

# PROCESS ANALYSIS AND SYNTHESIS OF LASER FORMING OF VARYING THICKNESS PLATE

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## KEYWORDS

Laser Forming, Varying Thickness, Process Analysis, Process Synthesis

## ABSTRACT

In order to apply laser forming (LF) process to complex structures such as compressor airfoils that are 3D geometries with large thickness variation, the analysis and process synthesis of LF varying thickness plate are conducted in this paper. In this paper, the bending mechanism and parametric effect on the deformation characteristics of varying thickness plate are investigated. An analytical model was proposed to predict the bending deformation. A strategy of LF process synthesis of varying thickness plate is also presented. The heating paths are determined by weighted averaging the in-plane strain and bending strain. A thickness-dependent database is established to determine the heating conditions. The analysis and the synthesis are validated by numerical simulation and experiments.

## INTRODUCTION

Laser forming (LF) is a non-traditional forming process that does not require hard tooling or external forces and hence, dramatically increases

process flexibility and reduces the cost of forming process. In order to advance the laser forming process further for realistic forming application, the understanding and application of laser forming of varying thickness plate need to be conducted.

Geometric effect, especially thickness effect, plays an important role in laser forming. Vollertsen (1994) proposed that the bending deformation is inversely proportional to the square of the plate thickness. Cheng and Yao (2004a) also proposed an analytical model that can predict the bending variation due to the change of sheet size (width and length). However, all the above analysis considered the thickness of plate as uniform and cannot be applied to the varying thickness plate.

In another aspect, the methodology of process design has been improved from simple empirical methods to the physics-based methods. However, no general process-planning scheme for general curved shapes is available due to the computational complexity of the inverse problem. Ueda et al. (1994) determined the heating paths based on the distribution of inherent strains obtained by FEM. Cheng and Yao (2004b), Liu and Yao (2003) developed strategies for process design of laser forming of doubly curved thin plates by considering both of the

in-plane and bending strains. However, their methods only concerned the uniform thickness plate, neglecting the effect of thickness on bending strain through the plate. In fact, whether a desired shape can be precisely formed or not depends on not only the shape curvature, but also its thickness.

In process analysis, the characteristics of bending deformation, and the evolution of bending variation under various scanning speeds and beam spot sizes are studied in detail. Rosenthal's solution to the moving point heat source is also applied to characterize the effect of varying thickness on the temperature field. In process synthesis, the typical desired shapes formed by varying thickness plate are used to investigate the characteristics of in-plane strain and bending strain. The heating paths are determined by weighted averaging the in-plane strain and bending strain. A thickness-dependent database is established by FEM and used to determine the heating conditions. The methodology is validated by laser forming experiments.

## PROCESS ANALYSIS

### Experiments and Simulation

The experiment and simulation was carried on in linearly varying thickness plates of AISI1010, with the thickness varies linearly from 1 mm in the edge to 3 mm in the center of the plate (shown in Fig.1). The simplified shape was chosen because the authors want to clarify the effect of laser forming process on varying thickness without inducing other complexity and at the same time without loss of the generality. The laser system is PRC-1500 CO<sub>2</sub> laser, which has a maximum output power of 1.5 kW. The laser operates in continuous wave (CW) mode and the power density distribution of the laser beam follows a Gaussian function (TEM<sub>00</sub>). The irradiated surface is coated with graphite to enhance the absorption of heat input.

The simulation was carried out similar to what was described in Cheng and Yao (2004a). In the simulation, material properties such as the modulus of elasticity, heat transfer properties, thermal conductivity, specific heat, and flow stress are all temperature dependent. A strain-hardening coefficient, which is also temperature dependent, is defined in order to consider strain

hardening of the material. No melting is involved in the forming process.

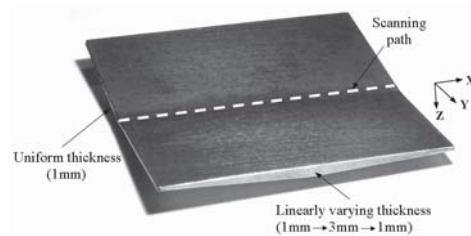


FIGURE 1. LASER FORMED VARYING THICKNESS PLATE SHOWING COORDINATE SYSTEM

## Results and Discussions

**Deformation Characteristics.** Fig. 2 shows the bending angle of varying thickness plate and equivalent-uniform thickness ( $h = 2\text{mm}$ ) plate under the condition of  $P=1000\text{W}$ ,  $V=20\text{mm/s}$  and spot size= $8\text{mm}$ . Compared with uniform-thickness plate, the variation of bending angle is more obvious. The nonuniformity of deformation is not only due to the variation of bending rigidity, but also due to the variation of temperature field. Fig. 3 shows comparison of peak temperature distribution between varying thickness plate and uniform thickness plate. For both cases, the peak temperature is lower at the entrance and higher at the exit of the scanning path due to the edge effect. For uniform thickness plate, the peak temperature is constant along the scanning path except for the two ends. For varying thickness plate, the peak temperature on both the scanned and opposite (unscanned) surface drops with increasing thickness due to an increasing heat sink effect. It is seen that the peak temperature drop on the unscanned surface is much larger than that of the scanned surface, and thus the temperature gradient through the thickness direction increases with the increasing plate thickness (see Fig 3).

From the bending variation shown in Fig.2, it is also seen that the bending angle increases first then decreases from the ends to the center of the scanning line. The reason is that from the ends to the center, the temperature gradient through the thickness increases. At the locations near the ends, the bending rigidity is smaller because of the thinner thickness. Therefore, the bending angle will increase with the increment of temperature gradient. If the thickness keeps increasing, the effect of bending rigidity becomes

more dominant so that the increment of bending rigidity will make the bending angle decrease. From the distribution of the generated plastic strain (Fig.4), it is clearly seen that the bending deformation increases first then decreases from the ends to the center, which agrees well with the analysis from the temperature field.

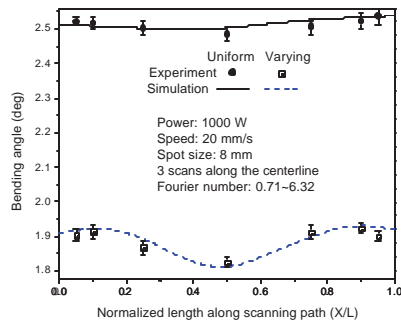


FIGURE 2. BENDING ANGLE VARIATION ALONG SCANNING PATH

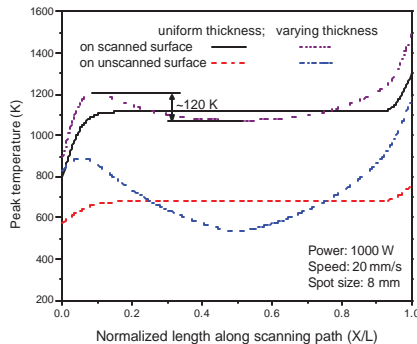


FIGURE 3. COMPARISON OF THE TEMPERATURE DISTRIBUTION ALONG SCANNING PATH

If the modified Fourier number ( $F_0 = \alpha d / (h^2 v)$ , where  $\alpha$  is thermal diffusivity,  $d$  the beam diameter,  $h$  the plate thickness and  $v$  the scanning speed) is used to approximately estimate the laser forming mechanism, it will be found that  $F_0$  varies from 6.3 to 0.7 when the thickness varies from 1 mm to 3 mm. This means there is a transition of mechanism along the scanning line. From the temperature distribution, it is already found that there is no steep temperature gradient through the thickness at the locations with thinner thickness. It is also found that at the ends of the varying thickness plate, much more plastic strain in the thickness direction occurs, which means the thickness increment at the thinner region is much larger than that of the thicker region. Therefore, it can be concluded that the upsetting mechanism (UM)

dominates at the ends with thinner thickness and the temperature gradient mechanism (TGM) dominate at the center with thicker thickness.

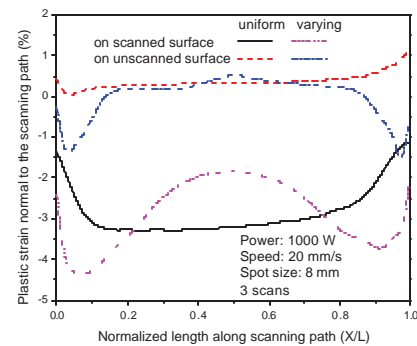


FIGURE 4. DISTRIBUTION OF PLASTIC STRAIN

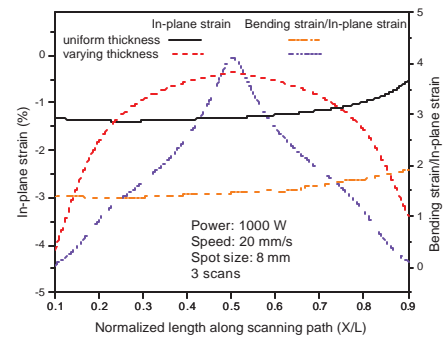


FIGURE 5. DISTRIBUTION OF IN-PLANE STRAIN AND STRAIN RATIO

The strain in the middle surface can be defined as in-plane strain which expands the middle surface. The difference of strain between the scanned surface and the middle surface is defined as bending strain which bends the plate towards the scanned surface. Fig.5 shows the distribution of in-plane strain and the ratio of bending strain to in-plane strain between uniform and varying thickness plate. It is seen that the strain ratio increases with the thickness, which means the effect of bending strain plays more important role in bending the thicker regions of the plate. This is consistent with the previous analysis that TGM is dominant in the thicker regions.

**Parametric Study.** Fig.6 shows the effect of different scanning speed. In general, the average bending angle decreases with the scanning speed. The reason is due to the decrease of effective heat input with the increasing scanning speed, so that both the peak temperature and temperature gradient through the thickness de-

crease. It also shows that the bending variations decrease with the increasing scanning speeds. The reason is the difference of temperature gradient between thicker locations and thinner locations also decreases with scanning speed. With speed increasing, the difference of Fourier number ( $F_0 = \alpha d / (h^2 v)$ ) decreases and the laser forming mechanism is more uniform to TGM along the scanning path. It also helps to achieve more uniform bending deformations in the varying thickness plate.

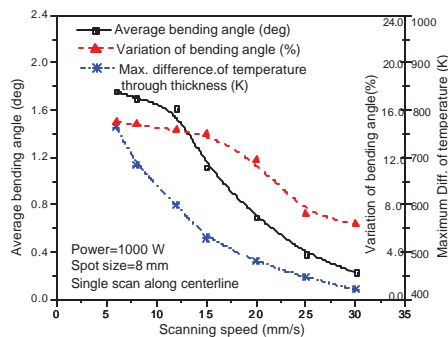


FIGURE 6. BENDING ANGLE AND ITS VARIATION WITH VARIOUS SCANNING SPEED

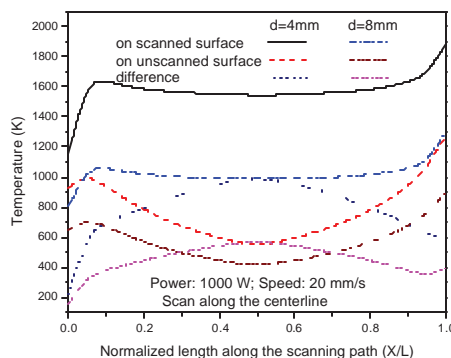


FIGURE 7. TEMPERATURE FIELD UNDER VARIOUS BEAM SPOT SIZES

It is also found that the variation of bending angle increases with the increasing spot size. From Fig. 7, it is found that both the peak temperature and temperature gradient through the thickness decreases with the increasing beam spot size. The temperature gradient through the thickness in the thicker locations decreases much more than that of the thinner locations. For the thicker locations where the laser forming mechanism is primarily in TGM, the larger decrease of temperature gradient causes the bending deformation to drop much more than that of the thinner locations. Since the bending

angle in thicker locations is already smaller than that of the thinner locations due to the larger bending rigidity, the variations of bending angle will increase with beam spot size.

### Further Analysis

To characterize the effect of varying thickness on the temperature field, Rosenthal's solution of moving heat source is applied to the varying thickness plate. The plate can be divided into a number of segments along the scanning directions and the thickness of each segment is  $h(x)$ . Assuming that there is a moving point heat source of  $q$  heat units per unit time along  $x$  direction with velocity of  $v$ , and there is no heat loss from the plate surface, the corresponding temperature  $\theta(x, y, z)$  is determined as

$$\theta = \frac{q}{2\pi K h} \left\{ K_0 \left( \frac{vr}{2\alpha} \right) + 2 \sum_{n=1}^{\infty} K_0 \left[ \frac{vr}{2\alpha} \left( 1 + \frac{4\pi^2 \alpha^2 n^2}{v^2 h^2} \right)^{1/2} \right] \cos \frac{n\pi z}{h} \right\} e^{-\frac{v\xi}{2\alpha}} \quad (1)$$

where  $\xi = x - vt$ ,  $r = \sqrt{\xi^2 + y^2 + z^2}$ ,  $K$  is the thermal conductivity,  $K_0$  the zero order modified Bessel function,  $\alpha$  the diffusivity.

Figure 8 shows the typical temperature field in the cross-section along the scanning path in the varying thickness plate obtained by FEM and the analytical model, respectively. Due to the quasi-static assumption, the temperature gradient in analytical model is a little smaller than that of the FEM results. The magnitude of temperature field in analytical model is a little higher than numerical values due to the neglecting of heat loss. By the analytical model, the effect of varying thickness on temperature field can be clearly illustrated: with the thickness increasing, the peak temperature drops down but the temperature difference between the scanned and unscanned surfaces increases.

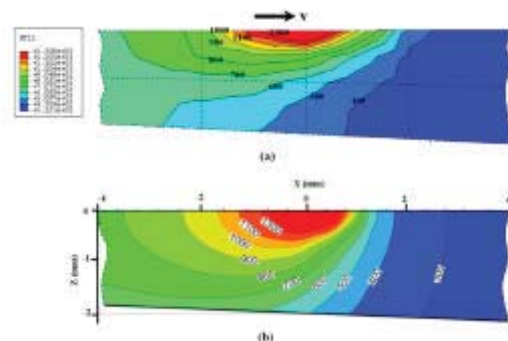


FIGURE 8. TEMPERATURE FIELD AT THE CROSS SECTION ALONG THE SCANNING PATH (A) BY FEM (B) BY ANALYTICAL MODEL

## PROCESS SYNTHESIS

In the laser forming process analysis of varying thickness plate, it is found that the strain distribution varies due to the variation of heat sink and the bending rigidity. Bending mechanism may also change with varying thickness between the buckling mechanism (UM) and the temperature gradient mechanism (TGM), and the bending strain plays more important role in the thicker locations. Therefore, for the process synthesis of varying thickness plate, the scanning paths should be determined by considering the variation of bending-strain effect. The heating conditions should be determined from a thickness-dependent heating-condition database.

### Problem Description

The process design of laser forming on varying thickness plate differs from the process applied on uniform thickness plate. Not only the shape curvature, but also the thickness variations need to be considered in determining the scanning path. The strain or strain ratio is not only the function of power and scanning velocity if the beam spot size is given, but also the function of thickness. In varying thickness plates, even under the same heating condition (laser power, scanning velocity and beam spot size) the bending mechanism may vary along the scanning path, therefore the effect of different type of strain will vary along the scanning path.

An overall strategy for process design of laser forming on varying thickness plates is presented which involves three steps. The first step is to determine the required strain field from planar shape to desired shape and then decomposed it into in-plane and bending strain components. The second step is to determine the scanning paths by locate the path perpendicular to the weighted averaged-strain. From the part of process analysis, it is found that the bending mechanism may vary with the thickness change and the effect of bending strain plays more important role in the thicker locations. Therefore, the bending strain needs to carry more weight in the thicker locations because under TGM bending strain is more significant to achieve the deformation. Both the shape curvature and the thickness should be considered in the weight number. The third step is to determine the heating conditions from the thickness-dependent databases established by laser forming simulation.

By matching the strain and strain ratio between the required values and those of database, the laser power and scanning speed can be determined uniquely if the beam spot size is assumed constant. Figure 9 summarizes the overall strategy for these steps.

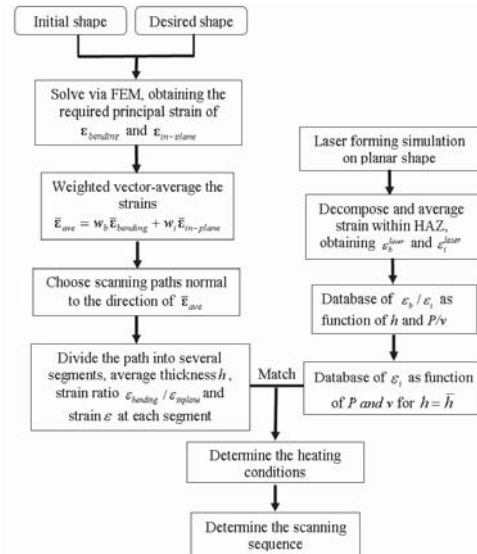


FIGURE 9. PROCESS DESIGN PROCEDURE FOR LF OF VARYING THICKNESS PLATE

The experiment and simulation were carried on in two typical shapes (shown in Fig.10). One is a singly curved shape with thickness varies linearly from one side of 1 mm to the other side of 3 mm. Another is a doubly curved shape (pillow) with thickness varies quadratically from the edge of 1 mm to the center of 3 mm. In the present study, although the plate thickness is not uniform, the plate is still in the range of thin plate, which is defined as the ratio of typical planar dimension of the plate and plate thickness ( $10 < w/h < 100$ ). Large deflection is considered so that stretching may exist in the middle surface.

The middle surface of the desired shapes is defined as the surface which has the same distance to both top and bottom surfaces. It can be specified by  $S(x,y)=A(x)+B(y)$ , where  $A(x)$  and  $B(y)$  are cubic-spline function,  $x, y \in [0,80]$ . For the parabolic shape,  $B(y)$  is a straight line with two ends:  $(-40,40,0)$  and  $(40,40,0)$ . For the pillow shape,  $B(y)$  is a cubic-spline defined by the three points:  $(-40,-40,0)$ ,  $(-40,0,3.5)$  and  $(-40,40,0)$ . For the two shapes,  $A(x)$  is same and defined by  $(-40,0,0)$ ,  $(0,0,5)$  and  $(40,0,0)$ .



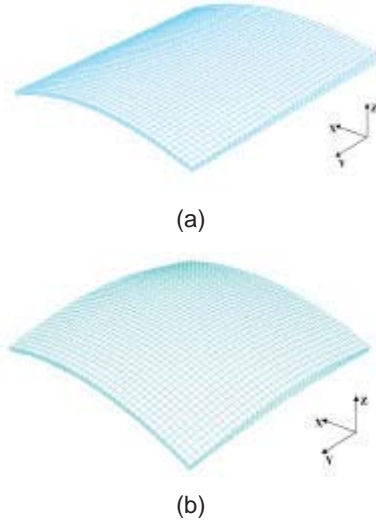


FIGURE 10. DESIRED SHAPES OF (A) SINGLY CURVED SHAPE AND (B) DOUBLY CURVED SHAPE, BOTH WITH VARYING THICKNESS

### Strain Field Determination

The strains can be expressed as

$$\begin{aligned}\epsilon_{xx} &= \frac{\partial u_0}{\partial x} + \frac{1}{2} \left( \frac{\partial w_0}{\partial x} \right)^2 - z \frac{\partial^2 w_0}{\partial x^2} = \epsilon_{xx}^I + \epsilon_{xx}^B \\ \epsilon_{yy} &= \frac{\partial v_0}{\partial y} + \frac{1}{2} \left( \frac{\partial w_0}{\partial y} \right)^2 - z \frac{\partial^2 w_0}{\partial y^2} = \epsilon_{yy}^I + \epsilon_{yy}^B \\ \epsilon_{xy} &= \frac{1}{2} \left( \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x} \frac{\partial w_0}{\partial y} \right) - z \frac{\partial^2 w_0}{\partial x \partial y} = \epsilon_{xy}^I + \epsilon_{xy}^B\end{aligned}\quad (2)$$

where  $u_0$ ,  $v_0$ , and  $w_0$  are displacements of points at middle plane.  $\epsilon^I$  and  $\epsilon^B$  are defined as in-plane strain and bending strain, respectively.

Since the bending strain is equal to a product of position in the thickness direction  $z$  and the curvature at that point, the distribution of bending strain through the thickness is linearly. While the in-plane strain is uniform through the thickness, therefore, the strain field in the middle surface is the required in-plane strain. The difference of the strain field between the top surface and the middle surface is the maximum bending strain for the required shape, expressed by

$$\mathbf{E}_B = \mathbf{E} - \mathbf{E}_I \quad (3)$$

where  $\mathbf{E}$  is the 3x3 strain tensor at the top surface and  $\mathbf{E}_B$  is the 3x3 strain tensor in the middle surface. Then the corresponding principal strains  $\mathbf{e}_B$  and  $\mathbf{e}_I$  can be calculated.

Figure 11 shows the bending strain of the singly curved shape. For the singly curved surface, in-plane strain is almost zero. The bending strain increases in Y-direction due to the increment of thickness along that direction. From the in-plane strain and bending strain of the doubly curved shape (pillow) with varying thickness, it is found that the in-plane strain does not change when the thickness varies, but the bending strain increases with the increasing thickness. To fulfill the requirement of both in-plane and bending strain, the effect of thickness must be considered.

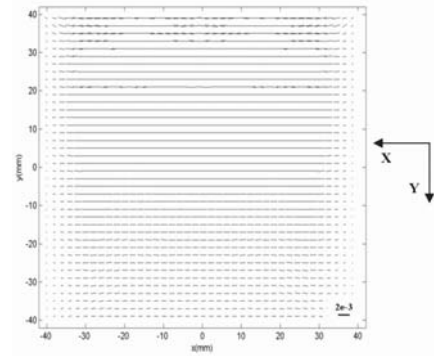


FIGURE 11. BENDING STRAIN OF SINGLY CURVED SHAPE WITH VARYING THICKNESS

(1)

### Scanning Path Determination

Due to the simplicity of singly curved shape, in this section we only focus on the doubly curved shapes (pillow) to investigate the scanning path determination strategy. As we know, the type of given doubly curved shapes requires both in-plane and bending strain and laser forming generally results both in-plane and bending strains. Furthermore, the highest compressive strains occur in a direction perpendicular to a scanning path. Therefore the scanning path should be perpendicular to the direction which is determined by both in-plane and bending strain. For varying thickness plate, sequentially determine the scanning paths according to the directions of bending strain and in-plane strain is not reasonable any more. It is more reasonable to vector average the direction of principal in-plane strain and principal bending strain. Since the effect of bending strain and in-plane strain varies with thickness and curvature, the different weight of each component should be considered during the vector averaging.

Ueda (1994) investigated the relation between in-plane strain and bending strain and pointed out that bending strain is proportional to the product of thickness and curvature, and the in-plane strain is proportional to the square of curvature. Therefore, the ratio of bending strain to in-plane strain can be expressed as

$$\varepsilon_B/\varepsilon_I \propto t/\rho \quad (4)$$

where  $t$  is the thickness and  $\rho$  the curvature. From above relation, it is illustrated that the effect of bending strain is more significant at the locations with large thickness, and the in-plane strain plays more important role in the locations with large curvature. To quantify this effect variation, a weighted vector-averaging scheme can be adopted as

$$\bar{\mathbf{e}}_{ave} = w_B \mathbf{e}_B + w_I \mathbf{e}_I = \left[1 + \frac{(t - \bar{t}) \bar{\rho}}{\bar{t} \rho}\right] \mathbf{e}_B + \left[1 - \frac{(t - \bar{t}) \bar{\rho}}{\bar{t} \rho}\right] \mathbf{e}_I \quad (5)$$

where  $\mathbf{e}_B$  and  $\mathbf{e}_I$  are principal bending strain and in-plane strain, respectively.  $\bar{t}$  and  $\bar{\rho}$  are average thickness and Gaussian curvature, respectively. For the uniform thickness ( $t = \bar{t}$ ) shape, the weight number is equal to 1 and above equation turns to be the normal vector averaging method. Fig.12 shows the determined paths on the weighted average strain field. It should be noted that the initial path spacing is chosen as the width of plastic deformed zone and can be simplified as the beam spot size.

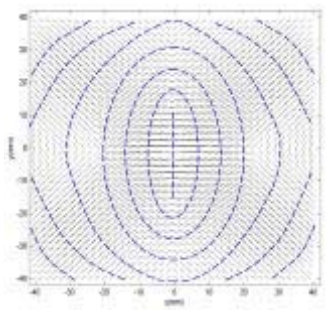


FIGURE 12. DETERMINED SCANNING PATHS ON AVERAGED PRINCIPAL MINIMUM STRAIN FOR PILLOW SHAPE WITH VARYING THICKNESS

### Heating Conditions Determination

After determination of the scanning paths, the heating conditions need to be determined that include laser power and laser scanning speed if the beam spot size and work material are given. While it is possible to continuously vary them to generate the strain field required to form the de-

sired shape, this study adopts the strategy of constant power and piecewise constant speed for a given path in favor of implementation simplicity. A path is broken down to a few segments such that within each segment, the range of strain variation is about the same as in other segments. Since the thickness varies through the segments, averaged thickness is adopted so that each segment has an equivalent constant thickness.

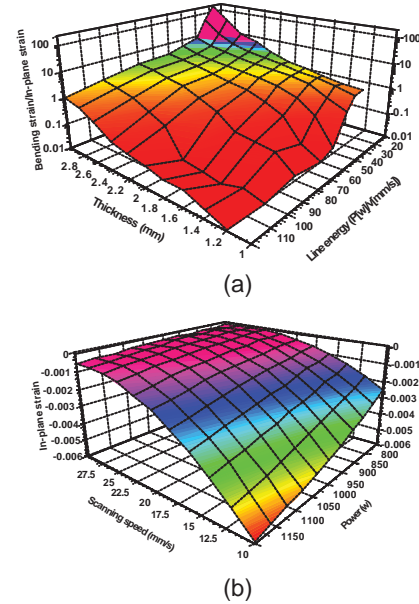


FIGURE 13. DATABASE OF (A) STRAIN RATIO AS A FUNCTION OF THICKNESS AND LINE ENERGY (B) IN-PLANE STRAIN AS A FUNCTION OF POWER AND SCANNING SPEED

For a scanning path, the equivalent thickness, strain and strain ratio for each segment can be determined. The segment having the largest strain, which has the strongest influence on the final shape, is first chosen. In determining the strain, strains between adjacent scanning paths are lumped together because all these strains are to be imparted by the paths. From the database of strain ratio as a function of thickness and line energy (P/V) (Fig.13a), the line energy can be determined for this segment. Then from the database of in-plane strain as the functions of power and velocity for an equivalent thickness (Fig.13b), a group of combinations of power and velocity can be determined. The combination of power and velocity which fulfills the requirement of line energy is the proper heating conditions. The determined P value is also adopted for the entire path. For other segments in this path, since the power has been de-

terminated, the corresponding scanning velocity can be determined from the database of strain ratio as a function of thickness and line energy.

Figure 14 shows the formed pillow shape and the deviations under these conditions. A general agreement can be seen from the figures. Possible sources contributing to the discrepancy include the lumped method used to sum strains between adjacent paths; finite number of paths to approximate a continuous strain field; and constant power within each path and constant velocity within each segment.

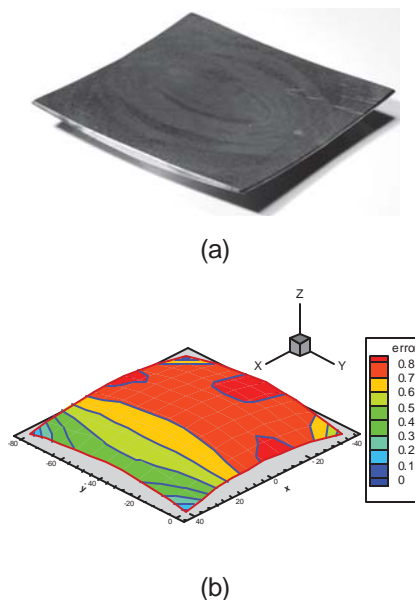


FIGURE 14. (A) FORMED PILLOW SHAPE (B) DEVIATIONS BETWEEN FORMED SHAPE AND DESIRED SHAPE (UNIT OF DEVIATION IS MM)

## CONCLUSIONS

For laser forming of varying thickness plates, the variation of bending angle is primarily due to the variation of heat sink and the bending rigidity. With the thickness increasing, temperature gradient mechanism (TGM) dominants and the bending strain plays more important role than in-plane strain. The higher scanning speed may decrease the bending variation because of the larger increments of temperature gradient through the thickness direction in the thicker locations on the plate. Rosenthal's solution to the moving point heat source on infinite plate was applied to characterize the effect of varying thickness on the temperature field

For the process design of thin plates with varying thickness, the scanning paths are located perpendicular to the directions obtained by weighted averaging the principal minimum in-plane and bending strains. The weighted vector-averaging method is capable to describe the variation of bending strain effect with the thickness. In determining heating conditions, the relationship between the ratio of bending strain to in-plane strain and the thickness, laser power and scanning speed was investigated. The proposed methodology is validated by the experiment.

## ACKNOWLEDGEMENT

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## REFERENCES

- Cheng, P. and Yao, Y. L., (2004a), "Analysis and Prediction of Size Effect on Laser Forming of Sheet Metal," *Transactions of NAMRI/SME*, Vol. 32, pp.439-446.
- Cheng, J. and Yao, Y. L., (2004b), "Process design of laser forming for three-dimensional thin plates," *ASME Transactions J. Manu. Science and Engineering*, Vol. 126, pp. 217-225.
- Liu, C., and Yao, Y. L., (2003), "FEM Based Process Design for Laser Forming of Doubly Curved Shapes," *ASME Transactions J. Manu. Science and Engineering*, submitted.
- Rosenthal, D., (1947), "The Theory of Moving Sources of Heat and Its Applications to Metal Treatments," *Trans. ASME, J. Heat Transfer*, Vol. 68, pp. 849-866.
- Timoshenko, S. and Woinowsky-Krieger, S., (1959), *Theory of Plates and Shells*, McGraw-Hill Book Company, Inc., New York.
- Ueda, K., et al., (1994), "Development of Computer-Aided Process Planning System for Plate Bending by Line Heating (Report I)-Relation Between Final Form of Plate and Inherent Strain", *Journal of Ship Production*, Vol. 10, No. 1, pp. 59-67.
- Vollertsen, F., (1994), "Mechanisms and Models for Laser Forming," *Laser Assisted Net Shape Engineering, Proceedings of the LANE'94*, Vol. 1, Meisenbach Bamberg, pp. 345-360.