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Design for Reliability: Case Studies in Manufacturing Process Synthesis

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Abstract: The task of manufacturing process design is to determine a set of process parameters for a manufacturing task. The design is normally carried out to optimize process performance, such as, to minimize discrepancy from a prescribed shape or to maximize production efficiency. An optimal design, however, is often not the most robust or reliable design because inevitable uncertainties in input variables may cause unacceptable deviations from the optimal point. Robust design is therefore needed to achieve compromise between optimization and robustness. Such a methodology is illustrated in case studies involving process design of laser forming of sheet metal, in which a set of parameters including laser scanning paths, laser power, and scanning speed is determined for a prescribed shape. Response surface methodology is used as an optimization tool. The propagation of error technique is built into the design process as an additional response to be optimized via desirability function and hence make the design robust. The results are also validated experimentally.

1. Introduction

Compared with conventional forming techniques, laser forming of sheet metal does not require hard tooling or external forces and hence, can increase process flexibility and reduce the cost of the forming process when low to medium production volume is concerned. Many efforts have been made on studying the mechanisms and modeling of the process. Magee, et al. (1998) reviewed literatures available up to 1998. More recently, selected issues related to extending laser forming to more practical applications started being addressed. For instance, repeated scanning is necessary to obtain the magnitude of deformation that practical parts require, and hence cooling effects during and between consecutive scans become critical (Cheng & Yao, 2001).

A vast majority of work on laser forming including the ones mentioned above can be considered as solving the direct problem. Such a problem is typically formulated based on physical laws such as heat transfer and elasticity/plasticity theories. To apply the laser forming process to real world problems, however, the inverse problem needs to be addressed. Solving the inverse problem analytically or numerically is difficult, if not impossible, for the following reasons. Firstly, no physical laws are readily available to establish governing equations leading to inverse solution. Secondly, to manipulate either the solution to the direct problem or the underlying differential equations leading to a solution to the inverse problem is also impossible because of the complexity involved. Thirdly, while the solution to the direct problem is unique, the solution to the inverse problem is certainly multi-valued. Given the understanding that numerical or analytical solutions to the inverse problem are less likely, empirical and heuristic approaches have been attempted. A genetic algorithm (GA) based approach was proposed by Shimizu, (1997) as an optimization engine to solve the inverse problem of the laser forming process. In his study, a set of arbitrarily chosen heat process conditions for a dome shape was encoded into strings of binary bits, which evolve over generations following the natural selection scheme. One of the important process parameters, heating path positions, was assumed given. To apply GA, it is necessary to specify crossover rate and mutation rate but their selection suffers from lack of rigorous criteria.

The objective of this paper is to develop a more systematic and reliable methodology to solve the inverse problem in laser forming for a class of shapes. A response surface methodology (RSM) based approach is attempted as an optimization tool. Discrete design variables are properly dealt with in the optimization process. Propagation of error (POE) technique is built into the design process as an additional response to be optimized via a desirability function and hence make the design more robust. Experiments and at places finite element method (FEM) are used to enable and validate the optimization process. The proposed approach is applied to two cases to demonstrate its validity.

2. Problem Description

As shown in Fig. 1, rectangular sheet metal is to be formed into 3D shapes by parallel laser irradiation. The 3D shape can be viewed as shape generated by a 2D generatrix in the y-z plane extruded in the x direction Therefore for this class of 3D shapes, the inverse design problem can be treated as a 2D curve design problem.

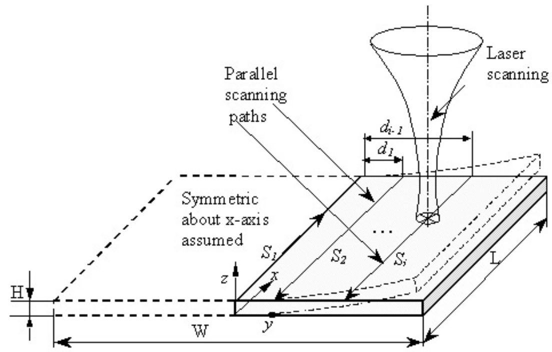

Fig.1 Schematic of a class of shapes to be laser formed, linear parallel scanning paths on a rectangular plate.

As shown in Fig.1, the parameters needs to be determined include number of scanning paths, *N*, positions of laser scanning paths d_i, laser powers p_i , beam scanning velocity v_i and laser spot diameter D_b. The approach presented in this paper, however, is not restricted to monotonic cases. A non-monotonic generatrix can be similarly dealt with using different beam spot sizes for different paths. The objective function is to minimize the difference between a possible solution shape and the prescribed shape, that is

$$\text{Minimize: } h = \sqrt{\sum_{i=1}^{k} (z_{si} - z_{pi})^2 / k} \quad (1)$$

where z_s and z_p are the *z* coordinates of corresponding points on the generatrix of the possible solution shape and the prescribed shape, respectively, and *k* is the number of points. It will be seen that values of the objective function *h*, in fact, serves as responses in the optimization process. In addition, if a point in the possible solution shape has a smaller *z* value than the corresponding point of the prescribed shape, the distance is defined as negative; otherwise it is positive. When the sum of the distances is positive, the objective (or response) is considered positive; otherwise, it is negative.

The final design of a product requires not only to be optimal, but also robust, namely, insensitive to the variation of input variables. In laser forming process, the achievable accuracy of forming is limited by numerous uncertainties. Hennige, et al. (1997) investigated influencing uncertainties in laser forming based on analyzing error propagation and found variations in power and coupling coefficient are most influential factors on the variation of deformation. In this paper, the influence of laser power on the robustness of the optimal design will be addressed in a robust design phase based on desirability consideration.

In this study, square plates of size $80 \times 80 \times 0.89$ mm are used. The material used is AISI1012 low carbon steel. The laser system used is a 1500W CO_2 laser. In all the experiments, the laser beam diameter is set to 4 mm, the beam moving velocity is kept constant at 50mm/s. A coordinate measuring machine (CMM) is used to measure the coordinates of the deformed plates.

3. RESPONSE SURFACE METHODOLOGY AND OPTIMAL DESIGN

Response surface methodology (RSM) is a collection of statistical and mathematical techniques useful for developing, improving, and optimizing process (Myers, et al. 1995). Applications of RSM comprise two phases. In the first phase the response surface function is based on a factorial design, approximated by a first-order regression model (Eq. 2), and complete with steepest ascent/descent search, until it shows significant lack of fit with experiment data. After reaching the vicinity of the optimum, the second phase of the response surface function is approximated by a higher order regression function such as a second-order one shown in Eq. 3.

$$\hat{y} = b_0 + \underline{b}_1^T \underline{x} + \varepsilon \quad (2)$$

$$\hat{y} = b_0 + \underline{b}_1^T \underline{x} + \underline{x}^T \underline{\underline{b}}_2 \underline{x} + \varepsilon \quad (3)$$

where $\hat{y}$ and $\underline{x} = [x_1, x_2, ..., x_n]^T$ are estimated response and decision variable vector, respectively, ε is the fitting error which is assumed to be normally distributed, and b_o, $\underline{b}_1$, and $\underline{\underline{b}}_2$ are coefficients determined using the least square method. If one or more of the decision variables are integers, the optimum problem is considered as a discrete problem, which can be dealt with methods such as the branch-and-bound method (Taha, 1987). The steepest line search starts with the initial design points of *q* integer variables and *n-q* non-integer variables. The next iteration starts by arbitrarily choosing a design point from one of the *q* integers variables, the this remaining *q-1* design points of integer variables of next movement are determined based on the coefficient from Eq. 2 and the branch-and-bound method. After reaching the vicinity of optimum, the response surface is approximated by a higher regression order with *q* integers determined from previous iteration and *n-q* non-integer factors. At this stage, the problem can be solved as a regular one with *n-q* continuous variables.

4. DESIRABILITY AND ROBUST DESIGN

Desirability based robust design is a tool to find controllable factor settings that optimize the objective yet minimize the response variation of the design (Kraber, et al., 1996). The transmitted variation of responses from input variables can then be reduced by moving the optimal solution to a flatter part of the response surface. The variation transmitted to the response can be determined by the error propagation equation *POE* =

$$\hat{\sigma}_y = \left(\sum_{i=1}^{n}\left(\frac{\partial \hat{y}}{\partial x_i}\right)^2 \sigma_{ii}^2 + \sum_{i<j}^{n} 2\frac{\partial \hat{y}}{\partial x_i}\frac{\partial \hat{y}}{\partial x_j}\sigma_{ij}^2 + \sigma_e^2\right)^{1/2} \tag{4}$$

where $\hat{\sigma}_y$ is the model-predicted standard deviation of the response or known as propagation of error (POE), σ_{ii}^2 is the variance of decision variable x_i, σ_{ij}^2 is the covariance between x_i and x_j, and σ_e^2 is the residual variance, $\sigma_e^2 = \sum_{j=1}^{k} \varepsilon_j^2 / df$, *df* is the residual degree of freedom. To reduce variance in the response, POE (Eq. 4) should be minimized therefore can be treated as an additional response built into the design process. The simultaneous optimization of several responses (in this case, *y* in Eq. 3 and $\hat{\sigma}_y$ in Eq. 4) is the essence of the desirability based robust design (Kraber, et al., 1996 and Derringer et al., 1980). For each response $\hat{y}_i$, a desirability function $D_i(\hat{y}_i)$ assigns a value between 0 and 1 to the possible values of $\hat{y}_i$, with $D_i(\hat{y}_i) = 0$ representing a completely undesirable value of $\hat{y}_i$ and $D_i(\hat{y}_i) = 1$ representing the ideal response value. The individual desirabilities are then combined using the geometric mean, which gives the overall desirability

$$D = (D_1(\hat{y}_1) \times D_2(\hat{y}_2) \times \ldots \times D_m(\hat{y}_m))^{1/m} \tag{5}$$

where *m* is the number of responses. The maximum of *D* represents the highest combined desirability of the responses. Depending on whether a particular response $\hat{y}_i$ is to be maximized, minimized, or assigned to a target value, different desirability functions $D_i(\hat{y}_i)$ are to be used. Let L_i, U_i and T_i be the lower, upper, and target values desired for response $\hat{y}_i$, where $L_i \le T_i \le U_i$. If a response is of the "target is best", its desirability function is expressed as:

$$D_i(\hat{y}_i) = \begin{cases} \left[\dfrac{\hat{y}_i - L_i}{T_i - L_i}\right]^r & L_i \le \hat{y}_i \le T_i \\ \left[\dfrac{\hat{y}_i - U_i}{T_i - U_i}\right]^s & T_i \le \hat{y}_i \le U_i \\ 0 & \hat{y}_i > U_i \quad or \quad \hat{y}_i < L_i \end{cases} \tag{6}$$

where the exponents *r* and *s* determine how strictly the target value is desired. The desirability function can be defined similarly if a response is to be minimized or maximized.

As seen from Eqs. 3 and 4, $\hat{y}_i$ is a continuous function of x_i, it follows that D_i and *D* are piecewise, continuous function of x_i. The above numerical optimization problem reduces to a general non-linear problem. However, as seen from equation (5) and (6), the derivative of *D* is not continuous. Therefore direct search methods need to be applied to find the optimal value of *D*. Downhill simplex method (Miller, 2000) is chosen in this study. An implementation of the algorithm is Design-Expert® by Stat-Ease, Inc, which is used in the paper.

5. APPROACH TO BENDING ANGLE ATTAINMENT

In the steepest ascent/descent search and RSM process, a large number of experiments are required to obtain bending angles under different conditions, which is time consuming and costly and thus poses a serious limitation to the method. If the total deformation of a sheet generated by the parallel laser scans can be obtained by summing deformations generated by these scans, a much smaller number of experiments will suffice. In other words, if deformations caused by scans at different d_i (Fig. 1) can be considered independent each other, only experiments with single scanning paths are needed. This is experimentally and numerically validated by Liu & Yao (2002). Fig. 2, adopted from Cheng & Yao. (2001), is for the square plate under different laser power level at scanning velocity of 50 mm/s. The scanning was done at y=0 and the bending angle is assumed to be the same if scanning is done at a non-zero y value. A line is fitted through the data points to allow interpolation in between.

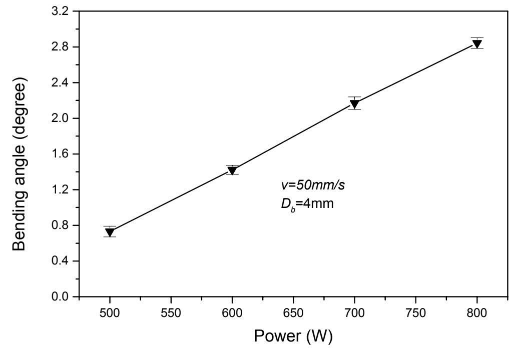

Fig. 2 Experimental results of bending angle induced by a single scan on a square plate (Cheng & Yao, 2001).

6. RESULTS AND DISCUSSIONS

6.1 <u>Case 1:</u> In this case, the desirable shape (Fig. 3) is given in terms of the following process parameters: number of scan paths N=6 and laser power p_i=p=700W. The scan paths are equally spaced. The way the prescribed shape is specified is to facilitate in this first case comparison of design result with the prescription. The task here is to find power p and number of scan paths N to minimize the difference between the shape formed using the found condition and that using the prescribed condition (Eq. 1).

<u>Optimal design</u> To apply RSM, an initial design point, N=4 and p=620W, is arbitrarily chosen. The corresponding initial shape is shown in Fig. 3. A two-level factorial design is conducted with half width ΔN=1 and Δp=30W. As outlined in Section 5, bending angles under the factorial design conditions are obtained from interpolating the experimental results shown in Fig. 2. To mimic the forming process repeatability characteristics, normally distributed random numbers are generated and added to each bending angle value. The standard deviation of the random numbers are chosen as the same as that shown in Fig. 2 in the form of error bars. A first-order regression model is fitted based on the factorial design and the direction of the steepest descent is determined from the coefficients of the regression model. At each movement along the steepest descent direction, the response obtained from regression model is compared with that based on the experimental result to examine if this model is still valid. The percentage discrepancies of the comparison at the initial point and first movement along the path are 0.42% and 2.22%, respectively, which are considered to indicate that the direction of steepest descent path is valid at these points. The responses h at these points are -1.316 and -0.928, respectively according to Eq. 1. After the next movement along the path (N=6 and p=664W), however, the percentage discrepancy increases to 5.87%, which indicates that the direction is no longer valid at this point (a tolerance of 5% is chosen as a lack of fit). Hence, another 2-level factorial design is conducted based on this point and a new steepest descent path is calculated. The new initial point (N=6 and p=664W) and the next point along the new path (N=7 and p=696W) have responses h = -0.5 and 0.21, respectively. The change of sign is clearly indicative of "overshooting", that is, the possible solution shape is bent more than the prescribed shape. This indicates that the optimum condition is in the vicinity of the last movement and normally a 3-level factorial design needs to be considered. In this case, however, the design variable N is subject to integer constraint and the solution must lie on either N=6 or 7. Therefore the branch-and-bound approach is applied. Three-level single-factor (p) designs are conducted separately at N=6 and N=7. The quadratic equation for N=6 is found as

$$h = 3.572 \times 10^{-5} p^2 - 0.03933 p + 10.02 \quad (7)$$

Solving the equation for zero response gives the optimal solutions p=701W for N=6 and p=671W for N=7. This indicates that multiple solutions are possible. An additional objective function, such as, one minimizes production time screens out the solution (p=671W for N=7). The optimization result (N=6 and p=701W) agrees very well with the prescribed value, (N=6 and p=700W). The optimization process towards the prescribed shape is shown in Fig. 3, which includes prescribed shape, initial design, some intermediate shapes and final design.

<u>Robust design</u> To make the design more robust propagation of error (POE) is calculated based on Eqs. 4 and 7, and assumed power standard deviation σ_p=6W as $POE = 4.29 \times 10^{-4} p - 0.236$. The response h is scaled to a desirability function D_1 ranged from [0,1] according to Eq. 6 because the response problem is of the "target is the

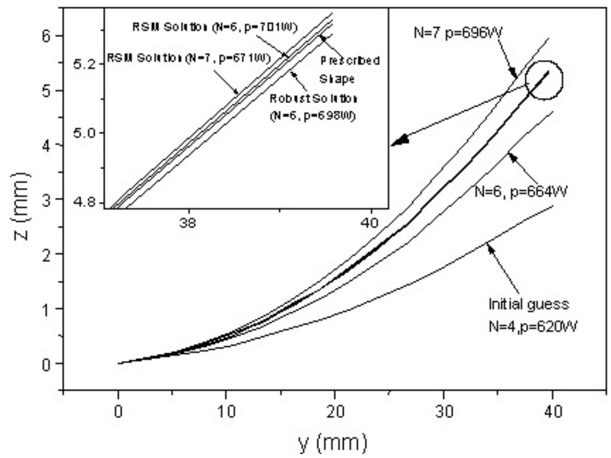

Fig. 3 Case1 Design evolution towards the prescribed shape

best" type, where $T_1 = 0$ (target), L_1 is set as –0.15, and U_1 +0.15 to ensure the robust design is not off the optimal solution too much. The *POE* is also scaled to a desirability function D_2 according to Eq. 7 because the *POE* problem is of the "minimization" type, where T_2 is set as 0.042 (for p=650W) and U_2 0.068 (for p=710W) according to Eq. 7. The overall desirability D is then expressed as the geometry mean of D_1 and D_2 using Eq. 7. As seen in Fig. 4, the overall desirability function D is a continuous, nonlinear, piecewise function, and the maximum value of desirability, 0.4, is found at p=698W, where the *POE* value is lower than that at the optimal solution (p=701W), and the response value is –0.003. At the optimal solution, the desirability is about 0.33 and response is obviously zero. The robust design therefore balances between the response and *POE*. In this case, the robust design does not differ much from the optimal solution but the design process is generally applicable.

6.2 <u>Case 2:</u> In this case, the desired shape is prescribed in terms of a generatrix that is a second order polynomial $z = 4(\frac{y}{40})^2 + 2(\frac{y}{40})$, as shown in Fig. 5, its curvature increases with y. It is obvious that evenly spaced scanning paths are no longer appropriate, this resulting in a large number of design variables and making the RSM based optimal design less feasible. As seen from Fig. 5, however, the curvature of the given profile decreases monotonically. Since the trend of spacing between adjacent laser paths is closely related to the curvature of the prescribed shape, the following control function is proposed to relate all d_i's,

$$d_i = 40(\frac{i}{N})^m \tag{8}$$

where d_i specifies the position of the i^{th} laser path, m is the design variable to be determined, and N is number of laser scan paths. In this case, N is set as *7*. The problem, therefore, becomes to determine the value of laser power p and exponent m to achieve the given profile. The same process as in the previous case starts with arbitrarily choosing an initial design at m=1.2 and p=650W with half width of 0.2 and 30W, respectively. The response function is found as h=1.58-5.81m+0.0073p-1.13m^2-1.14*$10^{-6}p^2$ +0.015mp. Since multi-solutions corresponding to zero response. Thus, another objective function, *POE*, is used to obtain the most desired solution.

Suppose the variations in m and p are 0.02 and 6W, respectively, the *POE* is constructed as *POE*=(9.38*$10^{-8}p^2$-6.87*$10^{-5}p$-3.02*$10^{-5}mp$+ 2.29* $10^{-3}m$+0.011m^2+0.02)$^{0.5}$, in which the residual variance of 0.0725 is included. h and *POE* are again scaled to desirability functions ranged between [0,1] with h constrained between [-0.15, 0.15]. The overall desirability D is then calculated. The desirability in two-dimensional cont-our is plotted in Fig. 9. As seen from Fig. 9, the most desirable value is 0.8013 corresponding to p=692W and m=1.17. With this m value and Eq. 8, laser path positions, d_i's, are calculated and plotted in form of crosses in Fig. 5, along with the final robust solution and prescribed shape. As expected, laser path locations become coarser with decreasing curvature of the prescribed shape. The profile based on the robust solution agrees with the prescribed profile. Laser forming experiments were conducted under the condition determined by the robust design and the results were plotted in Fig. 5. As seen, the experimental result shows a good fit with the predicted one.

7. CONCLUSIONS

It is shown that the proposed optimal and robust design schemes are feasible and effective for the class of shapes considered. Integer design variables are effectively dealt with by using standard methods such as the

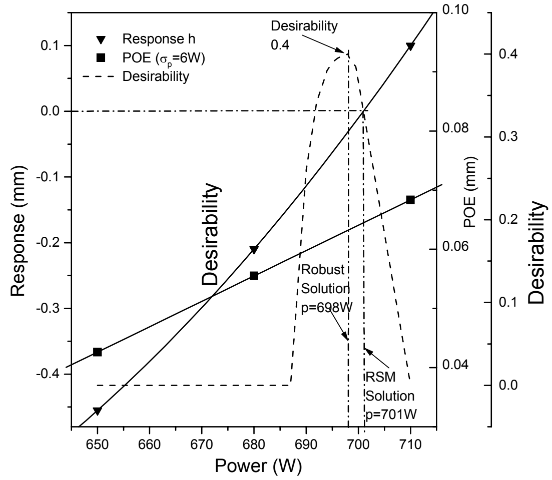

Fig. 4 Relationship of response, POE and desirability. (Standard deviation in laser power $\sigma_p = 6W$).

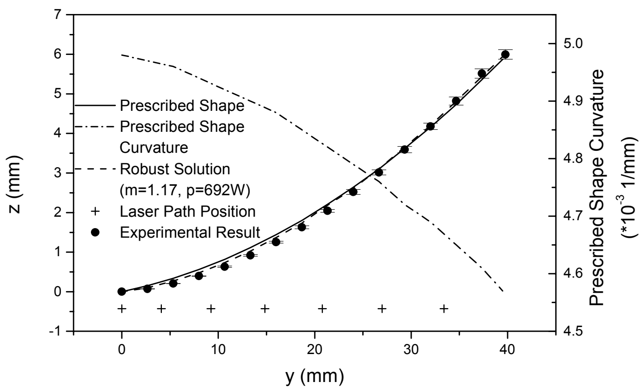

Fig. 5 Comparison of the robust solution and the prescribed shape (case 2).

branch-and-bound method and by integrating with RSM. To reduce the number of design variables, the laser path positions are specified by proper selection of control functions. The predicted results agree with the experimental ones.

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