Matching, Quality, and Comparative Advantage: A Unified Theory of Heterogeneous Firm Trade*†

Yoichi Sugita

Job Market Paper

November 22, 2009

Abstract

The theoretical literature on heterogeneous firm trade arose to explain a few key facts about exporting firms. The literature following Melitz (Econometrica, 2003) has addressed a number of new facts including those on importing firms and product prices, but the development of theories has mostly taken the form of one model per fact, with each model ignoring other established facts. Thus, this paper has three aims. The first is to unify the literature by developing a simple model that incorporates the central facts on exporting firms, importing firms, and product prices. Second, I provide the first account for the Amiti and Konings (AER, 2007) puzzle that liberalization of trade in intermediates improves the productivity even of firms that do not use imported intermediates. The model shows that nonimporting large firms using liberalized intermediates raise their empirical measure of productivity relatively to firms using non-liberalized intermediates. Finally, I demonstrate a new source of productivity gains from liberalization through resource reallocation toward some large and productive nonglobalized firms as well as globalized firms. All results are derived from only three key elements of the model: the assortative matching of firms by quality; fixed trade costs for intermediate goods; and Ricardian comparative advantage in production of intermediates.

*I am grateful to Donald Davis for his guidance and constant encouragement. I also thank Ronald Findlay, Amit Khandelwal, Eric Verhoogen, Jonathan Vogel, and David Weinstein for their suggestions, and JaeBin Ahn, Kyle Bagwell, Takakazu Honryo, Jota Ishikawa, Alexander McQuoid, Hitoshi Sato, Guru Sethupathy, Hitoshi Shigeoka, Kensuke Teshima, Catherine Thomas, and seminar participants at Columbia University, Hitotsubashi University, and Waseda University for their comments.

†E-mail: ys2176@columbia.edu. Website: http://www.columbia.edu/~ys2176/

1 Introduction

The past decade has witnessed an explosion of research on heterogeneous firms and trade (Bernard, Jensen, Redding, and Schott, 2007). The theoretical literature started as a way to explain two central facts about firms that engage in exporting (Melitz, 2003; Bernard, Eaton, Jensen, and Kortum, 2003). The first is the positive correlation between export status, size, and productivity (e.g. Bernard and Jensen, 1995, 1999; Roberts and Tybout, 1997). The second fact is the existence of aggregate productivity gains through resource reallocation among firms in the face of trade liberalization (e.g. Pavcnik, 2002; Trefler, 2004).

The empirical literature has identified a number of additional facts that have encouraged further theoretical development. These include: that many large and productive firms do not export; that exported goods are typically more expensive than local sales; that both output prices and input prices are correlated with firm size; that firms that import intermediates (importers) are larger and more productive than those that do not (nonimporters); that importers use more expensive inputs than nonimporters; and that importers receive larger productivity gains on average than nonimporters in liberalization of input trade (see Table 1).[1] However, the development of theories has mostly taken the form of one model per fact, with each model ignoring other established facts. More specifically, the literature lacks a model that can simultaneously address the heterogeneous characteristics of the three aspects of the global market: exporters, importers, and prices of traded products.

[1] See Bernard et al. (2003) and Hallak and Sivadasan (2009) for the existence of a number of large and productive nontrading firms. Baldwin and Harrigan (2007), Bernard et al. (2007), Helble and Okubo (2008), Johnson (2008), Mandel (2009) and Manova and Zhang (2009) find positive correlations between unit prices and trade costs from trade flow data and conclude more expensive products are more likely to be exported. Hallak and Sivadasan (2009), Iacovone and Javorcik (2009), and Kugler and Verhoogen (2008) report positive correlations between product prices and firms' exporting status from plant-level data. Kugler and Verhoogen (2008) also observe correlations among firm size, output prices, and input prices. See Bernard, Jensen, and Schott (2007, 2009), Halpern, Koren, and Szeidl (2009), and Kasahara and Lapham (2008) for the large size and productivity of importers; Kugler and Verhoogen (2009) for the use of expensive inputs by importers; Amiti and Konings (2007) for the larger productivity gains of importers.

Thus, this paper has three aims. The first is to develop a simple model that incorporates all of the facts that I have mentioned. Second, the model accounts for additional facts not explained in the previous literature, including the Amiti and Konings fact that liberalization of trade in intermediates raises the productivity even of firms that do not themselves use the imported intermediates (Amiti and Konings, 2007). The model shows that nonimporting large firms using liberalized intermediates increases their measured productivity relatively to firms using non-liberalized intermediates. Finally, the model demonstrates a new source of productivity gains from trade through resource reallocation toward some large and productive nontrading firms as well as trading firms. I derive all of the results from only three key elements of the model: the assortative matching of final producers and intermediate producers by quality; fixed trade costs for intermediate goods; and Ricardian comparative advantage in production of intermediates.

The model has the following features. There exist three types of quality-differentiated firms: final producers and producers of two different intermediates. One final producer and one each of the two types of intermediate producers form a team that produces a quality-differentiated final good. The quality of team members complements each other in developing the quality of the final products; therefore, in equilibrium, firms assortatively match by quality as in the traditional marriage market models of Becker (1973) and Sattinger (1979). Because consumers value the quality of final goods, high quality firms, which buy and sell expensive products, are large in terms of sales and employment and productive in terms of value-added per worker (Fact 5 in Table 1).[2] Firms become heterogeneous at entry by drawing random quality parameters as in the quality version of the Melitz (2003) model (e.g. Verhoogen, 2004, 2008; Baldwin and Harrigan, 2007).

By introducing trade in intermediates, the model explains stylized facts on the trade patterns of firms.[3] I consider two countries that are a mirror image of each other and differ only in

[2] In the current one-factor model with Leontief technology, the value-added per worker corresponds to revenue-based total factor productivity used in many empirical studies (Foster, Haltiwanger, and Syverson, 2008). See Section 3.

[3] This paper mainly focuses on trade in intermediate goods for three reasons. First, the majority of traded goods are intermediate goods rather than final goods. For example, Broda, Greenfield, and Weinstein (2009) estimate that they account for 68% of the total trade in a median country. Second, as will be discussed in the next section, the main difficulty in the prior literature is to simultaneously explain facts on prices of products traded between

entry costs for intermediates.[4] The difference in entry costs creates the difference in autarky matching patterns across countries. A final producer matches with a relatively higher quality intermediate producer in a sector with lower entry costs (the comparative advantage sector) and a lower quality one in a sector with higher entry costs (the comparative disadvantage sector). After the opening of trade in intermediate goods, some final producers match with foreign intermediate producers from the foreign comparative advantage sector to exploit the arbitrage opportunity of matching. However, not all of them match internationally for two reasons. First, only high quality large firms can afford fixed trade costs for intermediates. Therefore, on average, exporters and importers of intermediates transact more expensive products and are larger and more productive than nontrading firms (Facts 1, 4, 6, and 9 in Table 1). Second, many of these large firms do not trade (Fact 3 in Table 1) because they can also match with higher quality domestic partners that used to match with trading firms.

The change in firms' matching patterns creates a new gain from trade. In the short run after the opening of trade, in which re-matching occurs among surviving firms in autarky, all final producers including nonimporters improve the quality of final goods. The re-matching of firms reduces the quality gap of two intermediates: on one hand, new opportunities of trading with foreign high quality firms allow final producers to match with higher quality producers in the comparative disadvantage sectors; on the other hand, new competition with foreign final producers over intermediate producers make final producers to match with lower quality producers in the comparative advantage sectors. As a result, the quality of final goods rises when it is quasi-concave in the quality of intermediates, reflecting consumer or technology preference over a moderate combination of the quality of intermediate goods. The degree of quality improvement is larger for high quality final producers than low quality ones because the former can directly access the

heterogeneous exporters and importers. So far, all facts on importers are about importers of intermediates and no study documents stylized facts on importers of final goods. Finally, when trade in final goods is allowed in Appendix, the model explains facts on exporters of final goods based on mechanisms already known in the prior literature. Therefore, to focus on the novel mechanisms of the current model, I focus on trade in intermediate goods.

[4]The difference in entry costs captures the difference in technology of quality draws across countries. An alternative approach assuming the first order stochastic dominance in the distributions of quality draws yields isomorphic results.

global matching market.

This quality improvement explains the Amiti and Konings puzzle: liberalization of trade in intermediates improves the productivity even of final producers that do not use the imported intermediates (Fact 8 in Table 1). Their finding was puzzling in the conventional models of trade in intermediate goods such as the love of variety model (Ethier, 1982) and the quality-ladder model (Grossman and Helpman, 1991) because in these models, firms must use foreign intermediates to raise productivity.[5] In an extended two-industry model, I compare measured productivity (value-added per worker) of final producers in an industry that trade in intermediates (the liberalized industry) and that of final producers in an industry that do not trade (the non-liberalized industry).[6] The quality improvement in the liberalized industry after the opening of trade causes reallocation of labor from the non-liberalized industry to the liberalized industry. This reallocation reduces the profit and value-added per worker of final producers in the non-liberalized industry. As a result, large non-importers raise their measured productivity relatively to final producers in the non-liberalized industry. Furthermore, while Amiti and Konings attributed these productivity gains of nonimporters to externalities such as learning or technological spillovers that justify policy intervention, the current mechanism requires no externality.

Finally, among the final producers and intermediate producers in the comparative advantage sectors, high quality firms receive larger gains in matching than do low quality firms because they can directly access the global matching market. This unequal distribution of gains among

[5] Amiti and Konings (2007) control two other potential sources of measured productivity gains for nonimporting firms: changes in prices of domestic intermediates and indirect imports through intermediary firms. First, they construct separate price deflators for imported and domestic intermediates to control changes in prices of intermediates when they measure total factor productivity. Second, their dataset asks whether firms use imported intermediates regardless of their sources; therefore, firms do not have incentives to report intermediates imported through intermediary firms as domestic intermediates. Furthermore, the available evidence suggests the scale of imports of intermediates through intermediary firms is small. For instance, the volume of imports through intermediary firms, which include both final and intermediate goods, is only 20–30% of total manufacturing imports (see Bernard et al., 2009, for the US and Ahn, Khandelwal, and Wei, 2009, for China).

[6] This comparison of the two industries tries to imitate the comparison of firms using liberalized intermediate and firms using liberalized intermediates by Amiti and Konings (2007).

firms explains two facts. First, on average, the productivity gain for importers is larger than that for nonimporters (Fact 7 in Table 1). Second, it creates resource reallocation toward high quality firms to raise the sectoral productivity (Fact 2 in Table 1) of the final and the comparative advantage sectors. The reallocation pattern is quite different from the ones in existing models such as Melitz (2003), Bernard et al. (2003) and Kasahara and Lapham (2008). While in these models production factors are reallocated from nontrading firms to trading firms, in the current model factors are reallocated from low quality firms to high quality firms regardless of trading status. Furthermore, the model offers a new implication of large nontrading firms observed in data for productivity gains from reallocation in liberalization. In contrast to the conventional models, which predict that reallocation from large nontrading firms to small trading firms reduces the aggregate productivity, the current model suggests that the existence of large nontrading firms in data does not necessarily weaken the aggregate productivity gains from trade liberalization.[7]

The rest of the paper consists of four sections. Section 2 reviews the theoretical literature and explains how the current model solves difficulties in the previous approaches. Section 3 sets up the model of a closed economy. Section 4 introduces trade in intermediate goods in a two-country model and derives the main results. Section 5 concludes the paper.

2 Related Literature

After the first models of heterogeneous exporters (Melitz, 2003; Bernard et al., 2003), the theoretical literature has developed to account for additional facts on exporting firms, importing firms, and product prices by extending the Melitz model (Table 1). To explain large nonexporters, Eaton, Kortum, and Kramarz (2008) and Hallak and Sivadasan (2009) introduce idiosyncratic trade costs and two-dimensional heterogeneity of firms in productivity and quality, respectively. Because models based on firm heterogeneity in productivity predict a negative relation between product prices

[7] Armenter and Koren (2009) introduce idiosyncratic trade costs into a Melitz-type model to calibrate large nontrading firms in US data. They estimate that, compared with the hypothetical case that all large firms engage in exporting, the introduction of idiosyncratic trade costs halves the potential productivity gain from reallocation in liberalization.

and firms' trading status, Verhoogen (2004; 2008) and Baldwin and Harrigan (2007) emphasize product quality as an alternative source of firm heterogeneity to explain the positive correlation between product prices and firms' exporting status. Finally, to address the facts on importers of intermediates, Halpern, Koren, and Szeidl (2009) and Kasahara and Lapham (2008) extend the Melitz model to include trade in intermediates.[8] However, the literature lacks a single model that addresses the heterogeneity of exporters, importers, and product prices simultaneously.

The assumptions used in the previous models create difficulty in modeling transactions of quality-differentiated intermediates between heterogeneous exporters and importers. Most models derive import demand functions from the CES utility or production functions, following the love of variety models (Krugman, 1980; Ethier, 1982).[9] Therefore, it might appear straightforward to combine these models to explain facts on exporters and importers. However, because this approach predicts the extreme transaction pattern among firms that all importers buy all goods with identical shares, it fails to predict positive correlations among firm size, output prices, and input prices reported by Kugler and Verhoogen (2008).[10] Kugler and Verhoogen (2008) explain their fact by introducing complementarity between final producers' productivity of developing high quality final goods and the quality of intermediates. However, their model assumes competitive supply curves of intermediates that aggregate domestic and foreign intermediate producers. This paper develops a fully structured model of the assortative matching of exporters and importers by quality.

Kremer and Maskin (2006) and Antràs, Garicano, and Rossi-Hansberg (2006) developed the

[8] Antràs and Helpman (2003) also model heterogeneous importers, emphasizing their choices of arm's length trade and intrafirm trade.

[9] The only exception is the Bertrand competition model of Bernard et al. (2003). However, it is difficult to introduce heterogeneous importers into this almost-perfect-competition structure.

[10] The introduction of fixed costs of importing per variety of intermediates allows variations in the average prices of imported intermediates across importers, but this extension exacerbates the problem. First, large firms have to produce high quality goods to explain positive correlations among output prices, firm size and trading status. Second, all final producers prefer high quality varieties of intermediates to low quality ones from the assumption of an identical CES aggregator of intermediates. Third, the larger a final producer, the more varieties it imports. These three patterns predict that the input basket of high quality and large final producers will include more low quality varieties of imported inputs. This yields a negative relation between input prices and firm size.

assortative matching models of workers and managers between north and south by their skills.[11] However, it is difficult to interpret their workers as firms. In particular, their entry-and-exit mechanism of workers would be strange for firms because, of the workers engaging in production jobs, only the best workers exit from their jobs to become managers and the worst ones never exit from their jobs. Furthermore, because of this mechanism, if managers are interpreted as final producers who import intermediates, the model will not be able to explain simultaneously two facts on importers: the larger size of importers and the larger productivity gains of importers relative to nonimporters. This is because in Antràs et al. (2006), the highest-skilled managers, i.e. the largest final producers, always reduce their productivity from trade because they lose the highest-skilled workers, who leave their jobs to become managers. To solve this anomaly, I combine the assortative matching model with the Melitz type entry-and-exit mechanism of firms, in which the smallest firms exit from the market.

3 Closed Economy

3.1 Basic Structure

Consider an economy with one production factor, labor. The wage is normalized to one.

Consumers value the quantity, variety, and quality of final goods. A representative consumer maximizes the following CES utility function:

$$U = \left[\int_{\omega \in \Omega} q(\omega) \, c(\omega)^{\rho} \, d\omega \right]^{1/\rho},$$

where Ω is the set of available varieties of final goods, ω is a particular variety, $c(\omega)$ is consumption of ω, $q(\omega)$ is the product quality of ω, and $\rho \in (0, 1)$ is a parameter. The demand function for ω

[11] A matching model is also becoming a popular tool to study trade between countries with different distributions of workers' skill. Grossman and Maggi (2000) model domestic matching between heterogeneous workers. Ohnsornge and Trefler (2007), Costinot (2008), and Costinot and Vogel (2008) study domestic matching between heterogeneous workers and different industries. Nocke and Yeaple (2008) analyze an assortative matching model of corporate assets and managers to model international M&A without modeling their entry.

is derived as:

$$c(\omega) = \frac{Iq(\omega)^{\sigma} p(\omega)^{-\sigma}}{P^{1-\sigma}},$$

where $p(\omega)$ is the price of ω, I is the aggregate income, $\sigma \equiv 1/(1-\rho) > 1$ is the elasticity of substitution, and $P \equiv \left[\int_{\omega\in\Omega} p(\omega)^{1-\sigma} q(\omega)^{\sigma} d\omega\right]^{1/(1-\sigma)}$ is the quality-adjusted price index. The role of quality is a demand shifter: a variety with higher q has a larger demand at a given price.

There exist three types of firms: final producers, X, and two types of producers of different intermediates, Z_1 and Z_2. A final producer, a Z_1 producer, and a Z_2 producer form a team that produces one variety of final goods. After a team is formed, intermediate producers tailor their products for a particular variety of final goods, e.g. an engine for a particular model of car; therefore, firms transact intermediates only within a team. Firms are heterogeneous in product quality. Let x, z_1, and z_2 be the quality parameters of final producers, Z_1 producers, and Z_2 producers, respectively.[12] In the following, I use subscripts i and j to denote the variables and functions of Z_1 producers and Z_2 producers. They always mean that $i, j \in \{1, 2\}$ and $i \neq j$ when i and j are used together.

Team members play symmetric roles in the production of quality and quantity in a team. The quality of a final good is given by a simple supermodular and quasi-concave function of the quality of team members:[13]

$$q = xz_1z_2. \tag{1}$$

Following Kremer (1993) and Kugler and Verhoogen (2008), supermodularity captures complementarity among the quality of parts and components. For instance, if a car-producing team upgrades the quality of its engine, then supermodularity implies that the marginal quality improvement of the car is positively related to the quality of other components such as the transmission, body, tires, etc. The quasi-concavity expresses consumers' preference for moderate combinations of the quality of parts and components over extreme combinations. For instance, consumers may prefer

[12] The quality parameters can be interpreted as the "productivity" of developing high quality products.

[13] An increasing twice-differentiable function is called (strictly) supermodular if it has positive cross-derivatives. The function (1) can be relaxed to a more general form, $q = (xz_1z_2)^{\beta}$ for $\beta > 0$. With the quality parameters of firms redefined as $\tilde{x} \equiv x^{\beta}$ and $\tilde{z}_i \equiv z_i^{\beta}$, all the results of the paper hold because $\tilde{x}$ and $\tilde{z}_i$ follow the Pareto distribution.

a standard-class car with normal equipment to a luxury-class car with a poor air conditioner. An alternative interpretation of the quasi-concavity is that a moderate combination of the quality of intermediates allows final producers to develop high quality final goods.

Quantity production follows a Leontief-type technology. When a team produces Y units of final goods with quality q, each Z_i producer requires $L_{Z_i}(q, Y)$ units of labor to produce Y units of Z_i; then a final producer combines them with $L_h(q, Y)$ units of labor. The labor requirement is symmetric across team members, including fixed and variable components and increasing in q:

$$L_h(q, Y) = \frac{qY + f}{3} \quad \text{for} \quad h = X, Z_1, \text{and } Z_2. \tag{2}$$

Variable costs increasing in q reflect costs of quality control. Because even one defective component destroys the whole product (Kremer, 1993), the production of high quality final goods involves extra costs of quality control.

The firm heterogeneity arises from random quality draws at entry. There exist infinitely many ex ante symmetric final producers and Z_i producers. At their entry, each of them independently draws its quality parameter from an identical Pareto distribution by paying entry costs; the cumulative distribution function is given by $G(s) \equiv 1 - (1/s)^k$ for $s \in [1, \infty)$, where $k > 3$ is assumed to ensure a finite GDP.[14] While the probabilities of the quality draws are symmetric across sectors, entry costs are asymmetric across sectors: entry requires f_{Xe} units of labor for final producers and f_{Z_ie} units of labor for Z_i producers. For expositional purposes, I assume $f_{Z_1e} < f_{Xe} < f_{Z_2e}$.[15] Firms are risk neutral so they enter until their expected profits become zero.

After discovering their quality parameters, firms form production teams under perfect information. I assume one-to-one matching, i.e. each firm can join at most one team.

The model has three stages: (i) entry stage: firms enter and draw quality parameters by paying fixed entry costs; (ii) matching stage: firms form production teams; (iii) production stage: teams compete in the final goods market under monopolistic competition and distribute team profits among members.

[14] The Pareto distribution is commonly used to characterize empirical distributions of firm size. See Axtell (2001) and Helpman, Melitz, and Yeaple (2004), for example.

[15] All the main results in the paper depend only on the difference in entry costs for the two intermediate goods sectors.

3.2 Equilibrium

I derive an equilibrium allocation by backward induction.

Production Stage Because the teams' marginal cost is q and the demand is increasing in q, the optimal price $p(q)$ of final goods, the sales $r(q)$ of final goods, and the teams' joint profit $\Pi(q)$ are all increasing in q:

$$p(q) = \frac{q}{\rho},\ r(q) = I(\rho P)^{\sigma-1} q, \text{ and } \Pi(q) = \frac{r(q)}{\sigma} - f. \tag{3}$$

The optimal output, $c = \rho^{\sigma} I P^{\sigma-1}$, is independent of q. This is because consumers' demand and marginal costs both increase in q and the two effects are balanced.

From (1) and (3), the teams' joint profit is supermodular in the quality of team members:

$$\Pi(x, z_1, z_2) = Axz_1z_2 - f, \tag{4}$$

where $A \equiv \sigma^{-1} I (P\rho)^{\sigma-1}$expresses the market condition exogenous to individual teams.

Matching Stage Firms choose their partners and decide the distribution of profits, taking A as given. Two types of functions, profit schedules, $\pi_X(x)$ and $\pi_{Z_i}(z_i)$, and matching functions, $m_i(x)$, characterize equilibrium matching. A final producer with quality x chooses Z_i producers with quality $m_i(x)$ and receives the residual profit $\pi_X(x)$ after paying profits $\pi_{Z_i}(m_i(x))$ for intermediate producers. Firms that do not join any team exit. Following the matching literature, I focus on stable matching that satisfies two conditions: (i) (*individual rationality*) no firm is willing to deviate from the current team to exit; (ii) (*pair-wise stability*) no trio of a final producer, a Z_1 producer, and a Z_2 producer is willing to deviate from their teams to form a new team.[16] These two conditions are mathematically stated as follows: (i') all firms earn nonnegative profit, $\pi_X(x) \geq 0$ and $\pi_{Z_i}(z_i) \geq 0$ for all x and z_i; (ii') each firm is the optimal partner for the other

[16] Roth and Sotomayor (1990) is an excellent textbook on the literature. Although this definition of stability is common in the literature, it implicitly assumes firms can write complete contracts on the distributions on profits. Therefore, the current model is silent on the boundaries of firms.

team members:

$$\begin{aligned}\pi_X(x) &= \Pi(x, m_1(x), m_2(x)) - \pi_{Z_1}(m_1(x)) - \pi_{Z_2}(m_2(x)) \\ &= \max_{z_1, z_2} \Pi(x, z_1, z_2) - \pi_{Z_1}(z_1) - \pi_{Z_2}(z_2) \text{ and} \qquad (5) \\ \pi_{Z_i}(m_i(x)) &= \Pi(x, m_i(x), m_j(x)) - \pi_X(x) - \pi_{Z_j}(m_j(x)) \\ &= \max_{x', z_j} \Pi\left(x', m_i(x), z_j\right) - \pi_X(x') - \pi_{Z_j}(z_j). \qquad (6)\end{aligned}$$

The first order conditions for the maximization of (5) and (6):

$$\pi'_X(x) = Am_1(x)m_2(x) > 0 \text{ and } \pi'_{Z_i}(m_i(x)) = Axm_j(x) > 0, \tag{7}$$

prove that profit schedules increase in the quality parameters.

Because of the supermodularity of team profit, the stable matching is assortative by quality, i.e. $m'_i(x) > 0$. Because a high quality firm has a higher willingness to pay for the extra quality of its partners, high quality firms match with high quality firms while low quality firms match with low quality firms.

Production fixed costs force the lowest quality teams to exit from the market. The lowest quality teams must break even as follows:

$$Ax_L z_{1L} z_{2L} - f = 0, \tag{8}$$

where x_L and z_{iL} are the minimum quality thresholds for the survival of final producers and Z_i producers, respectively. All firms with lower quality than the thresholds exit. They satisfy:

$$\pi_X(x_L) = \pi_{Z_1}(z_{1L}) = \pi_{Z_2}(z_{2L}) = 0. \tag{9}$$

Profit schedules of firms are obtained by integrating the first order conditions (7) with initial conditions (9):

$$\pi_X(x) = A\int_{x_L}^{x} m_1(t)\, m_2(t)\, dt \text{ and } \pi_{Z_i}(m_i(x)) = A\int_{x_L}^{x} t m_j(t)\, m'_i(t)\, dt, \tag{10}$$

for all $x \geq x_L$. Profits are increasing in three factors: the firm's quality advantage over the lowest quality firms, the quality of the partners, and the market condition A.

Matching functions are derived from the assortative one-to-one matching of firms. Let M_{Xe}, M_{Z_1e}, and M_{Z_2e} be the mass of entrants of final producers, Z_1 producers, and Z_2 producers, respectively. The assortative one-to-one matching implies:

$$M_{Xe}\left[1-G\left(x\right)\right]=M_{Z_ie}\left[1-G\left(m_i\left(x\right)\right)\right] \text{ for all } x\geq x_L. \tag{11}$$

The left-hand side of (11) is the mass of final producers with higher quality than x, and the right-hand side is the mass of Z_i producers with higher quality than $m_i\left(x\right)$. Because the two sets of firms match with each other, the equality in (11) must hold for all $x\geq x_L$. I call equation (11) the "matching-market-clearing" condition.

Equilibrium matching patterns reflect the relative size of entrants across sectors. Matching functions solved from (11):

$$m_i\left(x\right)=x\left(\frac{M_{Z_ie}}{M_{Xe}}\right)^{1/k} \text{ for all } x\geq x_L, \tag{12}$$

imply that a firm from a sector with more entrants has higher quality than the other team members. Figure 1 describes this relation. Each of the three largest rectangles in the figure draws the distribution of final producers, Z_1 producers, and Z_2 producers, respectively, by assuming $M_{Z_1e}>M_{Xe}>M_{Z_2e}$. The width of each rectangle is equal to the mass of entrants in each sector. The height of the largest rectangles is set to one; the area of each of the largest rectangles is the total mass of entrants in that sector. The vertical axis expresses the value of $G(s)$, that is, the share of firms with lower quality than s in each sector.[17] Then, the three gray areas express the mass of final producers with higher quality than the given x (the left area), that of Z_1 producers with higher quality than $m_1\left(x\right)$ (the center area), and that of Z_2 producers with higher quality than $m_2\left(x\right)$ (the right area), respectively. Each of these three areas must be equalized under the assortative one-to-one matching. Therefore, a given final producer matches with a relatively higher quality intermediate producer from the sector with more entrants and a lower quality one from the sector with fewer entrants.

[17] For instance, if x_{median} is the median quality of entrants in the final sector, then $G\left(x_{median}\right)$ is at the midpoint on the vertical axis.

Entry Stage The minimum quality thresholds and the mass of entrants are determined through firms' entry. Because risk-neutral firms enter until their expected profits become zero, their free entry conditions are:

$$[1 - G(x_L)]\,\bar{\pi}_X = f_{Xe} \text{ and } [1 - G(z_{iL})]\,\bar{\pi}_{Z_i} = f_{Z_i e}, \tag{13}$$

where $\bar{\pi}_X$ and $\bar{\pi}_{Z_i}$ are the average profits of firms in the market, $\bar{\pi}_X = [1 - G(x_L)]^{-1} \int_{x_L}^{\infty} \pi_X(t)\, g(t)\, dt$ and $\bar{\pi}_{Z_i} = [1 - G(z_{iL})]^{-1} \int_{z_{iL}}^{\infty} \pi_{Z_i}(t)\, g(t)\, dt$. A straightforward manipulation from (10) and (12) shows that the average profits are constant as follows:[18]

$$\bar{\pi}_X = \bar{\pi}_{Z_i} = \frac{f}{k-3}. \tag{14}$$

I assume $f/(k-3) \geq \max\{f_{Xe}, f_{Z_i e}\}$ to ensure firms' entry. Then, the minimum quality thresholds are solved from (13) and (14) as follows:

$$x_L = \left[\frac{f}{f_{Xe}(k-3)}\right]^{1/k} \text{ and } z_{iL} = \left[\frac{f}{f_{Z_i e}(k-3)}\right]^{1/k}. \tag{15}$$

The minimum quality thresholds decrease in entry costs and increase in production fixed costs. The intuition will be clear below after the mass of consumption varieties and the mass of entrants in each sector are obtained.

Because firms earn zero expected profits, the aggregate revenue of teams must be equal to the aggregate income, $M\bar{r} = \bar{L}$, where $\bar{r}$ is the average revenue of surviving teams. From $\bar{r} = \sigma(\bar{\pi}_X + \bar{\pi}_{Z_1} + \bar{\pi}_{Z_2} + f)$ and (14), the mass of consumption varieties is proportional to the ratio of labor endowment to production fixed costs as in the standard Dixit–Stiglitz model:

$$M = \frac{(k-3)}{k\sigma}\left(\frac{\bar{L}}{f}\right). \tag{16}$$

Under the assortative one-to-one matching, the mass of teams is equal to the mass of surviving firms in each sector. Therefore, the mass of entrants is solved as:

$$M_{Xe} = \frac{M}{1 - G(x_L)} = \frac{\bar{L}}{f_{Xe} k\sigma} \text{ and } M_{Z_i e} = \frac{M}{1 - G(z_{iL})} = \frac{\bar{L}}{f_{Z_i e} k\sigma}. \tag{17}$$

[18] The constant average profit is similar to a known property of the Melitz-type (2003) model with a Pareto distribution. Teams' quality q follows the Pareto distribution.

The relative size of labor endowment to entry costs determines the mass of entrants in each sector. From the assumption on entry costs, the mass of entrants follows $M_{Z_1 e} > M_{Xe} > M_{Z_2 e}$.

The minimum quality thresholds in (15) link the mass of consumption varieties and the mass of entrants. Lower entry costs attract more entrants, but the total mass of surviving firms (16) is independent of the size of entry costs. Therefore, a sector with lower entry costs has a higher quality threshold for survival.

Positive Relations between Firm Characteristics and Quality The ranking of several characteristics of firms in each sector such as firm size, prices and an empirical measure of productivity follows the ranking of the quality of firms. First, high quality firms are large in terms of employment and sales. Employment and sales for final producers are given by $L_X(q_X(x), c)$ and $r(q_X(x))$, respectively, while those for Z_i producers are given by $L_{Z_i}(q_{Z_i}(z_i), c)$ and $[\pi_{Z_i}(z_i) + L_{Z_i}(q_{Z_i}(z_i), c)]$, respectively, where $q_X(x)$ and $q_{Z_i}(z_i)$ are the team quality of firms.[19] Because both the team quality and the profits of firms are increasing in firms' quality, all of these variables are increasing in firms' quality. Second, unit prices, which are given by $p(q_X(x))$ for final goods and by sales per output, $[\pi_{Z_i}(z_i) + L_{Z_i}(q_{Z_i}(z_i), c)]/c$, for intermediates, are also increasing in quality. Therefore, high quality large final producers buy expensive inputs and sell expensive outputs, which is consistent with positive correlations among firm size, output prices, and input prices reported by Kugler and Verhoogen (2008).

Finally, high quality firms have high measured productivity. Most empirical studies estimate firms' total factor productivity (TFP) without fully controlling for individual product prices and profits; then, estimated TFP is isomorphic to what Foster, Haltiwanger, and Syverson (2008) call "revenue-TFP": the residual of a regression of a firm's revenue on inputs. As shown by Foster et al. (2008), revenue-TFP corresponds to value-added per worker in the current one-factor model with Leontief-type technology:

$$V_X(x) \equiv \frac{\pi_X(x) + L_X(q_X(x), c)}{L_X(q_X(x), c)} \text{ and } V_{Z_i}(z_i) \equiv \frac{\pi_{Z_i}(z_i) + L_{Z_i}(q_{Z_i}(z_i), c)}{L_{Z_i}(q_{Z_i}(z_i), c)}. \tag{18}$$

Notice that value-added per worker is positively related to profit per worker. Therefore, if f is

[19] They are formally expressed as $q_X(x) = x m_{Z_1}(x) m_2(x)$ and $q_{Z_i}(z_i) = z_i m_i^{-1}(z_i) m_j(m_i^{-1}(z_i))$.

sufficiently large, high quality firms earning large profits exhibit large profit per worker and large value-added per worker reflecting the scale economy (the formal demonstration is in Appendix).[20]

4 Open Economy

In this section, I analyze trade in intermediate goods, i.e. international matching of firms in a two-country framework. I focus on trade in intermediate goods for three reasons. First, the majority of traded products are intermediate goods rather than final goods. Second, as discussed in Section 2, the main theoretical challenge is to reconcile stylized facts on importers of intermediates with facts on exporters and product prices of intermediates. Finally, because the final good market of this model is the same as that of existing models (Baldwin and Harrigan, 2008; Kugler and Verhoogen, 2008), the model can easily explain facts on exporters of final goods by introducing trade in final goods and idiosyncratic fixed trade costs as in Eaton et al. (2008) and Armenter and Koren (2009).[21] Therefore, in the following, I assume final goods are non-tradable and analyze trade in final goods in Appendix.

4.1 Comparative Advantage

I introduce another country, Foreign, as a mirror image of Home on entry costs for intermediate sectors:

$$f_{Z_1e} = f^*_{Z_2e} < f_{Z_2e} = f^*_{Z_1e}, \tag{19}$$

where foreign variables and functions are labeled by asterisks. The other aspects of these two countries are identical. Introducing this difference in entry costs is one of the simplest ways to formulate technological differences across countries, which gives rise to Ricardian comparative

[20] An alternative way to derive positive relations between quality and measured productivity is to assume linear demand functions shifting in q, following Melitz and Ottaviano (2008). This model predicts that the markup for final goods will increase in the quality of teams and exhibits larger measured productivity. Because team profits will be increasing in the quality of final goods and be supermodular, the main results of this paper will hold in this alternative model.

[21] To my knowledge, no empirical study documents the characteristics of importers of final goods.

advantage.[22] I call Z_i sectors with lower entry costs *the comparative advantage sectors* and those with higher entry costs *the comparative disadvantage sectors*. This mirror-image structure greatly simplifies the analysis: equilibrium functions and variables in the Home Z_i-sector are the same as those in the Foreign Z_j-sector and the other aspects are identical between Home and Foreign.[23] Because the two countries have the same wage, I keep normalizing it to one. From the mirror-image structure, in the following, I mainly analyze the consequences of trade on Home.

Forming international teams requires fixed trade costs. Each member in an international team hires $f_T/3$ units of labor for trade costs, which include transportation costs, communication costs, and costs of adopting foreign standards and regulations, etc. The profit of an international team is given by:

$$\Pi = Aq - f - f_T.$$

I assume sufficiently large f_T so that both countries produce both Z_1 and Z_2.[24] For simplicity, I ignore any variable trade cost.

The opening of trade creates two margins of adjustments among firms: (1) re-matching of existing firms and (2) new entry and exit of firms. Because the latter adjustment has a long-run nature, I first analyze a short-run equilibrium in which re-matching occurs only among firms that survive in autarky and, then, a long-run equilibrium in which firms adjust entry and exit to satisfy the free entry conditions.

4.2 Short-run Equilibrium

In this section, I analyze patterns of re-matching among existing firms in the short run after the opening of trade and its effects on the quality of final goods, measured productivity of firms, and

[22] Ricardian comparative advantage is one of the major determinants of world trade (see e.g. Costinot and Komunjer, 2007, as the most recent evidence). Another way to capture the difference in technology across countries is to allow a first order stochastic difference in G across countries. These two approaches yield isomorphic results under the Pareto distribution.

[23] The symmetric model also captures trade between developed countries at a similar income level that heavily depend on quality-differentiated products.

[24] If f_T is too small, both countries specialize in the comparative advantage sector in the long run and all firms engage in trade.

intra-sector reallocation of labor.

Patterns of International Matching Ricardian comparative advantage creates gains from international matching. Consider Home and Foreign final producers with the same quality x. In autarky, they match with relatively higher quality intermediate producers from the comparative advantage sectors and relatively lower quality ones from the comparative disadvantage sectors. After the opening trade, therefore, final producers and foreign intermediate producers from the foreign comparative advantage sectors are willing to form international teams to exploit the arbitrage opportunity in matching arising from the difference in the quality of two intermediate producers across countries.

Because of fixed trade costs, however, only high quality firms can match internationally. The following lemma shows that trade costs create two types of matching markets: an integrated global market for high quality firms and segmented local markets for low quality firms.

Lemma 1 *There exist threshold quality parameters x_T and z_T: (i) profit schedules for intermediate producers satisfy:*

$$\pi^*_{Z_1}(z) - \pi_{Z_1}(z) = \pi_{Z_2}(z) - \pi^*_{Z_2}(z) \begin{cases} = f_T & \text{if} \quad z \geq z_T \\ < f_T & \text{otherwise.} \end{cases} \tag{20}$$

(ii) matching functions for Home final producers $m_i(x)$ satisfy:

$$\left(M^a_{Xe} + M^{*a}_{Xe}\right)\left[1 - G(x)\right] = \left(M^a_{Z_ie} + M^{*a}_{Z_ie}\right)\left[1 - G(m_i(x_T))\right] \text{ for all } x \geq x_T, \tag{21}$$

$$M^a_{Xe}\left[G(x_T) - G(x)\right] = M^a_{Z_ie}\left[G(z_T) - G(m_i(x))\right] \text{ for all } x \in [x_L, x_T], \tag{22}$$

and $z_T = m_i(x_T)$. Matching functions for Foreign final producers satisfy similar conditions.

Proof. In Appendix. ■

The intuition for Lemma 1 is as follows. International matching occurs between high quality final producers and foreign high quality intermediate producers in foreign comparative advantage sectors. Figure 2 draws this process for Home. Among Z_1 producers in Home's comparative advantage sector, some of those with higher quality than a threshold z_T leave the Home matching market to match with foreign final producers; at the same time, Foreign Z_2 producers with the same distribution flow into the Home matching market.[25] This process continues until the difference in the quality of two intermediate producers in each of the high quality teams disappears, i.e. the mass of high quality Z_1 producers choosing domestic matching is equal to the sum of the mass of high quality local Z_2 producers and Foreign Z_2 producers exporting to Home, as shown in Figure 3. In equilibrium, the difference in the profits of Z_i producers between two countries is equal to trade costs as in (20). Then, final producers are indifferent between matches with domestic and foreign Z_i producers; therefore, the matching market for high quality firms behaves as a single global market that pools Home and Foreign high quality firms and equilibrium matching functions satisfy condition (21) in Lemma 1. The integration of matching markets also eliminates the difference in the quality of intermediate producers: each of the two intermediate producers in the high quality teams has the same quality. On the other hand, the matching markets for low quality firms are segmented across countries. Each team still employs a higher quality intermediate producer from the comparative advantage sector and a lower quality one from the comparative disadvantage sector.

Lemma 1 also implies that not all of the high quality firms engage in international trade. From (21), for given quality $x \geq x_T$, only a share $s_X \in (0,1)$ of final producers participate in importing while for given quality $z_i \geq z_T$, only $s_{Z_1} \in (0,1)$ of intermediate producers in the comparative advantage sectors participate in exporting, where $s_X = \left(M^a_{Z_1 e} - M^a_{Z_2 e}\right) / \left(M^a_{Z_1 e} + M^a_{Z_2 e}\right)$ and $s_{Z_1} = \left(M^a_{Z_1 e} - M^a_{Z_2 e}\right)/2$. Figure 4 draws the distributions of Home exporters and importers. These high quality firms transact (buy or sell) expensive products in f.o.b. prices, which does not include variable trade costs, and are large and productive.

Proposition 1 *(i) On average, exporting firms produce more expensive products in f.o.b. prices and are larger and more productive than nonexporting firms. (ii) On average, importing firms use more expensive intermediates in f.o.b. prices and are larger and more productive than nonimporting*

[25] Under the assumption of one-to-one matching, exporters give up domestic sales to serve a foreign market. These exporters' switch across markets is consistent with the empirical finding by Blum et al. (2007) that new exporting firms reduce their domestic sales relative to other firms. The current one-to-one matching model captures this pattern in an extreme form.

firms. (iii) However, there exist nontrading firms that transact more expensive products and are larger and more productive than some trading firms.

Proposition 1 assembles stylized facts on exporting firms, importing firms, and prices of traded goods. The assortative matching of firms by quality simultaneously explains the concentration of both exporting and importing into a small number of large and productive firms producing expensive goods.[26] Furthermore, Proposition 1 (ii) is consistent with the previously unexplained fact found by Kugler and Verhoogen (2009) for Colombian firms that importers use more expensive inputs than nonimporters.

Finally, in contrast to the standard Melitz-type models, in which the most productive firms always engage in trade, many large and productive firms do not trade, as observed by Bernard et al. (2003) for the US and by Hallak and Sivadasan (2009) for Chile, Colombia, India, and the US. The logic is novel compared with the previous explanations relying on Bertrand competition (Bernard et al., 2003) and idiosyncratic trade costs (e.g. Eaton et al., 2008; Armenter and Koren, 2009). While in these models, large and productive firms do not trade because they face tougher competition with foreign firms or larger trade barriers than other equally productive firms, in the current model, nontrading firms are simply indifferent between engaging in international trade and not doing so.

Final Goods Quality Re-matching among firms causes two opposite effects on the quality of final goods. While final producers gain new opportunities of trading with high quality intermediate producers in the comparative disadvantage sectors, they lose high quality intermediate producers in the comparative advantage sectors from new competition with foreign final producers. In the

[26] Bernard and Jensen (1995, 1999) and Roberts and Tybout (1997) are the first studies that report the concentration of exporting in a small number of large and productive firms for the US and Colombia, respectively. Bernard et al. (2009) and Kasahara and Lapham (2008) find similar patterns for importers from US and Chilean data, respectively. Baldwin and Harrigan (2007), Bernard et al. (2007), Johnson (2008), Mandel (2009), and Manova and Zhang (2008) report that products with high (f.o.b.) prices are more likely to be exported from trade flow data. From data on Colombian manufacturing firms, Kugler and Verhoogen (2008) find exporting plants produce more expensive products than nonexporting plants.

following, I analyze the net of these two effects on final goods quality.

Following Lemma 1, I first consider high quality final producers that can participate in the global matching market. From condition (21), Home matching functions are obtained as follows:

$$m_1(x) = m_2(x) = x\left(\frac{M^a_{Z_ie} + M^{*a}_{Z_ie}}{M^a_{Xe} + M^{*a}_{Xe}}\right)^{1/k} \text{ for } x \geq x_T. \tag{23}$$

For a given x, matching functions (23) are increasing and concave in the relative mass of entrants in each intermediate sector to final sector as autarky matching functions (12). Figure 5 draws this positive relation between the quality of Z_1 producers matched with a final producer with a given x, i.e. $m_1(x)$, and the mass of entrants in the Z_1 sector relative to the final sector. The curve in the figure shows that as entrants into the Z_1 sector increase relative to those in the final sectors, final producers with a given x match with higher quality Z_1 producers, but the marginal improvement in their matching is diminishing.[27] The quality of Z_1 producers matched with final producers with x in Home autarky, Foreign autarky, and in a short-run trade equilibrium is determined by the relative mass of entrants in the Z_1 sector to the final sector in Home autarky, in Foreign autarky, and in the world, respectively. Under trade, final producers match with lower quality producers in the comparative advantage sector, $m_1(x) < m^a_1(x)$, and with higher quality ones in the comparative disadvantage sector, $m_2(x) > m^a_2(x)$; the concave curve implies that the quality of Z_i producers $m_i(x)$ under trade is higher than the average of two partners in autarky, $m^a_1(x)$ and $m^a_2(x)\,(= m^{*a}_1(x))$. Figure 6 draws $m_i(x)$ (Point S) and $m^a_i(x)$ (Point A) with iso-$q_X(x)$ curves for a given x. Under the quasi-concavity of q, the reduction in the quality gap of the two intermediate goods improves the overall quality of final goods.

Furthermore, re-matching improves even low quality goods. From condition (22), the Home matching functions for low quality final producers are:

$$m_i(x) = \left(\frac{M^a_{Z_ie} + M^{*a}_{Z_ie}}{M^a_{Xe} + M^{*a}_{Xe}}\right)^{1/k} x\lambda_i(x) \text{ for } x \in [x_L, x_T), \tag{24}$$

where $\lambda_i(x) \equiv \left[1 + \left(1 - (x/x_T)^k\right)\left(M^a_{Z_je} - M^a_{Z_ie}\right)(2M^a_{Zie})^{-1}\right]^{-1/k}$ satisfies that $\lambda_1(x_T) = \lambda_2(x_T)$, $\lambda'_1(x) < 0$, and $\lambda'_2(x) > 0$ for $x < x_T$. As x becomes smaller than x_T, $m_1(x)$ in (24) becomes

[27] This concave relationship holds under a wide class of distributions including uniform, normal, exponential, and other frequently used distributions and other distributions with the nondecreasing hazard rate $g(x)/(1 - G(x))$.

larger and $m_2(x)$ becomes smaller than the values that functions (21) would predict for $x \leq x_T$. Therefore, in low quality teams, the reduction in the quality gap of two intermediates and the overall quality improvement are smaller relative to high quality teams. Nonetheless, the next proposition shows that even the lowest quality final producers improve their quality of final goods.

Proposition 2 *In the short run after the opening of trade: (i) all surviving final producers improve the quality of their final goods; (ii) the ratio of the quality of individual final goods under trade to autarky is weakly increasing in the quality of final producers:* $q_X(x)/q_X^a(x) = \Theta(x) > 1$, $\Theta'(x) > 0$ *and* $\Theta''(x) > 0$ *for* $x \in [x_L, x_T)$; $q_X(x)/q_X^a(x) = \Theta(x_T)$ *for* $x \geq x_T$.

Proof. In Appendix. ■

Figure 7 summarizes Proposition 2. Notice that Proposition 2 is only applied for surviving final producers. Two effects operate simultaneously to raise the minimum quality threshold for final producers. The first is the *replacement effect*, which is the result of a lack of new entrants in the short run. Under the assortative one-to-one matching, the outflow of mass M_T of Home Z_1 producers from the Home matching market and the inflow of the same mass of Foreign Z_2 producers automatically force M_T of the lowest quality final producers and $2M_T$ of the lowest quality Z_2 producers to exit. The second is the *selection effect*, from a reduction in the quality-adjusted price index, which also exists in Melitz (2003) and Kasahara and Lapham (2008). The aggregate improvement in the quality of final goods reduces the quality-adjusted price index and shifts the demand function for each variety of final goods downward for a given q. Because the quality gains are skewed toward high quality teams, the lowest quality teams face an inward shift of the demand curve, which raises the threshold quality for survival. The larger of these two effects determines the levels of the minimum quality thresholds.

Proposition 3 *In the short run after the opening of trade: (i) the minimum quality thresholds for final producers and intermediate producers in the comparative disadvantage sector rise; (ii) the threshold for intermediate producers in the comparative advantage sector rises if the selection effect is larger than the replacement effect; otherwise, the threshold stays at the autarky level.*

Proof. In Appendix. ■

From Propositions 1 and 2, I obtain relations between the quality of final goods and their use of imported intermediates.

Corollary 1 *(i) The opening of trade in intermediate goods improves the quality even of final goods that do not use imported intermediates in the short run. (ii) The quality improvement is on average greater for final goods that use imported intermediates than for those that do not.*

Measured Productivity Gains of Nonimporters This subsection analyzes the effect of trade on measured productivity of final producers, using Corollary 1. The model explains two findings of Amiti and Konings (2007) on Indonesian firms in the face of liberalization of trade in intermediates: (i) firms that use imported intermediates receive on average larger productive gains than firms that do not; (ii) even firms that do not use imported intermediates improve productivity. While the first finding is consistent with the conventional gains from imports of new foreign varieties of intermediates in the love of variety model (Ethier, 1982) and high quality foreign intermediate goods in the quality-ladder model (Grossman and Helpman, 1991), the second finding is puzzling in these two models, in which firms receive productivity gains only through using imported intermediates.

An extended version of the current model solves this empirical puzzle. Amiti and Konings (2007) measure productivity gains of firms using liberalized intermediates by comparing them with those of firms using non-liberalized intermediates to control the economy wide effects unrelated to liberalization such as business cycles.[28] Therefore, I also extend the model into a two-industry framework and compare firms in an industry that trade in intermediate goods (the liberalized industry) and those in an industry that does not trade in intermediate goods (the non-liberalized industry). The other aspects of the two industries are identical. The consumer preference is given by

$$\begin{aligned} U &= \left[C^{(\varepsilon-1)/\varepsilon} + \tilde{C}^{(\varepsilon-1)/\varepsilon}\right]^{\varepsilon/(\varepsilon-1)}, \\ C &= \left[\int_{\omega\in\Omega} q(\omega)\, c(\omega)^{\rho}\, d\omega\right]^{1/\rho}, \text{ and } \tilde{C} = \left[\int_{\omega\in\tilde{\Omega}} q(\omega)\, \tilde{c}(\omega)^{\rho}\, d\omega\right]^{1/\rho}, \end{aligned}$$

where variables in the non-liberalized industry are labeled by tilde. Parameter ε is the elasticity of substitution across industries. I assume goods are more substitutable within industry than across

[28]The authors use year-region fixed effects to conduct this comparison.

industry, i.e. $\varepsilon \leq \sigma$ and $\varepsilon \geq 1$. When the aggregate quality in the liberalized industry improves after the opening of trade, labor is reallocated from the non-liberalized industry to the liberalized industry. This inter-industry reallocation of labor tends to reduce the profit and value-added per worker (measured productivity) of final producers in the non-liberalized industry more than those of final producers in the liberalized industry. However, not all final producers in the liberalized industry increase their measured productivity relatively to those in the non-liberalized industry because of two negative effects on the measured productivity of final producers in the liberalized industry. The first effect is a fall in the quality-adjusted price index, which reduces A, i.e. the profit of teams at given q. The second is a fall in the share of final producer's profit in each team. These negative effects on the profit of final producers create a threshold for a rise in measured productivity.

Proposition 4 *In the short run after the opening of trade in the extended two industry model: there is a threshold $\hat{x}$ such that final producers with higher quality than $\hat{x}$ in the liberalized industry raise measured productivity relatively to the average final producers in the non-liberalized industry.*

Proof. In Appendix. ■

From Propositions 1 and 4, I obtain relations between the measured productivity of final producers in the liberalized industry and their use of imported intermediates.

Corollary 2 *In the short run after the opening of trade in the extended two industry model: (i) large nonimporting final producers using liberalized intermediates raise measured productivity relatively to the average final producers using non-liberalized intermediates; (ii) the gain in measured productivity of importing final producers using liberalized intermediates relative to the average final producers using non-liberalized intermediates is larger on average than that of nonimporting final producers using liberalized intermediates.*

Corollary 2 is consistent with the above two facts of Amiti and Konings (2007). Notice that this "spillover-like" effect on the productivity of nontrading firms from trade liberalization is fully internalized by frictionless market transactions. Amiti and Konings attributed the unexplainable productivity gain of nonimporting firms to technological externalities or spillovers. In contrast to these external effects, the current mechanism based on a systematic economic logic does not justify policy intervention. Therefore, one suggestion for empirical studies from this model is that spillover effects measured without controlling firm's transaction partners might overestimate the scope for policy intervention.

Matching of Intermediate Producers Now I go back to the one-industry model and analyze matching of intermediate producers. The opening of trade causes asymmetric consequences on matching of intermediate producers in the comparative advantage sectors and the comparative disadvantage sectors. The change in matching patterns of final producers discussed above is straightforwardly translated into those of intermediate producers: intermediate producers in the comparative advantage sectors receive higher quality matches with both partners, while those in the comparative disadvantage sectors receive lower quality matches with both partners. Furthermore, because the adjustment of matching is larger among high quality teams, both the gains and the losses in matching are increasing in quality of firms. Figure 8 draws the ratio of team quality of intermediate producers in trade to autarky, $q_{Z_i}(z_i)/q^a_{Z_i}(z_i)$.

Lemma 2 *In the short run after the opening of trade: (i) all surviving intermediate producers in the comparative advantage sectors improve their team quality, while all surviving intermediate producers in the comparative disadvantage sectors improve their team quality; (ii) their gains and losses are weakly increasing in the quality of firms.*

Sectoral Productivity Gains from Reallocation The short-run re-matching affects aggregate sectoral productivity by reallocating labor among firms. First, the exit of the lowest quality firms in each sector from Proposition 3 tends to raise sectoral productivity by reallocating labor toward high quality firms. Second, reallocation also occurs among surviving firms. Because firms' employment increases in the team quality of firms, Proposition 2 and Lemma 2 imply that labor is reallocated toward high quality firms in the final and comparative advantage sectors, while it is reallocated toward low quality firms in the comparative disadvantage sector. From these two effects, liberalization raises the aggregate sectoral productivity through reallocation of firms, as observed by Pavcnik (2002) for Chile and Trefler (2004) for Canada, in the final and comparative advantage sectors, but the two opposite effects make the net effect ambiguous in the comparative disadvantage sector.

Proposition 5 *In the short run after trade liberalization, the sectoral productivity rises in the final and comparative advantage sectors, while it ambiguously changes in the comparative disadvantage sector.*

Proposition 5 demonstrates a new mechanism for productivity gains from resource reallocation among firms in the face of liberalization. This productivity gain from reallocation is quite different from those in the existing models such as Melitz (2003), Bernard et al. (2003) and Kasahara and Lapham (2008). Whereas in these models labor flows from nontrading firms to trading firms, in the final and comparative advantage sectors in the current model, it flows from low quality firms to high quality firms regardless of trading status.

Furthermore, the model provides a new implication of large nontrading firms, which are frequently observed in data, for productivity gains from trade. The conventional Melitz-type models explain large and productive nontrading firms by introducing idiosyncratic trade costs or demand shocks; therefore, they conclude that the existence of nontrading firms weakens productivity gains from trade liberalization because resources flow from more productive nontrading firms to less productive trading firms. Armenter and Koren (2009) introduce idiosyncratic trade costs into a Melitz-type model to calibrate large nontrading firms in US data. They estimate that, compared with the hypothetical case that all large firms engage in exporting, the introduction of idiosyncratic trade costs halves the potential productivity gain from reallocation in liberalization. However, the current model suggests that these models might underestimate gains from reallocation.

4.3 Long-run Equilibrium

This section analyzes the long-run equilibrium. Because re-matching among firms analyzed in the previous sections still exists in the long-run equilibrium, I mainly focus on the difference between the short-run and the long-run equilibriums.

In the long run, the mass of entrants are adjusted to satisfy the free entry conditions. Because intermediate producers improve their matching in the comparative advantage sectors and lose in the comparative disadvantage sectors in the short run, entry shifts from the latter sectors to the former sectors.[29] This worldwide specialization into sectors with lower entry costs increases the mass of entrants of Z_i producers in the world.

Proposition 6 *In the long run after the opening of trade: (i) the mass of entrants of final producers remains at the autarky level, $M_{Xe} = M_{Xe}^a$; (ii) the mass of entrants of intermediate producers in the comparative advantage sector rises while that of intermediate producers in the comparative disadvantage sector falls:*

$$M_{Z_1e} > M_{Z_1e}^a > M_{Z_2e}^a > M_{Z_2e}, \tag{25}$$

(iii) the mass of entrants of intermediate producers in the world increases:

$$M_{Z_ie} + M_{Z_ie}^* > M_{Z_ie}^a + M_{Z_ie}^{*a}. \tag{26}$$

Proof. In Appendix. ■

Notice that Lemma 1 still holds, with the mass of entrants replaced by new amounts of entry, because it is derived from the difference in the mass of entrants between the two intermediate sectors. Therefore, all short-run results regarding trade patterns, which are derived from Lemma 1, still hold in the long-run equilibrium.

The long-run adjustment of firms' entry and exit enhances the unequal effects of re-matching on the quality of final goods. It creates another source of quality improvement for high quality final goods, but at the same time, it reduces the quality of final goods produced by the lowest quality teams. From Lemma 1, the long-run matching functions for high quality final producers that can access the global matching market take a form similar to (23) as follows:

$$m_1(x) = m_2(x) = \left(\frac{M_{Z_ie} + M_{Z_ie}}{2M_{Xe}}\right)^{1/k} x \text{ for } x \geq x_T. \tag{27}$$

From Proposition 6 (iii) and (27), the increase in entry of intermediate producers in the world further improves the quality of final goods, which is shown as Point L in Figure 9. In sum,

[29] The average profits of intermediate producers in the comparative advantage sectors rise from autarky to the short-run equilibrium, while those in the comparative disadvantage sector fall.

trade liberalization improves the quality of high quality final goods in two steps: it eliminates the mismatch of the quality of intermediates in the short run, and then it increases the quality of intermediates in the long run.

The specialization toward the comparative advantage sector, however, works against the low quality final producers. Because a country is already abundant in entrants in the comparative advantage sector, the shift of entry from the comparative disadvantage sectors to the comparative advantage sectors widens the imbalance of entrants in the two sectors. Although the outflow of high quality producers in the comparative advantage sector and the inflow of foreign high quality producers in the comparative disadvantage sector mitigate this effect, it is not sufficient to offset the negative effect from the long-run entry and exit for the lowest quality final producers. The next proposition shows there exists a threshold quality parameter regarding quality gains or losses for final producers.

Proposition 7 *In the long run after the opening of trade, there exist thresholds* $\tilde{x} > x_L$, $\tilde{z}_1 > z_{1L}$ *and* $\tilde{z}_2 > z_{2L}$ *: (i) firms at these thresholds match with the same quality of partners as in autarky,* $\tilde{z}_1 = m_1(\tilde{x}) = m_1^a(\tilde{x})$ *and* $\tilde{z}_2 = m_1(\tilde{x}) = m_1^a(\tilde{x})$*; (ii) the convergence in the quality of intermediate producers in teams,* $m_1(x) < m_1^a(x)$ *and* $m_2(x) > m_2^a(x)$*, occurs if final producers have higher quality than* $\tilde{x}$ *and the divergence occurs if they have lower quality than* $\tilde{x}$*; (iii) The quality of final goods improves,* $q_X(x) > q_X^a(x)$*, if* $x > \tilde{x}$ *and falls if* $x < \tilde{x}$*; (iv) The team quality of* Z_1 *producers improves,* $q_{Z_1}(z_1) > q_{Z_1}^a(z_1)$*, if* $z_1 > \tilde{z}_1$ *and falls if* $z_1 < \tilde{z}_1$*; (v) The team quality of* Z_2 *producers falls,* $q_{Z_2}(z_2) > q_{Z_2}^a(z_2)$*, if* $z_2 > \tilde{z}_2$ *and rises if* $z_2 < \tilde{z}_2$*.*

Proof. In Appendix. ■

The changes in team quality in each sector are summarized by Figures 10 and 11.

The last qualitative difference between the short-run equilibrium and the long-run equilibrium is the level of the minimum quality thresholds for survival. Because the thresholds affect the expected profit by changing the survival probability, the entry conditions play central roles in determining them in the long run.

Proposition 8 *In the long run after the opening of trade: (i) the minimum quality thresholds for* *final producers and intermediate producers in the comparative advantage sector rise compared with the autarky levels; (ii) the threshold for intermediate producers in the comparative disadvantage sector falls.*

Proof. In Appendix. ■

The thresholds for final producers and intermediate producers in the comparative advantage sector rise through a similar mechanism as in Melitz (2003). Because in these two sectors firms' gains from matching are larger for high quality firms, tougher competition with high quality firms forces the lowest quality firms to exit. The fall in the threshold in the comparative disadvantage sector is derived from the free entry condition. Because intermediate producers lose team quality and profits, the decrease in the expected profit must be compensated by an increase in the survival probability.

5 Conclusion

This paper has developed a model of heterogeneous firm trade based on the assortative matching of firms by quality, fixed trade costs, and Ricardian comparative advantage. These three elements are simple but rich enough to unify key facts on exporting firms, importing firms, and prices of traded goods in a single framework. In addition to the unification of existing facts, the model solves the Amiti–Konings puzzle about the productivity gains of nonglobalized firms without relying on any technological externality. Finally, the model demonstrates an unexplored mechanism of productivity gains through resource reallocation toward nontrading productive firms. An empirical investigation on international matching of firms including the new hypotheses is left for future research.

References

Ahn, JaeBin, Amit K. Khandelwal, and Shang-Jin Wei. 2009. "The Role of Intermediaries in Facilitating Trade." mimeo, Columbia University.

Amiti, Mary, and Jozef Konings. 2007. "Trade Liberalization, Intermediate Inputs, and Productivity: Evidence from Indonesia." *American Economic Review*, 97(5): 1611–38.

Antràs, Pol, Luis Garicano, and Esteban Rossi-Hansberg. 2006. "Offshoring in a Knowledge Economy." *Quarterly Journal of Economics*, 121(1): 31–77.

Antràs, Pol, and Elhanan Helpman. 2004. "Global Sourcing." *Journal of Political Economy*, 112(3): 552–80.

Armenter, Roc and Miklós Koren. 2009. "Economies of Scale and the Size of Exporters." Working Papers 09-15, Federal Reserve Bank of Philadelphia.

Axtell, Robert L. 2001. "Zipf Distribution of U.S. Firm Sizes." *Science*, 293(5536): 1818–20.

Baldwin, Richard, and James Harrigan. 2007. "Zeros, Quality and Space: Trade Theory and Trade Evidence." NBER Working Paper No. 13214.

Becker, Gary S. 1973. "A Theory of Marriage: Part I." *Journal of Political Economy*, 81(4): 813–46.

Bernard, Andrew B., Jonathan Eaton, J. Bradford Jensen, and Samuel Kortum. 2003. "Plants and Productivity in International Trade." *American Economic Review*, 93(4): 1268–90.

Bernard, Andrew B., and J. Bradford Jensen. 1995. "Exporters, Jobs, and Wages in U.S. Manufacturing: 1976-1987." *Brookings Papers on Economic Activity. Microeconomics,*1995: 67-119.

Bernard, Andrew B., and J. Bradford Jensen. 1999. "Exceptional Exporter Performance: Cause, Effect, or Both?" *Journal of International Economics*, 47(1): 1–25.

Bernard, Andrew B., J. Bradford Jensen, Stephen J. Redding, and Peter K. Schott. 2007. "Firms in International Trade." *Journal of Economic Perspectives*, 21(3): 105–30.

Bernard, Andrew B., J. Bradford Jensen, and Peter K. Schott. 2009. "Importers, Exporters and Multinationals: A Portrait of Firms in the U.S. that Trade Goods." In *Producer*

Dynamics: New Evidence from Micro Data, ed. Timothy Dunne, J. Bradford Jensen, and Mark J. Roberts, 513–52. Chicago: University of Chicago Press.

Blum, Bernardo, Sebastian Claro, and Ignatius Horstmann. 2007. "Why do Firms Export? Dumping Marginal Costs and the Gains from Trade." mimeo, University of Tronto.

Costinot, Arnaud. 2009. "An Elementary Theory of Comparative Advantage." *Econometrica*, 77(4): 1165-92.

Costinot, Arnaud and Ivana Komunjer. 2008. "What Goods Do Countries Trade? A Structural Ricardian Model." http://econ-www.mit.edu/faculty/costinot/papers.

Costinot, Arnaud, and Jonathan Vogel. 2009. "Matching and Inequality in the World Economy." http://econ-www.mit.edu/faculty/costinot/papers.

Eaton, Jonathan, Samuel Kortum, and Francis Kramarz. 2008. "An Anatomy of International Trade: Evidence from French Firms." NBER Working Paper No. 14610.

Ethier, Wilfred J. 1982. "National and International Returns to Scale in the Modern Theory of International Trade." *American Economic Review*, 72(3): 389–405.

Foster, Lucia, John Haltiwanger, and Chad Syverson. 2008. "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?" *American Economic Review*, 98(1): 394–425.

Grossman, Gene M., and Elhanan Helpman. 1991. *Innovation and Growth in the Global Economy.* Cambridge: MIT Press.

Grossman, Gene M., and Giovanni Maggi. 2000. "Diversity and Trade." *American Economic Review*, 90(5): 1255–75.

Hallak, Juan Carlos, and Jagadeesh Sivadasan. 2009. "Firms' Exporting Behavior under Quality Constraints." NBER Working Paper No. 14928.

Halpern, László, Miklós Koren, and Adam Szeidl. 2009. "Imported Inputs and Productivity." http://miklos.koren.hu/view/list/.

Helble, Matthias and Toshihiro Okubo. 2008. "Heterogeneous Quality and Trade Costs." The World Bank Economic Policy Research Paper 4550.

Helpman, Elhanan, Marc J. Melitz, and Stephen R. Yeaple. 2004. "Export Versus FDI with Heterogeneous Firms." *American Economic Review*, 94(1): 300–16.

Iacovone, Leonardo, and Beata S. Javorcik. 2009. "Shipping Good Tequila Out: Investment, Domestic Unit Values and Entry of Multi-product Plants into Export Markets." http://www.economics.ox.ac.uk/members/beata.javorcik/working.html.

Johnson, Robert C. 2008. "Trade and Prices with Heterogeneous Firms." http://robjohnson41.googlepages.com/.

Kasahara, Hiroyuki, and Beverly Lapham. 2008. "Productivity and the Decision to Import and Export: Theory and Evidence." CESifo Working Paper Series No. 2240.

Kremer, Michael. 1993. "The O-Ring Theory of Economic Development." *Quarterly Journal of Economics*, 108(3): 551–75.

Kremer, Michael, and Eric Maskin. 2006. "Globalization and Inequality." http://www.economics.harvard.edu/faculty/kremer/.

Krugman, Paul. 1980. "Scale Economies, Product Differentiation, and the Pattern of Trade." *American Economic Review*, 70(5): 950–9.

Kugler, Maurice, and Eric Verhoogen. 2008. "The Quality-Complementarity Hypothesis: Theory and Evidence from Colombia" NBER Working Paper No. 14418.

Kugler, Maurice, and Eric Verhoogen. 2009. "Plants and Imported Inputs: New Facts and an Interpretation." *American Economic Review Papers and Proceedings*, 99(2): 501–7.

Mandel, Benjamin R. 2008. "Heterogeneous Firms and Import Quality: Evidence from Transaction-Level Prices." http://www.econ.ucdavis.edu/graduate/brmandel/.

Manova, Kalina and Zhiwei Zhang. 2009. "Export Prices and Heterogeneous Firm Models." NBER Working Paper No. 15342.

Melitz, Marc J. 2003. “The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity.” *Econometrica*, 71(6): 1695–725.

Melitz, Marc J., and Gianmarco I. P. Ottaviano. 2008. “Market Size, Trade, and Productivity.” *Review of Economic Studies,* 75(1): 295–316.

Nocke, Volker, and Stephen R. Yeaple. 2008. “An Assignment Theory of Foreign Direct Investment.” *Review of Economic Studies*, 75(2): 529–57.

Ohnsorge, Franziska and Daniel Trefler. 2007. “Sorting It Out: International Trade with Heterogeneous Workers.” *Journal of Political Economy*,115(5): 868–92.

Pavcnik, Nina. 2002. “Trade Liberalization, Exit, and Productivity Improvements: Evidence from Chilean Plants.” *Review of Economic Studies*, 69(1): 245-76.

Roberts, Mark J. and James R. Tybout. 1997. **“**The Decision to Export in Colombia: An Empirical Model of Entry with Sunk Costs.”*American Economic Review*, 87(4): 545–64.

Roth, Alvin E., and Marilda A. Oliveira Sotomayor. 1990. *Two-sided Matching: A Study in Game-theoretic Modeling and Analysis,* Cambridge University Press, Cambridge.

Sattinger, Michael. 1979. “Differential Rents and the Distribution of Earnings.” *Oxford Economic Papers*, 31(1): 60–71.

Trefler, Daniel. 2004. “The Long and Short of the Canada-US Free Trade Agreement.” *American Economic Review*, 94(4): 870-95.

Verhoogen, Eric A. 2004. “Trade, Quality Upgrading and Wage Inequality in the Mexican Manufacturing Sector: Theory and Evidence from an Exchange-Rate Shock.” Center for Labor Economics, University of California, Barkeley, Working Paper No.67.

Verhoogen, Eric A. 2008. “Trade, Quality Upgrading, and Wage Inequality in the Mexican Manufacturing Sector.” *Quarterly Journal of Economics*, 123(2): 489–530.

6 Appendix 1: Trade in Final Goods

In this appendix, I consider trade in final goods in addition to trade in intermediate goods. Because the final goods market has the same structure as Baldwin and Harrigan (2008) and Kugler and Verhoogen (2008), the pattern of trade in final goods is the same as in these models.

Trade in final goods requires F_T of fixed costs and no variable costs. The team profit functions become:

$$\Pi = \begin{cases} Aq - f + \max\{Aq - F_T, 0\} & \text{for domestic teams,} \\ Aq - f - f_I + \max\{Aq - F_T, 0\} & \text{for international teams.} \end{cases}$$

Assume $F_T > f$. Then, there exists a threshold team quality $q_T = F_T/A$ such that teams with $q \geq q_T$ export final goods, while teams with $q < q_T$ do not. Therefore, only the largest final producers producing expensive products can export final goods. Furthermore, by assuming that F_T is an idiosyncratic random variable that teams draw after their formation, the model shows that some large final producers do not trade as in Eaton et al. (2008) and Armenter and Koren (2009). Finally, because team profits are supermodular, all qualitative results for trade in intermediate goods discussed in the paper still hold.

7 Appendix 2: Proofs

Proof for the Positive Relation between Value-Added Per Worker and Quality of Firms The value-added per worker of final producers is expressed as:

$$\begin{aligned} V_X(x) &= \frac{3\pi_X(x)}{q_X(x)c + f} + 1 \\ V_X'(x) &= \frac{3\pi_X(x)q_X(x)c}{(q_X(x)c + f)^2}\left[\frac{\pi_X'(x)}{\pi_X(x)} - \frac{q_X'(x)}{q_X(x)} + \frac{\pi_X'(x)f}{q_X(x)c}\right]. \end{aligned}$$

Those of intermediate producers are expressed in similar forms. In autarky, $\pi_X'(x)/\pi_X(x) = q_X'(x)/q_X(x)$ holds from $\pi_X(x) = (f/3)[(x/x_L)^3 - 1]$ and $q_X(x) = \left(M_{Z_1e}M_{Z_2e}/(M_{Xe})^2\right)^{2/k} x^3$. In a trade equilibrium, if f is sufficiently large, high quality final producers show larger value-added per worker than low quality ones.

Proof for Lemma 1 First, I introduce the concept of exportability of intermediate producers.

Definition 1 *Home Z_i producers with quality z_i are called "exportable" if $\pi_{Z_i}(z_i) + f_T = \pi^*_{Z_i}(z_i)$ and "nonexportable" if $\pi_{Z_i}(z_i) + f_T > \pi^*_{Z_i}(z_i)$. Similarly, Foreign Z_i producers with quality z_i are called exportable if $\pi^*_{Z_i}(z_i) + f_T = \pi_{Z_i}(z_i)$ and nonexportable if $\pi^*_{Z_i}(z_i) + f_T > \pi_{Z_i}(z_i)$.*

The exportability is the necessary condition for intermediate producers to export. When Home Z_i producers with quality z_i are exportable, Foreign final producers are indifferent between Home Z_i producers with quality z_i and Foreign Z_i producers with the same quality. When Home Z_i producers with quality z_i are nonexportable, Foreign final producers strictly prefer Foreign Z_i producers with z_i to Home Z_i producers with the same quality. Therefore, nonexportable Z_i producers never export. Notice that the exportability is the necessary condition for exporting, but not a sufficient condition and that all Z_i producers are classified as either exportable or nonexportable. This is because neither $\pi_{Z_i}(z_i) + f_T < \pi^*_{Z_i}(z_i)$ nor $\pi^*_{Z_i}(z_i) + f_T < \pi_{Z_i}(z_i)$ holds; otherwise, e.g. in the former case, no final producer would choose Foreign Z_i producers with z_i.

The proof for Lemma 1 consists of claims 1 to 5.

Claim 1 *A Z_1 producer matches with a Z_2 producer with the same quality if either one of them is exportable.*

Proof. It is sufficient to consider the following two cases: (i) an exportable Home Z_i producer matches with a Foreign Z_j producer; (ii) an exportable Home Z_i producer matches with a Home Z_j producer.

Case (i): Suppose an exportable Home Z_i producer with z_i matches with a Foreign Z_j producer with $z_j \neq z_i$ and a Home final producer with x. From the stability condition, the final producer chooses optimal partners and receives:

$$\begin{aligned} & Axz_iz_j - \pi_{Z_i}(z_i) - \pi^*_{Z_j}(z_j) - f_T - f \\ \geq \; & \max_{z'_i, z'_j} Axz'_iz'_j - \pi_{Z_i}(z'_i) - \pi^*_{Z_j}(z'_j) - f_T - f \; . \end{aligned}$$

From the mirror-image structure of Home and Foreign, $\pi_{Z_i}(z_i) = \pi^*_{Z_j}(z_i)$. Then, the above inequality becomes:

$$\begin{aligned} & Axz_iz_j - \pi^*_{Z_j}(z_i) - \pi^*_{Z_j}(z_j) - f_T - f \\ \geq \; & \max_{z'_i, z'_j} Axz'_iz'_j - \pi^*_{Z_j}(z'_i) - \pi^*_{Z_j}(z'_j) - f_T - f \ . \end{aligned} \tag{28}$$

Because the second order condition for maximization (28) requires $\pi^{*\prime\prime}_{Z_j}(z) > 0$, $\bar{z} \equiv (z_i + z_j)/2$ satisfies

$$\begin{aligned} Axz_iz_j - \pi^*_{Z_j}(z_i) - \pi^*_{Z_j}(z_j) - f_T &< Ax(\bar{z})^2 - 2\pi^*_{Z_j}(\bar{z}) - f - f_T \\ &= Ax(\bar{z})^2 - \pi_{Z_i}(\bar{z}) - \pi^*_{Z_j}(\bar{z}) - f - f_T \text{ (from } \pi_{Z_i}(\bar{z}) = \pi^*_{Z_j}(\bar{z})\text{)}. \end{aligned}$$

The last inequality implies that the final producer is willing to form a new team with a Home Z_i producer with $\bar{z}$ and Foreign Z_j producer with $\bar{z}$, which contradicts stable matching. A similar proof is applied for the case that the two intermediate producers match with a foreign final producer.

Case (ii): an exportable Home Z_i producer with z_i matches with a Home Z_j producer with z_j. Suppose $z_i \neq z_j$ and they match with a Home final producer with x. The final producer chooses optimal partners and receives:

$$\begin{aligned} & Axz_iz_j - \pi_{Z_i}(z_i) - \pi_{Z_j}(z_j) - f \\ \geq \; & \max_{z'_i, z'_j} Axz'_iz'_j - \pi_{Z_i}(z'_i) - \pi_{Z_j}(z'_j) - f \ . \end{aligned} \tag{29}$$

From the mirror-image structure, Foreign Z_j producer with z_i is also exportable, i.e. $\pi^*_{Z_j}(z_i) + f_I = \pi_{Z_j}(z_i)$. Because $\pi_{Z_i}(z_i) = \pi^*_{Z_j}(z_i)$, this implies $\pi_{Z_i}(z_i) = \pi_{Z_j}(z_i) - f_I$. Then, the inequality (29) becomes:

$$\begin{aligned} & Axz_iz_j - \pi_{Z_j}(z_i) - \pi_{Z_j}(z_j) - f - f_I \\ \geq \; & \max_{z'_i, z'_j} Axz'_iz'_j - \pi_{Z_j}(z'_i) - \pi_{Z_j}(z'_j) - f - f_I \ . \end{aligned}$$

From the second order condition $\pi''_{Z_j}(z) > 0$, $\bar{z} \equiv (z_i + z_j)/2$ satisfies:

$$\begin{aligned} Axz_iz_j - \pi_{Z_j}(z_i) - \pi_{Z_j}(z_j) - f - f_T &< Ax(\bar{z})^2 - 2\pi_{Z_j}(\bar{z}) - f - f_T \\ &= Ax(\bar{z})^2 - \pi_{Z_i}(\bar{z}) - \pi_{Z_j}(\bar{z}) - f. \end{aligned}$$

The inequality implies that the Home final producer with x forms a new team with a Home Z_i producer with $\bar{z}$ and Home Z_j producer with $\bar{z}$, which contradicts stable matching. ■

Claim 2 *All Home Z_2 producers and Foreign Z_1 producers are nonexportable.*

Proof. Suppose Home Z_2 producers with quality z are exportable. Under the assortative one-to-one matching, the market clearing condition for matching between Home Z_1 producers and Home Z_2 producers is expressed as:

$$M_{Z_1 e}^a \int_z^\infty \theta_{Z_1}^D(t)\, g(t) dt = M_{Z_2 e}^a \int_z^\infty \theta_{Z_2}^D(t)\, g(t) dt, \tag{30}$$

where $\theta_{Z_i}^D(z_i)$ is the share of Home Z_i producers with quality z_i domestically matching with Home Z_j producers. By definition, $\theta_{Z_i}^D(z_i) < 1$ holds only when Home Z_i producers with quality z_i are exportable. From Claim 1, Home Z_i producers with quality z match with Home Z_j producers with the same quality.

Differentiation of (30) with respect to z leads to:

$$M_{Z_1 e}^a \theta_{Z_1}^D(z) = M_{Z_2 e}^a \theta_{Z_2}^D(z). \tag{31}$$

Notice that if Home Z_i producers with quality z are exportable, i.e. $\pi_{Z_i}^*(z) = \pi_{Z_i}(z) + f_T$, then Home Z_j producers with quality z are nonexportable because under the mirror-image structure, it follows that $\pi_{Z_j}(z) = \pi_{Z_j}^*(z) + f_T$. Therefore, only one of either $\theta_{Z_1}^D(z)$ or $\theta_{Z_2}^D(z)$ can be smaller than unity. From $M_{Z_1 e}^a > M_{Z_2 e}^a$, only a combination of $\theta_{Z_2}^D(z) = 1$ and $\theta_{Z_1}^D(z) = M_{Z_2 e}^a / M_{Z_1 e}^a < 1$ satisfies condition (31). Therefore, Home Z_2 producers are all nonexportable. From the mirror-image structure, Foreign Z_1 producers are also nonexportable. ■

Claim 3 $m_1(x) \geq m_2(x)$ *for all* $x \geq x_L$.

Proof. Because $\theta_{Z_2}^D(z) = 1$ for all $z \geq z_{2L}$ from Claim 2, the market clearing condition for matching between Home Z_1 producers and Home Z_2 producers (see (30)) becomes:

$$M_{Z_1 e}^a \int_{m_1(x)}^\infty \theta_{Z_1}^D(t)\, g(t) dt = M_{Z_2 e}^a [1 - G(m_2(x))] \text{ for all } x \geq x_L. \tag{32}$$

A straightforward manipulation yields:

$$\frac{M^a_{Z_1e}}{M^a_{Z_2e}}\int_{m_1(x)}^{\infty}\left(\theta^D_{Z_1}(t)-\frac{M^a_{Z_2e}}{M^a_{Z_1e}}\right)g(t)dt = G(m_1(x)) - G(m_2(x)) \text{ for all } x \geq x_L. \tag{33}$$

Because $\theta^D_{Z_1}(z) \geq M^a_{Z_2e}/M^a_{Z_1e}$ for all $x \geq x_L$ from the proof for Claim 2, the right-hand side of (33) is nonnegative for all $x \geq x_L$. ■

Claim 4 *There exists a threshold z_T: (i) $\pi^*_{Z_1}(z) - \pi_{Z_1}(z) = \pi_{Z_2}(z) - \pi^*_{Z_2}(z) = f_T$ for all $z \geq z_T$. (ii) $0 \leq \pi^*_{Z_1}(z) - \pi_{Z_1}(z) = \pi_{Z_2}(z) - \pi^*_{Z_2}(z) < f_T$ for all $z \in [z_{1L}, z_T)$.*

Proof. Consider two teams with bundles of quality parameters, (x, z_1, z_2) and (x', z'_1, z'_2), respectively. Suppose $z_2 = z'_1 (\equiv \hat{z})$. Claim 3 implies that $x \geq x'$ and $z_1 \geq z_2 = z'_1 \geq z'_2$. Therefore, from the first order conditions (7), we obtain:

$$\pi'_{Z_2}(\hat{z}) = Axz_1 \geq \pi'_{Z_1}(\hat{z}) = Ax'z'_2. \tag{34}$$

In a trade equilibrium, there exists some exportable Home Z_1 producer with quality $\tilde{z} \geq z_L$ such that $\pi^*_{Z_1}(\tilde{z}) - \pi_{Z_1}(\tilde{z}) = f_T$. Suppose there exists $z > \tilde{z}$ such that $\pi^*_{Z_1}(z) - \pi_{Z_1}(z) < f_T$ on the contrary. Because $\pi^*_{Z_1}(z) = \pi_{Z_2}(z)$ for all z, the difference in the profit schedules satisfies:

$$\begin{aligned}\pi^*_{Z_1}(z) - \pi_{Z_1}(z) &= \pi^*_{Z_1}(\tilde{z}) - \pi_{Z_1}(\tilde{z}) + \int_{\tilde{z}}^{z}\left[\pi^{*\prime}_{Z_1}(u) - \pi'_{Z_1}(u)\right]du \\ &= f_T + \int_{\tilde{z}}^{z}\left[\pi'_{Z_2}(u) - \pi'_{Z_1}(u)\right]du.\end{aligned}$$

The second term in the right-hand side must be nonnegative from (34), which contradicts $\pi^*_{Z_1}(z) - \pi_{Z_1}(z) < f_T$. Therefore, if $\pi^*_{Z_1}(z') - \pi_{Z_1}(z') = f_T$ holds for some z', then $\pi^*_{Z_1}(z) - \pi_{Z_1}(z) = f_T$ holds for all $z \geq z'$.

Notice that $z_{2L} = z^*_{1L} < z_{1L} = z^*_{2L}$ from $M^a_{Z_2e} > M^a_{Z_1e}$. From $\pi^*_{Z_1}(z) = \pi_{Z_2}(z)$ for all z, the difference in the profit schedules is:

$$\begin{aligned}\pi^*_{Z_1}(z) - \pi_{Z_1}(z) &= \pi^*_{Z_1}(z_{1L}) - \pi_{Z_1}(z_{1L}) + \int_{z^*_{1L}}^{z}\left[\pi^{*\prime}_{Z_1}(u) - \pi'_{Z_1}(u)\right]du \\ &= \begin{cases}\pi_{Z_2}(z_{1L}) + \int_{z_{1L}}^{z}\left[\pi'_{Z_2}(u) - \pi'_{Z_1}(u)\right]du & \text{if} \quad z \in [z_{1L}, z_T) \\ \int_{z_{2L}}^{z}\pi'_{Z_2}(u)\,du & \text{if} \quad z \in [z_{2L}, z_{1L}).\end{cases}\end{aligned}$$

From (34), $\pi^*_{Z_1}(z) - \pi_{Z_1}(z)$ is monotonically increasing in z and $\pi^*_{Z_1}(z_{2L}) - \pi_{Z_1}(z_{2L}) = 0$. Therefore, there exists a threshold z_T: $\pi^*_{Z_1}(z) - \pi_{Z_1}(z) = \pi_{Z_2}(z) - \pi^*_{Z_2}(z) = f_T$ for all $z \geq z_T$ and $\pi^*_{Z_1}(z) - \pi_{Z_1}(z) = \pi_{Z_2}(z) - \pi^*_{Z_2}(z) < f_T$ for all $z \in [z_{1L}, z_T)$. ■

Claim 5 *Matching functions for Home final producers $m_i(x)$ satisfy:*

$$\begin{aligned} \left(M^a_{Xe} + M^{*a}_{Xe}\right)[1 - G(x)] &= \left(M^a_{Z_ie} + M^{*a}_{Z_ie}\right)[1 - G(m_i(x_T))] \text{ for all } x \geq x_T \text{ and} && (35) \\ M^a_{Xe}[G(x_T) - G(x)] &= M^a_{Z_ie}[G(z_T) - G(m_i(x))] \text{ for all } x \in [x_L, x_T]. && (36) \end{aligned}$$

Matching functions for Foreign final producers satisfy similar conditions.

Proof. From Claims 3 and 4, Home Z_1 producers with $z_1 \geq z_T$ are exportable, but other Home Z_1 producers and Home Z_2 producers are nonexportable. Because $\theta^D_{Z_1}(z_1) = M^a_{Z_2e}/M^a_{Z_1e}$ for $z_1 \geq z_T$ from the proof for Claims 2, the market clearing condition for matching between Home final producers and Home Z_1 producers is:

$$\begin{aligned} M^a_{Xe}[1 - G(x)] &= M^a_{Z_1e}\int_{m_1(x)}^{\infty} \theta^D_{Z_1}(t)\, g(t)dt && (37) \\ &= M^a_{Z_2e}[1 - G(m_1(x))] \text{ for } x \geq x_T, && (38) \end{aligned}$$

while the condition between Foreign final producers and Foreign Z_1 producers is:

$$\begin{aligned} M^{a*}_{Xe}[1 - G(x)] &= M^{a*}_{Z_1e}[1 - G(m_1(x))] + M^a_{Z_1e}\int_{m_1(x)}^{\infty}\left(1 - \theta^D_{Z_1}(t)\right) g(t)dt \\ &= M^{a*}_{Z_1e}[1 - G(m_1(x))] + \left(M^a_{Z_1e} - M^a_{Z_2e}\right)[1 - G(m_1(x))] && (39) \end{aligned}$$

for $x \geq x_T$. Condition (35) is derived from (38) and (39). From the mirror-image structure, a similar condition holds for Z_2 producers. The market clearing condition for matching among low quality firms is given by (36). From the mirror-image structure, Foreign matching functions satisfy similar conditions. ■

Proof for Proposition 2 The ratio of the quality of final goods produced by a final producer with x in the short-run equilibrium to that in autarky is:

$$\frac{q_X(x)}{q^a_X(x)} = \begin{cases} \left(M^a_{Z_1e} + M^a_{Z_2e}\right)^{2k}\left(4M^a_{Z_1e}M^a_{Z_2e}\right)^{-k} \equiv K & \text{if} \quad x \geq x_T \\ \Theta(x) & \text{if} \quad x < x_T, \end{cases}$$

where

$$\Theta(x) \equiv K\left[1+\left\{1-\left(\frac{x}{x_T}\right)^{2k}\right\}\frac{\left(M^a_{Z_1e}-M^a_{Z_2e}\right)^2}{4M^a_{Z1e}M^a_{Z_2e}}\right]^{-1/k} < 1,$$

and $\Theta'(x) > 0$ and $\Theta''(x) > 0$. From (23) and (24), $m_i(x)$ satisfies:

$$\left(\frac{1}{m_i(x)}\right)^k = \frac{M^a_{Xe}}{M^a_{Z_ie}}\left(\frac{1}{x}\right)^k + \left(\frac{1}{z_T}\right)^k\left(\frac{M^a_{Z_ie}-M^a_{Z_je}}{2M^a_{Zie}}\right) \text{ if } x \leq x_T.$$

From $M^a_{Z_1e} > M^a_{Z_2e}$, it follows that

$$\begin{aligned}\left(\frac{1}{m_1\left(x^a_L\right)m_2\left(x^a_L\right)}\right)^k &= \frac{\left(M^a_{Xe}\right)^2}{M^a_{Z_1e}M^a_{Z_2e}}\left(\frac{1}{x^a_L}\right)^{2k} - \left(\frac{1}{z_T}\right)^{2k}\left(\frac{\left(M^a_{Z_1e}-M^a_{Z_2e}\right)^2}{4M^a_{Z1e}M^a_{Z_2e}}\right) \\ &< \frac{\left(M^a_{Xe}\right)^2}{M^a_{Z_1e}M^a_{Z_2e}}\left(\frac{1}{x^a_L}\right)^{2k} = \left(\frac{1}{z^a_{1L}z^a_{2L}}\right)^k.\end{aligned}$$

Therefore, $q_X\left(x^a_L\right) = x^a_L m_1\left(x^a_L\right)m_2\left(x^a_L\right) > x^a_L z^a_{1L}z^a_{2L} = q^a_X\left(x^a_L\right)$. Since $q_X(x)/q^a_X(x)$ is increasing in x, all final producers improve the quality of their final good. ■

Proof for Proposition 3 I show the minimum quality threshold determined by the selection effect is higher than the autarky threshold.

The new threshold is obtained from the labor market clearing condition for the residual labor subtracted from sunk entry costs. The residual labor supply is

$$L^S = \bar{L} - M^a_{Xe}f_{Xe} - M^a_{Z_1e}f_{Z1e} - M^a_{Z_2e}f_{Z2e} = \bar{L}\left(\frac{k\sigma-3}{k\sigma}\right).$$

Since Home and Foreign are symmetric, the labor demand by Home firms is equal to the labor demand from production of final goods that Home consumes

$$\begin{aligned}L^D &= M^a_{Xe}\left[1-G(x_L)\right]\left(\bar{c}\bar{q}_X\left(x_L\right)+f\right)/3 \\ &\quad + \sum_{i=1,2} M^a_{Z_ie}\left[1-G(z_{iL})\right]\left(\bar{c}\bar{q}_{Z_i}\left(z_{iL}\right)+f\right)/3 + M_T f_T,\end{aligned}$$

where $\bar{q}_X\left(x_L\right)$ and $\bar{q}_{Z_i}\left(z_{iL}\right)$ are the average quality of final goods and M_T is the mass of international teams. Since

$$\begin{aligned}2M^a_{Xe}\left[1-G(x_L)\right] &= M^a_{Z_1e}\left[1-G(z_{1L})\right] + M^a_{Z_2e}\left[1-G(z_{2L})\right] \text{ and} \\ 2M^a_{Xe}\left[1-G(x_L)\right]\bar{q}_X\left(x_L\right) &= \sum_{i=1,2} M^a_{Z_ie}\left[1-G(z_{1L})\right]\bar{q}_{Z_i}\left(z_{iL}\right),\end{aligned}$$

the labor demand is rewritten as

$$
\begin{aligned}
L^D &= M_{Xe}^a [1 - G(x_L)] (\bar{c}\bar{q}_X(x_L) + f) + M_T f_T \\
&= \bar{L}\left(\frac{k\sigma - 3}{k\sigma}\right)\left[1 + \left(\frac{(\sigma-1)(k-3)}{k\sigma - 3}\right)\left(\frac{\bar{q}_X(x_L)}{q_X(x_L)} - \frac{k}{k-3}\right)\right]\left(\frac{x_L^a}{x_L}\right) + M_T f_T
\end{aligned}
$$

Define

$$
\Phi(x_L) \equiv \left(\frac{x_L}{x_L^a}\right)^k \text{ and } \Psi(x_L) \equiv 1 + \left(\frac{(\sigma-1)(k-3)}{k\sigma - 3}\right)\left(\frac{\bar{q}_X(x_L)}{q_X(x_L)} - \frac{k}{k-3}\right).
$$

Define $q_X(x) = x m_1(x) m_2(x)$ even for $x < x_L$, $z'_{1L} \equiv m_1(x_L^a)$ and $z'_{2L} \equiv m_2(x_L^a)$ from matching functions (24). The average quality is

$$
\begin{aligned}
\bar{q}(x_L^a) &= \frac{1}{1 - G(x_L^a)}\left[\int_{x_L^a}^{\infty}\left(\frac{z'_{1L}z'_{2L}}{(x_L^a)^2}\right)x^3 g(x)dx + \int_{x_L^a}^{\infty}\left[q_X(x) - \left(\frac{z'_{1L}z'_{2L}}{(x_L^a)^2}\right)x^3\right]g(x)dx\right] \\
&= \left(\frac{k}{k-3}\right)q_X(x_L^a) + \frac{1}{1 - G(x_L^a)}\int_{x_L^a}^{\infty}\left[q_X(x) - \left(\frac{z'_{1L}z'_{2L}}{(x_L^a)^2}\right)x^3\right]g(x)dx \\
&> \left(\frac{k}{k-3}\right)q_X(x_L^a). \qquad (40)
\end{aligned}
$$

The inequality is from $q_X(x) \geq \left(z'_{1L}z'_{2L}/(x_L^a)^2\right)x^3$ for all $x \geq x_L^a$. Therefore, $1 = \Phi(x_L^a) < \Psi(x_L^a)$ and $x_L > x_L^a$. ■

Proof for Proposition 4 The value-added per worker for final producers in the liberalized industry is rewritten as

$$
V_X(x) = 3\xi_X(x)[V(q_X(x)) - 1] + 1, \qquad (41)
$$

where $\xi_X(x) = \Pi(q_X(x))/\pi_X(x)$ is the profit share of final producers in their teams and $V(q_X(x)) \equiv [1 + \Pi(q(x))/(q(x)c + f)]$ is the value-added per worker of their teams. The value-added per worker of final producers in autarky $V_X^a(x)$ and that of those in the non-liberalized industry under trade $\tilde{V}_X(x)$ are given by similar equations. I show that there exists a threshold $\hat{x} \in (x_L, x_T)$ such that

$$
V_X(x) - V_X^a(x) > \gamma \equiv \int_{\tilde{x}_L}^{\infty}[V_X(x) - V_X^a(x)]\left(\frac{g(x)}{1 - G(\tilde{x}_L)}\right)dx \text{ for all } x \geq \tilde{x},
$$

where γ expresses the average change in the value-added per workers of final producers in the non-liberalized industry.[30]

[30] Alternatively, γ can be defined by the integral over a range $[x_L^a, \infty)$.

The proof consists of the following seven claims.

Claim 6 *Let* $\eta_i(x) \equiv x m'_{Z_i}(x)/m_i(x)$*. (i) In autarky,* $\eta_i(x) = 1$ *for all* $x \geq x_L$*. (ii) Under trade,*

$$\eta_i(x) = 1 \text{ if } x > x_T \text{ and } \eta_i(x) = \frac{M_{Xe}[1 - G(x)]}{M_{Z_ie}[1 - G(m_i(x))]} \text{ if } x \in [x_L, x_T).$$

Proof. From (27), $\eta_i(x) = 1$ for $x > x_T$. From (22), $m_i(x)$ satisfies:

$$\left(\frac{1}{m_i(x)}\right)^k = \frac{M_{Xe}}{M_{Z_ie}}\left(\frac{1}{x}\right)^k + \left(\frac{1}{z_T}\right)^k - \frac{M_{Xe}}{M_{Z_ie}}\left(\frac{1}{x_T}\right)^k \quad \text{for } x \in [x_L, x_T].$$

From the implicit function theorem, $\eta_i(x)$ is solved as:

$$\eta_i(x) = \frac{M_{Xe}}{M_{Z_ie}}\left(\frac{m_i(x)}{x}\right)^k = \frac{M_{Xe}[1 - G(x)]}{M_{Z_ie}[1 - G(m_i(x))]} \quad \text{for } x \in [x_L, x_T]. \tag{42}$$

■

By using this claim, I prove properties of firms' profit shares.

Claim 7 *(i) In autarky,* $\xi_X^a(x) = 1/3$ *for all* $x \geq x_L^a$*. (ii) Under trade,* $\xi_X(x) < 1/3$ *for all* $x \geq x_L$*,* $\xi'_X(x) > 0$ *and* $\lim_{x\to\infty} \xi_X(x) = 1$ *in the liberalized industry and* $\tilde{\xi}_X(x) = 1/3$ *for all* $x \geq \tilde{x}_L$ *in the non-liberalized industry.*

Proof. (i) Both in autarky and under trade, the profit schedules of firms are rewritten as:

$$\pi_X(x) = A\int_{x_L}^{x} m_1(t)\, m_2(t)\, dt \text{ and } \pi_{Z_i}(m_i(x)) = A\int_{x_L}^{x} m_1(t)\, m_2(t)\, \eta_i(t) dt. \tag{43}$$

Because $\eta_1(x) = \eta_2(x) = 1$ in autarky and in the non-liberalized industry from Claim 6, $\xi_X^a(x) = 1/3$ for all $x \geq x_L^a$ and $\tilde{\xi}_X(x) = 1/3$ for all $x \geq \tilde{x}_L$.

From (43), the profit share of final producers in the liberalized industry is

$$\begin{aligned} \xi_X(x) &= \frac{\pi_X(x)}{\pi_X(x) + \pi_{Z_1}(m_1(x)) + \pi_{Z_2}(m_2(x))} \\ &= \int_{x_L}^{x} m_1(t)\, m_2(t)\, dt \Bigg/ \left[\int_{x_L}^{x} m_1(t)\, m_2(t)\, (1 + \eta_1(x) + \eta_2(x))\, dt\right]. \end{aligned} \tag{44}$$

For $x \leq x_T$, from Claim 6, $\eta_1(x)$ and $\eta_2(x)$ satisfy

$$\eta_1(x) + \eta_2(x) = \frac{1}{1 + \delta(x)} + \frac{1}{1 - \delta(x)} = \frac{2}{1 - (\delta(x))^2} > 2. \tag{45}$$

where $\delta(x) \equiv M_T/M_{Xe}(1 - G(x)) < 1$. Therefore, $\xi_X(x) < 1/3$ for all $x \geq x_L$. Since $\eta'_1(x) + \eta'_2(x) < 0$ and $\eta_1(x) + \eta_2(x) = 2$ for all $x \geq x_T$, $\xi'_X(x) > 0$ and $\lim_{x\to\infty} \xi_X(x) = 1$. ■

Claim 8 *The average gross profit of team rises after the opening of trade:* $A\bar{q}_X > A^a\bar{q}_X^a$.

Proof. From (8), it follows that

$$\begin{aligned} A\bar{q}_X &= f\frac{\bar{q}_X}{q_L} = \frac{kf}{k-3} + \frac{f}{q_L\left[1-G(x_L)\right]}\int_{x_L}^{\infty}\left[q_X(t) - \left(\frac{z_{1L}z_{2L}}{(x_L)^2}\right)t^3\right]g(t)dt \\ &> \frac{kf}{k-3} = f\frac{\bar{q}_X^a}{q_L^a} = A^a\bar{q}_X^a, \end{aligned}$$

where $q_L = q_X(x_L)$ and $q_L^a = q_X^a(x_L)$ are the lowest quality of surviving teams in a short-run trade equilibrium and in autarky. The inequality holds because $q_X(x) > x^3(z_{1L}z_{2L})/(x_L)^2$ holds for all $x > x_L$ from Proposition 2 and $x_L > x_L^a$ from Proposition 3. ■

The next claim is that high quality teams improve their measured productivity.

Claim 9 *There exists a threshold* $x' \in (x_L, x_T)$ *such that* $V(q(x)) > V^a(q^a(x))$ *for all* $x \geq x'$.

Proof. Suppose $V(q(x)) < V^a(q^a(x))$ for all $x \geq x_T$. The value-added per worker of teams in the liberalized industry can be written as:

$$V(q(x)) = \frac{\Pi(q_X(x))}{(\sigma-1)\Pi(q_X(x)) + \sigma f} + 1.$$

Then, it follows that

$$V(q(x)) - V^a(q^a(x)) = \frac{\sigma\left(Aq_X(x) - A^a q_X^a(x)\right)}{\left[(\sigma-1)\Pi(q_X(x)) + \sigma f\right]\left[(\sigma-1)\Pi^a\left(q_X^a(x)\right) + \sigma f\right]}. \tag{46}$$

Therefore, $Aq(x) < A^a q_X^a(x)$ for all $x \geq x_T$. Since

$$Aq_X(x) = Aq_X(x_T)\left(\frac{x}{x_T}\right)^3 \text{ and } A^a q_X^a(x) = A^a q_X^a(x_T)\left(\frac{x}{x_T}\right)^3 \text{ for all } x \geq x_T,$$

$Aq_X(x) \leq A^a q_X^a(x)$ must hold for all $x \geq x_T$. Notice that from Proposition 2,

$$q_X'(x) - q_X^{a\prime}(x) = m_1(x)m_2(x)\left[1 + \eta_1(x) + \eta_2(x)\right] - 3m_1^a(x)m_2^a(x) > 0 \text{ for all } x \geq x_L.$$

Therefore, $Aq_X(x) \leq A^a q_X^a(x)$ holds for all $x \geq x_L$. Since $x_L > x_L^a$, $A\bar{q}_X < A^a\bar{q}_X^a$, which contradicts with Claim 8. ■

Claim 10 *All final producers in the non-liberalized industry reduces measured productivity if* $\tilde{x}_L > x_L^a$.

Proof. Notice that in the non-liberalized industry, matching patterns do not change after the opening of trade, i.e. $\tilde{q}_X(x) = q_X^a(x)$. From $\tilde{\xi}_X(x) = \xi_X^a(x) = 1/3$ for all x, it follows that

$$\tilde{V}(\tilde{q}_X(x)) - V^a(q_X^a(x)) = \frac{\sigma q_X^a(x)\left(\tilde{A} - A^a\right)}{\left[(\sigma-1)\tilde{\Pi}\left(q_X^a(x)\right) + \sigma f\right]\left[(\sigma-1)\Pi^a\left(q_X^a(x)\right) + \sigma f\right]}. \tag{47}$$

From $\tilde{A}q^a(\tilde{x}_L) = A^a q^a(x_L^a) = f$, the claim holds. ∎

Claim 11

$$[1 - G(x_L)]\frac{\bar{q}_X}{q_L} - [1 - G(x_L^a)]\frac{\bar{q}_X^a}{q_L^a} \leq G(x_L) - G(x_L^a) - \frac{f_T}{f}[1 - G(x_T)] \tag{48}$$

Proof. Notice that in the long-run equilibrium, the aggregate profit including entry fixed costs is maximized and equal to zero for a given A, which is exogenous to individual firms, because of the frictionless matching and the free entry. The amount of entry in a short-run equilibrium, which was determined in autarky, is suboptimal under trade in the long run. Therefore, the aggregate profit in a short-run equilibrium including sunk entry costs is not positive.

$$\begin{aligned} AM\bar{q}_X - Mf - M_T f_T &\leq M_{Xe}^a f_{Xe} + M_{Z1e}^a f_{Z1e} + M_{Z2e}^a f_{Z2e} \\ &= A^a M^a \bar{q}_X^a - M^a f. \end{aligned} \tag{49}$$

From $A = f/q_L$, dividing inequality (49) by $M_{Xe}f$ leads to:

$$[1 - G(x_L)]\frac{\bar{q}_X}{q_L} - [1 - G(x_L^a)]\frac{\bar{q}_X^a}{q_L^a} \leq G(x_L) - G(x_L^a) - \frac{f_T}{f}[1 - G(x_T)].$$

the inequality (48). ∎

Claim 12 *All final producers in the non-liberalized industry reduces measured productivity; therefore,* $\gamma < 1$.

Proof. The thresholds x_L and $\tilde{x}_L$ are determined by two conditions. The first condition is the labor market clearing condition. From $c = (\sigma - 1)A = (\sigma - 1)f/q_L$, where the labor demand by the liberalized industry L and that by the non-liberalized industry $\tilde{L}$ are given by:

$$\begin{aligned} L &= M(c\bar{q}_X + f) + M_T f_T = Mf\left((\sigma-1)\frac{\bar{q}_X}{q_L} + 1\right) + M_T f_T \text{ and} \\ \tilde{L} &= \tilde{M}\left(\tilde{c}\tilde{\bar{q}}_X + f\right) = \tilde{M}f\left((\sigma-1)\frac{\tilde{\bar{q}}_X}{\tilde{q}_L} + 1\right). \end{aligned}$$

Similarly, the labor demand by each industry in autarky L^a is given by

$$L^a = M^a\left(c^a\bar{q}_X^a + f\right) = M^a f\left((\sigma-1)\frac{\bar{q}_X^a}{q_L^a}+1\right).$$

The market clearing condition $L + L = 2L^a$ is equivalent with

$$\begin{aligned} & [1-G(x_L)]\left((\sigma-1)\frac{\bar{q}_X}{q_L}+1\right)+[1-G(\tilde{x}_L)]\left((\sigma-1)\frac{\bar{\tilde{q}}_X}{\tilde{q}_L}+1\right) \\ = \;& 2\left[1-G(x_L^a)\right]\left((\sigma-1)\frac{\bar{q}_X^a}{q_L^a}+1\right)-\frac{f_T}{f}\left[1-G(x_T)\right]. \end{aligned} \tag{50}$$

Equation (50) implicitly determines a combination of x_L and $\tilde{x}_L$ satisfying $x_L = \chi(\tilde{x}_L)$ and $\chi'(\tilde{x}_L) < 0$.

The second condition is the ratio of output of individual firms in the two industry:

$$\frac{c}{\tilde{c}} = \left(\frac{P}{\tilde{P}}\right)^{\sigma-\varepsilon}$$

Since $c = (\sigma-1)A = (\sigma-1)f/q_L$ and $\rho P = \left(M_{Xe}\int_{x_L}^{\infty} q_X(x)\, g(x)dx\right)^{1/(1-\sigma)}$, this equation can be rewritten as:

$$\frac{q_L}{\tilde{q}_L} = \left(\frac{[1-G(x_L)]\,\bar{q}_X}{[1-G(\tilde{x}_L)]\,\bar{\tilde{q}}_X}\right)^{(\sigma-\varepsilon)/(\sigma-1)}, \tag{51}$$

Equation (51) implicitly determines a combination of x_L and $\tilde{x}_L$ satisfying $x_L = \psi(\tilde{x}_L)$ and $\psi'(\tilde{x}_L) > 0$. Equation (51) is rewritten as:

$$[1-G(x_L)]\left(\frac{\bar{q}_X}{q_L}\right) = [1-G(\tilde{x}_L)]\left(\frac{\bar{\tilde{q}}}{\tilde{q}_L}\right)\left(\frac{q_L}{\tilde{q}_L}\right)^{\gamma} \tag{52}$$

where $\gamma = (\varepsilon-1)/(\sigma-\varepsilon) > 0$.

Suppose $\tilde{x}_L = \tilde{x}_L^a$. Since $\psi'(\tilde{x}_L) > 0 > \chi'(\tilde{x}_L)$, it is sufficient to show $\psi(x_L^a) < \chi(x_L^a)$ to prove $\tilde{x}_L > x_L^a$. The condition (50) can be rewritten as:

$$\begin{aligned} & [1-G(x_L)]\frac{\bar{q}_X}{q_L}-[1-G(x_L^a)]\frac{\bar{q}_X^a}{q_L^a} \\ = \;& -\frac{1}{(\sigma-1)}\left[G(x_L^a)-G(x_L)+\frac{f_T}{f}[1-G(x_T)]\right] \\ \leq \;& \left[G(x_L^a)-G(x_L)+\frac{f_T}{f}[1-G(x_T)]\right] \end{aligned}$$

The inequality is from Claim 11. Therefore, $G(x_L^a) - G(x_L) + \frac{f_T}{f}[1-G(x_T)] \geq 0$ and

$$[1-G(\chi(x_L^a))]\left.\frac{\bar{q}_X}{q_L}\right|_{x_L=\chi(x_L^a)} \leq [1-G(x_L^a)]\frac{\bar{q}_X^a}{q_L^a}.$$

On the other hand, since $(q_L/\tilde{q}_L)^{\gamma} > 1$ and $\tilde{x}_L = \tilde{x}_L^a$, the condition (52) with implies

$$[1 - G(\psi(x_L^a))] \left. \frac{\bar{q}_X}{q_L} \right|_{x_L = \psi(x_L^a)} > [1 - G(x_L^a)] \frac{\bar{q}_X^a}{q_L^a}.$$

Since $[1 - G(x_L)]\,\bar{q}_X = \int_{x_L}^{\infty} q_X(x)\,g(x)dx$ is decreasing in x_L, $\psi(x_L^a) < \chi(x_L^a)$ and $\tilde{x}_L > x_L^a$. Hence, from Claim 10, all final producers in the non-liberalized industry reduces measured productivity and $\gamma < 1$. ■

Claim 13 *There exists $\hat{x}$ such that $V_X(x) - V_X^a(x) > \gamma$ for all $x > \hat{x}$.*

From (41) and $3\xi_X^a(x) = 1$, it follows that

$$\begin{aligned} & V_X(x) - V_X^a(x) - \gamma \\ = \; & [3\xi_X(x) - \gamma]\,[V(q_X(x)) - 1] + V(q_X(x)) - V^a(q_X^a(x)). \end{aligned}$$

From $\xi_X'(x) > 0$, $\lim_{x\to\infty} \xi_X(x) = 1$, and Claim 12, there exists a threshold $\hat{x}$ such that $V_X(x) - V_X^a(x) > \gamma$ for all $x > \hat{x}$.

Proof for Lemma 6 (i) From $k[1 - G(x)] = xg(x)$ and integration by parts, the free entry condition can be rewritten as:

$$\begin{aligned} \frac{f_{Xe}}{1 - G(x_L)} &= \frac{1}{1 - G(x_L)} \int_{x_L}^{\infty} \pi_X(t)\,g(t)dt \\ &= \frac{1}{1 - G(x_L)} \int_{x_L}^{\infty} \pi_X'(t)\,[1 - G(t)]\,dt \\ &= \frac{A}{k[1 - G(x_L)]} \int_{x_L}^{\infty} t m_1(t)\,m_2(t)\,g(t)\,dt \\ &= \frac{A}{k}\bar{q}_X, \end{aligned} \tag{53}$$

where $\bar{q}_X$ is the average quality of final goods. Because $A = \bar{L}/(\sigma M \bar{q})$ from the aggregate zero profit, it follows that:

$$M_{Xe} = \frac{M}{1 - G(x_L)} = \frac{\bar{L}}{f_{Xe} k \sigma} = M_{Xe}^a.$$

(ii)(iii) The proof for (2) and (3) consists of claims 6 to 15.

Claim 14 *The mass of entrants satisfies:*

$$M_{Z_1e}f_{Z_1e} + M_{Z_2e}f_{Z_2e} = 2M_{Xe}f_{Xe}. \tag{54}$$

Proof. From integration by parts and the first order condition, the free entry condition for final producers is:

$$\begin{aligned} f_{Xe} &= \int_{x_L}^{\infty} \pi_X(t)\, g(t)dt \\ &= \int_{x_L}^{\infty} \pi'_X(t)\,[1-G(t)]\,dt \\ &= A\int_{x_L}^{\infty} m_1(t)\, m_2(t)\,[1-G(t)]\,dt. \end{aligned} \tag{55}$$

From claim 6, the free entry condition for Z_i producers is:

$$\begin{aligned} f_{Z_ie} &= \int_{x_L}^{\infty} \pi_{Z_i}(m_i(t))\, m'_i(t)\, g(t)dt \\ &= A\int_{x_L}^{\infty} m_1(t)\, m_2(t)\, \eta_i(t)\,[1-G(m_i(t))]\,dt \qquad (56) \\ &= A\int_{x_T}^{\infty} m_1(t)\, m_2(t)\,[1-G(m_i(t))]\,dt \\ &\quad + \frac{M_{Xe}}{M_{Z_ie}} A\int_{x_L}^{x_T} m_1(t)\, m_2(t)\,[1-G(t)]\,dt. \qquad (57) \end{aligned}$$

Because $2M_{Xe}[1-G(x)] = \sum_{i=1,2} M_{Z_ie}[1-G(m_i(x))]$ for $x \geq x_L$, it follows that:

$$\begin{aligned} M_{Z_1e}f_{Z_1e} + M_{Z_2e}f_{Z_2e} &= 2M_{Xe}A\int_{x_L}^{\infty} m_1(t)\, m_2(t)\,[1-G(t)]\,dt \\ &= 2M_{Xe}f_{Xe} \text{ (from (55))}. \end{aligned}$$

■

Claim 15

$$M_{Z_1e} > M^a_{Z_1e} > M^a_{Z_2e} > M_{Z_2e} \text{ and } \frac{M_{Z_1e}+M_{Z_2e}}{2M_{Xe}} > \frac{M^a_{Z_1e}+M^a_{Z_2e}}{2M^a_{Xe}}.$$

Proof. From $f_{Z_1e}/f_{Xe} = M^a_{Xe}/M^a_{Z_1e}$, (11) and (12), condition (55) is rewritten as:

$$\begin{aligned} f_{Z_1e} = f_{Xe}\frac{f_{Z_1e}}{f_{Xe}} = &\quad A\int_{x_T}^{\infty} m_1(t)\, m_2(t)\left(\frac{M^a_{Xe}}{M^a_{Z_1e}}\right)[1-G(t)]\,dt \\ &+ \frac{M^a_{Xe}}{M^a_{Z_1e}} A\int_{x_L}^{x_T} m_1(t)\, m_2(t)\,[1-G(t)]\,dt \\ = &\quad A\int_{x_T}^{\infty} m_{Z1}(t)\, m_{Z2}(t)\,[1-G(m^a_{Z1}(t))]\,dt \\ &+ \frac{M^a_{Xe}}{M^a_{Z1e}} A\int_{x_L}^{x_T} m_{Z1}(t)\, m_{Z2}(t)\,[1-G(t)]\,dt \end{aligned} \tag{58}$$

Because $m_1^a(x) > m_1(x)$ for $x \geq x_T$, the comparison of (57) and (58) proves that $M_{Z_1e} > M_{Z_1e}^a$. From (54), we also obtain $M_{Z_2e}^a > M_{Z_2e}$. Under the constraint of (54), it follows that:

$$\frac{M_{Z_1e} + M_{Z_2e}}{2M_{Xe}} > \frac{M_{Z_1e}^a + M_{Z_2e}^a}{2M_{Xe}^a} \text{ since } \frac{M_{Z_1e}}{M_{Xe}} > \frac{M_{Z_1e}^a}{M_{Xe}^a}.$$

■

Proof for Proposition 7 By defining $\tilde{x}_1$, $\tilde{x}_2$, $\tilde{z}_1$, and $\tilde{z}_2$ such that $\tilde{z}_1 = m_1(\tilde{x}_1) = m_1^a(\tilde{x}_1)$ and $\tilde{z}_2 = m_2(\tilde{x}_2) = m_2^a(\tilde{x}_2)$, I will show $\tilde{x}_1 = \tilde{x}_2 \equiv \tilde{x}$ and $\tilde{x} \geq x_L$. From (27), $\tilde{x}_1$ and $\tilde{x}_2$ are smaller than x_T. By definition, these variables satisfy:

$$\begin{aligned}
M_{Xe}[1 - G(\tilde{x}_1)] &= M_{Z_1e}[1 - G(\tilde{z}_1)] - M_T \\
&= M_{Z_1e}^a[1 - G(\tilde{z}_1)] \text{ and} \\
M_{Xe}[1 - G(\tilde{x}_2)] &= M_{Z_2e}[1 - G(\tilde{z}_2)] + M_T \\
&= M_{Z_2e}^a[1 - G(\tilde{z}_2)],
\end{aligned}$$

where M_T is the mass of exporting Home Z_1 producers and Foreign Z_2 producers. From straightforward manipulations, we obtain:

$$\begin{aligned}
M_T &= M_{Z_1e}^a[1 - G(\tilde{z}_1)]\left(\frac{M_{Z_1e}}{M_{Z_1e}^a}\right) - M_{Xe}[1 - G(\tilde{x}_1)] \\
&= M_{Xe}[1 - G(\tilde{x}_1)]\left[\frac{M_{Z_1e}}{M_{Z_1e}^a} - 1\right],
\end{aligned}$$

and

$$\begin{aligned}
M_T &= M_{Xe}[1 - G(\tilde{x}_2)] - M_{Z_2e}^a[1 - G(\tilde{z}_2)]\left(\frac{M_{Z_2e}}{M_{Z_2e}^a}\right) \\
&= M_{Xe}[1 - G(\tilde{x}_2)]\left[1 - \left(\frac{M_{Z_2e}}{M_{Z_2e}^a}\right)\right].
\end{aligned}$$

From (11) and (54), we have:

$$\begin{aligned}
\left(\frac{M_{Z_1e}}{M_{Z_1e}^a} - 1\right) - \left(1 - \frac{M_{Z_2e}}{M_{Z_2e}^a}\right) &= \frac{M_{Z_1e}}{M_{Xe}}\frac{M_{Xe}^a}{M_{Z_1e}^a} + \frac{M_{Z_2e}}{M_{Xe}}\frac{M_{Xe}^a}{M_{Z_2e}^a} - 2 \\
&= \frac{M_{Z_1e}}{M_{Xe}}\frac{f_{Z_1e}}{f_{Xe}} + \frac{M_{Z_2e}}{M_{Xe}}\frac{f_{Z_2e}}{f_{Xe}} - 2 \\
&= 0.
\end{aligned}$$

Therefore, $\tilde{x}_1 = \tilde{x}_2 \equiv \tilde{x}$. From the proof of Proposition 8 below, $z_{2L} < z_{2L}^a$ and $x_L > x_L^a$; therefore, $\tilde{x} > x_L$.■

Proof for Proposition 8 The minimum quality threshold for final producers is derived from the labor market clearing condition:

$$\bar{L} = L_e + L_p + L_T,$$

where L_e, L_p, and L_T are labor demands for entry costs, production, and trade costs, respectively. First, from Claim 14 and $M_{Xe} = M_{Xe}^a$, the labor demand for entry costs is the same as in autarky:

$$\begin{aligned} L_e &\equiv M_{Xe} f_{Xe} + M_{Z_1e} f_{Z_1e} + M_{Z_2e} f_{Z_2e} \\ &= 3M_{Xe}^a f_{Xe} = L_e^a. \end{aligned}$$

Second, the labor demand for production is given by:

$$L_p = M_{Xe} \left[1 - G\left(x_L\right)\right] \left(\bar{c}\bar{q}_X + f\right)/3 + \sum_{i=1,2} M_{Z_ie} \left[1 - G\left(z_{iL}\right)\right] \left(\bar{c}\bar{q}_{Z_i} + f\right)/3,$$

where $\bar{q}_X$ and $\bar{q}_{Z_i}$ are the average team quality of final producers and Z_i producers. From $c = (\sigma - 1) A$, (53) and similar derivations for Z_i producers, we have $\bar{c}\bar{q}_X = k(\sigma - 1)\left[1 - G\left(x_L\right)\right] \bar{\pi}_X$ and $\bar{c}\bar{q}_{Z_i} = k(\sigma - 1)\left[1 - G\left(z_{iL}\right)\right] \bar{\pi}_{Z_i}$. From the free entry conditions and $M_{Xe}\left[1 - G\left(x_L\right)\right] = \sum_{i=1,2} M_{Z_ie}\left[1 - G\left(z_{iL}\right)\right]$, L_p becomes:

$$\begin{aligned} L_p &= M_{Xe} \left[1 - G\left(x_L\right)\right] f/3 + \sum_{i=1,2} M_{Z_ie} \left[1 - G\left(z_{iL}\right)\right] f/3 \\ &\quad + k(\sigma - 1)\left[M_{Xe} f_{Xe} + M_{Z_1e} f_{Z_1e} + M_{Z_2e} f_{Z_2e}\right]/3 \\ &= M_{Xe}^a \left[1 - G\left(x_L\right)\right] f + k(\sigma - 1) L_e^a/3. \end{aligned}$$

From the labor demand for trade costs $L_T = M_T f_T$ and (17), the labor market clearing condition is:

$$\begin{aligned} \bar{L} &= L_e^a + M_{Xe}^a \left[1 - G\left(x_L\right)\right] f + k(\sigma - 1) L_e^a/3 + M_T f_T \\ &\Leftrightarrow M_X^a \left[1 - G\left(x_L^a\right)\right] f = M_{Xe}^a \left[1 - G\left(x_L\right)\right] f + M_T f_T. \end{aligned}$$

Because $M_T > 0$, $x_L > x_L^a$.

The minimum quality thresholds for Z_i producers are obtained from their free entry conditions. Let $\bar{\pi}_{Z_i}\left(z_{iL}\right)$ be the average profit of Z_i producers when the lowest quality threshold is z_{iL}. From integration by parts and the first order condition, $\bar{\pi}_{Z_i}(z_{iL})$ is:

$$\begin{aligned}\bar{\pi}_{Z_i}(z_{iL}) &= \int_{z_{iL}}^{\infty} \pi'_{Z_i}(t)\left(\frac{1-G(t)}{1-G(z_{iL})}\right)dt \\ &= A\int_{z_{iL}}^{\infty} m_i^{-1}(t)\, m_j\left(m_i^{-1}(t)\right)\left(\frac{1-G(t)}{1-G(z_{iL})}\right)dt,\end{aligned}$$

where $m_1(\cdot)$ is the inverse function of $m_1(\cdot)$. From the cutoff condition (4), it follows that:

$$\begin{aligned}\pi_{Z_1}(z_{1L}) &= A\int_{z_{1L}}^{\infty} \frac{x_L z_{2L} t^2}{(z_{1L})^2}\left(\frac{1-G(t)}{1-G(z_{1L})}\right)dt \\ &\quad + A\int_{z_{1L}}^{\infty}\left[m_1^{-1}(t)\, m_2\left(m_1^{-1}(t)\right) - \frac{x_L z_{2L} t^2}{(z_{1L})^2}\right]\left(\frac{1-G(t)}{1-G(z_{1L})}\right)dt \\ &= \frac{f}{k-3} + A\int_{z_{1L}}^{\infty}\left[m_1^{-1}(t)\, m_2\left(m_1^{-1}(t)\right) - \frac{x_L z_{2L} t^2}{(z_{1L})^2}\right]\left(\frac{1-G(t)}{1-G(z_{1L})}\right)dt.\end{aligned}$$

From the matching functions (23) and (24), it follows that if $z_{1L} < z_T$, $m_1^{-1}(z_1)\, m_2\left(m_1^{-1}(z_1)\right) > \left(z_{1L} z_{2L} z_1^2\right)/(z_{1L})^2$ for all $z_1 \geq z_{1L}$, while if $z_{1L} \geq z_T$, $m_1^{-1}(z_1)\, m_2\left(m_1^{-1}(z_1)\right) = \left(z_{1L} z_{2L} z_1^2\right)/(z_{1L})^2$ for all $z_1 \geq z_{1L}$. Therefore, we have:

$$\bar{\pi}_{Z_1}(z_{1L}^a) > \frac{f}{k-3} = \frac{f_{Z_1 e}}{1-G\left(z_{1L}^a\right)} \text{ and } \bar{\pi}_{Z_1}(z_T) = \frac{f}{k-3} < \frac{f_{Z_1 e}}{1-G(z_T)}.$$

Therefore, there exists $z_{1L} \in (z_{1L}^a, z_T)$ such that $\bar{\pi}_{Z_1}(z_{1L}) = f_{Z_1 e}\left[1-G(z_{1L})\right]^{-1}$. Similarly:

$$\begin{aligned}\bar{\pi}_{Z_2}(z_{2L}) &= A\int_{z_{2L}}^{\infty} \frac{x_L z_{1L} t^2}{(z_{2L})^2}\left(\frac{1-G(t)}{1-G(z_{2L})}\right)dt \\ &\quad + A\int_{z_{2L}}^{\infty}\left[m_2^{-1}(t)\, m_1\left(m_2^{-1}(t)\right) - \frac{x_L z_{1L} t^2}{(z_{2L})^2}\right]\left(\frac{1-G(t)}{1-G(z_{2L})}\right)dt \\ &= \frac{f}{k-3} + A\int_{z_{2L}}^{\infty}\left[m_2^{-1}(t)\, m_1\left(m_2^{-1}(t)\right) - \frac{x_L z_{1L} t^2}{(z_{2L})^2}\right]\left(\frac{1-G(t)}{1-G(z_{2L})}\right)dt.\end{aligned}$$

From the matching functions (23) and (24), it follows that if $z_{1L} < z_T$, $m_2^{-1}(z_2)\, m_1\left(m_2^{-1}(z_2)\right) < \left(x_L z_{1L} z_2^2\right)/(z_{2L})^2$ for all $z_2 \geq z_{2L}$, while if $z_{1L} \geq z_T$, $m_2^{-1}(z_2)\, m_1\left(m_2^{-1}(z_2)\right) = \left(x_L z_{1L} z_2^2\right)/(z_{2L})^2$ for all $z_2 \geq z_{2L}$. Therefore, we have:

$$\bar{\pi}_{Z_2}(z_{2L}^a) \leq \frac{f}{k-3} = \frac{f_{Z_2 e}}{1-G\left(z_{2L}^a\right)} \leq \frac{f_{Z_2 e}}{1-G(z_{2L})} \text{ if } z_{2L} \geq z_{2L}^a.$$

Therefore, $z_{2L} < z_{2L}^a$ must hold. ■

			Models							
	Stylized facts	Key empirical papers	Melitz (2003)	Bernard et al. (2003)	Eaton et al. (2008)	Verhoogen (2004,2008) Baldwin-Harrigan (2007)	Kugler-Verhoogen (2008)	Hallak-Sivadasan (2009)	Kasahara-Lapham (2007) Halpern et al. (2009)	This paper
1	Exporters are larger and more productive than nonexporters.	Bernard-Jensen (1995) Roberts-Tybout (1997)	●	●	●	●	●	●	●* (Final goods)	●
2	Liberalization raises the aggregate productivity through reallocation among firms.	Pavcknik (2002) Trefler (2004)	●	●	●	●	●	●	●* (Final goods)	●
3	Many large and productive firms do not export.	Bernard et al. (2003) Hallak-Sivadasan (2009)		●	●			●		●
4	Exporters produce more expensive goods than nonexporters.	Baldwin-Harrigan (2007) Bernard et al. (2007) Johnson (2008) Kugler-Verhoogen (2008) Mandel (2009) Manova-Zhang (2009)				●	●	●		●
5	Output prices and input prices are correlated with firm size.	Kugler-Verhoogen (2008)					●			●
6	Importers are larger and more productive than nonimporters.	Bernard et al. (2007, 2009) Kasahara-Lapham (2007) Halpern et al. (2009)							●* (Intermediates)	●
7	In liberalization of trade in intermediates, importers receive larger productivity gains than nonimporters.	Amiti-Konings (2007)							●* (Intermediates)	●
8	In liberalization of trade in intermediates, nonimporters improve productivity.	Amiti-Konings (2007)								●
9	Importers use more expensive intermediates than nonimporters.	Kugler-Verhoogen (2009)								●

● indicates stylized facts explained by the model in each column. *Kasahara and Lapham (2007) explain facts on exporters of final goods and importers of intermediates in two different markets; Halpern, Koren and Szeidl (2009) do not explain facts about exporting firms.

Table 1: Facts, empirical papers, and models in the literature

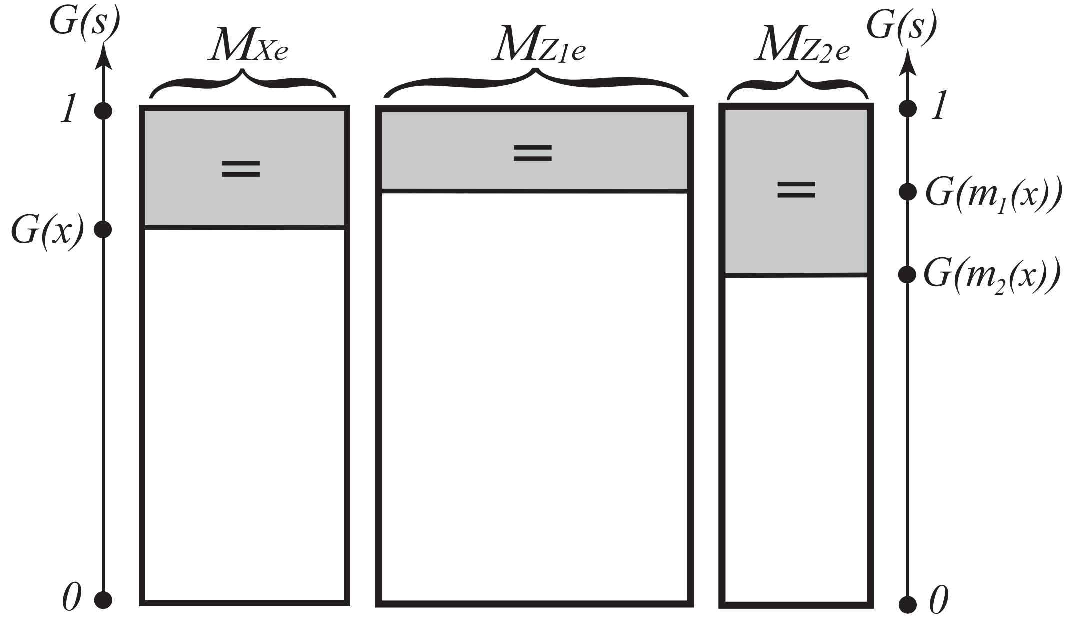

Figure 1: Matching patterns reflect the relative size of entrants across sectors. A producer from a sector with more entrants has higher quality in a team.

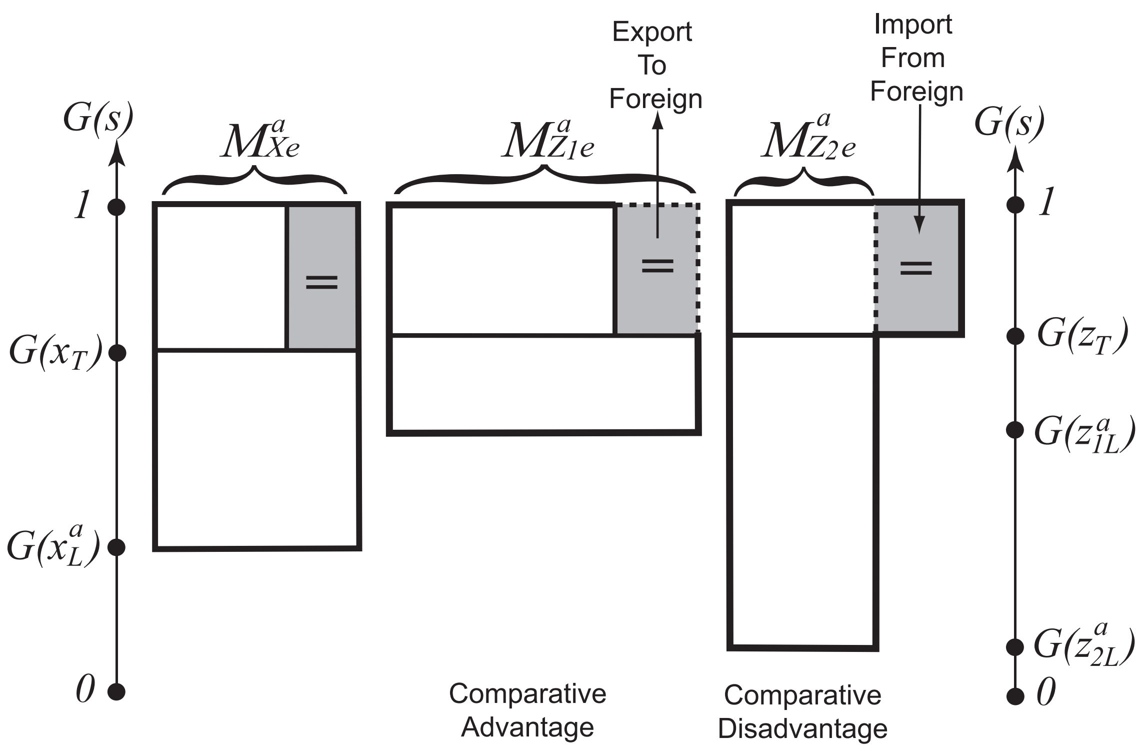

Figure 2: Importers and Exporters. Areas I and E express the distributions of Home importers and Home exporters, respectively. While only the highest quality firms engage in exporting and importing, many of them do not trade.

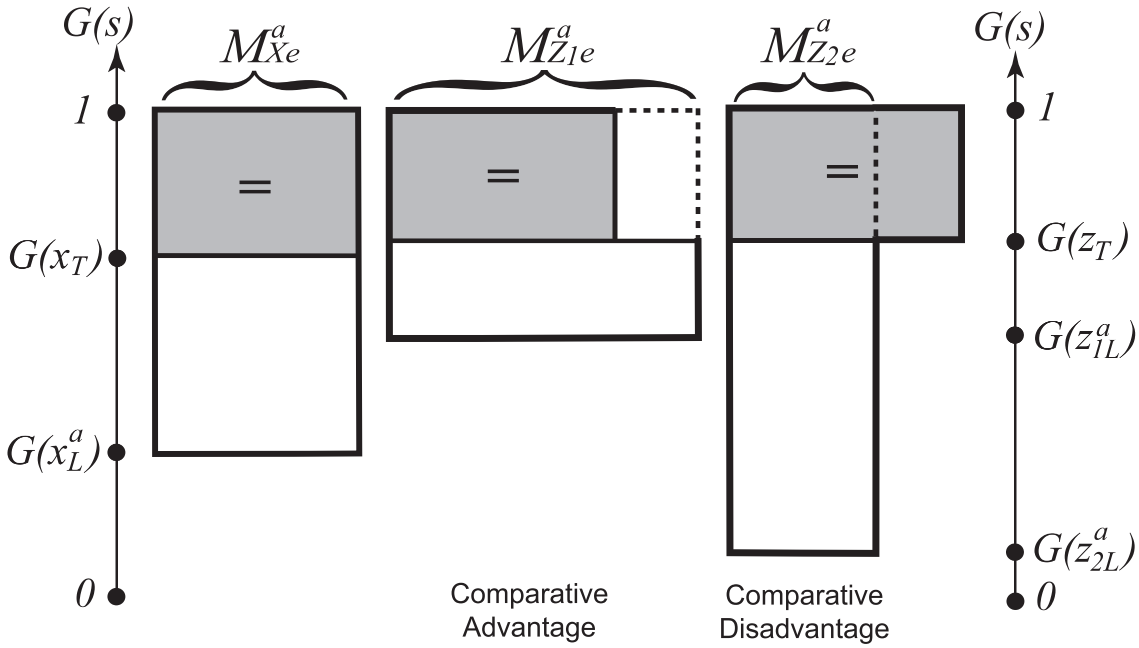

Figure 3: High quality firms in the Home matching market. In a trade equilibrium, the distributions of high quality Z_1 producers and Z_2 producers in the Home matching market are equalized.

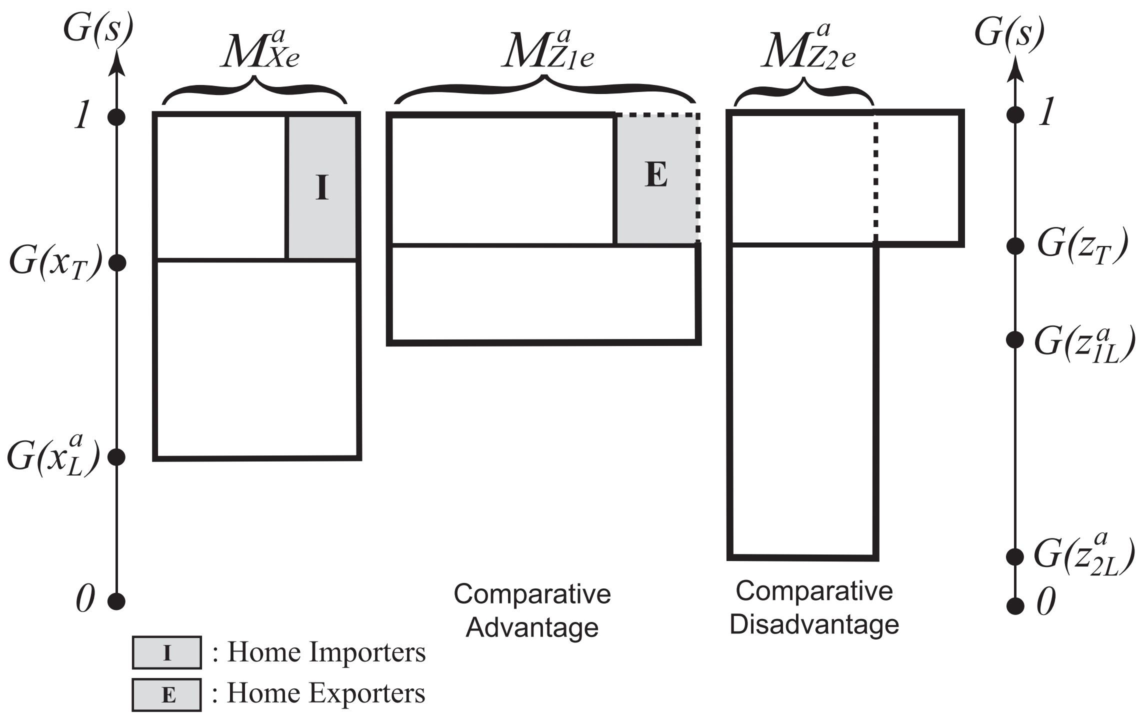

Figure 4: Importers and Exporters. Areas I and E express the distributions of Home importers and Home exporters, respectively. While only the highest quality firms engage in exporting and importing, many of them do not trade.

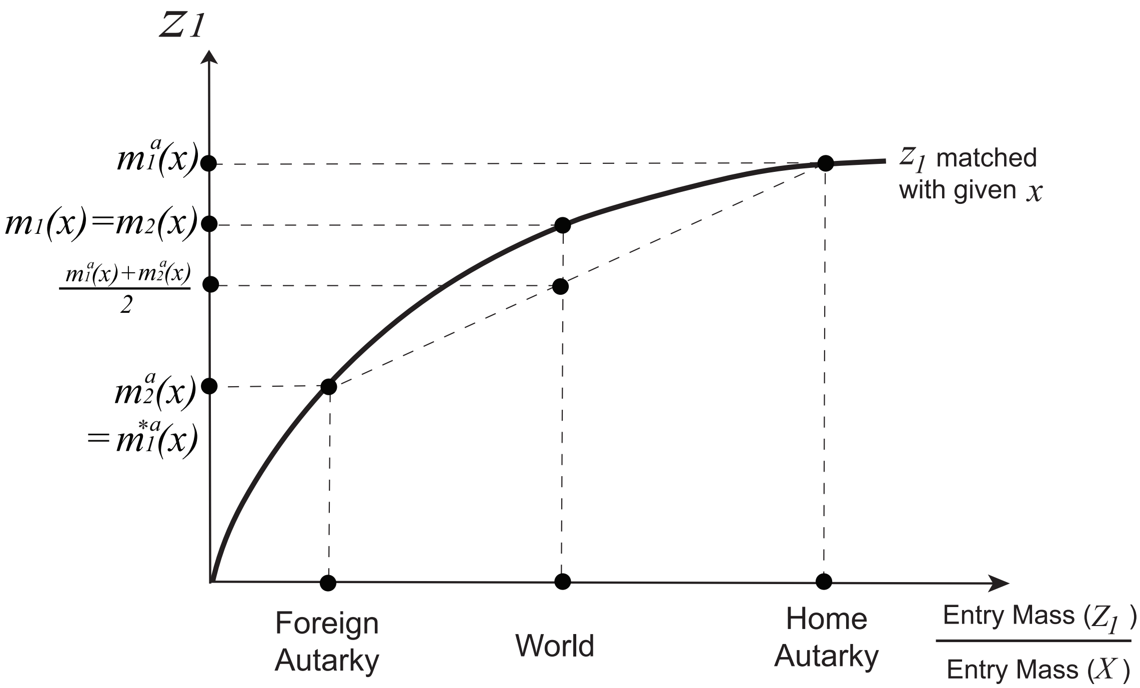

Figure 5: The quality of Z_1 producers matched with final producers with given $x \geq x_T$ in three regimes: Home autarky, Foreign autarky and a short run trade equilibrium. The quality of matched Z_1 producers increases in the mass of entrants of Z_1 producers relative to final producers in each of the three regimes. Under trade, a final producer matches Z_i producers with the same quality. They are higher quality than the average of two intermediate producers in autarky.

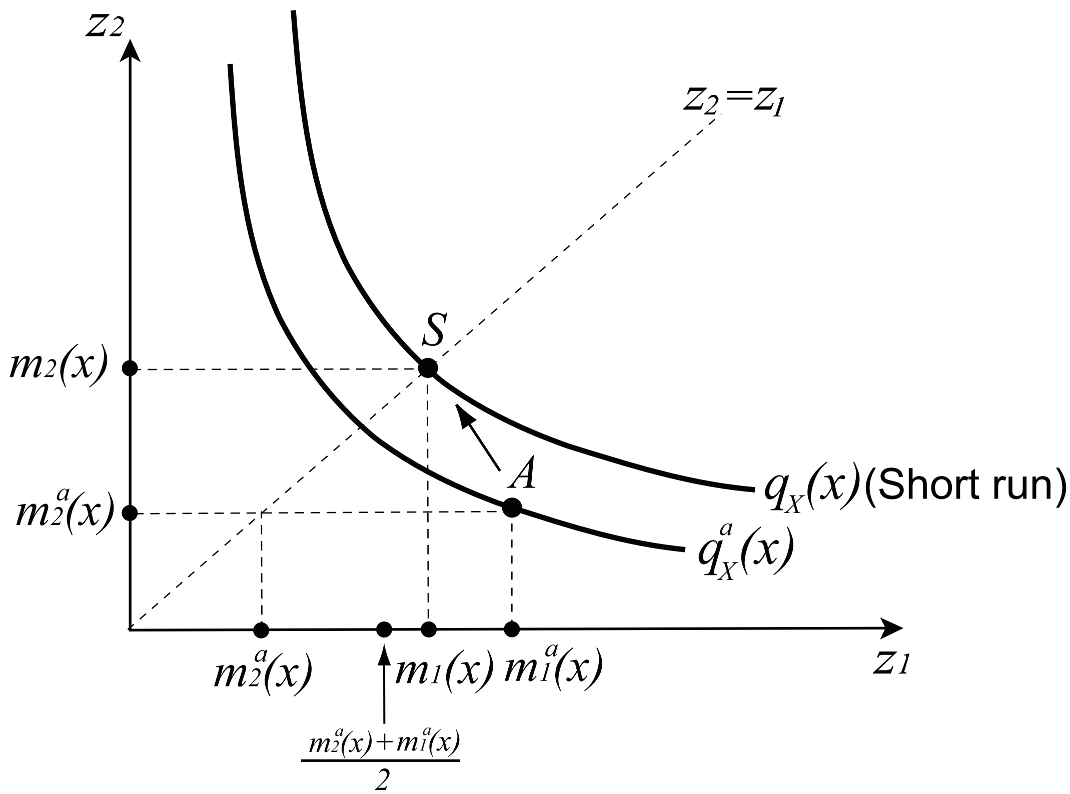

Figure 6: The short-run quality improvement of high quality final goods with $x \geq x_T$. Under trade, a final producer matches with Z_i producers with the same quality (Point S). From Figure 4, they are higher quality than the average of two intermediate producers in autarky (Point A); therefore, the quality of final goods rises from $q_X^a(x)$ to $q_X(x)$.

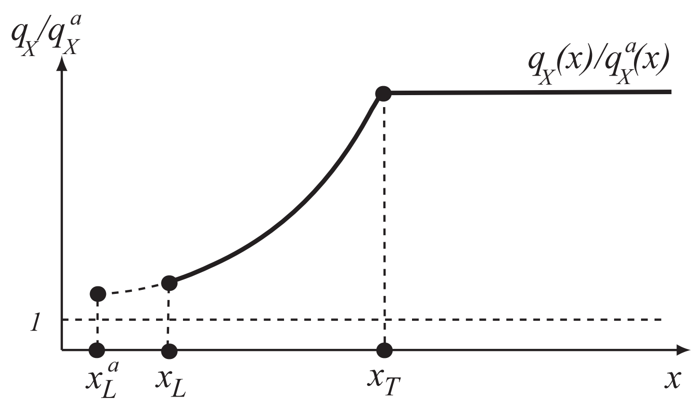

Figure 7: The quality of final goods produced by final producers in a short-run trade equilibrium relative to in autarky. All final producers improve the quality of final goods. The improvement is weakly increasing in the quality of firms.

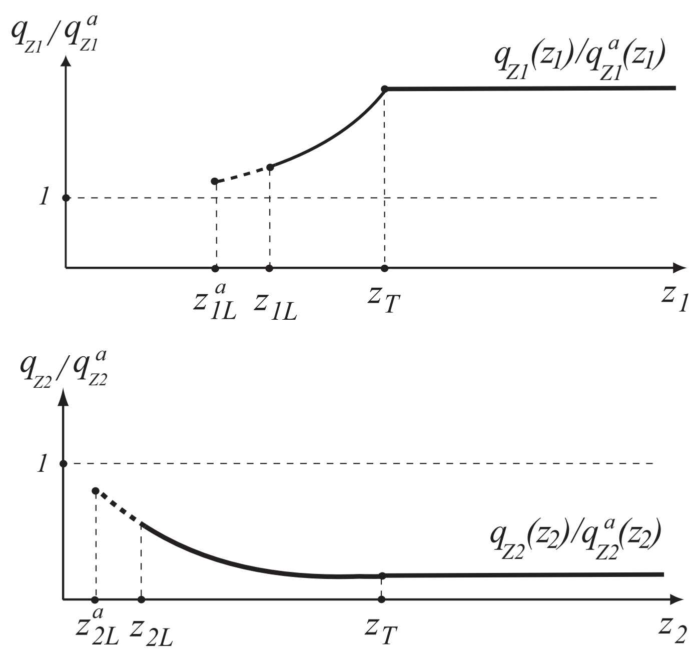

Figure 8: The team quality of Home intermediate producers in a short-run trade equilibrium relative to in autarky. All producers gain team quality in the comparative disadvantage sector and lose in the comparative disadvantage sector. The gains and losses are weakly increasing in the quality of firms.

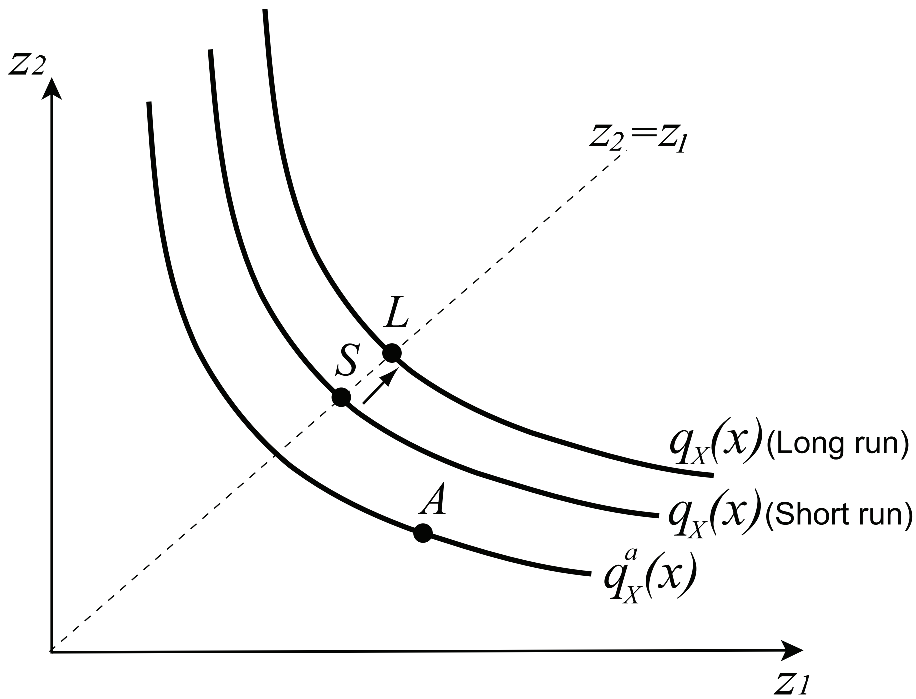

Figure 9: The long-run quality improvement of high quality final goods. The quality of intermediate producers rises in the long run to Point L.

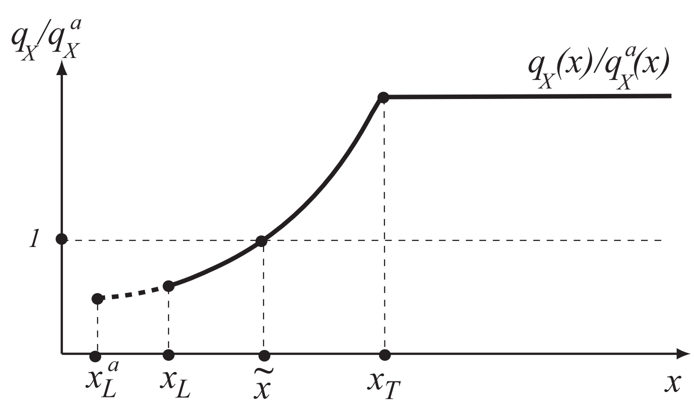

Figure 10: The quality of final goods in a long-run trade equilibrium relative to autarky. There exists the threshold quality for quality gains and losses.

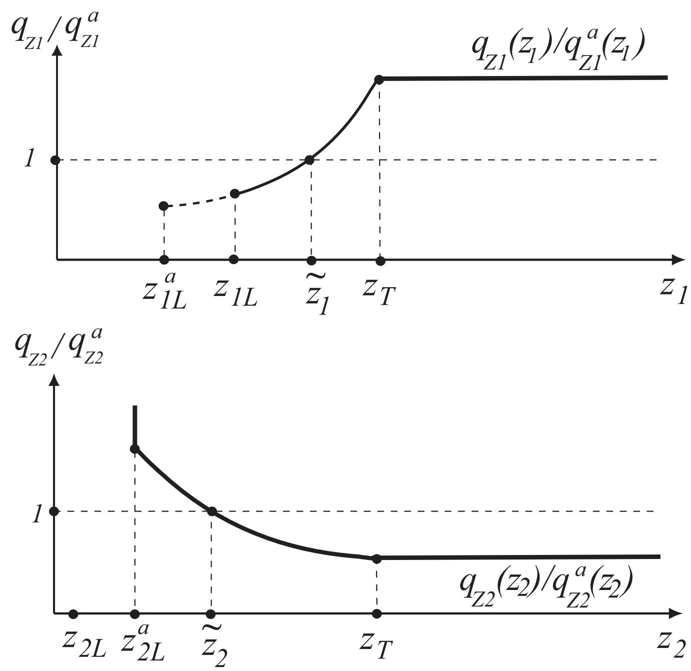

Figure 11: The team quality of Home intermediate producers in a long-run trade equilibrium relative to in autarky. There exist the threshold quality levels for gains and losses.