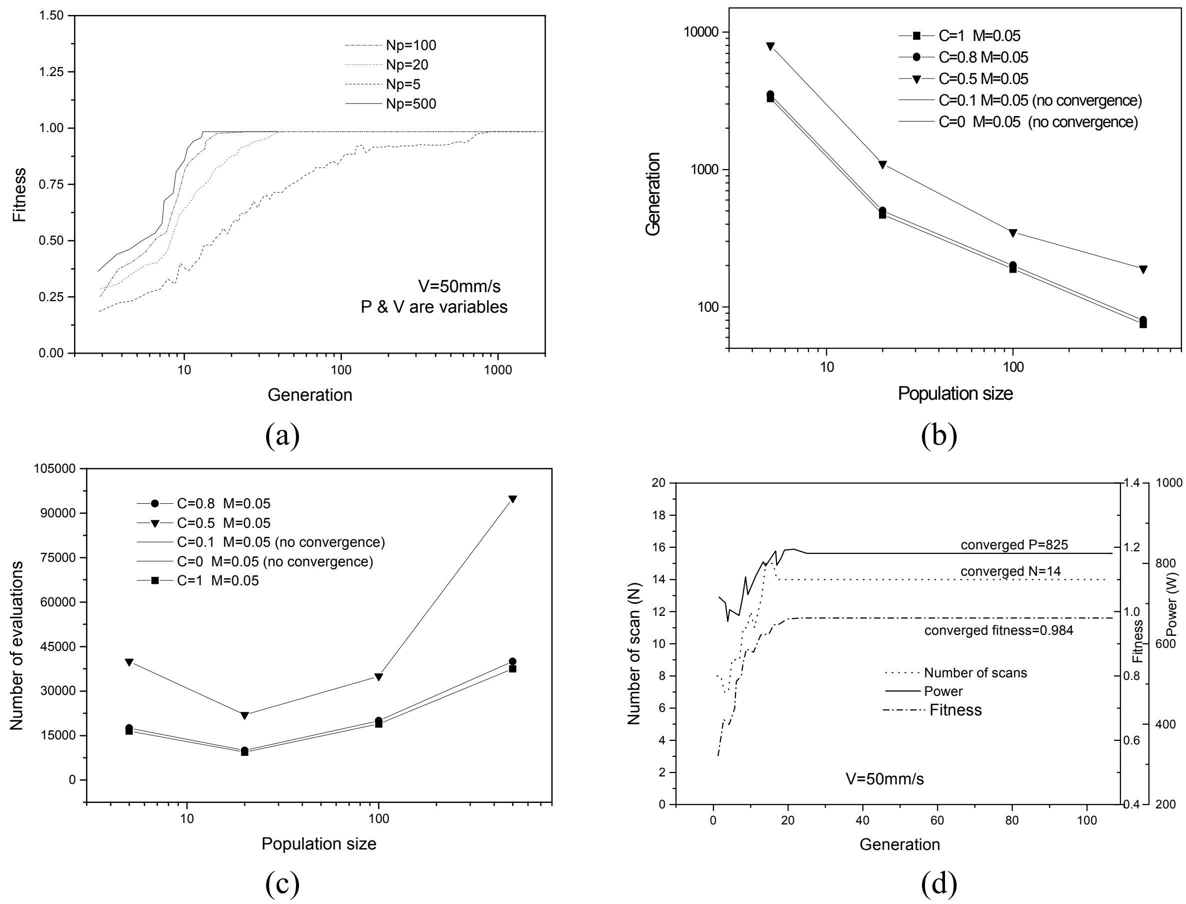


Figure 4. Effect of population size N_p on (a) fitness convergence, (b) number of generations, and (c) number of evaluations (=N_p*generations); (d) Convergence history of Case 1 (N_p=20, crossover rate C=0.8, and mutation rate C=0.05).

Fig. 4 (b) & (c) show the effect of crossover rate on the convergence at different population sizes. Mutation rate is set as 0.05. When a small crossover rate (C=0, 0.1 or 0.5) is used, the Gas' performance degrades dramatically. The number of evaluations when crossover rate is 0.5 is much larger than that when crossover rate is large (C=0.8 or 1) and the GA does not converge when crossover rate is 0 or 0.1. The performance of GAs falls with large population sizes due to smaller number of allowed generations, and the number of evaluations is large when population size is small because the convergence speed is small. Similar approach was applied to investigate the effect of mutation rate on the convergence. Crossover rate is set as 0.8 and mutation rate varies from 0 to1. The optimal mutation rate is about 0.05. This is consistent with the results from Bäck , (1993). These results show that if the problem is complex as in laser forming, GAs require a suitable population size to solve the problem. In this case, the best set of control parameter is population size about 20, high crossover rate of 0.8, and low mutation rate of 0.05.

With these control parameters, the GA optimization is implemented in four steps. (1) An initial set of 20 chromosomes is randomly generated within a range. Each chromosome represents a heat condition in terms of $3\times N$ decision variables as described in Section 3. For this case, there are only two decision variables P and N. The range for P is chosen as 200 to 900 W based on the prior knowledge of the deformation/power relationship and the desired shape. The range for N is chosen as 5 to 15 based on the fact that no significant overlapping of scan lines is desired, and the sheet metal half width and laser beam size D are 40 *mm* and 4 *mm*, respectively. (2) The fitness of the 20 chromosomes is evaluated using the fitness function shown in Eq. 9 (to be described in the subsequent section) and 10 chromosomes having the highest fitness values are chosen. Eight of these 10 chromosomes (C = 0.8) crossover with other 8 out of 10 randomly generated chromosomes to generate new chromosomes according to

$$E_{ij}^{new} = r_c E_{ij}^1 + (1 - r_c) E_{ij}^2 \tag{7}$$

where E_{ij}^1 and E_{ij}^2 are one of the 8 best chromosomes and one of the 8 randomly generated chromosomes, respectively (also see Eq. 4); and r_c is a random number from a (0,1) uniform distribution. (3) The new set of 20 chromosomes (16 crossovered ones and 4 un-crossovred ones) are then mutated according to

$$E_{ij}^{mutate} = U(a_i, b_i) \tag{8}$$

where a_i and b_i are the range of the i^{th} decision variable. $U(a_i, b_i)$ is a randomly generated value from (a_i, b_i) uniform distribution. (4) The new set of offsprings after selection, crossover and mutation operators, are chosen for the next generation. The steps 2 and 3 are repeated until a global convergence measure is achieved. The global convergence is assumed if no improvement in the fitness value takes place after a fixed number of trials, which is set as 20 in this study. With these steps, convergence to P = 825 *W* and N = 14 was achieved in about 20 generations with a fitness value of 0.984 (Fig. 4(d)) in the above case with circular profile as target shape (radius=0.1m) and V is set as 50mm/s.

5.2. Fitness function

The selection of fitness function has to be consistent with the nature of the problem at hand. In this study, the most concerned issues are geometry accuracy, total forming time, and energy consumption. Based on these considerations, the following two fitness functions are formulated.

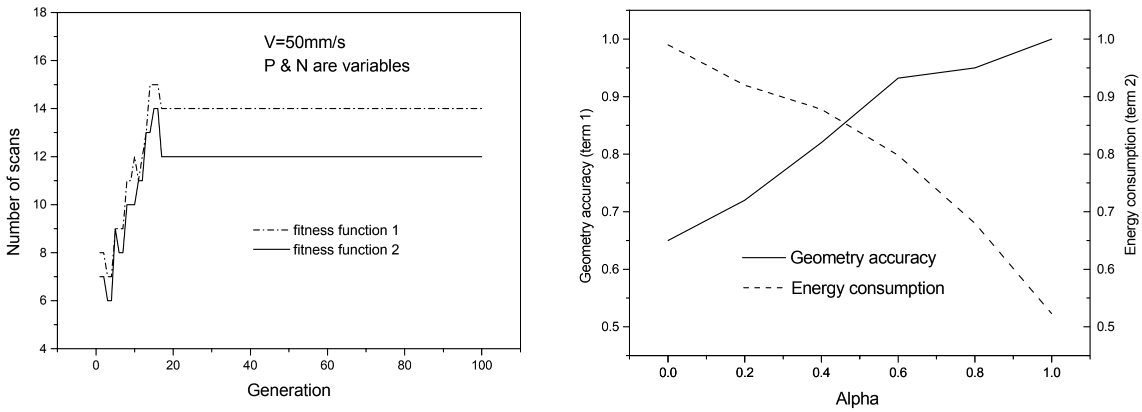
$$f_1 = \left(1 \Big/ \left(1 + \int_0^l \frac{|f(y) - f_0(y)| dy}{l S_0}\right) \right)^N \tag{9}$$

where $f(y)$ represents the formed shape function, $f_0(y)$ is the target shape function; S_0 is the sheet thickness and l the sheet length; and N is the number of scans. As seen, the optimal fitness value is 1. The fitness value decreases as the number of scans increases and the shape difference increases. A large number of N favors geometry accuracy but consumes more time.

$$f_2 = \alpha\, 1 \Big/ \left(1 + \int_0^l \frac{|f(y) - f_0(y)| dy}{l S_0}\right) + \beta \left[\sum_{i=1}^{N} \frac{P_i l_i}{V_i} \right]_{\min} \Big/ \sum_{i=1}^{N} \frac{P_i l_i}{V_i} \tag{10}$$

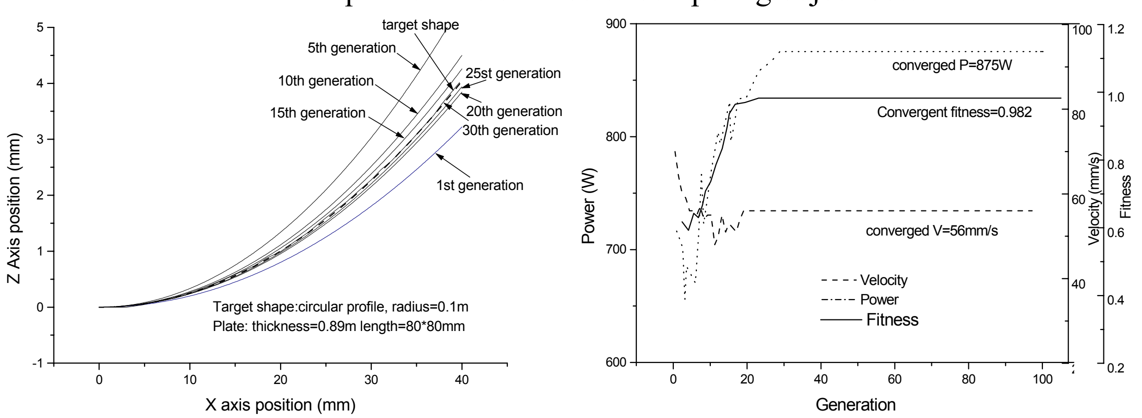
P_i, V_i, and l_i are power, scanning velocity and length of the i^{th} scanning path, respectively. $P_i l_i / V_i$ is the energy consumption of the i^{th} scan. $\left[\sum_{i=1}^{N} \frac{P_i l_i}{V_i} \right]_{\min}$ is the minimal total energy consumption among all the strings in a generation. The first item of this fitness function measures the fitness associated with geometry accuracy weighted by α; and the second item energy consumption weighted by β.

To compare the effect of optimization results using these two fitness functions, f_2 was applied to the same case as dealt with in Section 5.1. In Section 5.1, the result shown in Fig. 4 (d) was obtained using the fitness function f_1. Fig. 5 (a) compares the development of the number of scans with increase of generations using the two fitness functions. In the fitness function, f_2, α and β are both set as 0.5. The results show that N converged to 14 using f_1 (the same as in Fig. 4 (d), and converged to10 using f_2. The corresponding P converged to 825 W (Fig. 4 (d)) and 846 W (not shown), respectively. The converged fitness f_1 and f_2 are 0.994 (Fig. 4 (d)) and 0.84 (not shown), respectively. The result under f_2 naturally varies with the choice of α and β (= 1-α) value. Fig. 5 (b) shows the effect of α on the balance between the fitness associated with geometry accuracy (the first term in Eq. 10) and energy consumption (the second term in Eq. 10). The fitness function, therefore, should be carefully chosen to have results that best reflect the designers' priorities.



(a) (b)

Figure 5. (a) Comparisons of two fitness functions; and (b) effect of weighting coefficient α in Eq. 10 on the balance of competing objectives.



(a) (b)

Figure 6. Convergence history of (a) decision variables P and V, and fitness; and (b) shape towards the target shape (Case 2: number of scans N=10, and d_i=d=40/N=4 mm).

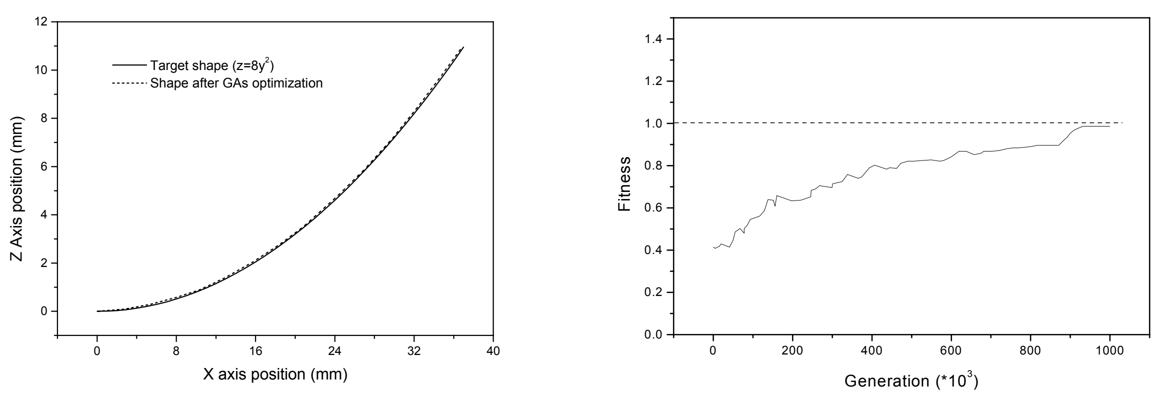
5.3 Two more case studies

To test the algorithm's applicability under other conditions, two more case studies were conducted. Case 2 is the same as the previous case 1 except velocity is made as variable, V_i=V and number of scans N is made constant equal to 10. Therefore, there are two decision variables P and V. d_i = half sheet width/N = 40/10 = 4 mm. The target shape remains as a circular profile with diameter of 0.1 m in the Y-Z plane (Fig. 6 (b)). Fig. 6 (a) shows the development of the decision variables, P and V, as well as fitness value during the GA optimization. Fig. 6 (b) shows the shape evolution during the optimization process of GA. The result shows that when the number of generations reaches about 30, the shape from GAs optimization is very close the target shape, and P and V converged to optimal values of 875 W and 56 mm/s, respectively. Fig. 6 (a) also shows that the fitness of the converged results is close to optimal, 0.982.

The third case assumes a parabolic target shape of $z = 8y^2$, whose radius of curvature increases with the X position (Fig. 7 (a)). As a result, the intervals between scans, d_i, and powers of each scan, P_i, need to be variables. The number of scans, N, is set to 15, and the scanning velocity of each scan are set as $V_i=V=50$ mm/s. The number of decision variables is 29, that is P_i and d_i (i=1 to 15) but summation of all d_i's equals to the half sheet width of 40 mm. Table 1 lists the design results from GA optimization. As seen, d_i increases and P_i decreases with N in order to approach the varying curvature of the target shape. Fig. 7 (a) shows a good agreement between the shape from GA optimization and the target shape. Figure 7 (b) shows that the fitness of the converged results is close to optimal, 0.978 but only after a much larger number of generations than in Cases 1 and 2, simply because of the larger number of decision variables in this case. This case shows that when the profile of the target shape is more complex and the number of decision variables is large, the algorithm converged but with a large time premium.

Table 1. Results from GAs optimization (Case 3)

No.	d_i (mm)	P_i (W)
1	2.67	682.8
2	2.67	679.2
3	2.68	675.8
4	2.69	672.6
5	2.72	670.6
6	2.74	668.8
7	2.67	666.9
8	2.8	665.2
9	2.84	663.8
10	2.88	662.5
11	2.92	661.4
12	2.97	660.3
13	3.02	659.0
14	3.08	657.8
15	0.65	656.1



(a) (b)

Figure 7. Convergence of (a) shape and (b) fitness (Case 3: 29 decision variables, and $N=15$).

5. Conclusions

In this paper, a synthesis process for laser forming of sheet metal based on Genetic Algorithms (GAs) is presented. It is shown that convergence to target shapes of constant and variable curvatures was achieved with good accuracy. Investigations showed that algorithm control parameters and fitness function type have significant effects on the GA synthesis results. It is shown that a proper form of fitness function is important to balance among competing objectives, such as geometrical accuracy, forming time, and energy consumption. It is also shown when the number of decision variables was close to 30, it took a large number of generations to achieve convergence.

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