ISOM 2700: Operations Management

Session 1. Introduction to Operations Management

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Course Contents

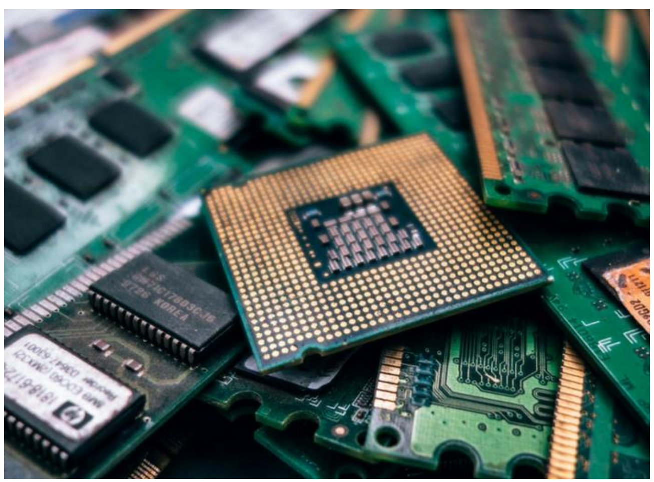
- Module I: Managing Process and Service System
  - Identify bottlenecks, handle variability, improve performance
- Module II: Quality and Resource Allocation
  - Measure quality, allocate resources, and make decisions
- Module III: Matching Supply with Demand
  - Manage inventory, benefit of centralization and pooling
- Module IV: Managing Supply Chain
  - Coordinate different players in supply chain by contract design

So…
What is Operations Management

Briefly Speaking, …

Operations Management:

To manage how a firm creates its products and/or services



Examples:

Intel – CPU

HSKH Medical – healthcare services

Ernst & Young – accounting services

HSBC – financial services

HKUST – education service

What is Operations Management?

Operations management is about:

- designing, analyzing, and improving …
- the whole transformation process …
- with the objective of creating and delivering the firm's products and services

What is Operations Management?

- This still sounds a bit abstract, isn't it?
- A better way is to think of operations management as the "physics" of the business world…
- Of course, it is not as difficult as physics, at least for this course :)

Key Features of OM

- Generality
- Abstraction in modeling
- Quantitative analysis
- Practical relevance

Features of OM: Generality

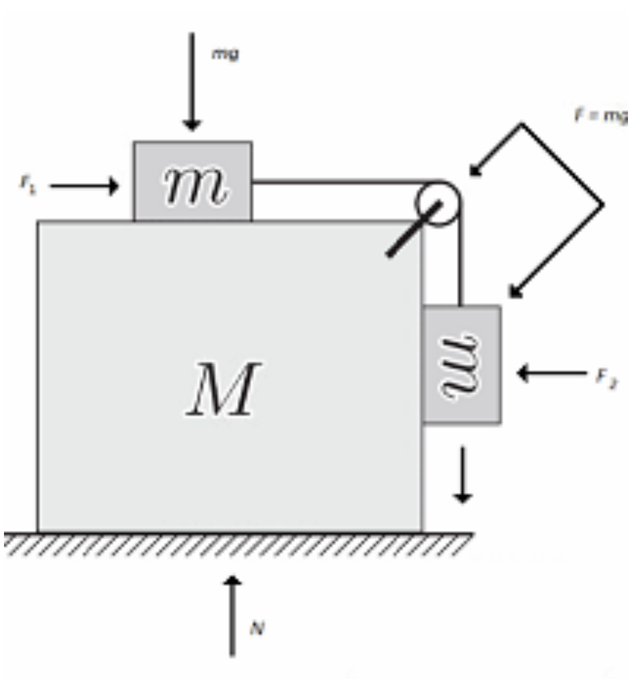
- Generality: the results from OM apply to many business settings
  - They can be regarded as "physics laws" in business world

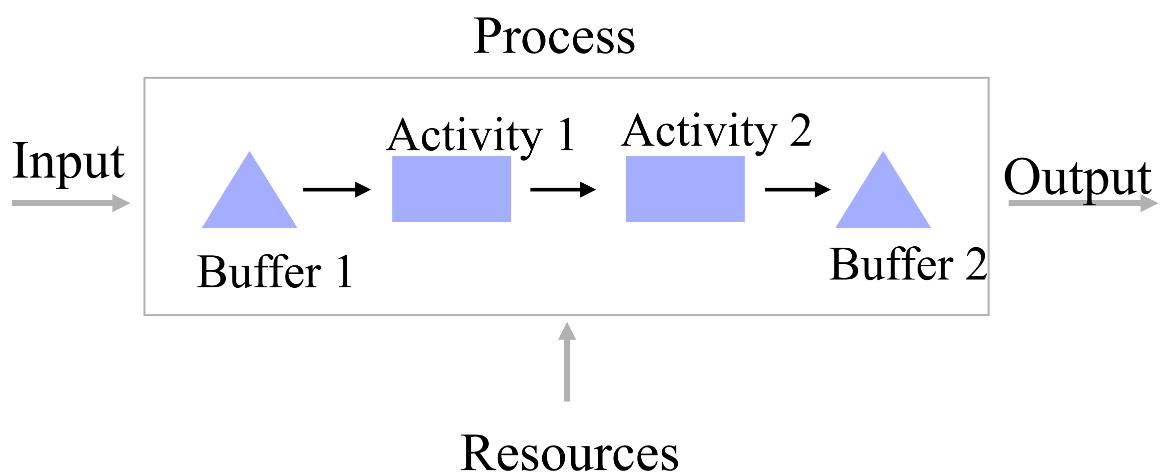
- For example, Little's Law tells you how to connect flow time, flow rate, and inventory level
  - You can use this for restaurants, colleges, company recruitment, emergency departments,…

- Newsvendor model tells you how to determine production and ordering level under uncertainty
  - You can use this for supermarkets, coffee shops, factories, supply chains, ride hailing platforms, operating rooms,…

Features of OM: Abstraction

- Abstraction in modeling: OM captures the key decision-making step and trade-offs from a complex setting
  - Then apply the results back in a proper way
  - Abstractions are needed --- it brings convenience but also limitation





Features of OM: Quantitativeness

- Quantitative analysis: math tools are needed for OM
- Relatively easy in this course, but can be difficult if you want to study more in this area (e.g., a data analytics job)
  - pillars of OM: stochastic analysis, optimization, probabilities & statistics
- In general, math is not the "bottleneck" for this course
  - once you figure out the correct "picture (story)", applying the formulas should be straightforward

Features of OM: Relevance

- Practical relevance: OM focuses on real-world business problems and has substantial implications on humans
- For example, during the pandemic, you may need to decide how to allocate ICU beds for emergent patients
    - Should we admit current patients or save bed for the future?
    - Life-or-death consequence for patients
- Good business acumen is needed: what are the key trade-offs? What are the managerial insights?
- Fairness, equity, and social responsibility also matter in addition to profit maximization

What do we do in OM?

- Problem-solving framework: Description + Optimization
- Description: understand the dynamics and performance of the actual system (like physics)
- E.g., what is the expected waiting time in the queue? What is the average inventory level in the system? What is our expected revenue given the demand level?
- Optimization: find ways to improve the system performance (like engineering)
- E.g., how many inventory should we order to maximize expected revenue? If we hire another staff, what job should be assigned to reduce waiting time?

Why we need OM: Reality is Complex

- Business problems in real world can be very complex, thus require deep understanding and powerful tools to solve

Suppose you run a taco store, what kinds of decisions you need to make?

Possible Decisions in a Taco Store

- Capacity: how many customers can we serve? what is the current “bottleneck”? Should we hire a new staff or buy a new machine?
- Demand: how many customers arrive per day? What is the intraday variation? Should we advertise to increase demand?
- Pricing: should we increase the price of our product? Will we make more money if we do so? How will our competitor respond?
- Inventory: how many inventory should we buy and hold each day? Can we sell them all before they deteriorate?
- Supply chain: is there risk related to our suppliers? should we find a new supplier? Should we share information and profit with them?

More Complexity for Big Business

Take a look at the operations of Heathrow Airport

Source:
Wendover Production

Video

Operations Management is Everywhere

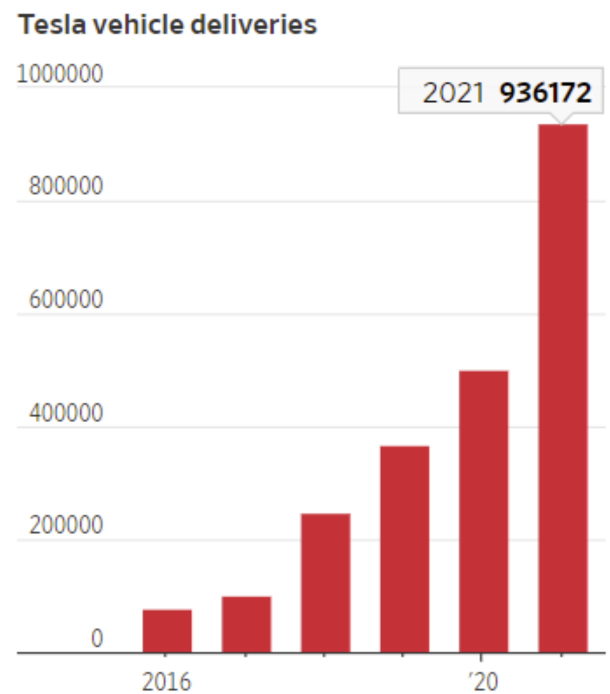
+ many more

Importance of OM

BUSINESS

Tesla Reports 87% Growth in Annual Vehicle Deliveries

The electric-vehicle maker's expansion comes despite parts shortages and other logistical problems that have hamstrung the global auto industry



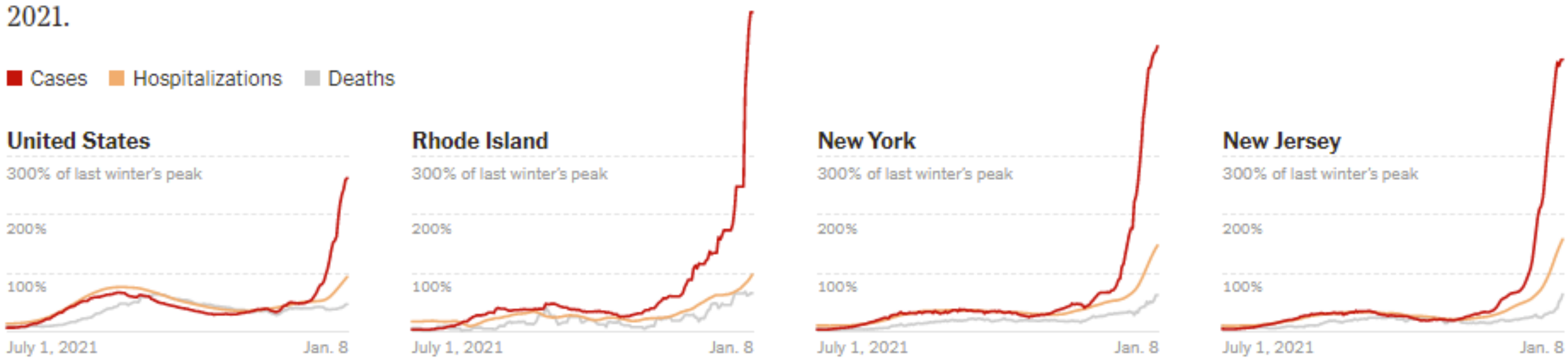
Source:
[The Wall Street Journal]

Importance of OM

ICU Beds in Shortage during COVID-19

How cases, hospitalizations and deaths are trending

Each chart shows how these three metrics compare to the corresponding peak level reached nationwide last winter. For example, a state's case line exceeds 100 percent on the chart when its number of cases per capita exceeds the highest number of U.S. cases per capita reached in January 2021.



Source: [The New York Times]

Importance of OM

TECH

U.S. Sanctions Cut Huawei's Revenue for Fourth Straight Quarter

Chinese telecom giant continues to suffer from plunging smartphone sales due to U.S. restrictions on suppliers

Source:
The Wall Street Journal

Importance of OM

Southwest Airlines Co + Add to myFT

Southwest Airlines operation failures during blizzard to cost up to $825mn

Outdated technology led to widespread disruption during December snowstorm while US rivals recovered fairly quickly

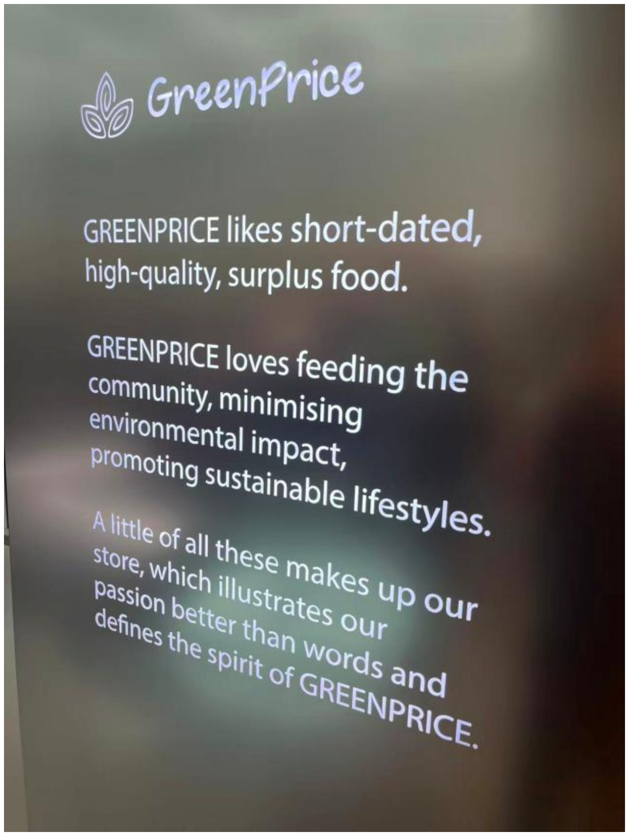
Failures with the flight network system used by Southwest Airlines meant thousands of passengers had to sleep in airports © David Zalubowski/AP

Source: [Financial Times]

Pricing Strategy of Restaurants

Restaurants set different prices for same product at same time on the delivery app and in the restaurant --- Why they do this?

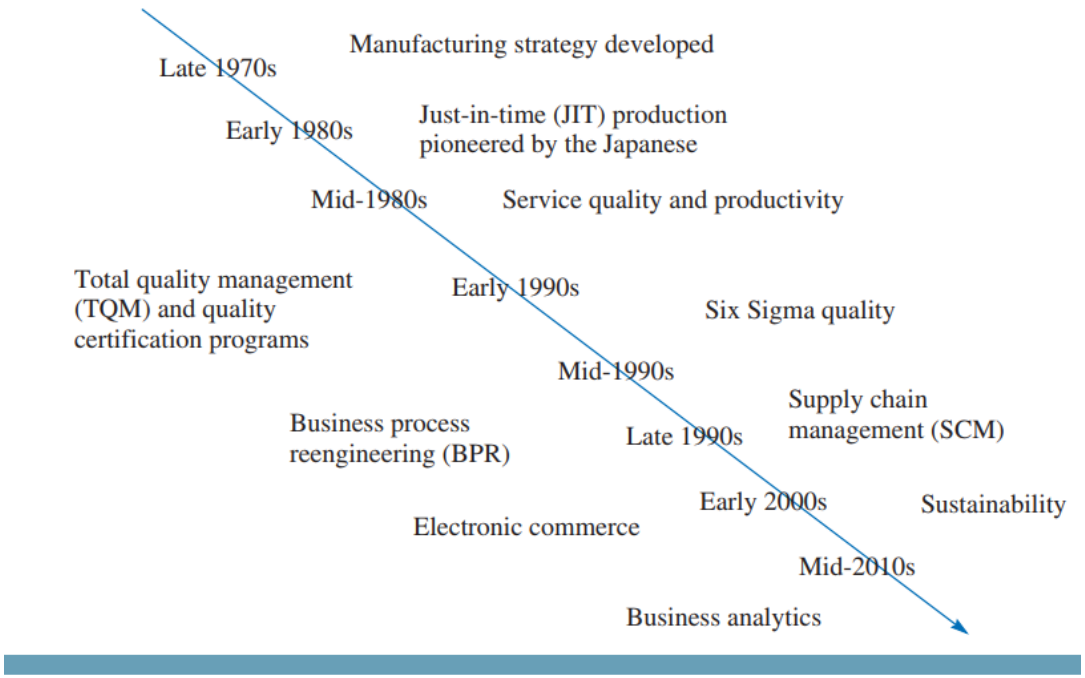
Popularity of GreenPrice



GreenPrice sells short-dated surplus food and cosmetics. What is the rationale behind it? What are its strategies and impacts?

How OM Shaped Today's Business World

exhibit 1.6 Time Line Depicting When Major OSCM Concepts Became Popular



- Manufacturing strategy: trade-off between performance measures
- JIT: high-volume production with minimal inventory and defects
- BPR: innovation by eliminating non-value-added activities
- Business analytics: data-driven decision making with advanced tools
- Sustainability: stakeholders instead of shareholders

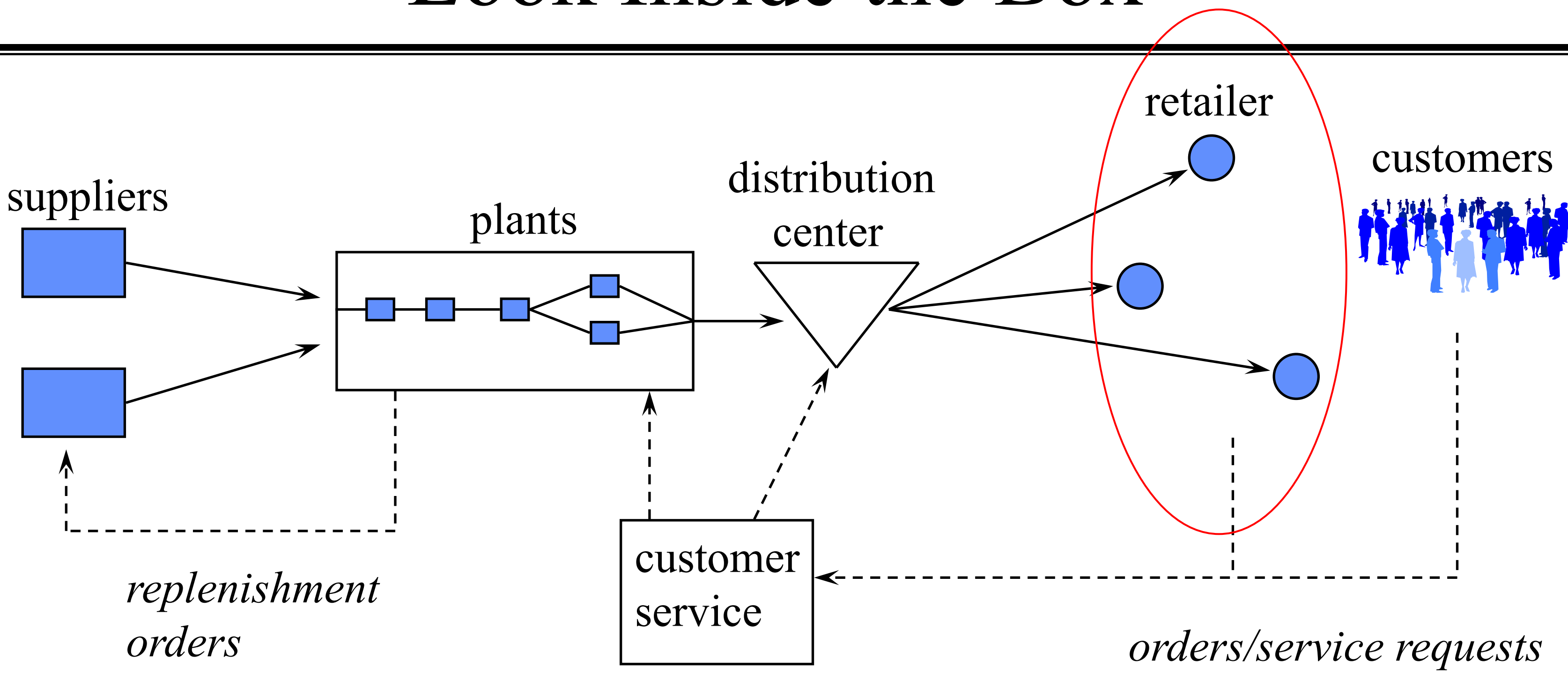
Current Challenges/Opportunities in OM

- Coordination of partners/stakeholders in an increasingly complex global supply network
  - Geopolitical risks, pandemic disruptions, supply chain robustness
- Managing customer touchpoints with technology innovation
  - Digital marketing, content management system, search engine optimization & marketing
- Enhancing corporate performance with data-driven methods
  - Business analytics, machine learning, big data
- Balancing profitability and sustainability goals
  - Environmental, sustainability, and governance (ESG), triple bottom line, fairness in operations

How could OM benefit you?

- Look inside the box: to understand how firms *actually* make decisions in real business world
- Rich opportunities in and out of the operations area
- For your career development, job hunting, future studies, interviews

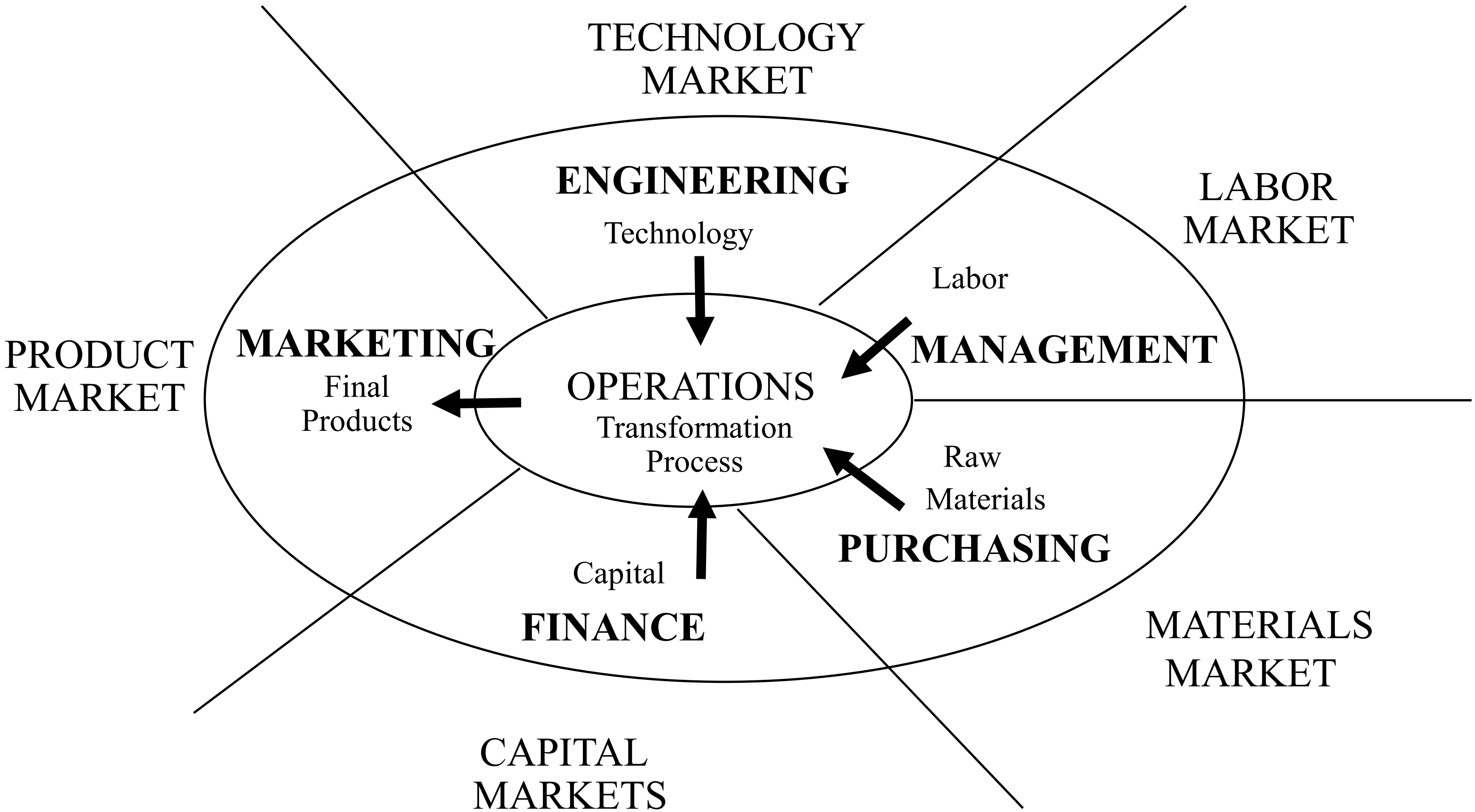
Look Inside the Box



OM can help you to know:

- What kind of decisions are made in real business environments
- How should we make theses decisions to optimize the performance
- What are their impacts for multiple players?

OM: Intersection of Business Functions



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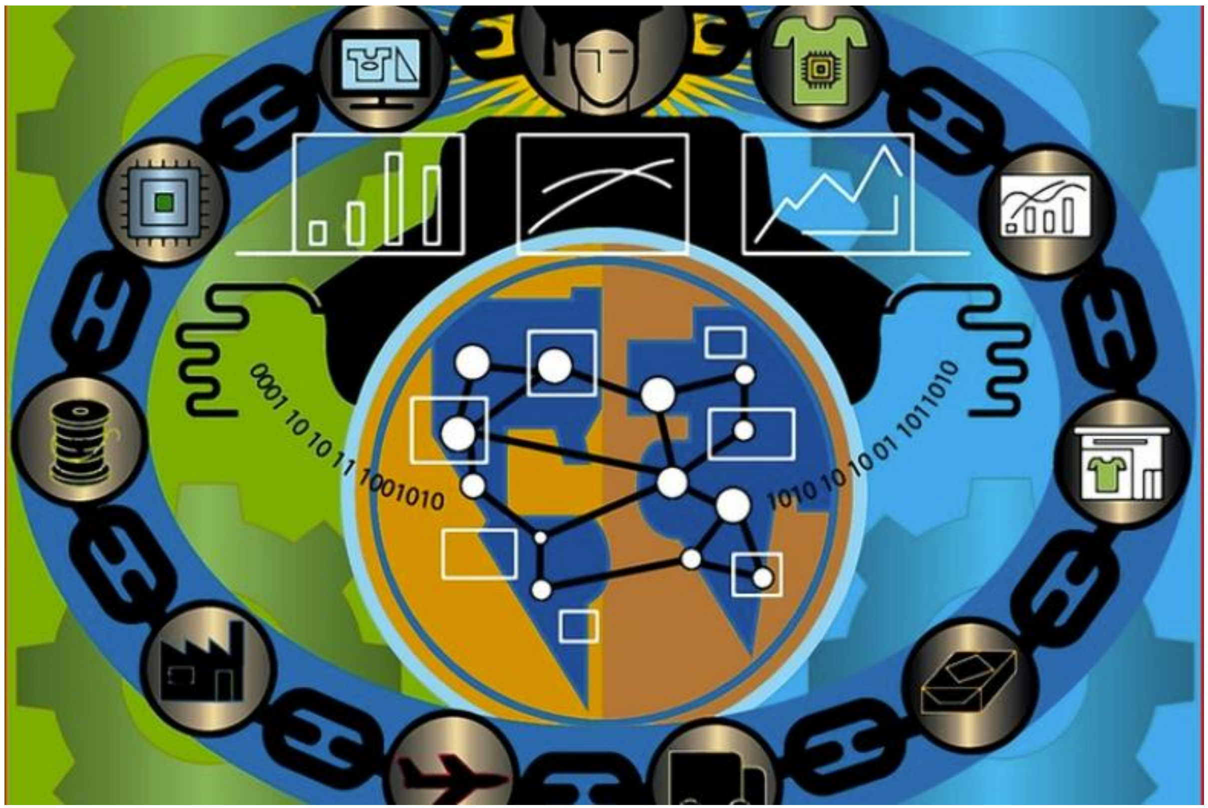
Career in Management Consulting

The Hot New M.B.A.: Supply-Chain Management

More Schools Are Ramping Up Their Programs, Adding Majors and Concentrations as Employer Demand Grows



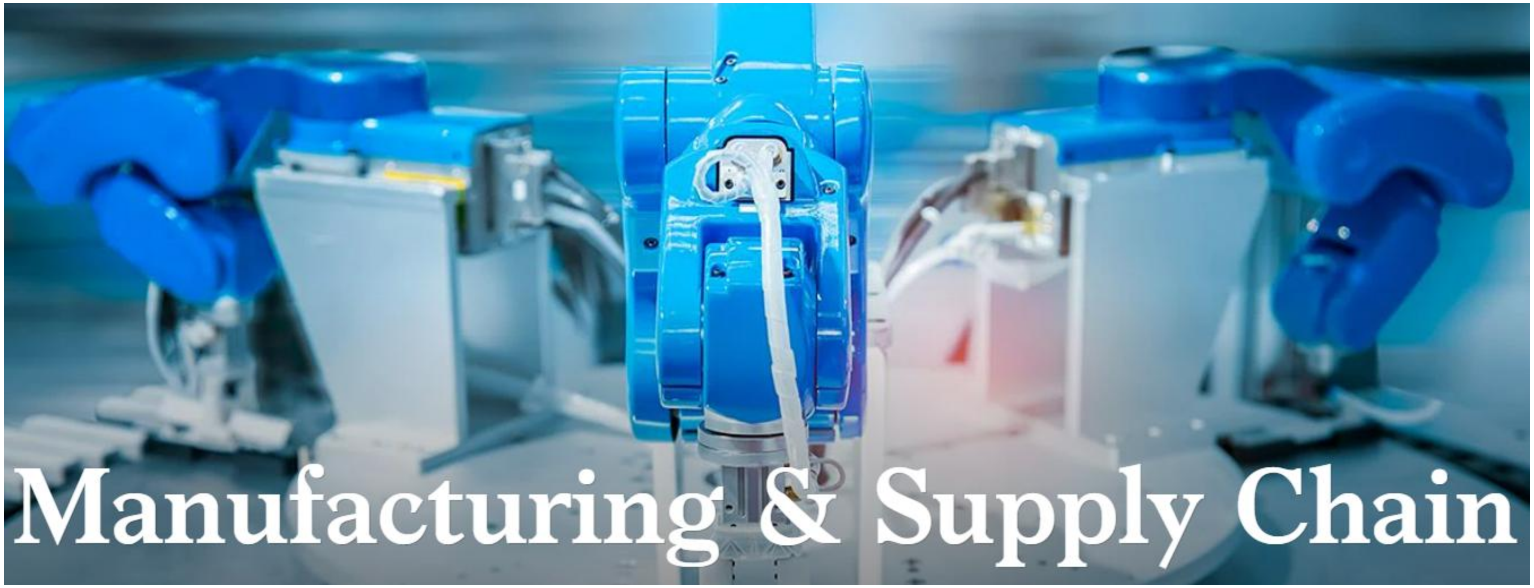
Source: The Wall Street Journal

Career in Management Consulting

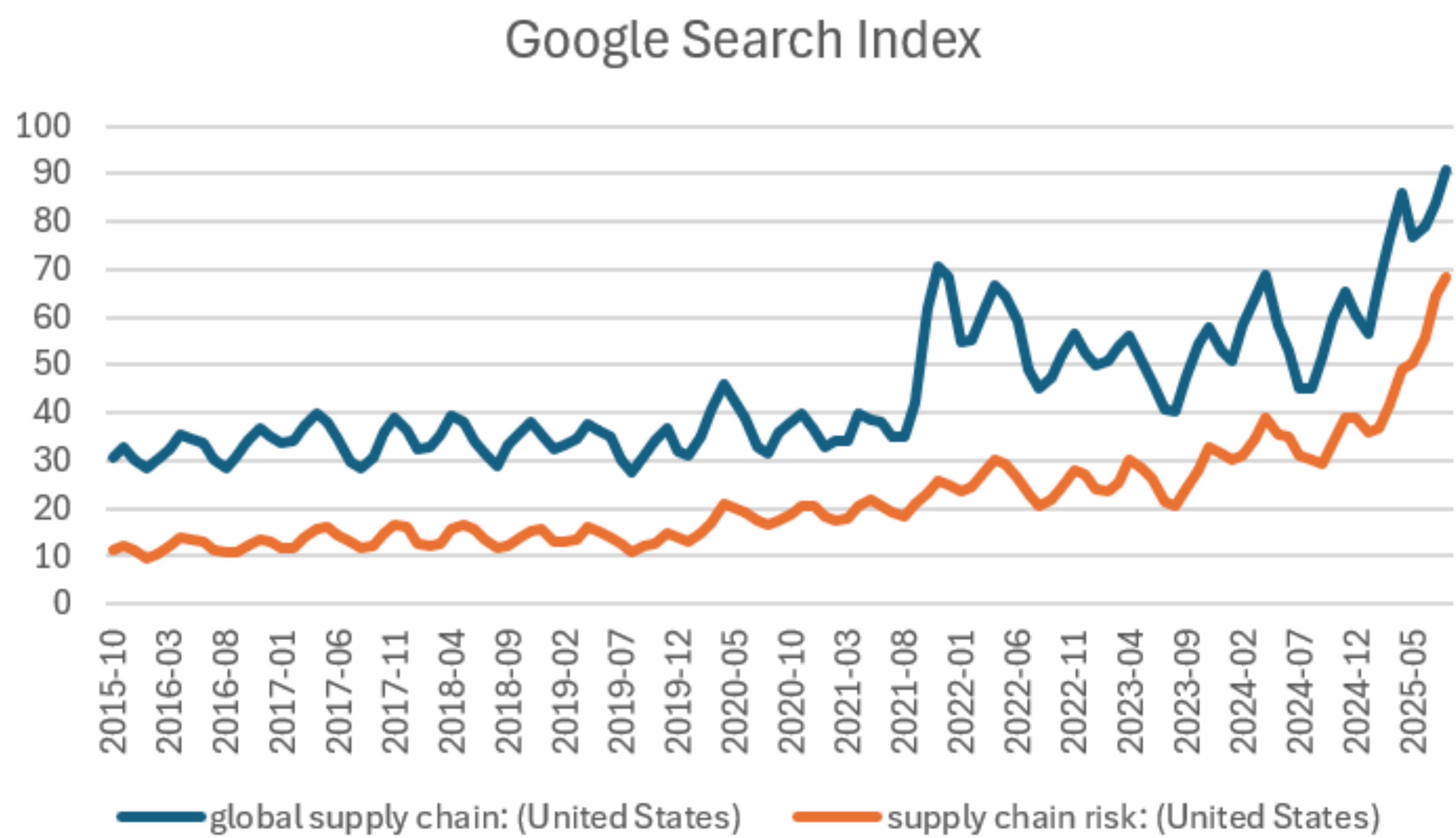
McKinsey & Company

Operations

Our Insights How We Help Clients Our People



Growing Attention on Supply Chains



Synergy with Other Areas: Operations+

- Operations + Finance/Accounting:
  - Find investment opportunities by analyzing a firm operational status and decisions
  - E.g., Morgan Stanley, Deloitte, McKinsey
- Operations + Technology:
  - Distribute products efficiently to the broad market
  - Manage supply/demand on a large scale in a fast way
  - E.g., Google, Facebook, Uber, Tesla
- Operations + Marketing:
  - Data-driven decision making in marketing strategies
  - New roles: digital marketing, marketing operations

How could OM benefit you?

- Look inside the box: to understand how firms actually make decisions in real business world
- Rich opportunities in and out of the operations area
- For your career development, job hunting, future studies, interviews

OM Helps You in Multiple Ways

- Stock-pitching/investment competitions:
  - Understand business model and process of companies
  - Improve your valuation model and/or business plan

- Case interviews:
  - Provide you with new ideas and analytical tools for real-world problems
  - Help you interact with interviewer and interviewees

- Future studies:
  - Quantitative skills, multidisciplinary knowledge, business problem contexts

Where Else Should You Learn OM

- Business magazines and news media
  - Real-world problems, challenges, and recent developments
  - Bloomberg Business, WSJ, Economist, CNBC, etc

ch Bloomberg Businessweek + Green

January 6, 2022, 5:00 PM GMT+8

Covid-19 Hit Supply Chains Hard. Climate Shocks May Hurt More

Business | One for the price of two

Airlines' favourite new pricing trick

Our analysis shows that some carriers have started charging more for solo travellers

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Tariffs will now be applied to all e-commerce packages entering the U.S.; expect to see prices go up—or your options shrink

The Economist

Menu | Weekly edition | Search

Leaders

Mar 31st 2021 edition ›

Message in a bottleneck

Global supply chains are still a source of strength, not weakness

ISOM 2700: Operations Management

Session 2. Process Analysis

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Fall, 2025

Agenda

- **Process View of Organization**
- Process Measures
- Process Analysis

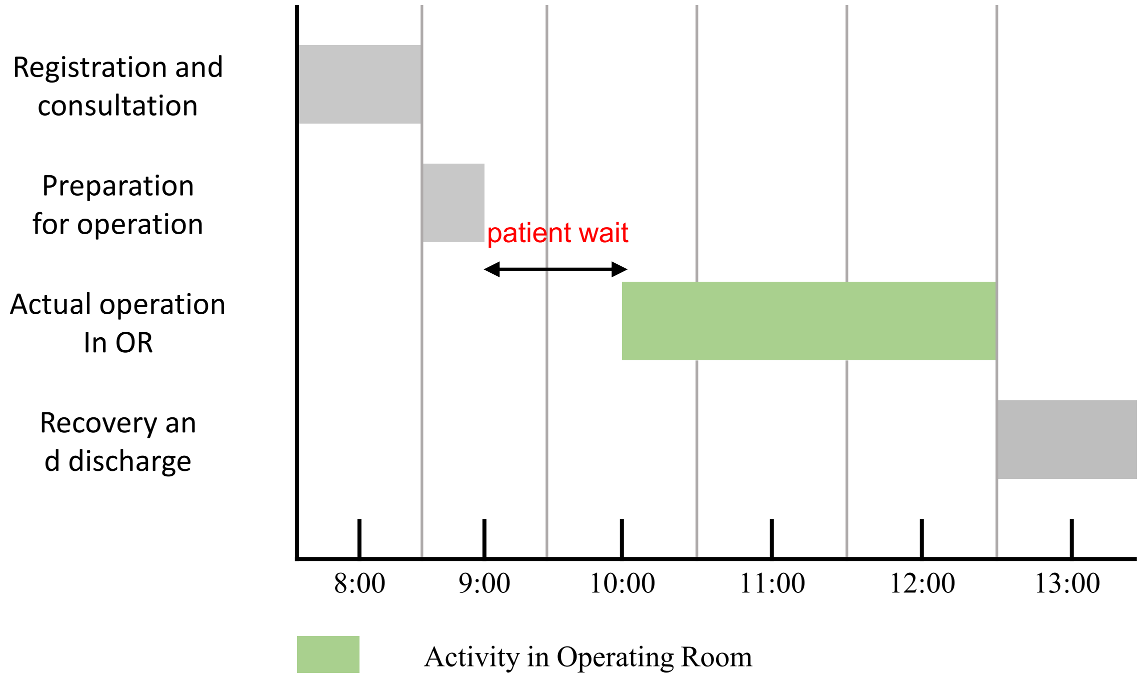
Operating Room in a Hospital

- The patients need to go through a set of activities
  - Registration and consultation
  - Preparation of procedure
  - Actual procedure
  - Recovery and discharge

- Suppose you are the unit manager, how would you measure its performance?

The Patient's View

- For a specific patient, a Gantt Chart can be used



- Information: sequence, duration, dependence

The Manager's View

- Suppose you are now the OR manager that wants to evaluate its performance and make improvement
- Can you do this with one Gantt chart for each patient? (There are perhaps hundreds of patients going to the OR each month)
- No! This would be too messy and complicated for meaningful analysis ---- and it is usually unnecessary!

The Manager's View

- Instead, you can view the OR and its procedures as a process for each patient to pass through

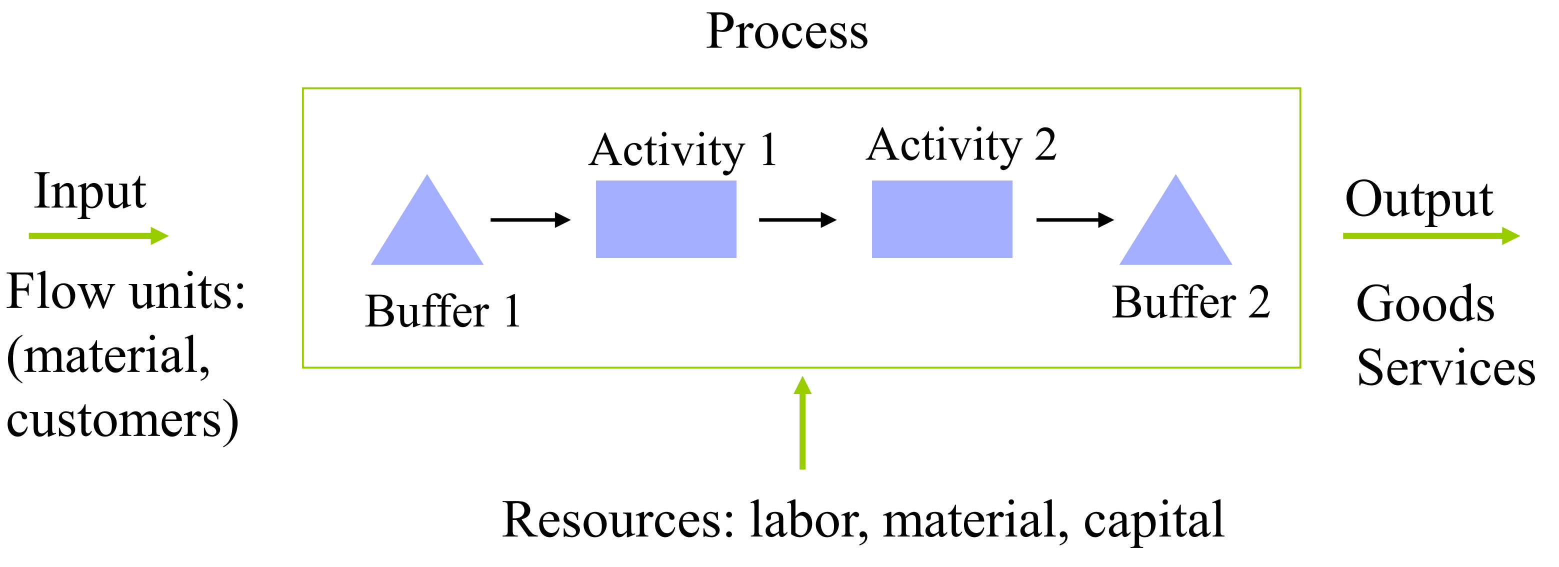
- Benefits: allow you to build a holistic/macro view of the system without diving into the details of each patients
  - Recall modelling and abstraction are needed in OM!

- Then, you can identify the key challenge and improvement opportunities for system performance

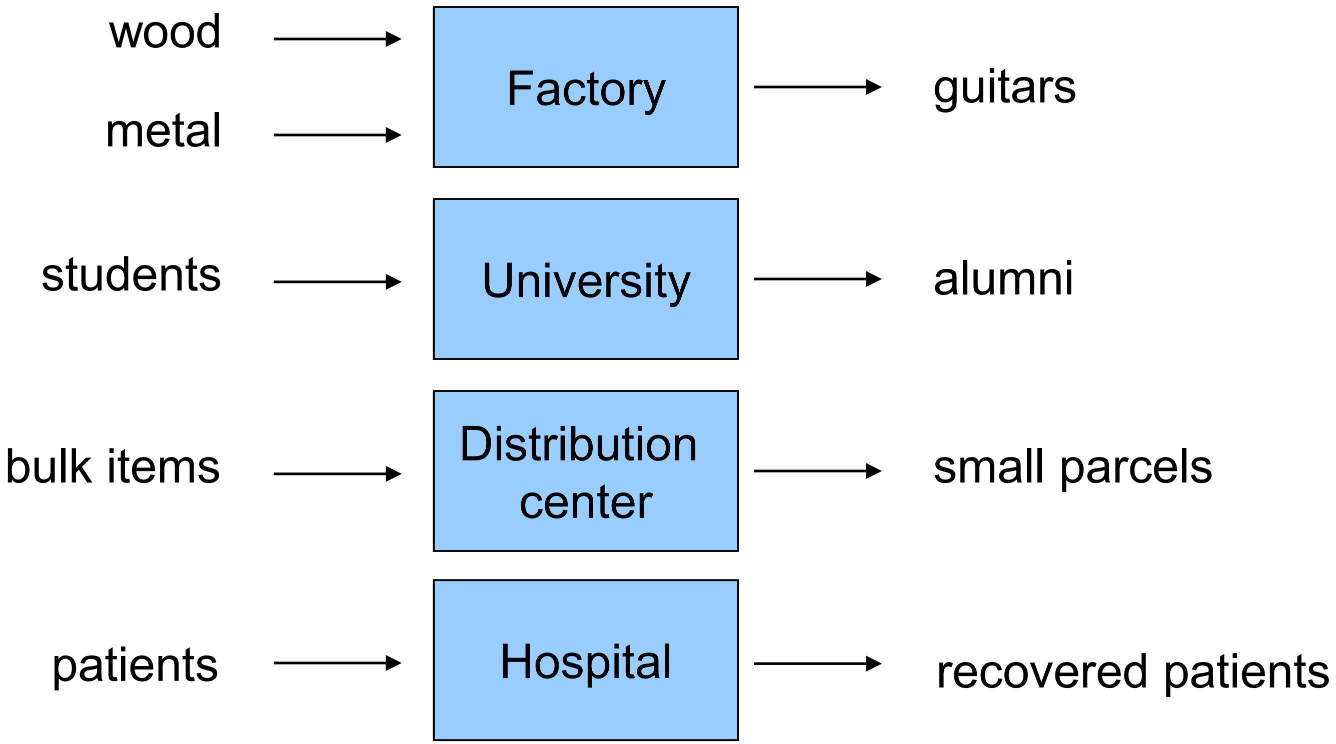
- Limitation: you will lose the granular information of each patient (this is usually fine for analysis purpose)

Process View

- A business process is a network of activities performed by resources that transform inputs into outputs
- It reflects a way of modeling and abstraction of reality



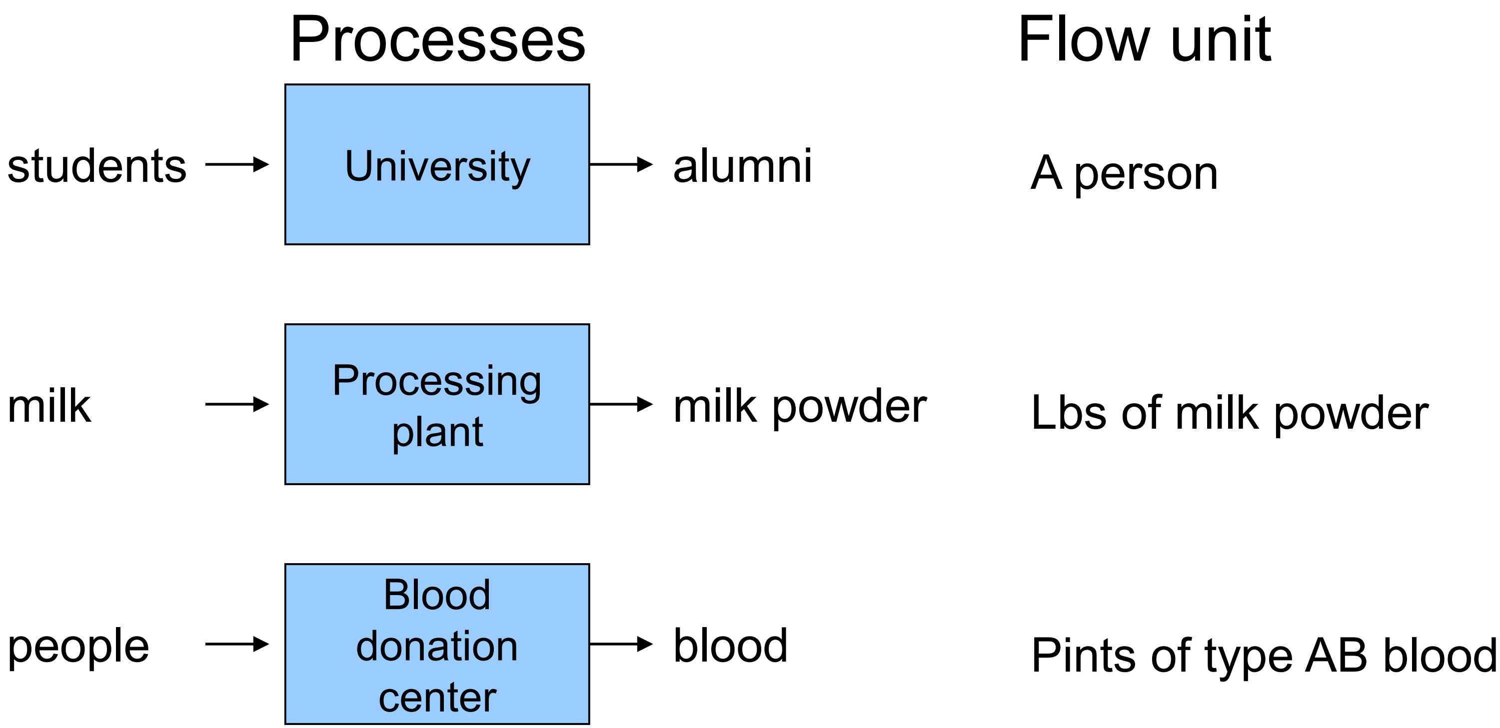
Examples of Processes



- Processes can involve both goods and services.
- Processes can have multiple inputs and/or multiple outputs.

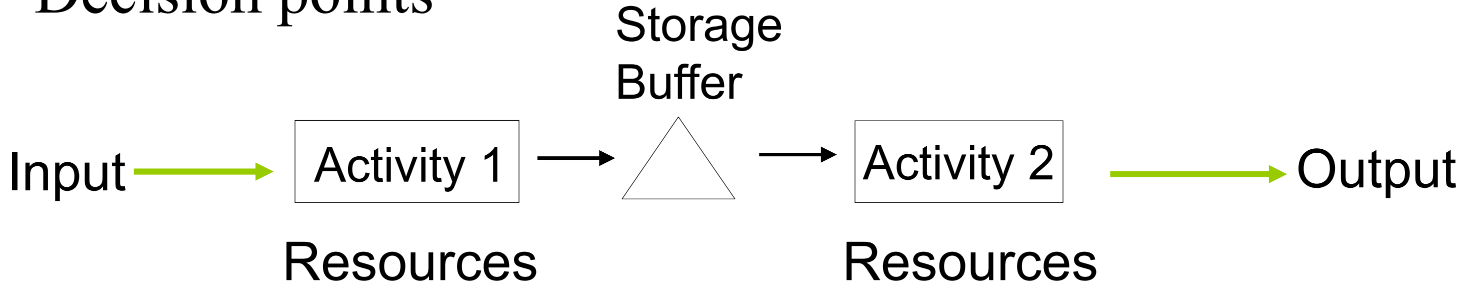
Defining a Process's Flow Unit

- The flow unit is what is tracked through the process and generally defines the process output of interest.



Process Flow Chart

- Process flow chart is the use of a diagram to present the major elements of a process
  - Activities or operations
  - Resources
  - Product flows
  - Decision points



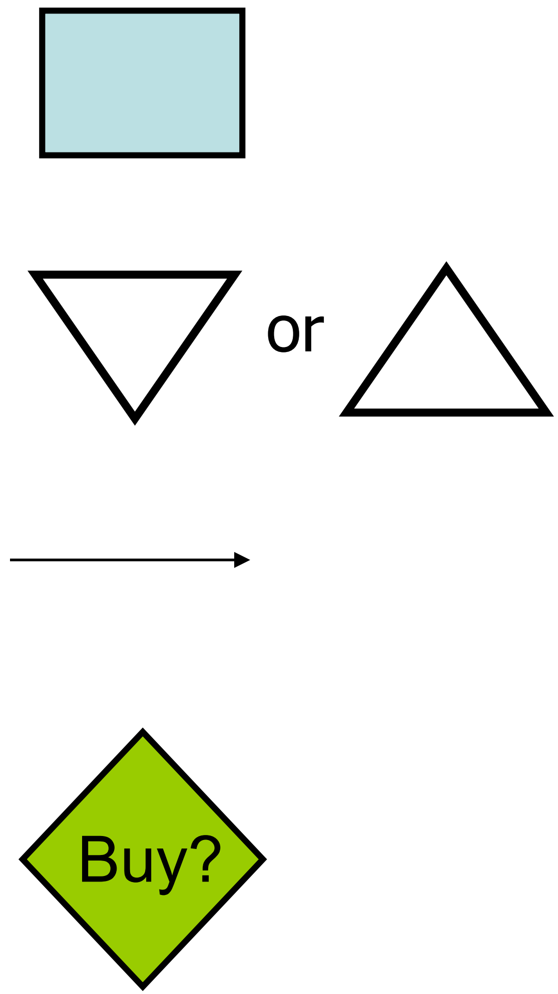
Process Flow Chart: Terminology

Tasks/activities

Buffer (Storage areas or queues)

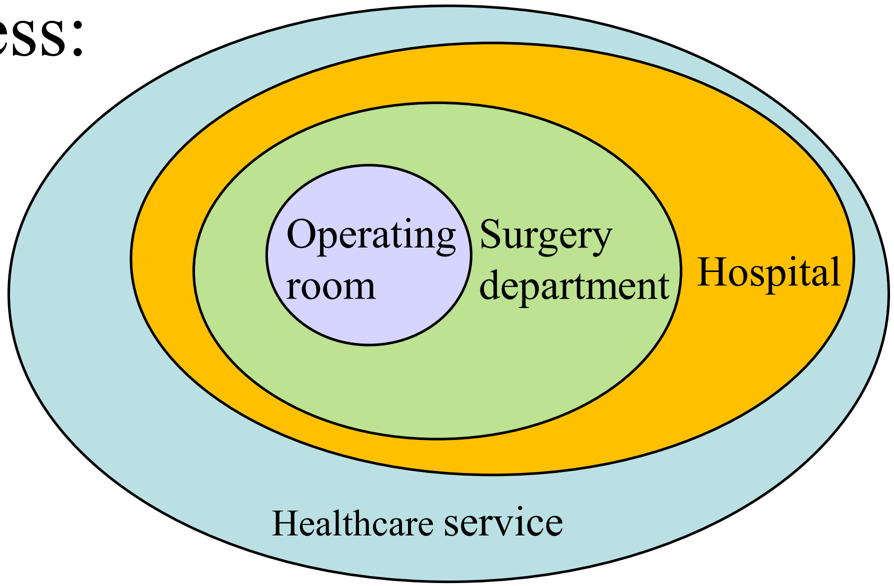
Flow of goods/materials

Decision



Process Flow: Considerations

- Boundaries of the process:



- Level of simplification:
  – Information granularity versus analytical convenience
- Both should depend on the goal/target of study
  – Problem-driven and goal-oriented

Agenda

- Process View of Organization
- **Process Measures**
- Process Analysis

T-shirt Production: Problem Description

Fabric →	Cutting →	Sewing →	Packing →	T-shirt
Resources:	Worker A	Worker B	Worker C	
Processing time:	3 min/unit	4 min/unit	3 min/unit	

- How long does it take to produce a unit?
  - How do you define “how long”?
- How many units can the process produce in an hour?
- What is the time between two successive production completions?
- How many T-shirts are in the system at a given time on average?

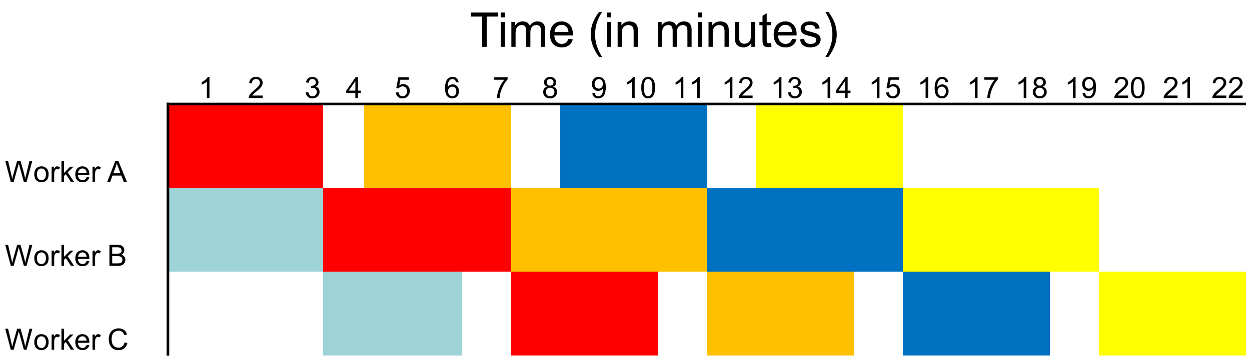
Performance Measures

- Flow time: the time spent by a given unit of product in the system
  - For a given unit, how long does it take for it to travel through the system?
  - E.g., it takes in total ten minutes to produce a shirt from beginning to end
- Cycle time: the time between two successive product completions
  - What is the time gap between two successive units being produced?
  - E.g., we see a shirt being produced from system every four minutes
- Flow/through rate: the rate at which the process is delivering output
  - How many units does the system actually produce per hour?
  - E.g., the system is producing 15 shirts per hour

Performance Measures

- Capacity: the maximum rate at which output can be delivered given sufficient inputs and demands
  - How many units can the system produce at most per hour?
  - Capacity is the maximum flow rate of the process
  - E.g., the system can produce at most 15 shirts per hour
- Work-in-process inventory: the number of units staying within the process at a given time
  - How many units of products are in the system right now?
  - E.g., there are 2.5 shirts in the system at different production stages
  - Why this matter? Think about the congestion in LG1 around lunch time!

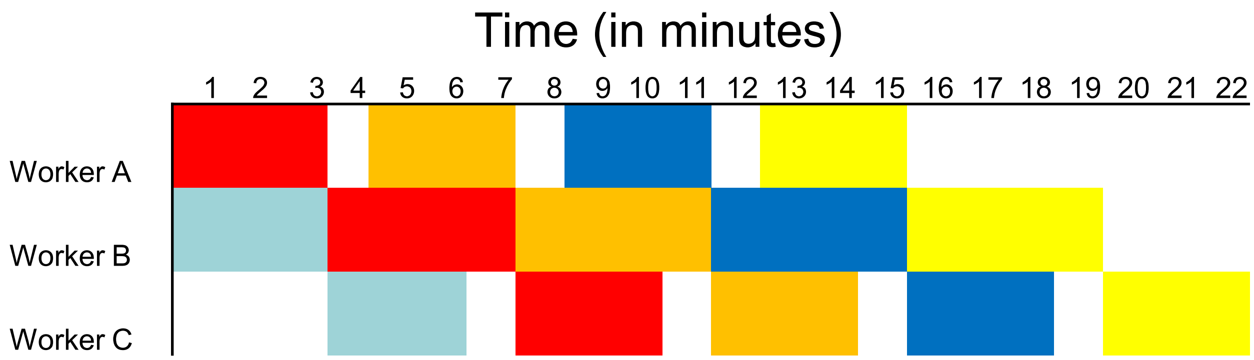
T-shirt Production: Gantt Chart



- Each colour bar represents a unit of product
- We start from the time that the red unit enters the system; assume we have enough inputs and demands for the system

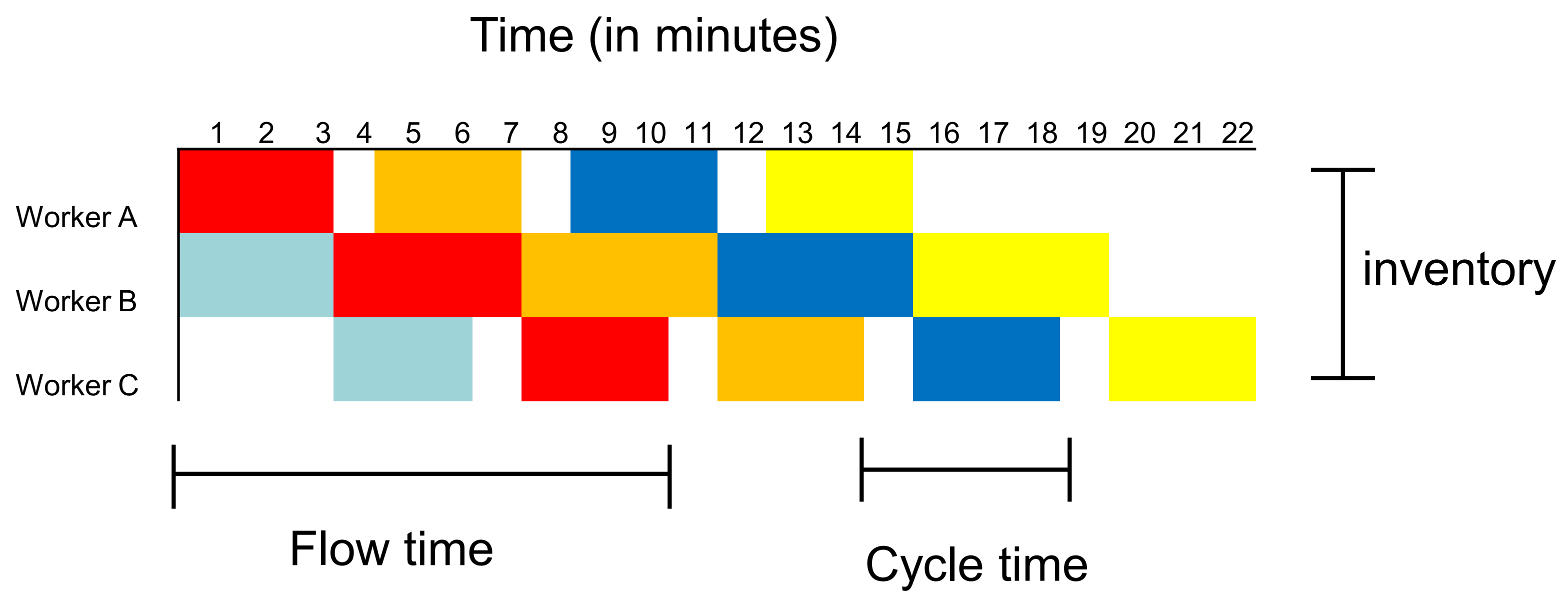
- After worker A finishes cutting, she waits an additional minute, why?
- After worker C finishes packing, she waits an additional minute, why?
- Worker B is always busy, why?

T-shirt Production: Gantt Chart



- It takes 10 minutes to produce 1 unit, i.e., flow time = 10 minutes
- The process can produce 60/4=15 units per hour, i.e., flow rate = capacity = 15 units per hour
- The time between two successive production completions is equal to 4 minutes, i.e., cycle time = 4 minutes
- The average work-in-process inventory is 3/4+1+3/4=2.5 units

T-shirt Production: Gantt Chart



- Be sure to differentiate between flow time and cycle time
- By definition, we always have

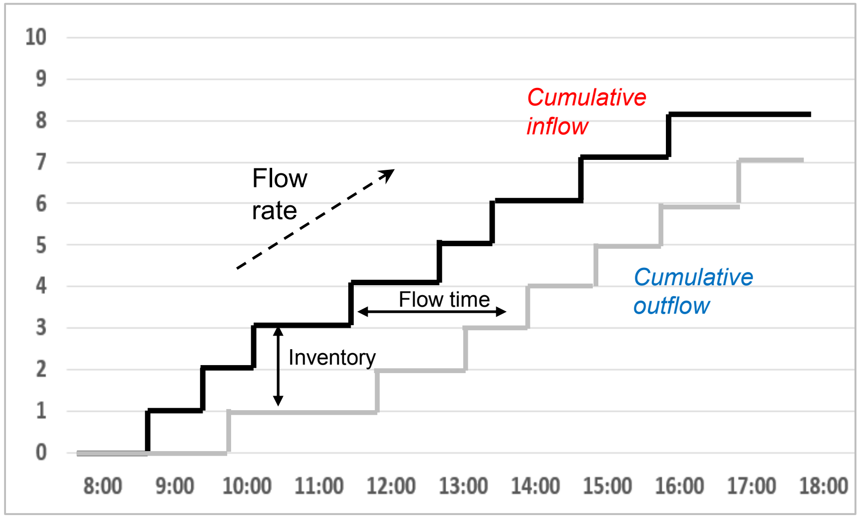
$$\text{Cycle time} = \frac{1}{\text{Flow rate}}$$

Agenda

- Process View of Organization
- Process Measures
- **Process Analysis**
  - **Little's Law**

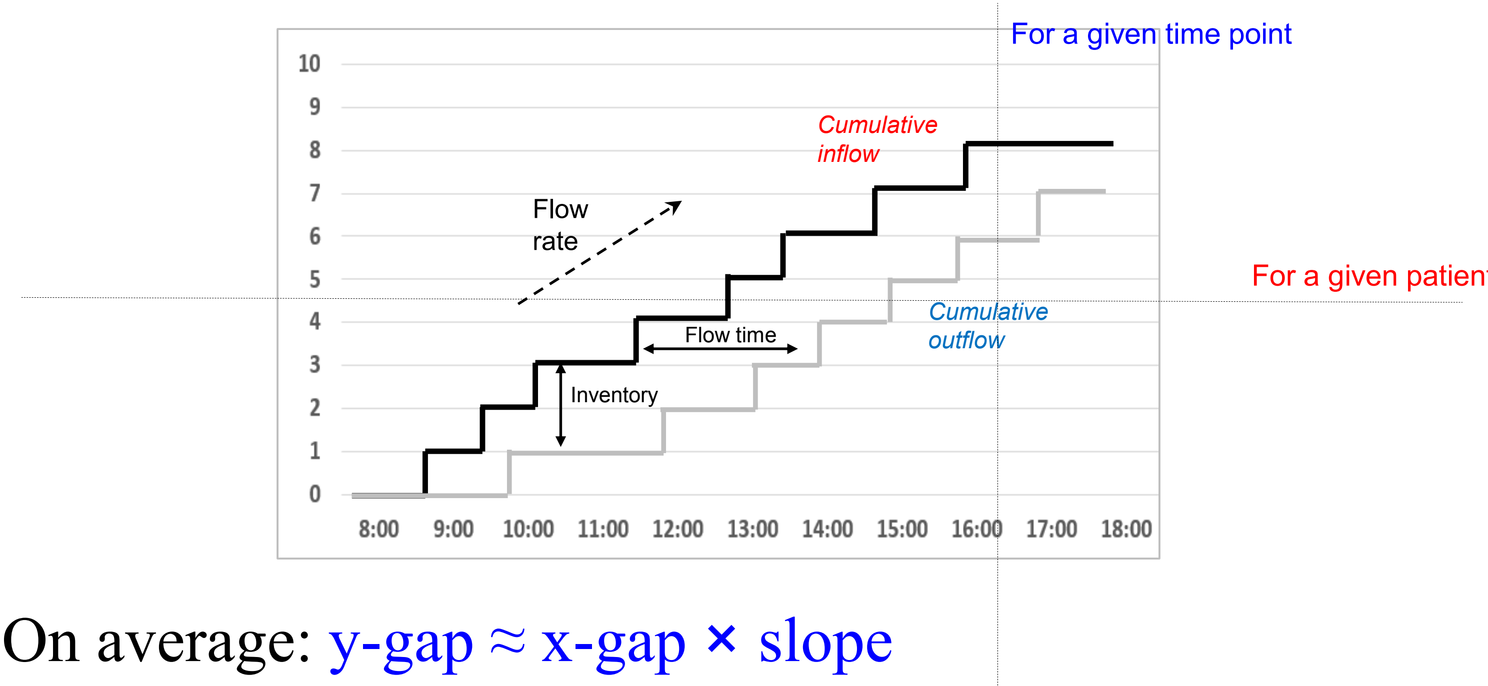
Little’s Law: Visualization

- Let us move back to the operating room
- As manager, you can draw a cumulative inflow/outflow chart (assume patients are served in sequence of their arrivals)



- Quantities shown on the chart: Flow time (x-gap), inventory (y gap), flow rate (overall slope)
  - How should they be connected?

Little's Law: Explanation



- On average: y-gap ≈ x-gap × slope
- Little's Law: Avg. Inventory = Avg. Flow rate × Avg. Flow time
- With common notations: $I = R \times T$
  - I: avg inventory, R: avg flow rate, T: avg flow time

Little's Law: Sub-processes

Fabric → Cutting → Sewing → Packing → T-shirt

	Cutting	Sewing	Packing
Resources:	Worker A	Worker B	Worker C
Processing time:	3 min/unit	4 min/unit	3 min/unit

- Little's law can be applied to both the full process as well as any subprocesses
  - E.g., the cutting, sewing, and packing stages in the T-shirt problem
- For subprocesses, we need to use their corresponding flow time (shorter) and inventory (fewer)

Little's Law: Implications

- Little's Law holds for all types of stable process, regardless of
  - the complexity of the process (multiple flow units/resources)
  - the sequence for the flow unites to be served (FIFO vs LIFO)
  - randomness in the arrival and service time
- Stable process: inflow = outflow, inventory does not accumulate to infinity as time goes by
- Implications:
  - finding the third measure when the other two are known
  - trade-off when setting management goals

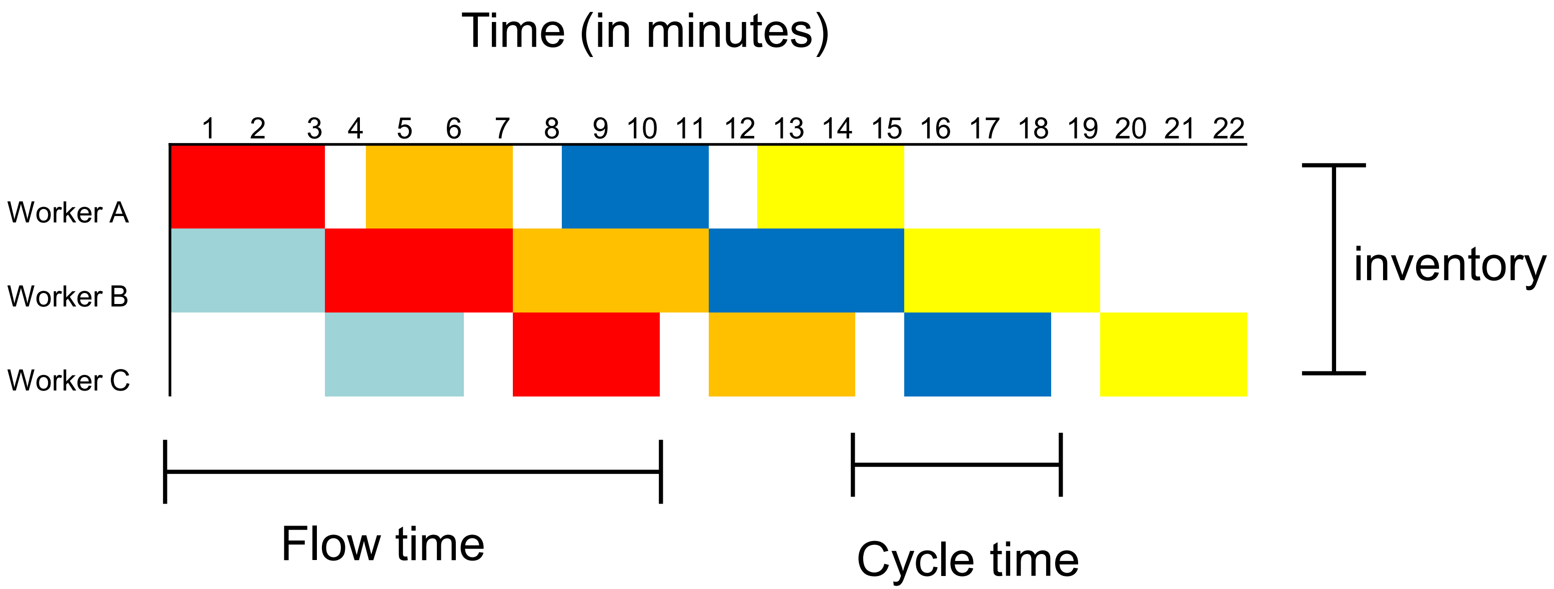
Little's Law: Calculations

- $\text{Inventory} = \text{Flow time} \times \text{Flow rate}$
  - To determine the how many units are in system

- $\text{Flow time} = \frac{\text{Inventory}}{\text{Flow rate}}$
  - To determine how much time a unit spends in the system

- $\text{Flow rate} = \frac{\text{Inventory}}{\text{Flow time}}$
  - To determine how many units flow through the system in a given time

Example 1: T-shirt Production



- For the process, flow rate R = 15 units per hour, flow time T = 10 minutes = 1/6 hours
- By Little's law: average inventory = R×T = 2.5 units

Example 2: Job Flow Problem

- A branch office of an insurance company processes 10,000 claims per year and the office works only 50 weeks per year

- Q1: What is the flow rate?
  - R = 10000/50 = 200 claims per week

- Q2: If the average processing time is three weeks for each claim, then how many claims are in different stages of the process at a given time?
  - Flow time: T = 3 weeks
  - Average inventory: $I = 200 \times 3 = 600$ claims

Example 2: Job Flow Problem

- A branch office of an insurance company processes 10,000 claims per year and the office works only 50 weeks per year

- Q3: Assume an improvement is made such that new processing time can be reduced by 80% (i.e., from 3 weeks to 0.6 week), then how many claims are in different stages of the process at a given time?
  - Now: R = 200 claims per week and T = 0.6 weeks
  - New inventory: $I = 200 \times 0.6 = 120$ claims

Customers in IKEA

Customers in IKEA

- The amount of time customers spend in a supermarket is of interest to the management.
- How would you go about finding the average time that customers spend in the store?

ISOM 2700: Operations Management

Session 3. Bottleneck Analysis

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Fall 2025

Agenda

- Process View of Organization
- Process Measures
- **Process Analysis**
  - Little's Law
  - **Bottleneck analysis**

Process Capacity

- Recall: process capacity is the maximum rate the flow units can be produced given enough inputs and outputs

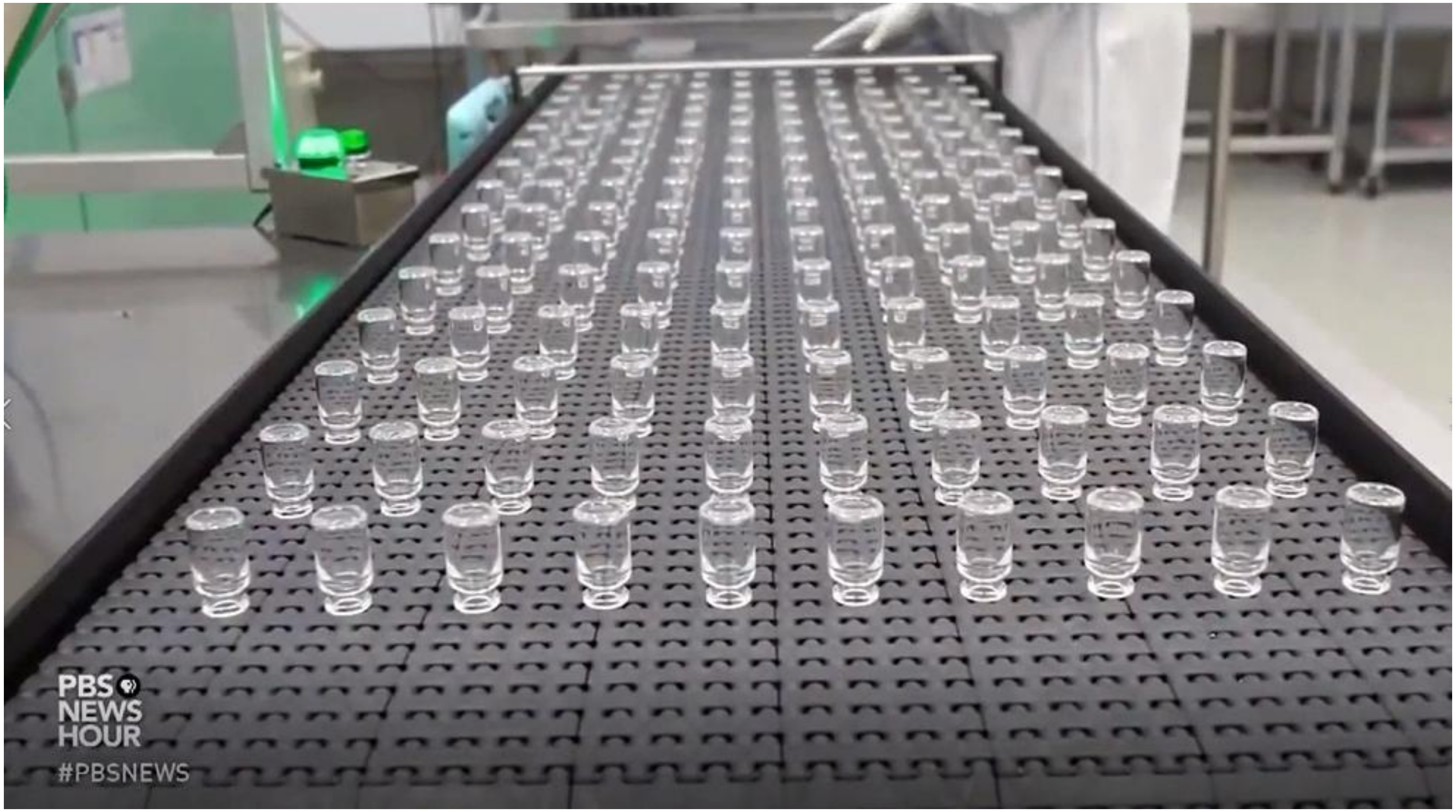
- Key question: what determines the capacity of a process?

- Relatedly, if you have additional resource, where should you invest it to improve process capacity?

- Consider a taco store. You need multiple steps to complete a gyro (meat, fries, vegies, sauce..)

- If you can increase the speed for one of them, which one you should prioritize?

Video: Bottleneck in Covid-19 Vaccine



Video

Video: Bottleneck in Covid-19 Vaccine

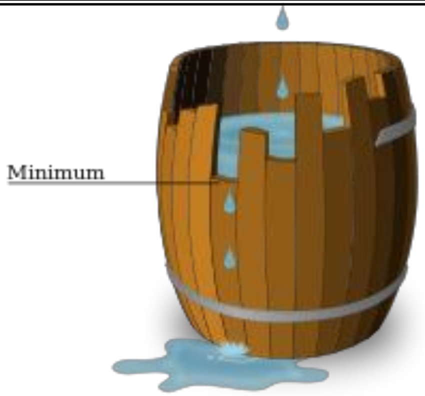
- For Covid-19 vaccine, you might think that the bottleneck should be the research stage…
- However, it shows that the bottleneck lies in the assembling (vaccine vials) and distribution (cold-chain logistics)
- This highlights the importance of understanding bottleneck, especially in large-scale implementation of policies

Law of the Minimum

- Key intuition: For a process to produce a unit, every resource must finish its own job
- Thus, the process capacity is determined by the slowest resource
- This gives the "Law of Minimum" for determining process capacity

Law of the Minimum

The capacity of a barrel with staves of unequal length is limited by the shortest stave



A chain is only as strong as its weakest link

The flow rate of an irregular pipe is limited by its narrowest gap

Process Capacity

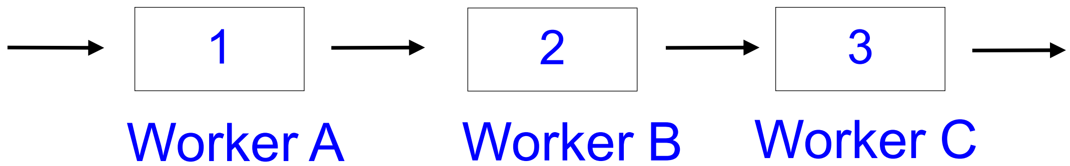
- Consider the process where flow unit has to visit every resource in the process (same flow rate for all)
- Law of minimum:
  - Process capacity = minimum{Capacity of resource 1, Capacity of resource 2, …, Capacity of resource n}
  - How much the process can produce given enough input and demand
- Bottleneck:
  - It is defined as the resource with the smallest capacity
  - By law of minimum, process capacity equals bottleneck capacity

Procedure to Identify Bottleneck Resource

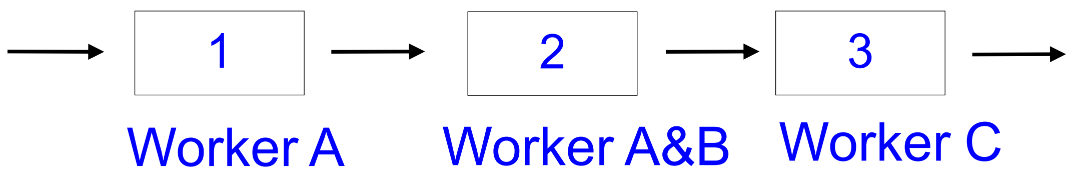
- **Step 1**. List activities, resources, and processing times
- **Step 2**. Categorize the process into one of the systems we consider in subsequent slides
- **Step 3**. For each resource, compute its workload and determine its capacity
- **Step 4**. Identify the **bottleneck resource** with the lowest capacity, which determines the capacity of the entire process

Identify Bottleneck: Scenarios

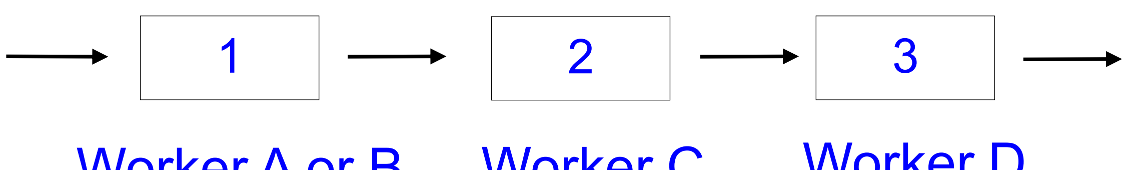
- One-to-one mapping between resource and activity



- One resource is needed in multiple activities
  - Sum up the multiple workload for the resource



- One activity can be performed by multiple resources
  - Pool the capacity by the multiple resources



Bottleneck Analysis: One-to-One Mapping

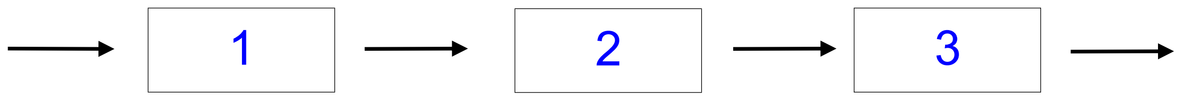
Fabric → Cutting → Sewing → Packing → T-shirt

	Cutting	Sewing	Packing
Resources:	Worker A	Worker B	Worker C
Processing time:	3 min/unit	4 min/unit	3 min/unit

- Capacity of Worker A: 60/3 = 20 units/hour
- Capacity of Worker B: 60/4 = 15 units/hour
- Capacity of Worker C: 60/3 = 20 units /hour

- Process capacity = 15 units /hour
- Worker B is the bottleneck!

Bottleneck Analysis: One Resource for Multi Tasks



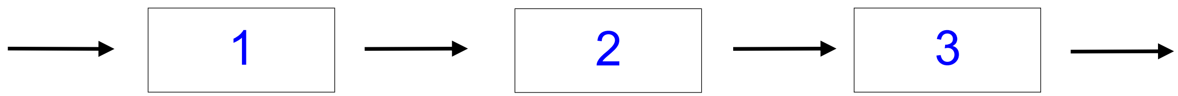
Resources:	Worker A	Worker A&B	Worker C
Processing time:	2 min/unit	3 min/unit for both	4 min/unit

It shows that to produce a unit product, Worker A must work in both step 1 and 2

Is the bottleneck the resource that is required to perform the activity with the longest time, i.e., Worker C?

Then who is the bottleneck?
It is the **resource** that has the highest amount of workload (per unit), i.e., Worker A!

Table Form Implementation of the Procedure

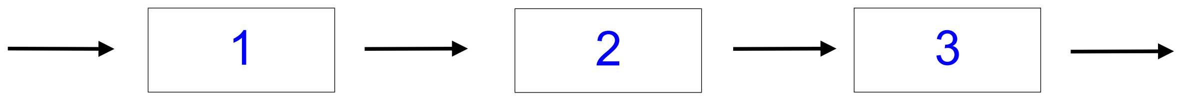


Resources:	Worker A	Worker A&B	Worker C
Processing time:	2 min/unit	3 min/unit for both	4 min/unit

Resource	Activities where the resource is needed	Time required per unit of work (i.e., workload)	Capacity (=1/workload)
Worker A			
Worker B			
Worker C			

Process capacity = ?

Table Form Implementation of the Procedure

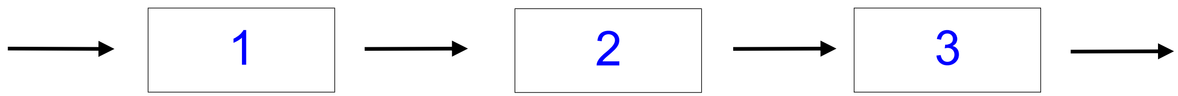


Resources:	Worker A	Worker A&B	Worker C
Processing time:	2 min/unit	3 min/unit for both	4 min/unit

Resource	Activities where the resource is needed	Time required per unit of work (i.e., workload)	Capacity (=1/workload)
Worker A	Activity 1&2		
Worker B	Activity 2		
Worker C	Activity 3		

Process capacity = ?

Table Form Implementation of the Procedure



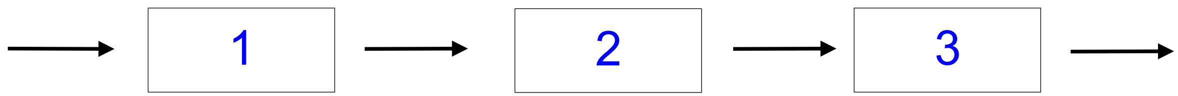
Resources: Worker A | Worker A&B | Worker C

Processing time: 2 min/unit | 3 min/unit for both | 4 min/unit

Resource	Activities where the resource is needed	Time required per unit of work (i.e., workload)	Capacity (=1/workload)
Worker A	Activity 1&2	2+3=5 mins	
Worker B	Activity 2	3 mins	
Worker C	Activity 3	4 mins	

Process capacity = ?

Table Form Implementation of the Procedure

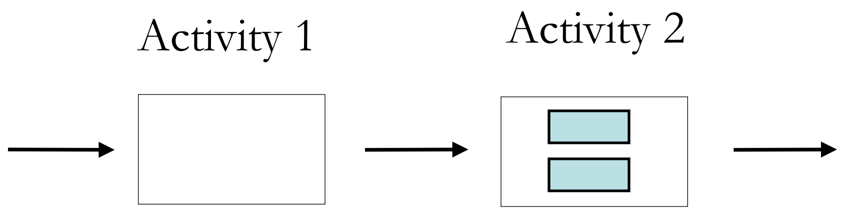


Resources:	Worker A	Worker A&B	Worker C
Processing time:	2 min/unit	3 min/unit for both	4 min/unit

Resource	Activities where the resource is needed	Time required per unit of work (i.e., workload)	Capacity (=1/workload)
Worker A	Activity 1&2	2+3=5 mins	1/5 units/min = 12 units/hour
Worker B	Activity 2	3 mins	1/3 units/min = 20 units/hour
Worker C	Activity 3	4 mins	1/4 units/min = 15 units/hour

Process capacity = capacity of resource with the smallest capacity
= 12 units/hour

Bottleneck Analysis: Pooled Resource



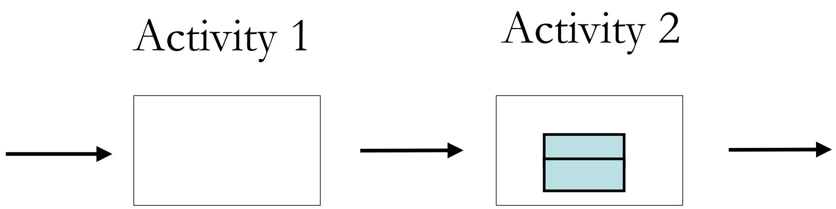
Resources:	machine A	machine B (two identical machines)
Processing time:	2 mins/unit	5 mins/unit (each machine)

The two Machine B can do activity 2 for two units simultanesouly

What is the capacity of this process?

Resources	Activities where needed	Time required per unit of work	Number of each resource	Capacity (number ×1/workload)
Machine A				
Machine B				

Bottleneck Analysis: Pooled Resource



Resources: machine A machine B (two identical machines)

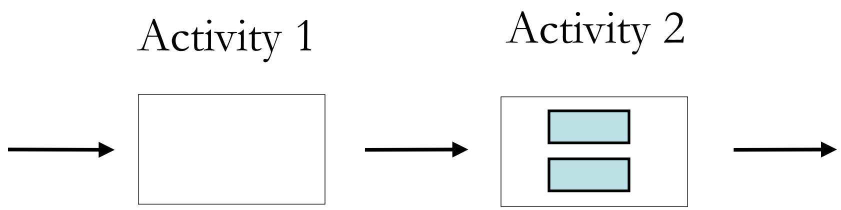
Processing time: 2 mins/unit 5 mins/unit (each machine)

What is the capacity of this process?

Resources	Activities where needed	Time required per unit of work	Number of each resource	Capacity (number ×1/processing time)
Machine A	Activity 1	2 mins	1	**1**×1/2 units/min = 30 units/hour
Machine B	Activity 2	5 mins	2	**2**×1/5 units/min =24 units/hour

Capacity of process = capacity of resource with smallest capacity
= 24 units/hour

Bottleneck Analysis: Pooled Resource



Resources: machine A machine B or C

Processing time: 2 mins/unit 5 mins/unit for B, 4 mins/unit for C

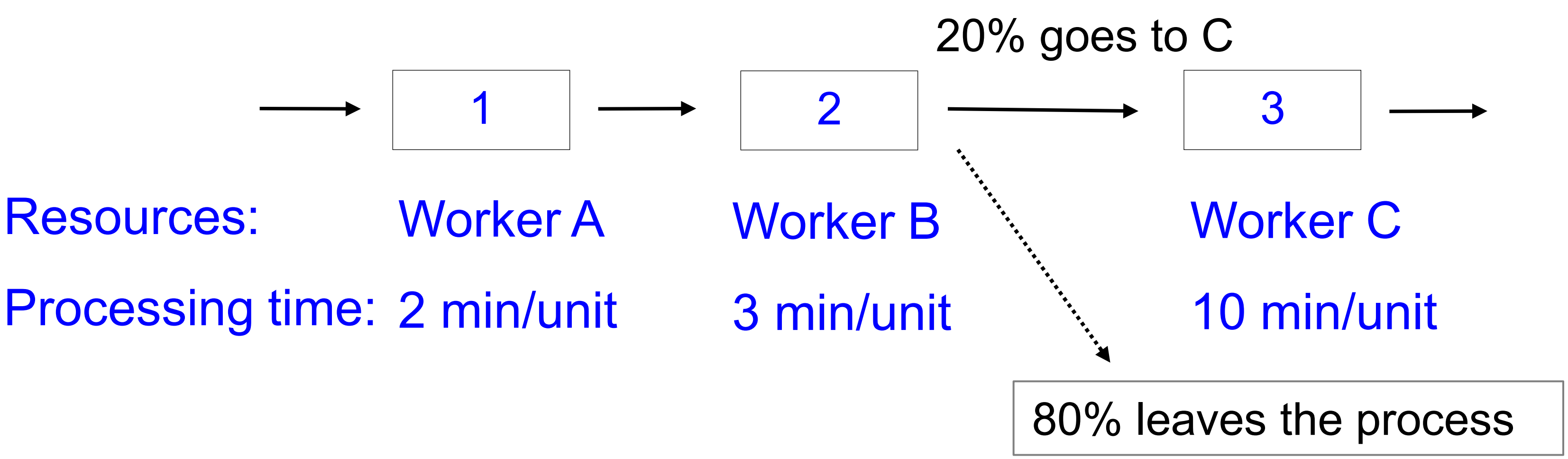
What is the capacity of this process?

Resources	Activities needed	Time per unit of work	Number of each resource	Capacity
Machine A	Activity 1	2 mins	1	1/2 units/min =30 units/hour
Machine B	Activity 2	5 mins	1	1/5 units/min =12 units/hour
Machine C	Activity 2	4 mins	1	1/4 units/min =15 units/hour

Pooled capacity = 27 units/hour

Capacity of process = capacity of resource with smallest capacity
= 27 units/hour (pooled resource for activity 2)

Process with Random Path



- The capacity of Worker C is 60/10 = 6 units/hour
- However, this capacity is for the units that finally arrive to C
- We convert it to the capacity for the original flow units
  - Capacity = 6/20% = 30 units/hour
- That is, the Worker C has a 30 units/hour "effective" capacity

Improve Performance: Line-balancing

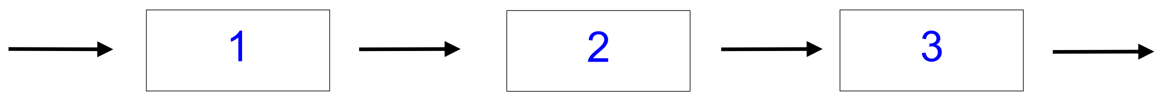
- The process capacity is determined by the bottleneck

- Moving capacity from other underutilized resources to the bottleneck may be able to improve the process capacity
  - Or move workload away from bottleneck to underutilized resources

- Line-balancing: balance the workload among the resources in the process

- Intuition: reduce supply-demand mismatch within the process

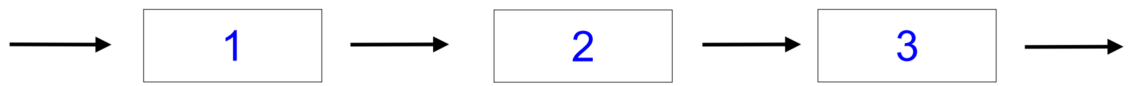
Line-balancing: Example



Resources:	Worker A	Worker A	Worker B
Processing time:	3 min/unit	1 min/unit	2 min/unit

- Bottleneck: Worker A, Process Capacity = 60/4 = 15 units/hour

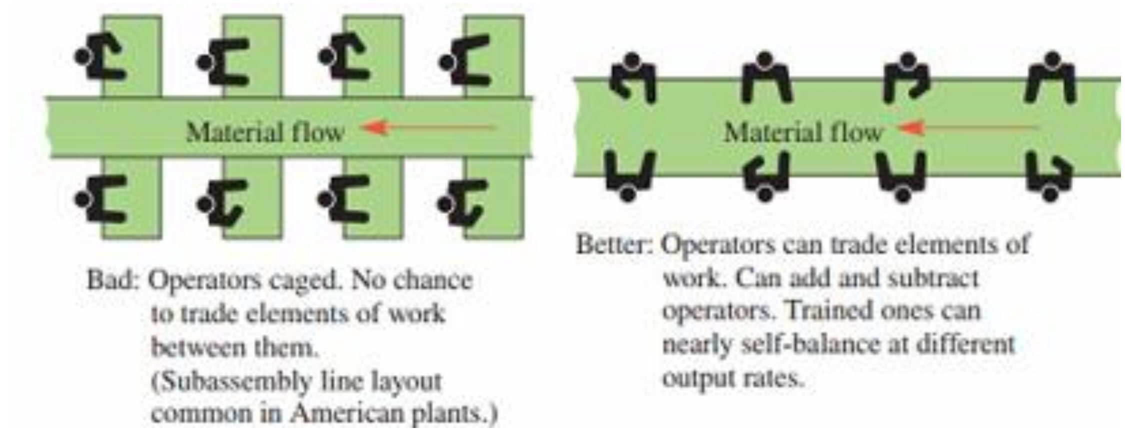
What if we move activity 2 to Worker B?

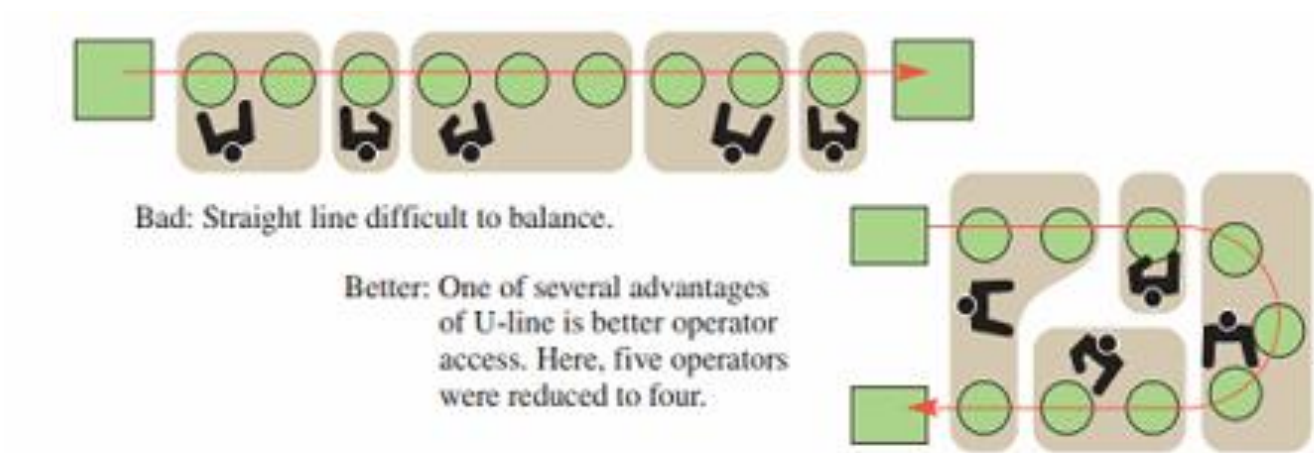


Resources:	Worker A	Worker B	Worker B
Processing time:	3 min/unit	1 min/unit	2 min/unit

- Bottleneck: Worker A or B (same capacity)
- Process Capacity = 60/3 = 20 units/hour > 15 units/hour

Line-balancing: Flexible Line Layouts





Flow Rate with Input and Demand

- When there are both input and demand constraints for the process, we have

$$\text{Flow rate} = \text{minimum}\{\text{Available input rate, Potential demand rate, Process capacity}\}$$

- This is how much the process actually produces in a stable state

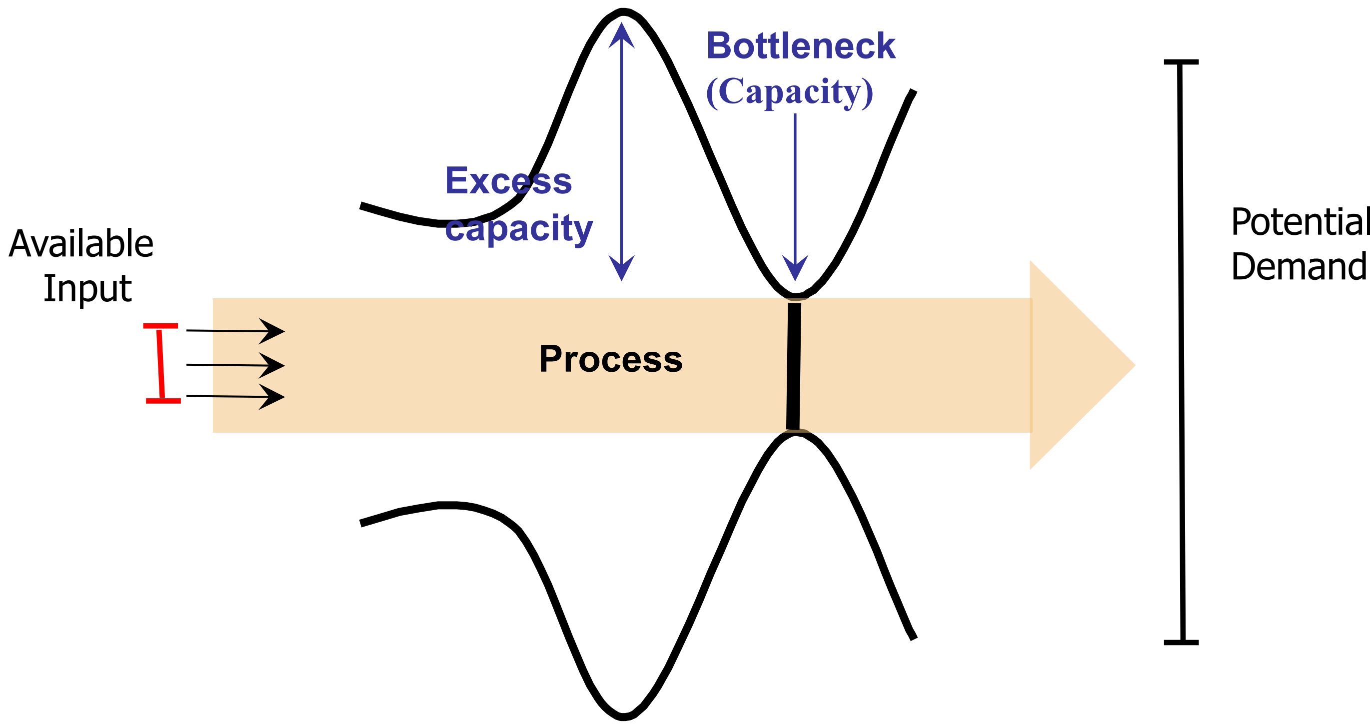
- Reason: the actual production speed is constrained by capacity, input, and demand

Three Types of Processes

- Three-types of process:
- Input-constrained: flow rate = input rate being smallest
  - To increase output, we need to increase the input
- Demand-constrained: flow rate = demand rate being smallest
  - To increase output, we need to increase the demand
- Capacity-constrained: flow rate = process capacity being smallest
  - To increase output, we need to increase the process capacity

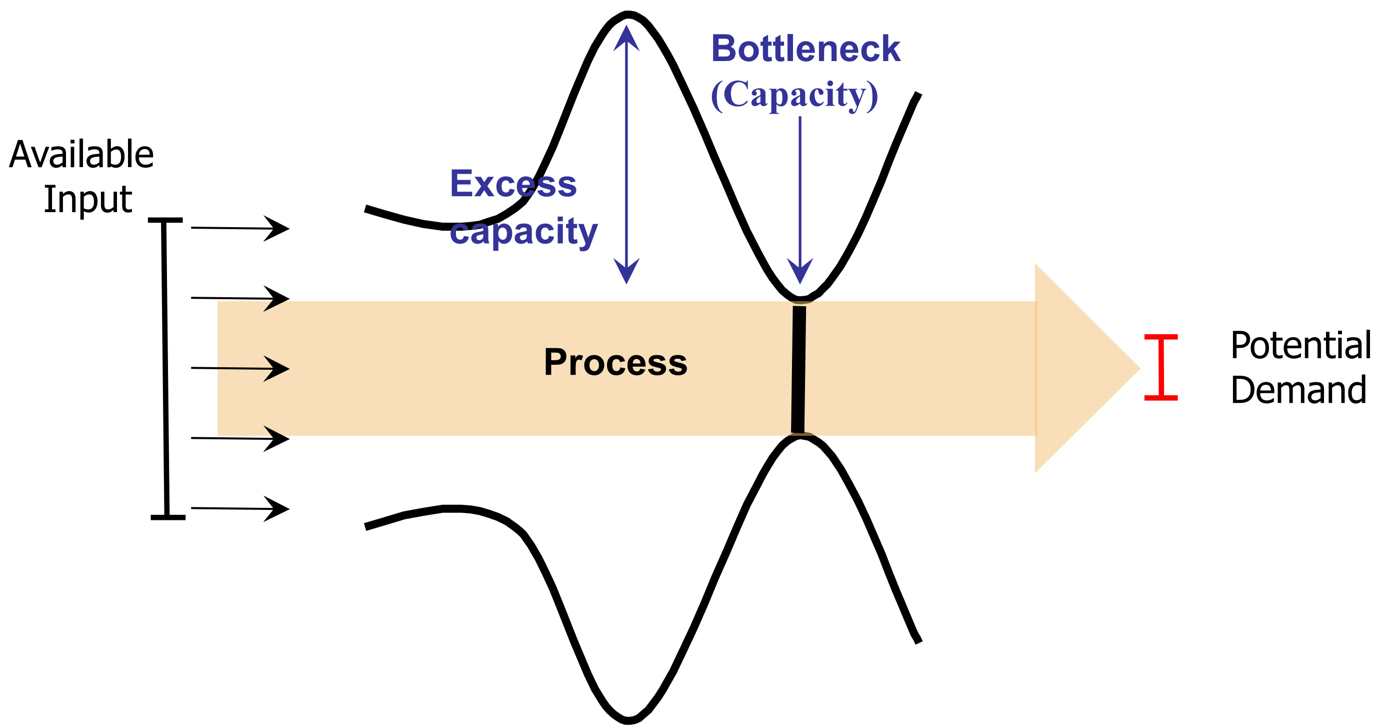
Input-constrained Process

- All available inputs are used
- Potential demands are not fully satisfied
- Process's capacity is not fully used



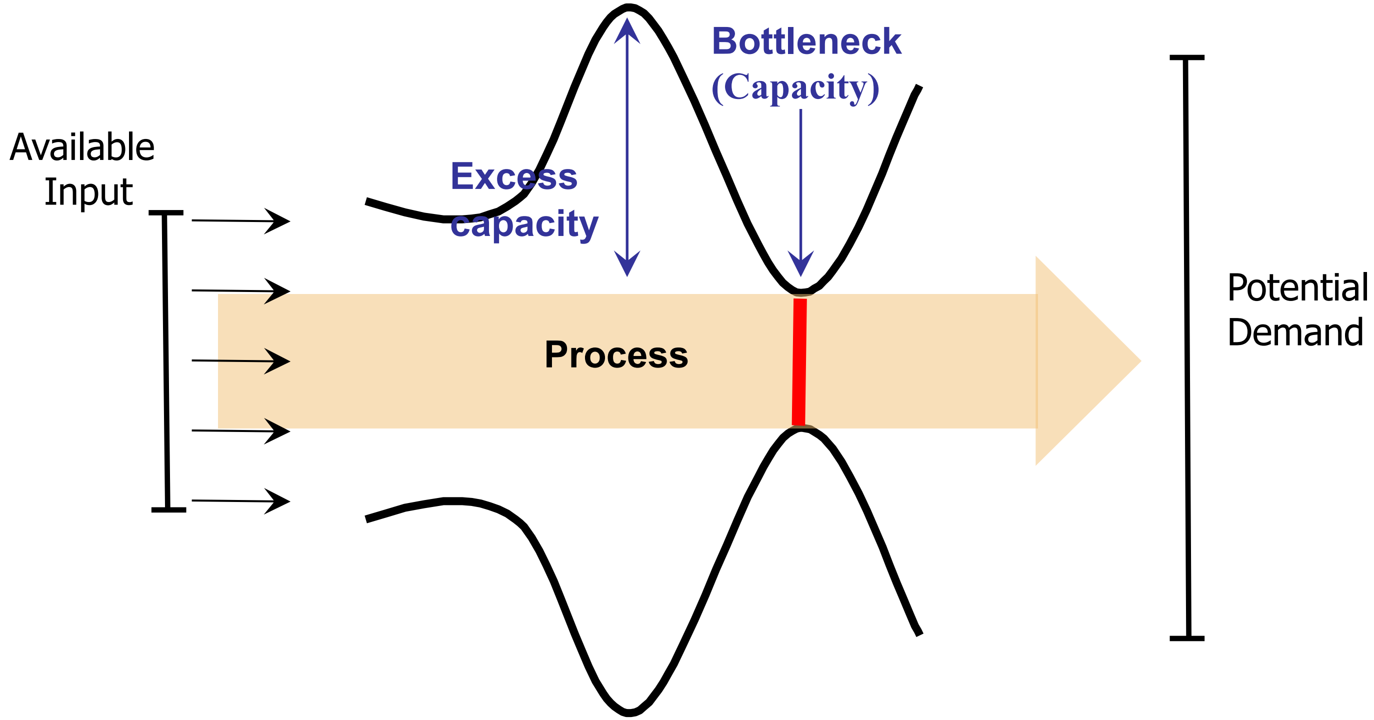
Demand-constrained Process

- All potential demands are satisfied
- Available inputs are not fully used
- Process's capacity is not fully used



Capacity-constrained Process

- Available inputs are not fully used
- Potential demands are not fully satisfied
- Process is running at full capacity



Process Flow Rate and Utilization

- Utilization of resource

$$\text{Resource Utilization} = \frac{\text{Flow rate}}{\text{Resource capacity}}$$

- If a resource runs at its capacity, we get 100% utilization
- Given flow rate being same for all resources, if a resource has a lower capacity, it will have a higher utilization
- Thus, bottleneck is the resource with the smallest capacity or highest utilization

Process Flow Rate and Utilization

- Utilization of process

$$\text{Process Utilization} = \frac{\text{Flow rate}}{\text{Process capacity}}$$

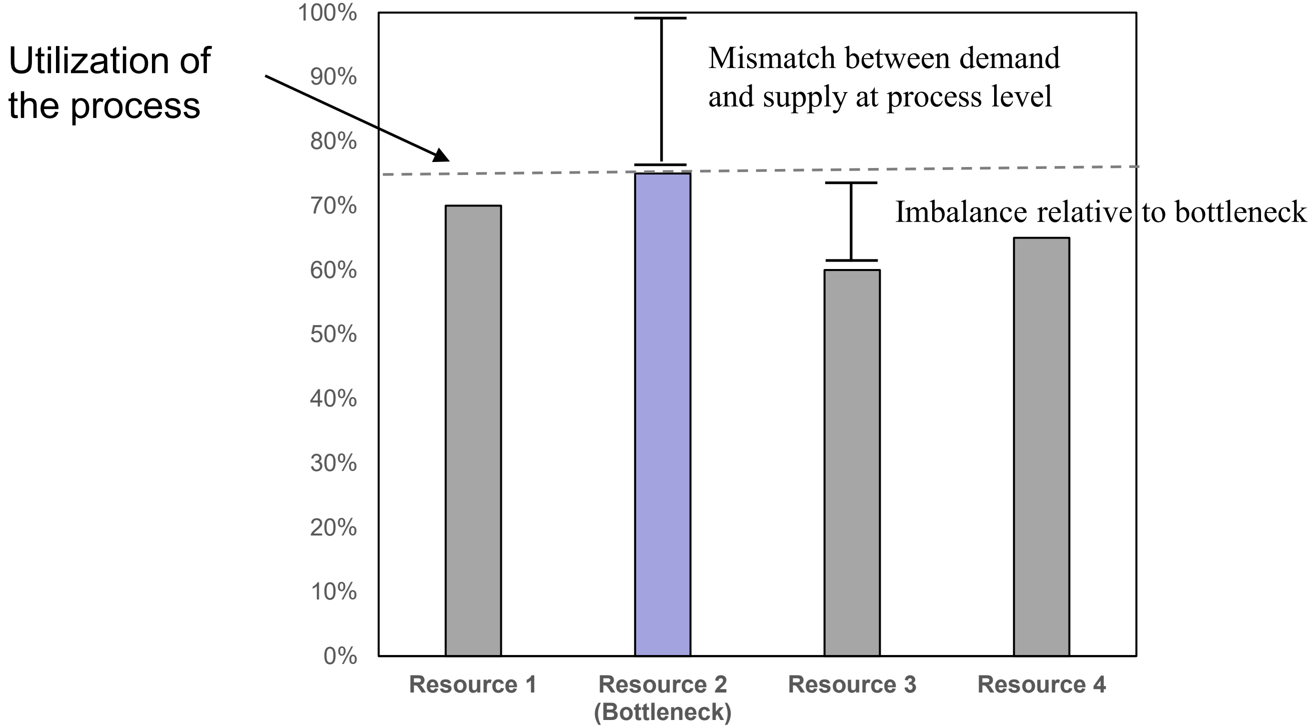
- It measures how many percents of the capacity is being used in actual production (flow rate)

- Since process capacity equals the bottleneck capacity (smallest one), process utilization equals the bottleneck utilization (highest one)

- Thus, utilization of process equals to the highest utilization of all resources

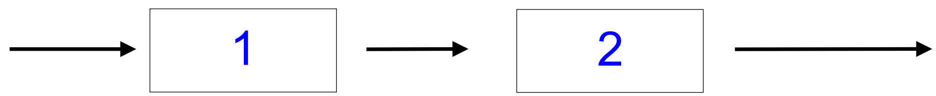
Utilization Profile Chart

- We can visualize the resource usage by plotting their utilization



- Idle capacity: mismatch between supply and demand
- Capacity gap relative to the bottleneck resource

Capacity and Utilization

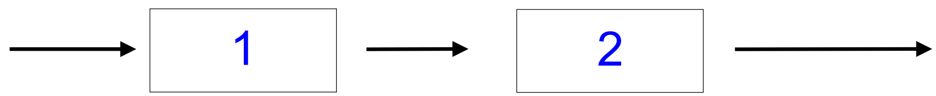


Resources:	Worker A	Worker B
Processing time:	2 min/unit	3 min/unit

What is the capacity of each worker and the process? What is the bottleneck?

- Capacity of worker A: 60/2 = 30 units/hour
- Capacity of worker B: 60/3 = 20 units/hour
- Process capacity = minimum{20,30} = 20 units/hour
- Bottleneck is worker B

Capacity and Utilization

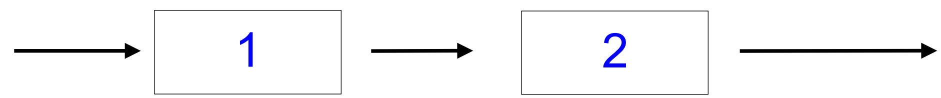


Resources:	Worker A	Worker B
Processing time:	2 min/unit	3 min/unit

Suppose the available input rate is 15 units/hour and potential demand is 25 units/hour, what is the process flow rate?

- Flow rate = minimum{input rate, process capacity, potential demand} = minimum{15,20,25} = 15 units/hour
- It is an input-constrained process

Capacity and Utilization

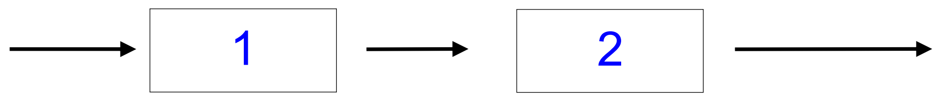


Resources:	Worker A	Worker B
Processing time:	2 min/unit	3 min/unit

At the above flow rate, what is the utilization of the process and the two workers?

- Given flow rate = 15 units/hour
- Utilization of worker A = 15/30 = 50%
- Utilization of worker B = 15/20 = 75%
- Utilization of process = 15/20 = 75%
- Process utilization equals the bottleneck utilization

Capacity and Utilization



Resources:	Worker A	Worker B
Processing time:	2 min/unit	3 min/unit

Suppose we have enough input and demand, and the process is running at its capacity, what is the utilization of worker A and B?

- At capacity, the flow rate = process capacity = 20 units/hour
- Utilization of worker A = 20/30 = 67%
- Utilization of worker B = 20/20 = 100%

- The utilization of non-bottleneck resource is smaller than 100% even the process runs at its capacity!

Formulas for Process Analysis

- Definitions of flow time, cycle time, flow rate, capacity, inventory
- $\text{Cycle time} = \frac{1}{\text{Flow Rate}}$
- Little's formula:

$$\text{Avg Inventory} = \text{Avg Flow time x Avg Flow rate}$$

- Process capacity = minimum{Capacity of resource 1, Capacity of resource 2, …, Capacity of resource n}

- $\text{Flow rate} = \text{minimum}\{\text{Available input rate, Potential demand rate, Process capacity}\}$

Summary of Process Analysis

- Process view of organization and activities
  - Process flow diagram and process measures
- Little's law:
  - Relationship between inventory, flow time, and flow rate
- Bottleneck and process capacity
  - Identify bottleneck in the process: the resource with the smallest capacity or highest utilization

Multiple Types of Flow Units

- Not required for this course

- A process can have multiple flow units
  - E.g., undergraduate/graduate students, patients of different types
  - Each type of flow units can follow different paths

- Bottleneck can always be identified as the resource with the highest implied utilization

- Need to compute the demand for each resource by summing up all the units types

Implied Utilization (Not required)

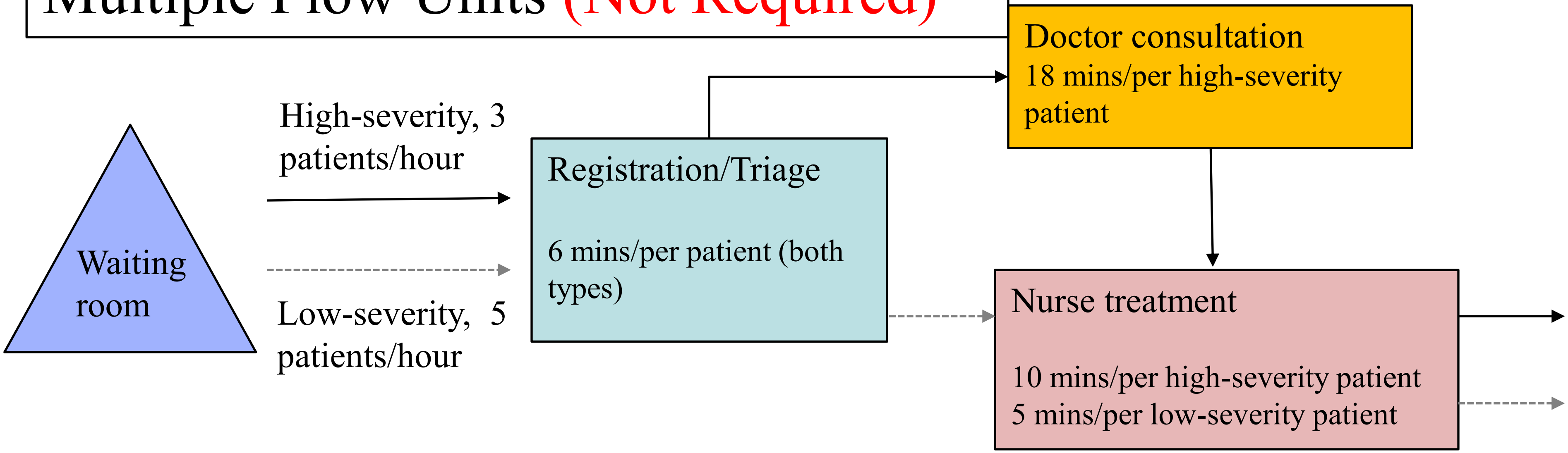
- Implied utilization:

$$\text{Implied utilization} = \frac{\text{Demand}}{\text{Capacity}}$$

- Implied utilization can be higher than 100%
  – The capacity can not meet the demand

- The bottleneck is the resource with highest implied utilization
  – Always true, even for complex processes
  – E.g. multiple types of flow units, multiple possible paths

Process Analysis: Example with Multiple Flow Units (Not Required)



	Process (mins/patient)		Input rate (patients/hour)		Demand/Workload (mins/hour)			(Implied) Utilization
	low	high	low	high	low	high	total	
Triage	6	6	5	3	30	18	48	48/60 = 80%
Doctor	0	18	0	3	0	54	54	54/60 = 90%
Nurse	5	10	5	3	25	30	55	55/60 = 91.7%

ISOM 2700: Operations Management

Session 4. Link between OM and Finance, Basic Statistics Concepts

Yiwen Shen
Dept. of ISOM, HKUST
Fall 2025

Agenda

- **Link between OM and Finance**

- Some basic concepts in statistics

OM in Finance and Accounting

- OM plays an important role in understanding and improving firm's financial performance
- Inventory turnovers and costs
- ROIC: Return on invested capital

Inventory: Purposes and Costs

- Inventory: the stock of items and goods to be sold later (e.g., think about the 711 store, Walmart)

- Purposes/benefits of inventory:
  - meet variation in demand and/or manufacturing process
  - provide safeguard/buffer for unexpected shocks
  - take advantage of economies of scale
  - More on this in later part of this course

- Costs of inventory:
  - holding (carrying) costs
  - ordering, set-up, or production change costs
  - shortage costs (lost sales)

Inventory and Out of Inventory

Inventory

Out of inventory

Inventory is everywhere

Markets | Economics

U.S. Economic Growth Quickened Last Quarter With Inventory Boost

- GDP expanded at 6.9% pace, exceeding economists' expectations
- Inventories accounted for most of the growth in fourth quarter

Source: Bloomberg

MARKETS | HEARD ON THE STREET

GM Signals an Easing of the Car Shortage

General Motors expects to boost production as chip supplies improve, giving dealers more to sell and possibly keeping a lid on inflation

A dealership in Libertyville, Ill., displays Cadillacs made by General Motors.

PHOTO: TANNEN MAURY/SHUTTERSTOCK

Source: WSJ

BUSINESS | EARNINGS

Candy Makers Say Expect Shortages This Valentine's Day

Hershey says it lacks manufacturing capacity and labor to meet demand; Oreo maker Mondelez also struggles with supplies

Hershey's fourth-quarter adjusted earnings topped analyst expectations.

PHOTO: LUKE SHARRETT/BLOOMBERG NEWS

Source: WSJ

Inventory: Financial Importance

- Inventory is a very important type of firm's asset
- By a survey, inventory is the largest asset on the balance sheet for 57% of publicly traded retailers (e.g., Walmart) in U.S.
- The average inventory/total assets ratio is 35.1%.
- Important signal for managers and analysts to determine how well a retailer is running its business

Source: Gaur & Kesavan, 2009

Inventory Turnover and Costs

- We want to compute (1) inventory turnover, (2) weeks of supply, (3) per-unit inventory cost

- Inventory turnover:

$$\text{Inventory turnover} = \frac{1}{\text{Flow Time}}$$

  - Flow time: how long does the flow unit spend within the firm before being sold
  - Inventory turnover measures how "fast" we can turn inventory to sales

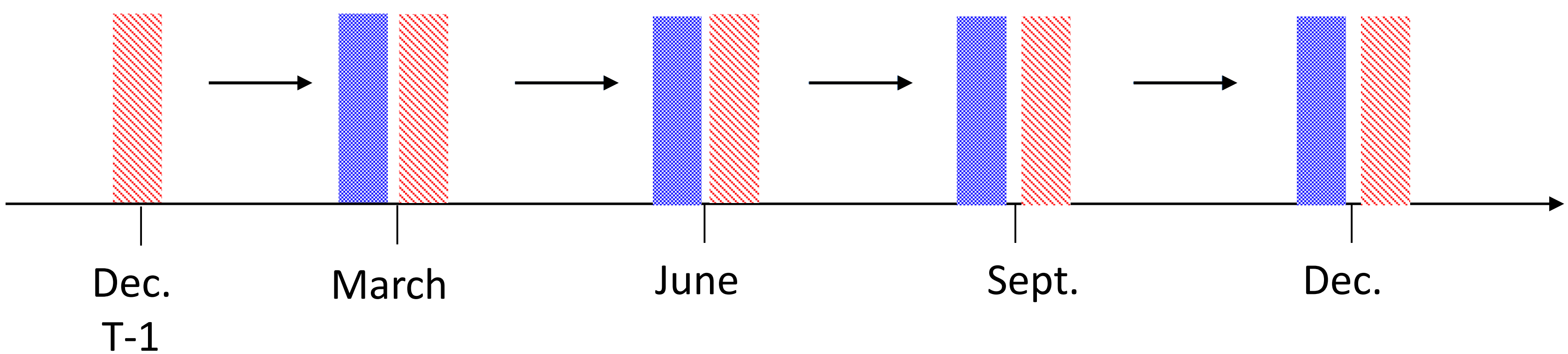
- Little's Law: Inventory = Flow time$\times$ Flow rate

Inventory Turnover and Costs

Buy 100k USD inventory

Sell 100k USD inventory

Assume the entire inventory is bought at the beginning, and sold at the end of each quarter



Sales of Inv = 400k USD
Avg. Inventory = 100k USD
Flow time = 1/4 year
Inventory turnover = 4

Inventory Turnover

- Little's Law:

$$\text{Flow time} = \frac{\text{Inventory}}{\text{Flow rate}} = \frac{\text{\# of Units as Inventory}}{\text{\# of Units Sold Each Year}}$$

- In general, we do not know the number of units as inventory and number of units sold each year (or there can be multiple types of products)

- We multiply both numerator and denominator by purchasing cost per unit

$$\text{Flow time} = \frac{\text{\# of Units as Inventory} \times \text{Purchase cost}}{\text{\# of Units Sold Each Year} \times \text{Purchase cost}} = \frac{\text{Avg. Inventory value}}{\text{COGS}}$$

- Here Avg. Inventory value is the money amount of inventory; COGS is the Cost of Goods Sold in a year, also measured in money amount

Estimate Inventory Turnover from Financial Statement

- We can get the information from the financial statement (measured in $):
  - Average inventory value
  - Flow rate: Cost of Good Sold (COGS)
  - Both COGS and inventory value are based on the purchasing cost of inventory
- Flow time:

$$\text{Flow time} = \frac{\text{Avg. Inventory value}}{\text{COGS}}$$

- Inventory turnover

$$\text{Inventory Turnover} = \frac{1}{\text{Flow Time}} = \frac{\text{COGS}}{\text{Avg. Inventory value}}$$

Weeks of Supply

- Weeks of supply: value of a firm's inventory in terms of weekly sales

$$\text{Weeks of supply} = \frac{\text{Avg. Inventory value}}{\text{COGS}} \times 52 \text{ weeks}$$

  – Inverse of inventory turnover times 52 weeks

- The weeks of supply measures how many weeks of sales we can sustain using our current inventory level

- A higher weeks of supply means we are more "robust" in the operation, but we may incur losses due to high inventory level

Per-unit Inventory Holding Cost

- Per-unit % inventory holding cost (measured as % of per-unit inventory value)

$$\text{Perunit \% inventory holding cost} = \frac{\text{Annual \% inventory holding cost}}{\text{Annual inventory turnover}}$$

  – Here we measure annual holding cost as % of average inventory value

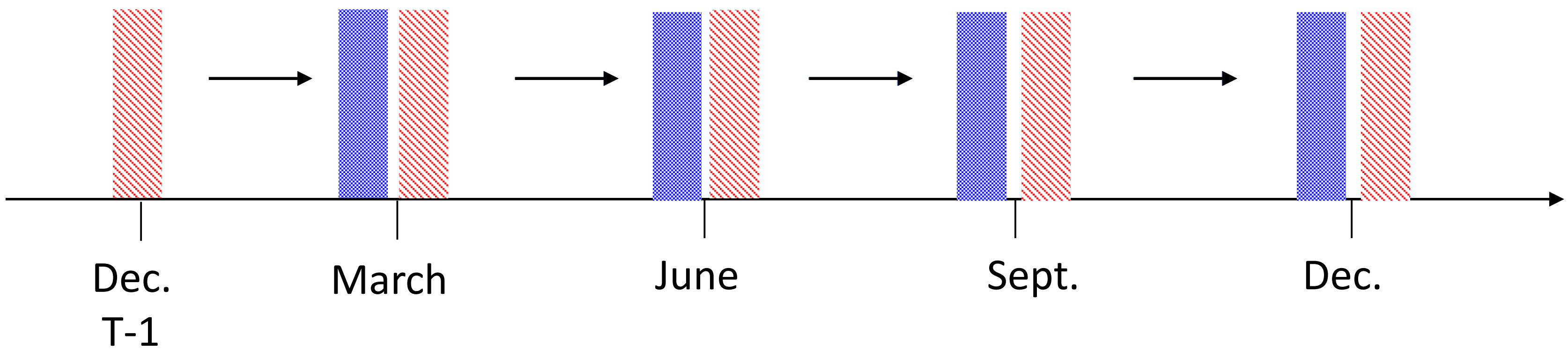
- Higher inventory turnover, fewer weeks of supply, lower per unit inventory cost
- Reason: we turn inventory to sales faster, thus we bear lower inventory cost for each unit

Inventory Turnover and Costs

Buy 100k USD inventory

Sell 100k USD inventory

Assume the entire inventory is bought at the beginning, and sold at the end of each quarter



Recall inventory turnover = 4

Weeks of supply = 1/4*52 = 13 weeks

Suppose annual holding cost equals 16% of inventory value, then per-unit holding cost = 16%/4 = 4%

An Example

- An US retailer company in 2016:
  - COGS = \$360,984 million/year, Avg. inventory = \$44,469 million

Flow time = \$44,469 million/\$360,984 million/year

= 0.123 year = 45 days

Inventory turnover = 1/45 turns/day = 8.12 turns/year

Weeks of supply = 1/8.12 × 52 = 6.4 weeks

- Assume annual inventory holding cost = 20% (of inventory value)

Per-unit inventory holding cost = 20%/8.12 = 2.46%

- The company on average turns its inventory eight times in a year
- Additional 2.46% of costs are imposed due to holding inventory

Inventory Turnover by Sectors

- Inventory turnover can vary substantially across industry sectors

Inventory Turnover Ratio by Economic Sector

Ranking	Industry Sector	Inventory Turnover Ratio (avg.)
1	Financial	48.76
2	Services	28.47
3	Transportation	14.15
4	Technology	11.21
5	Retail	10.86
6	Utilities	10.44
7	Energy	8.20
8	Consumer Discretionary	6.86
9	Basic Materials	6.77
10	Consumer Non Cyclical	6.70

Source: CSI Markets

We can evaluate a firm's performance by comparing its inventory turnover with firms in the same sector

Return on Invested Capital

- ROIC is perhaps the most important financial metrics
  - Economic value created = Capital×(ROIC - WACC)
  - WACC: weighted average cost of capital

- How is ROIC affected by OM measures, e.g., flow rate (units sold per year)?

- Analytical tools: DuPont model

Building an ROIC Tree

- ROIC:

$$\text{ROIC} = \frac{\text{Return}}{\text{Invested capital}}$$

- Decompose to two parts:

margin — asset turnover

$$\text{ROIC} = \frac{\text{Return}}{\text{Invested capital}} = \boxed{\frac{\text{Return}}{\text{Revenue}}} \times \boxed{\frac{\text{Revenue}}{\text{Invested capital}}}$$

- Return = Revenue – Fixed costs – Volume (Flow rate) × Variable costs
- Revenue = Flow rate × Price
- Plugging in, we can derive ROIC

$$\text{ROIC} = \left[1 - \frac{\text{Fixed costs}}{\text{Flow rate} \times \text{Price}} - \frac{\text{Variable costs}}{\text{Price}}\right] \times \frac{\text{Flow rate} \times \text{Price}}{\text{Invested capital}}$$

Flow Rate on ROIC

- ROIC

$$\text{ROIC} = \left[1 - \frac{\text{Fixed costs}}{\text{Flow rate} \times \text{Price}} - \frac{\text{Variable costs}}{\text{Price}}\right] \times \frac{\text{Flow rate} \times \text{Price}}{\text{Invested capital}}$$

- Its shows how ROIC is affected by flow rate

- Keeping other variables fixed, ROIC increases in flow rate
  - Higher margin as fixed costs are spread out
  - Higher asset turnover due to increase in revenue

An Example

- Price: $100
- Variable cost: $80
- Fixed costs: $1000/year
- Flow rate: 100 units/year
- Invested capital: $12,000

$$\text{ROIC} = \left[1 - \frac{1000}{100 \times 100} - \frac{80}{100}\right] \times \frac{100 \times 100}{12000} = 8.33\%$$

What-if Analysis

- What if we increase the flow rate by 10%?
  - New flow rate = 100×1.1 = 110 units/year

$$\text{New ROIC} = \left[1 - \frac{1000}{110 \times 100} - \frac{80}{100}\right] \times \frac{110 \times 100}{12000} \quad = 10\%$$

- Increase in ROIC:

$$\frac{10\% - 8.33\%}{8.33\%} = 20\%$$

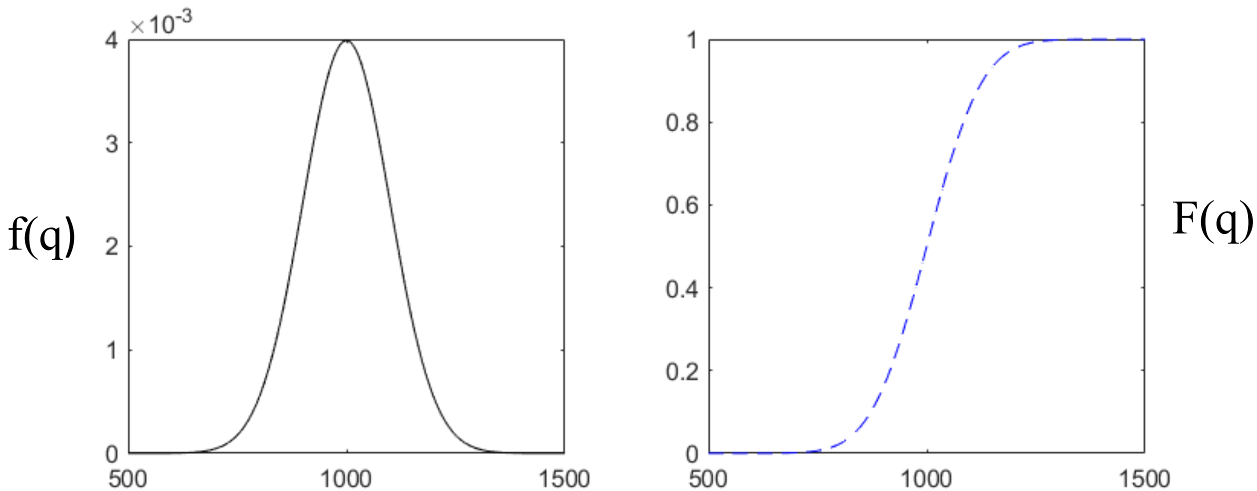
- Increase in return: (10%−8.33%) ×\$12,000 = \$200/year

Agenda

- Link between OM and Finance
- **Some basic concepts in statistics**

Random Variable

- A random variable X
  - mean, variance = (standard deviation)^2
- Cumulative probability F(Q):
  - F(Q) = Prob (X will be less than or equal to Q)
- Probability density f(q):
  - f(q) = Prob (X will be exactly q)



Random Variable: Demand

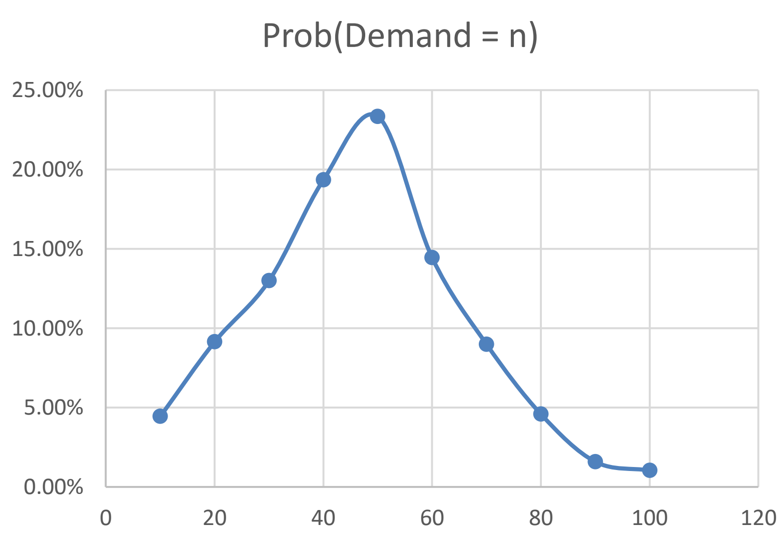
- Demand for a product/service is usually random, and we can only forecast it from its distribution

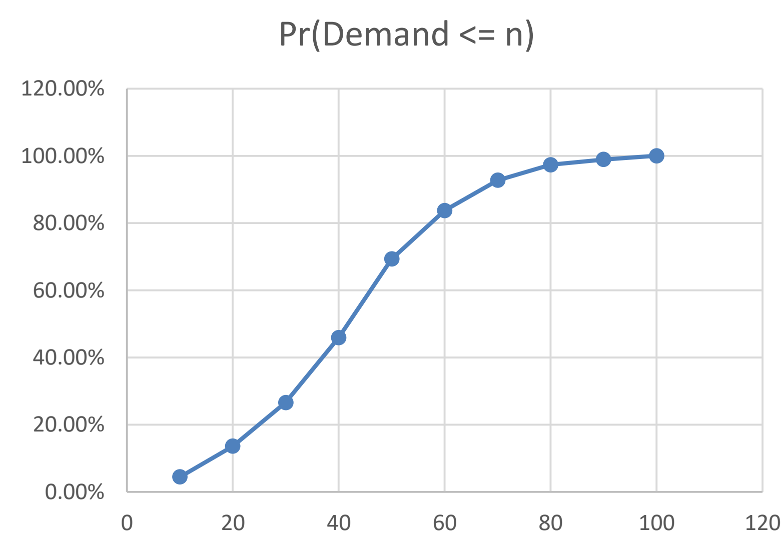
- Demand forecasting is very important in OM (we will learn more about it)

- We can obtain demand distribution from historical or survey data

Demand Distribution: Example

		Probability density	Cumulative probability
Demand	Frequency (out of 2000)	Prob(Demand = n)	Pr(Demand <= n)
10	89	4.45%	4.45%
20	183	9.15%	13.60%
30	260	13.00%	26.60%
40	387	19.35%	45.95%
50	467	23.35%	69.30%
60	289	14.45%	83.75%
70	180	9.00%	92.75%
80	92	4.60%	97.35%
90	32	1.60%	98.95%
100	21	1.05%	100.00%





Expected Value

- For a random variable X, we can calculate its expected value
  - Usually approximated by the average from its samples (LoLN)

$$\begin{aligned}\text{Expected value} &= \text{1st value} \times \text{Prob. of 1st value} \\ &+ \text{2nd value} \times \text{Prob. of 2nd value} \\ &+ \cdots + \text{nth value} \times \text{Prob. of nth value}\end{aligned}$$

- For example, suppose X takes value -1 with prob. 0.3, 0 with prob. 0.6, and +1 with prob. 0.1
  - Its expected value = $-1 \times 0.3 + 0 \times 0.6 + 1 \times 0.1 = -0.2$

- For continuous random variable, expected value is given by

$$E[X] = \int f(x)xdx$$

here f(x) is the probability density function (not required)

Coefficient of Variation

- Compare two distributions:
  - A: mean 100, standard deviation 10
  - B: mean 10, standard deviation 5

- Which one you think have larger variation?

- Coefficient of variation is a way to measure the degree of variation for different objectives
  - Widely used in OM
  - Idea of standardization

$$CV = \frac{standard\ deviation}{mean}$$

I.I.D Samples: Average

- Suppose X_1, X_2, …, X_n are independent observations with identical distribution (i.i.d.)
  - Each of them has mean E[X] and variance Var[X]
  - n is the number of samples
- Their sample average (also random) is given by

$$\bar{X} = \frac{X_1 + X_2 + \cdots + X_n}{n}$$

- Statistics of sample average:

$$E[\bar{X}] = E[X];\ Var[\bar{X}] = \frac{Var[X]}{n};\ Sd[\bar{X}] = \frac{Sd[X]}{\sqrt{n}}$$

I.I.D Samples: Summation

- Suppose X_1, X_2, …, X_n are independent observations with identical distribution (i.i.d.)
  – Each of them has mean E[X] and variance Var[X]

- Their sum is given by

$$\sum_{i=0}^{n} X_i = X_1 + X_2 + \cdots + X_n$$

- Statistics of sample sum:

$$E[\sum X_i] = nE[X];\ Var[\sum X_i] = nVar[X];\ Sd[\sum X_i] = \sqrt{n}Sd[X]$$

I.I.D Samples: Example

- Suppose X_1, X_2, …, X_{100} are i.i.d. variables. Each has a mean of 3 and a variance of 4 (a standard deviation of 2)

- Their sum has mean, variance, and standard deviation of (check it by yourself):

$$E[\sum X_i] = 300, \qquad Var[\sum X_i] = 400, \qquad Sd[\sum X_i] = 20$$

- Their average has mean, variance, and standard deviation of:

$$E[\bar{X}] = 3, \qquad Var[\bar{X}] = 0.04, \qquad Sd[\bar{X}] = 0.2$$

Benefit of Pooling

- Coefficient of variation for individual X_i

$$CV(X) = \frac{Sd[X]}{E[X]}$$

- Coefficient of variation for summation $\sum X_i$

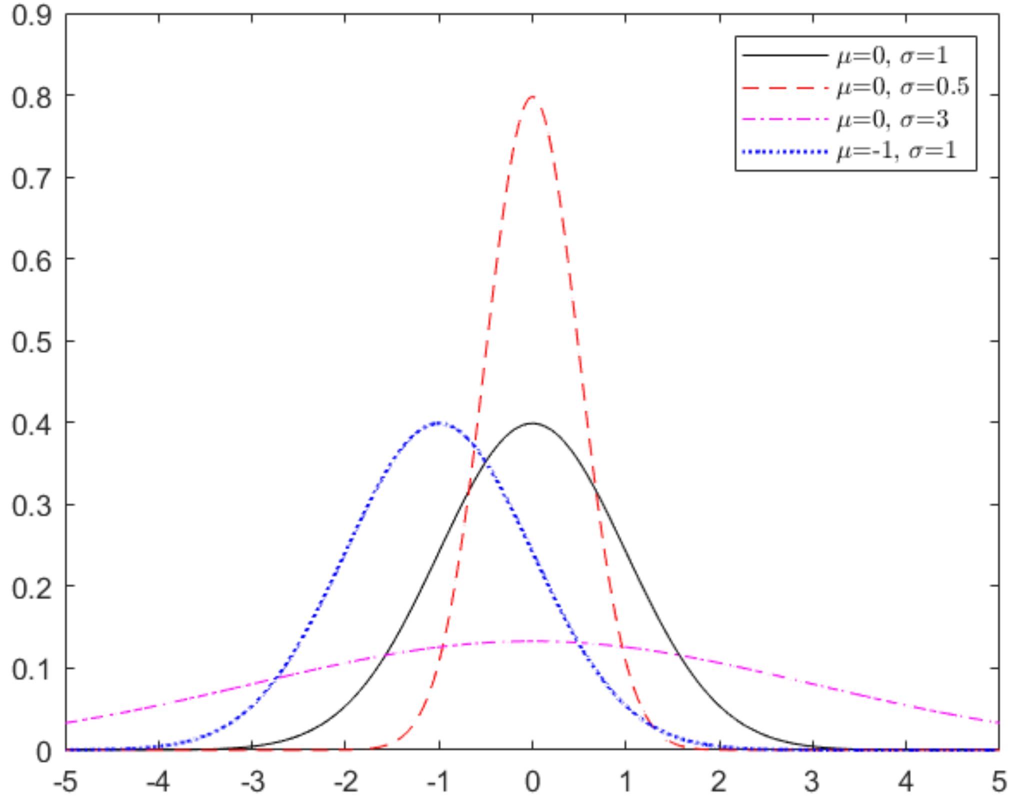
$$CV(\sum X_i) = \frac{Sd[\sum X_i]}{E[\sum X_i]} = \frac{\sqrt{n}\, Sd[X_i]}{n\, E[X_i]} = \frac{CV(X)}{\sqrt{n}}$$

- Pooling can effectively reduce variability
  - Retailing: consolidated distribution versus direct delivery
  - Queueing: pooled queue versus dedicated queues

Normal Distribution

- Normal distribution $X \sim N(\mu, \sigma^2)$:
  - Parameters: mean μ, standard deviation σ
  - Standard normal distribution: $\mu = 0$, $\sigma = 1$

Probability density function



Normal Distribution

- Suppose X ~ N(μ, σ^2), consider its transformation

$$Z = \frac{X - \mu}{\sigma}$$

- Then Z follows a standard normal distribution with mean zero and standard deviation of one
- We can convert

$$\mathrm{Prob}\{X \le Q\} = \mathrm{Prob}\left\{\frac{X-\mu}{\sigma} \le \frac{Q-\mu}{\sigma}\right\} = \mathrm{Prob}\left\{Z \le \frac{Q-\mu}{\sigma}\right\}$$

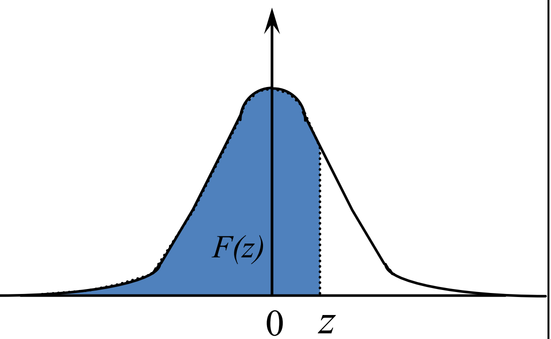
- Standard normal distribution function table:

$$\Phi(z) = \mathrm{Prob}(Z \le z)$$

The Standard Normal Distribution

Z follows standard normal distribution, find probability:

F(*z*) = Prob(*Z* ≤ *z*)



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997

Standard Normal Distribution Function Table

- Suppose X follows a normal distribution with mean $\mu=10$ and standard deviation $\sigma=10$. What is the probability that X is smaller or equal to 12.8?

- Using the transformation, we can compute the probability as

$$\text{Prob}\{X \leq 12.8\} = \text{Prob}\left\{\frac{X-10}{10} \leq \frac{12.8-10}{10}\right\} = \text{Prob}\{Z \leq 0.28\}$$

- Thus, we only need to find the probability that a standard normal variable is smaller or equal to ***z = 0.28***

- Use the standard normal distribution table!

Standard Normal Distribution Function Table

- Q: *What is the probability the outcome of a standard normal will be* ***z = 0.28*** *or smaller?*
  - Look for the intersection of the fourth row (with the header 0.2) and the ninth column (with the header 0.08) because 0.2+0.08 = 0.28, which is the z we are looking for
- A: The answer is 0.6103, which can also be written as 61.03%

Standard Normal Distribution Function Table (continued), $\Phi(z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224

ISOM 2700: Operations Management

Session 5. Variability of Service Systems

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Fall 2025

Agenda

- **Variability in Service System**
- Queueing Model
- Some Simulation Basics

Waiting in LG1

Motivating Example

- Consider the following hypothetical argument from the LG1 canteen manager:

- "*On average, the canteen operates ten hours a day with a total arrival of 500 students, thus the flow rate is 50 per hour*.
- *The canteen's capacity can serve 100 students per hour. Thus, the utilization is only 50%, meaning we are doing well*"

- Do you agree or disagree with the manager? What is missing in his arguments?

Importance of Variability

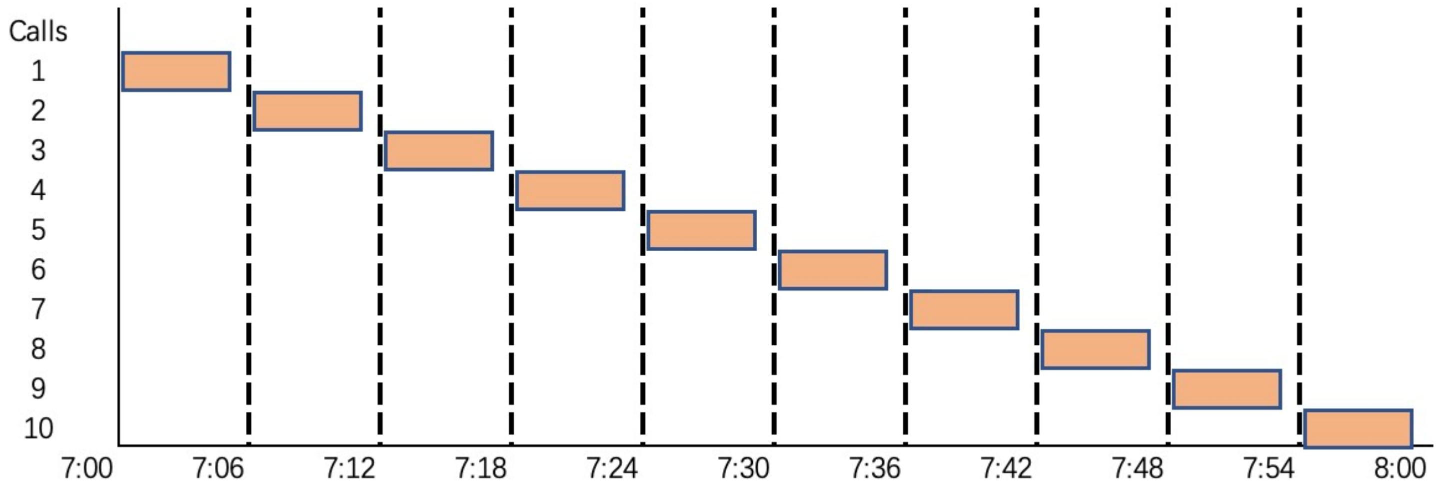
- Consider a call center with one staff; on average
  - 10 incoming calls per hour
  - each call takes 5 minutes to answer

- Capacity $= \frac{60}{5} = 12$ calls/hour

- Utilization $= \frac{10}{12} = 83.33\% < 100\%$
  - Demand < capacity

Ideal Case: No Variability

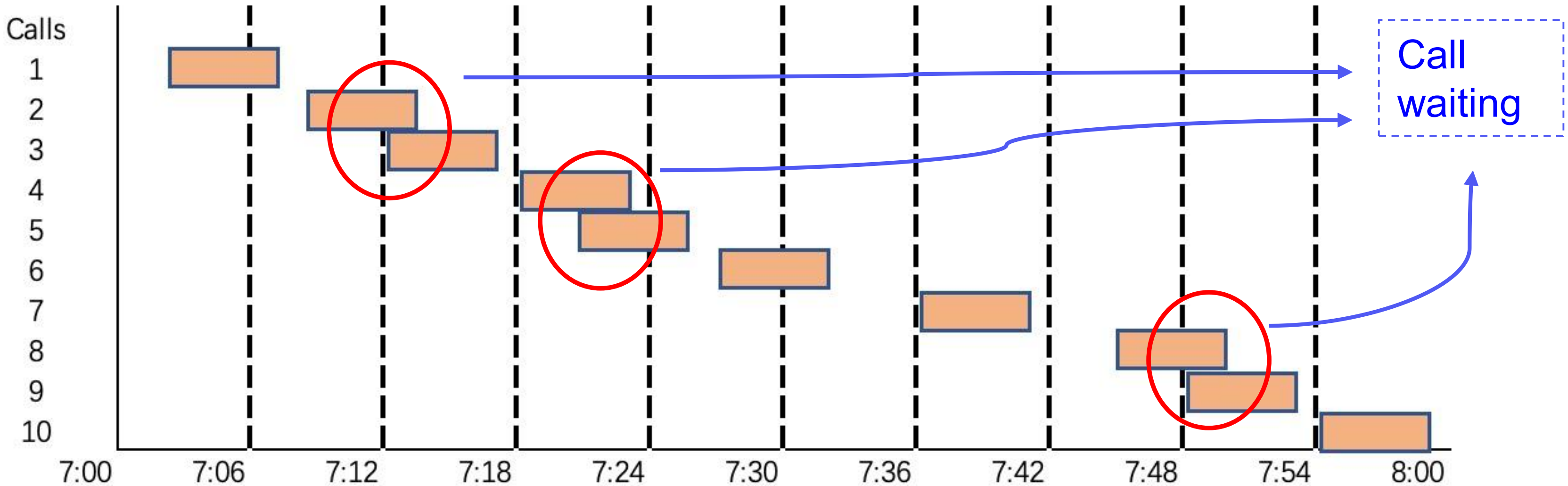
- Each call arrives exactly every six minutes; and takes exactly five minutes to answer



- No incoming call will be waiting

Variability in Arrival Times

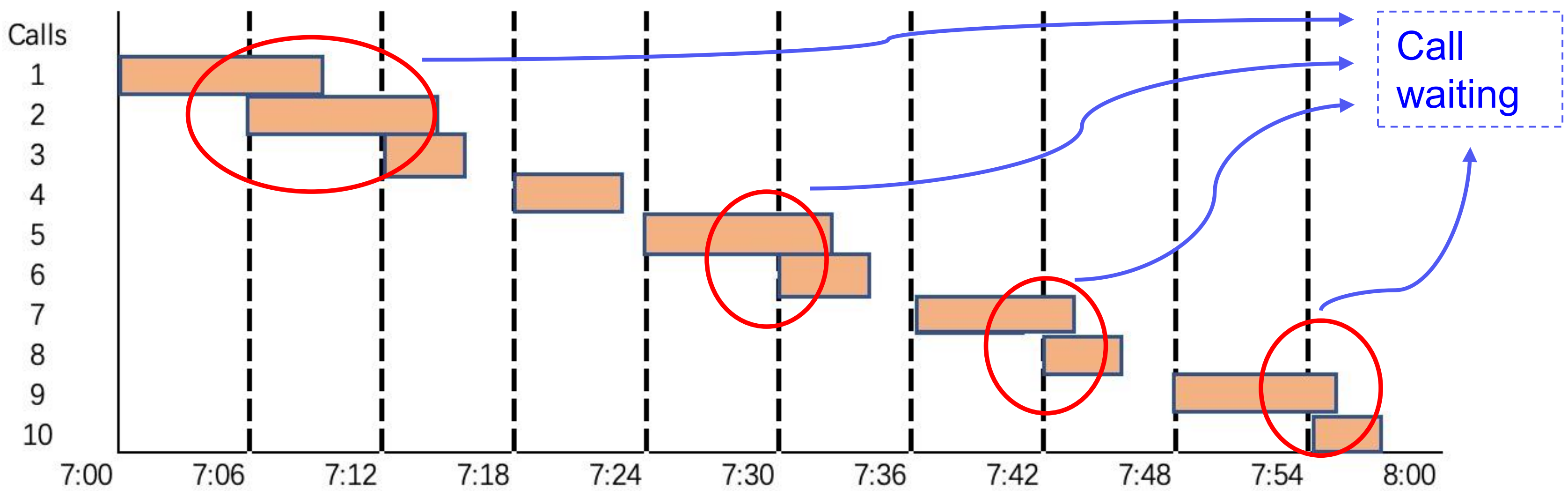
- Still, there are 10 incoming calls per hour
- But now, assume variability in their arrivals



- There will be waiting for some of the calls (call 3,5, and 9)

Variability in Service Times

- Still, the average time for a call is five minutes
- But now, assume variability in their service time



- There will be waiting for some of the calls (call 2, 3, 6, 8, and 10)

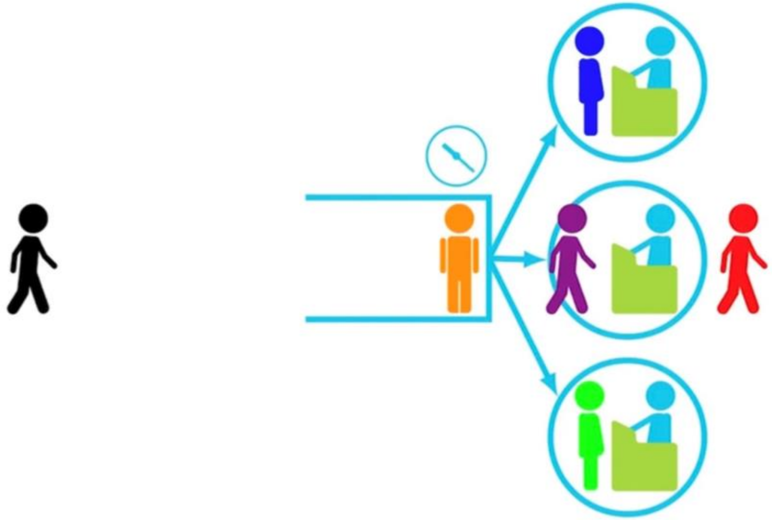
Root of Waiting: Variability

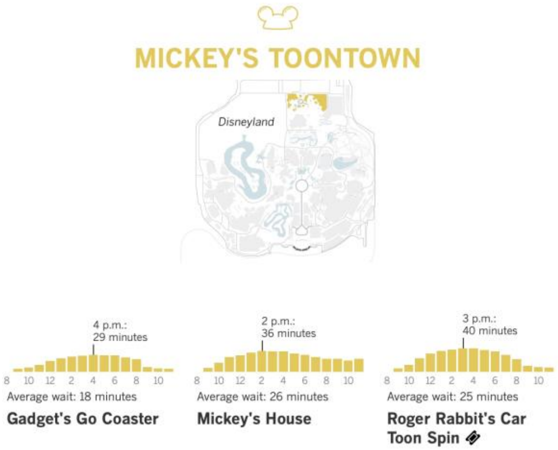
- Variability in arrival or service time leads to waiting of customers and longer queues
- This happens even when the demand is smaller than capacity (utilization < 100%)
- Reason: service can only start when there are both capacity and demand
  - However, demand can run ahead of capacity
  - Capacity of service cannot be stored or carried over

Short-term/temporary mismatch between demand and supply

Waiting Lines and Queuing System

- Where?
  - Call centers
  - Theme parks
  - Restaurants
  - Hospitals …





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Agenda

- Variability in Service System
- **Queueing Model**
  - Queueing models
  - Formulas for M/M/1 model
  - Excel solution for M/M/s models
- Some Simulation Basics

Why Study Queuing Models?

- Modeling of queueing systems

↓

- To assess system performance (description)
  - waiting time
  - number of customers in the system
- To design a “better” system (optimization)
  - reduce waiting time, queue length

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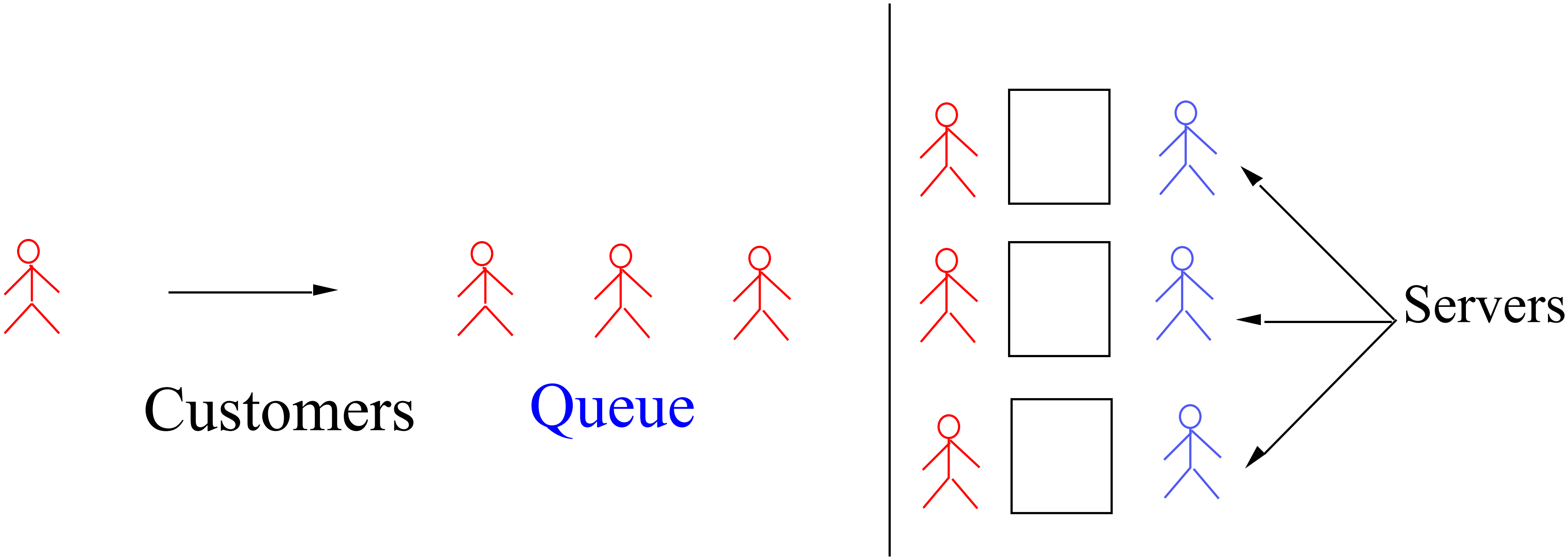
- To improve customer service
- Good service can
  - increase profit & market share
  - enhance customer satisfaction

Queuing Model



- Queueing models are characterized by customers arriving at a service facility
- Assumption: a customer can be served by any one of the servers

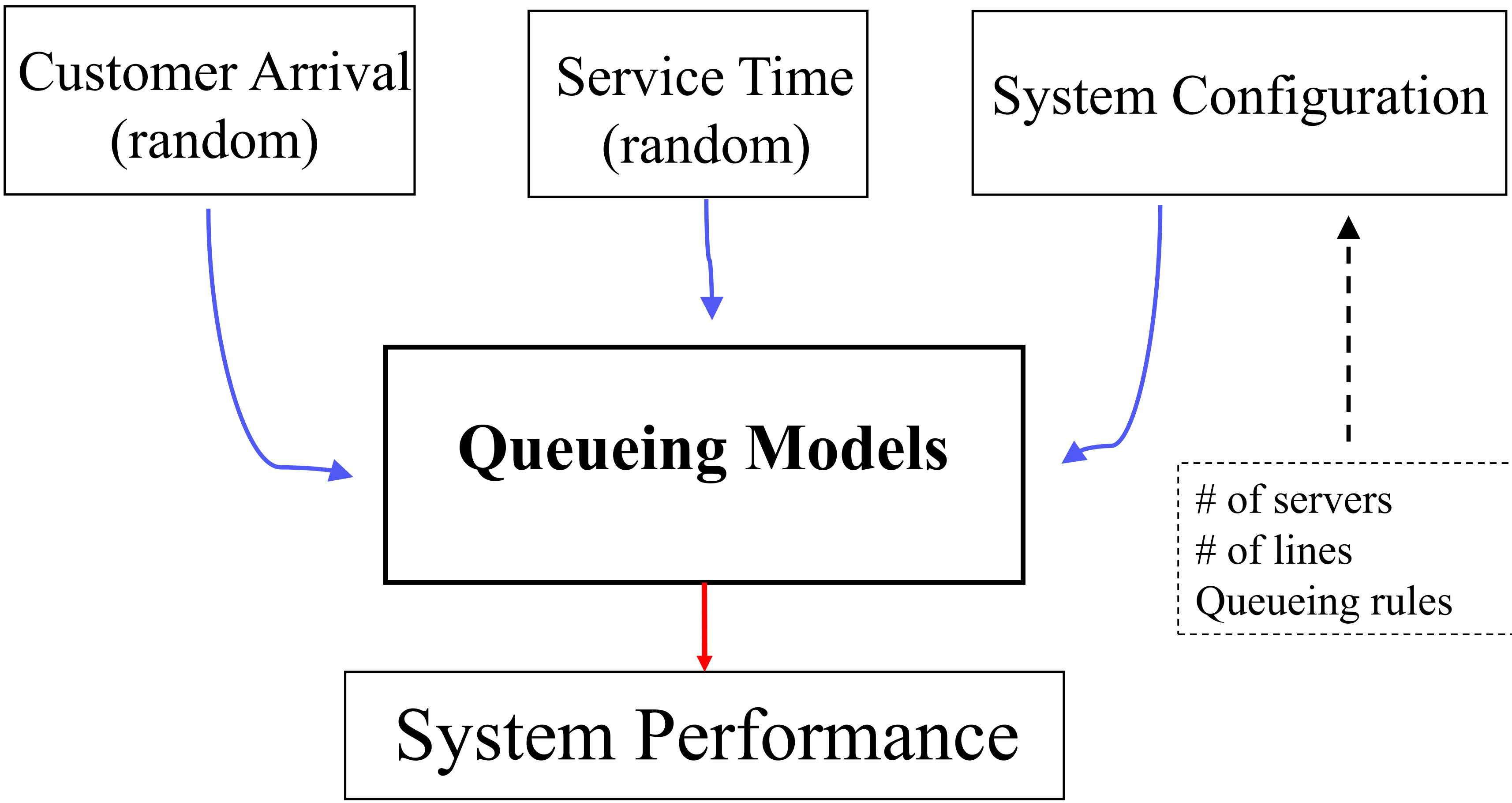
How to Define a Server

- A "server" in the queueing model is defined as the necessary resource (people or equipment) that can serve one customer independently at a time

- So, having *n* servers in the queueing models means that you can serve *n* customers at most simultaneously

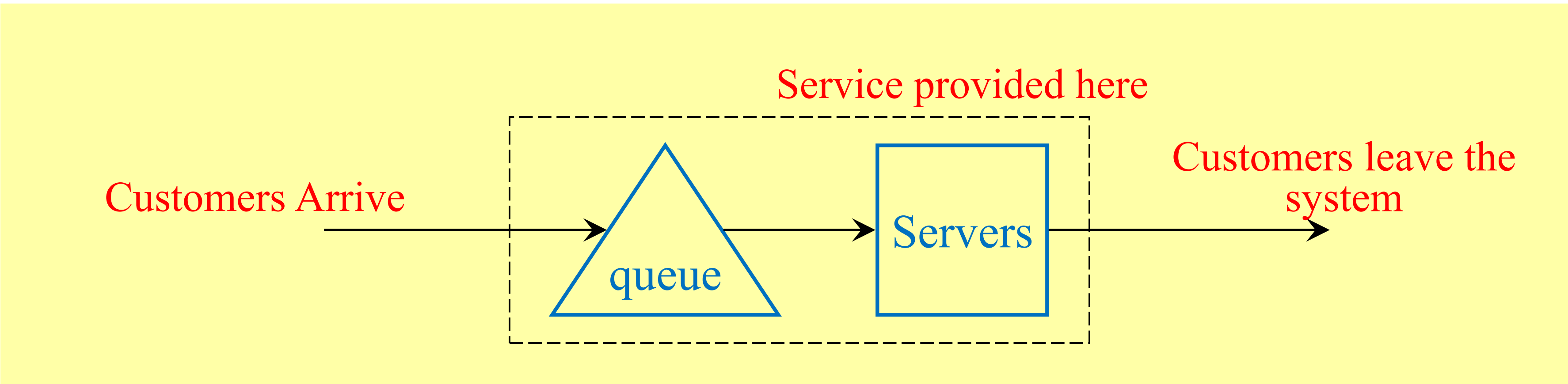
- Server does not necessarily refer to one staff or machine, as sometimes you need multiple staff to serve one customer at a time

Elements of Queuing Models

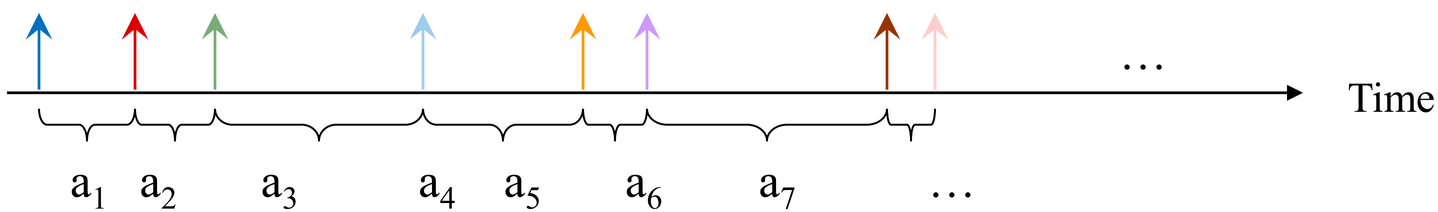


- Average # customers in the system (and queue)
- Average time spent in the system (and queue)

Elements of Queuing Models



- Inter-arrival time: time between two consecutive arrivals (random)



- Service time: time taken to serve one customer (random)

Interarrival Time and Total Arrivals

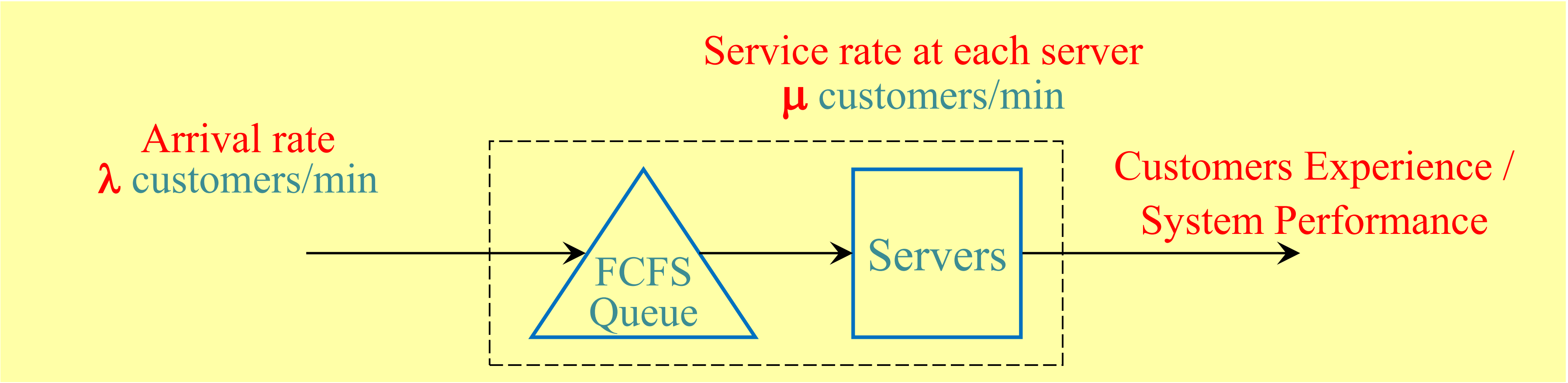
- Suppose the interarrival times are given as follows
  - Between 0th and 1st customer: 2 mins
  - Between 1st and 2nd customer: 5 mins
  - Between 2nd and 3rd customer: 1.5 mins
  - Between 3rd and 4th customer: 4 mins
  - Between 4th and 5th customer: 3 mins

- Then the arrival times are given by
  - 1st customer: 2 min
  - 2nd customer: 2 + 5 = 7 min
  - 3rd customer: 7+1.5 = 8.5 min
  - 4th customer: 8.5 + 4 = 12.5 min
  - 5th customer: 12.5 +3 = 15.5 min

It is equivalent to know:
(1) interarrival time between each two customers
(2) arrival time of each customer
(3) total arrivals at each time point

- Number of total arrivals:
  - First 5 minutes: one (customers 1)
  - First 10 minutes: three (customers 1, 2, and 3)
  - First 15 minutes: four (customers 1, 2, 3, and 4)

M/M/s Model



- **Input of the queue**
  - ❑ Exponential inter-arrival time: arrival rate = λ
  - ❑ Exponential service time: service rate at each server = μ
  - ❑ Number of servers = s
  - ❑ Assumption: one common queue; FCFS; $\lambda < s\mu$

This is called M/M/s queue

Name convention for M/M/s model

- M: exponential inter-arrival time
- M: exponential service time
- s: number of servers

Poisson Arrival Process

Inter-arrival time **Exponential Distribution**	⟷	# of Arrivals per Period **Poisson Distribution**

- Customers arrive at random, independently of one another, at the same average rate
  - Inter-arrival times are independent
  - Number of arrivals for different periods are independent

- When the inter-arrival time follows an exponential distribution, the number of arrivals in a given period follows a Poisson distribution
  - The mean of Poisson arrivals equals to the arrival rate × period length
  - This is not required for the course

Arrival and Service Rates

- λ = Average number of arrivals per time period
  - e.g., one customer arrives every 20 minutes
  - arrival = 3 units/hour

λ is also the (average) flow rate of the system

- μ = Average number of customers served per time period for each server
  - e.g., service time = 15 minutes/unit
  - $\mu = 60/15 = 4$ units/hour

μ is also the capacity of each server

Server Utilization ρ

$$\text{Server utilization} = \frac{\text{average arrival rate}}{\text{average potential service rate}}, \text{i.e.}, \rho = \frac{\lambda}{s\mu}$$

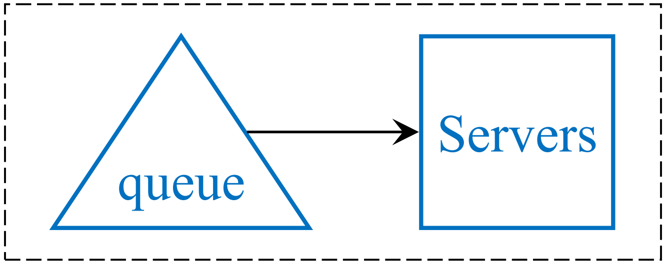
- Server utilization is the average fraction of time a server is busy (just like process utilization)
- The total capacity is $s \times \mu$: pooled resource case in Session 3
- Stable condition: $\rho = \frac{\lambda}{s\mu} < 1$, i.e., $\lambda < s\mu$

Example: There are two servers. The average arrival rate is 10 customers per hour. The average service time per customer is 10 minutes

$$\text{Server utilization} = \frac{10}{2 \times \frac{60}{10}} = 83.33\%$$

Queueing Metrics

- Utilization of the server ρ
- Average number of customers in the queue L_q (or in the system L_s)
  - System = queue + servers



- Average time spent in the queue W_q (or in the system W_s)
- Probability of waiting when arriving the queue
  - i.e., queue is nonempty

Process View of Queue

- We can view the queueing model through the process view

	Queueing models	**Process analysis**
s	Number of servers	Number of pooled resources
λ	Arrival rate	Flow rate
μ	Service rate	Capacity of each server
ρ	Server utilization	Utilization
L_s or L_q	Number of customers (in system or queue)	Inventory in the process
W_s or W_q	Waiting time	Flow time

Connections between measures

- Time in system = Time in queue + Service time

$$W_s = W_q + \frac{1}{\mu}$$

- Number in system = Number in queue + Number in servers

$$L_s = L_q + \rho \times s$$

- Little's Law for queue and system

$$L_q = \lambda \times W_q, \quad L_s = \lambda \times W_s$$

- These holds for general queueing models

Formulas for M/M/1 Model

Closed-formula for M/M/1 queue: need condition $\lambda < \mu$

Given: Mean arrival rate = λ, Mean service rate = μ

- Utilization: $\rho = \frac{\lambda}{\mu}$
  - Proportion of time the server is busy
- Probability of exactly *n* customers in the system: $P_n = \rho^n(1 - \rho)$
  - When a random customer arrives, the probability she sees *n* customers in the system
- Probability of delay = 1 – probability of zero customer in system = ρ
  - When a random customer arrives, the probability she needs to wait for the server

Formulas for M/M/1 Model

Closed-formula for M/M/1 queue: need condition $\lambda < \mu$

Given: Mean arrival rate = λ, Mean service rate = μ

- Mean number of customers in the system: $L_s = \frac{\lambda}{\mu-\lambda} = \frac{\rho}{1-\rho}$
    - Including both queue and server
- Mean number of customers in queue: $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{\rho^2}{1-\rho}$
- Mean time in system: $W_s = \frac{L_s}{\lambda} = \frac{1}{\mu-\lambda}$
    - Including both queue and server
- Mean time in queue: $W_q = \frac{L_q}{\lambda} = \frac{\rho}{\mu-\lambda}$

Connections between measures (verify in M/M/1 model)

- Recall utilization $\rho = \frac{\lambda}{\mu}$ in M/M/1 queue
- Time in queue + Service time = Time in system

$$W_q + \frac{1}{\mu} = \frac{\rho}{\mu - \lambda} + \frac{1}{\mu} = \frac{\lambda}{\mu(\mu - \lambda)} + \frac{1}{\mu} = \frac{1}{\mu - \lambda} = W_s$$

- Number in queue + Number in servers = Number in system

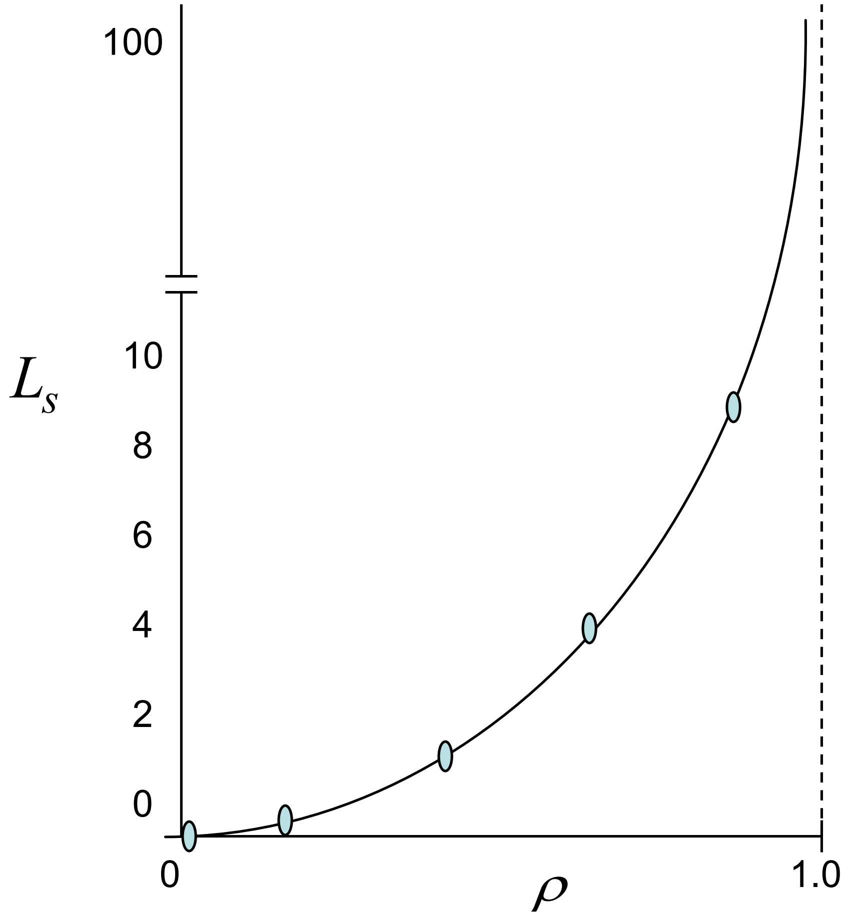
$$L_q + \rho = \frac{\rho^2}{1 - \rho} + \rho = \frac{\rho}{1 - \rho} = L_s$$

- Little's Law for queue and system

$$\lambda \times W_q = \frac{\lambda \times \rho}{\mu - \lambda} = \frac{\rho^2}{1 - \rho} = L_q \text{ and } \lambda \times W_s = \frac{\lambda}{\mu - \lambda} = \frac{\rho}{1 - \rho} = L_s$$

- Probability of delay = Probability of non-idle = ρ

Congestion vs. System Utilization



For the M/M/1 model, avg. number in the system is:

$$L_s = \frac{\rho}{1-\rho}$$

ρ	L_s
0	0
0.2	0.25
0.5	1
0.8	4
0.9	9
0.99	99

Queue length (waiting time) grows **super-linearly** in the utilization --- costly to have very high utilization

- This will not happen in if we do not have variation in the system

Example: Pizza Hut

At a take-away only Pizza Hut branch (assuming exponential interarrival and service time)

- Customers arrive randomly at the average rate of **25 per hour**
- There is one employee who can, on average, **serve one customer every 2 minutes**

M/M/1 Queue

Poisson Arrival

Exponential Service Time

1 Employee

Example: Pizza Hut (continued)

1) Average utilization of the employee

$$\rho = \frac{\lambda}{\mu} = \frac{25 \text{ cuts/hr}}{30 \text{ cust/hr}} = 0.8333$$

2) Average queue length

$$L_q = \frac{\rho^2}{1-\rho} = \frac{0.8333^2}{1-0.8333} = 4.167$$

3) Average number of customers in the system

$$L_s = \frac{\rho}{1-\rho} = \frac{0.8333}{1-0.8333} = 5$$

Example: Pizza Hut (continued)

4) Average waiting time in line

$$W_q = \frac{L_q}{\lambda} = 0.1667 \text{ hrs} = 10 \text{ mins}$$

5) Average amount of time spent in the system

$$W_s = \frac{L_s}{\lambda} = 0.2 \text{ hrs} = 12 \text{ mins}$$

Example: Pizza Hut (continued)

6) What is the probability that an arriving student will find at least one other student waiting in line?

When there is at least one student in line, it implies at least two will be in system (one being served and one waiting). The probability is $1 – (P_0 + P_1)$.

Compute $P_n = (1 - \rho)\times \rho^n \rightarrow P_0 = 0.1667$ and $P_1 = 0.1389$

The probability that an arriving student will find at least one other student waiting in line is 0.6944

ISOM 2700: Operations Management

Session 6. Queuing Models

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Dept. of ISOM, HKUST
Fall 2025

Agenda

- Variability in Service System

- **Queueing Model**
  - M/M/1 model
  - **M/M/s model**

- Some Simulation Basics

M/M/s Queue: Using Spreadsheet

When we have more than one server: M/M/s model

Model parameters

λ

μ

s

→ Performance measures

$\rho, W_q, W_s, L_q, L_s \ldots$

Inputs:

lambda	20
mu	15

Definitions of terms:

lambda = arrival rate
mu = service rate
s = number of servers
Lq = average number in the queue
Ls = average number in the system
Wq = average time spent in the queue (avg. wait in queue)
Ws = average time spent in the system (avg wait in system)
P(0) = probability of zero customers in the system
P(delay) = probability that an arriving customer has to wait

- Condition: $\rho = \frac{\lambda}{s\mu} < 1$
- Only input: λ and μ

Outputs:

s	Lq	Ls	Wq	Ws	P(0)	P(delay)	Utilization
0							
1	infinity	infinity	infinity	infinity	0.0000	1.0000	1.0000
2	1.0667	2.4000	0.0533	0.1200	0.2000	0.5333	0.6667
3	0.1446	1.4780	0.0072	0.0739	0.2542	0.1808	0.4444
4	0.0259	1.3592	0.0013	0.0680	0.2621	0.0518	0.3333
5	0.0046	1.3379	0.0002	0.0669	0.2634	0.0126	0.2667
6	0.0008	1.3341	0.0000	0.0667	0.2636	0.0026	0.2222
7	0.0001	1.3334	0.0000	0.0667	0.2636	0.0005	0.1905
8	0.0000	1.3333	0.0000	0.0667	0.2636	0.0001	0.1667
9	0.0000	1.3333	0.0000	0.0667	0.2636	0.0000	0.1481
10	0.0000	1.3333	0.0000	0.0667	0.2636	0.0000	0.1333
11	0.0000	1.3333	0.0000	0.0667	0.2636	0.0000	0.1212
12	0.0000	1.3333	0.0000	0.0667	0.2636	0.0000	0.1111
13	0.0000	1.3333	0.0000	0.0667	0.2636	0.0000	0.1026

Example: Call Center for Software Support

-- 3 representatives answer phones (each can handle all types of calls)
-- 20 calls per hour (λ = 20 calls/hour)
-- 4 minutes per call, on average (μ = 60/4 = 15 calls/hour)

Find:
utilization = 0.4444
probability that all rep. are idle = 0.2542
Probability of delay = 0.1808
average # customers in system = 1.4780
average queue length = 0.1446
average time spent in system = 0.0739 hour
average time spent in queue = 0.0072 hour

For M/M/s queue, only need to find the three parameters: λ, μ, and s

M/M/s Queueing Spreadsheet

Inputs:

lambda	20
mu	15

Definitions of terms:

lambda = arrival rate
mu = service rate
s = number of servers
Lq = average number in the queue
Ls = average number in the system
Wq = average time spent in the queue (avg. wait in queue)
Ws = average time spent in the system (avg wait in system)
P(0) = probability of zero customers in the system
P(delay) = probability that an arriving customer has to wait

Outputs:

s	Lq	Ls	Wq	Ws	P(0)	P(delay)	Utilization
0							
1	infinity	infinity	infinity	infinity	0.0000	1.0000	1.0000
2	1.0667	2.4000	0.0533	0.1200	0.2000	0.5333	0.6667
3	0.1446	1.4780	0.0072	0.0739	0.2542	0.1808	0.4444
4	0.0259	1.3592	0.0013	0.0680	0.2621	0.0518	0.3333
5	0.0046	1.3379	0.0002	0.0669	0.2634	0.0126	0.2667
6	0.0008	1.3341	0.0000	0.0667	0.2636	0.0026	0.2222
7	0.0001	1.3334	0.0000	0.0667	0.2636	0.0005	0.1905
8	0.0000	1.3333	0.0000	0.0667	0.2636	0.0001	0.1667
9	0.0000	1.3333	0.0000	0.0667	0.2636	0.0000	0.1481
10	0.0000	1.3333	0.0000	0.0667	0.2636	0.0000	0.1333
11	0.0000	1.3333	0.0000	0.0667	0.2636	0.0000	0.1212
12	0.0000	1.3333	0.0000	0.0667	0.2636	0.0000	0.1111
13	0.0000	1.3333	0.0000	0.0667	0.2636	0.0000	0.1026

Example: Pizza Hut

At a take-away only Pizza Hut branch

- Customers arrive randomly at the average rate of 25 per hour

- There is one employee who can, on average, serve one customer every 2 minutes

We have solved this by M/M/1 model:

$$L_q = 4.167, L_s = 5, W_\mathrm{q} = 10 \text{ mins}$$

Example: Pizza Hut (Extended)

At a take-away only Pizza Hut branch

- Customers arrive randomly at the average rate of 25 per hour.
- There is one employee who can, on average, serve one customer every 2 minutes

If an extra employee (with the same training and thus same service rate) works in this branch, what will happen?

Example: Pizza Hut (Extended)

- Customers arrive randomly at the average rate of 25 per hour.
- There are 2 employees who can, on average, serve one customer every two minutes.

M/M/2 Queue

Poisson Arrival

Exponential Service Time

2 Employees

Example: Pizza Hut Solution

We can solve the M/M/2 model from spreadsheet

mms.xls M/M/s Queueing Formula Spreadsheet

Inputs:

lambda	25
mu	30

Definitions of terms:

lambda = arrival rate
mu = service rate
s = number of servers
Lq = average number in the queue
Ls = average number in the system
Wq = average wait in the queue
Ws = average wait in the system
P(0) = probability of zero customers in the system
P(delay) = probability that an arriving customer has to wait

Outputs:

s	Lq	Ls	Wq	Ws	P(0)	P(delay)	Utilization
0							
1	4.1667	5.0000	0.1667	0.2000	0.1667	0.8333	0.8333
2	0.1751	1.0084	0.0070	0.0403	0.4118	0.2451	0.4167
3	0.0222	0.8555	0.0009	0.0342	0.4321	0.0577	0.2778
4	0.0029	0.8362	0.0001	0.0334	0.4343	0.0110	0.2083

Number of customers in queue/system

$$L_q \approx 0.175 \; and \; L_s \approx 1.008$$

Total time customers wait before being served

$$W_q = \frac{L_q}{\lambda} = \frac{0.175 \text{ cust}}{25 \text{ cust/hour}} = 0.007 \text{ hr or } 0.4 \text{ min}$$

Benefit of Adding Server

The probability of waiting, queue length, and waiting time significantly decreased

	1 Employee	2 Employees
Probability of waiting in line	0.833	0.245
Average queue length	4.167	0.176
Average number of customers in the system	5.000	1.009
Average waiting time	10 min	0.4 min

Queuing System Cost Tradeoff

Let:

C_w = Cost of waiting per customer per hour

L_q = Mean number of waiting customers

C_s = Cost of service per server per hour

S = Number of servers

Total cost per hour = service cost + customer waiting cost

$$= C_s S + C_w L_q$$

Number of server: trade-off between service and waiting costs

- Service cost: proportional to # of servers
- Waiting cost: proportional to queue length

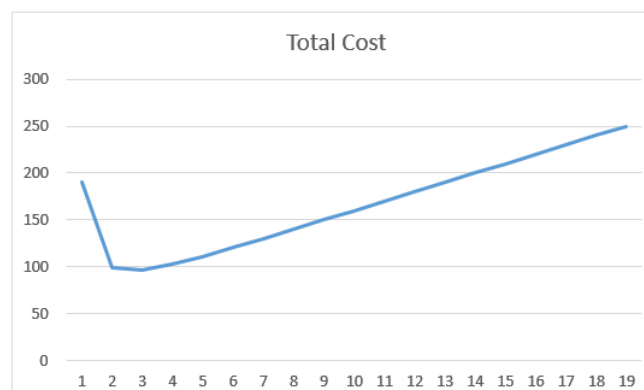
Example: Queueing Cost

Consider a restaurant serving HK dim sum. The customer arrival rate is 20 per hour. The service rate per waiter is three customers per hour. The hourly cost of hiring a waiter is 10 dollars, and the implicit cost of customer waiting is 7 dollars per customer per hour.

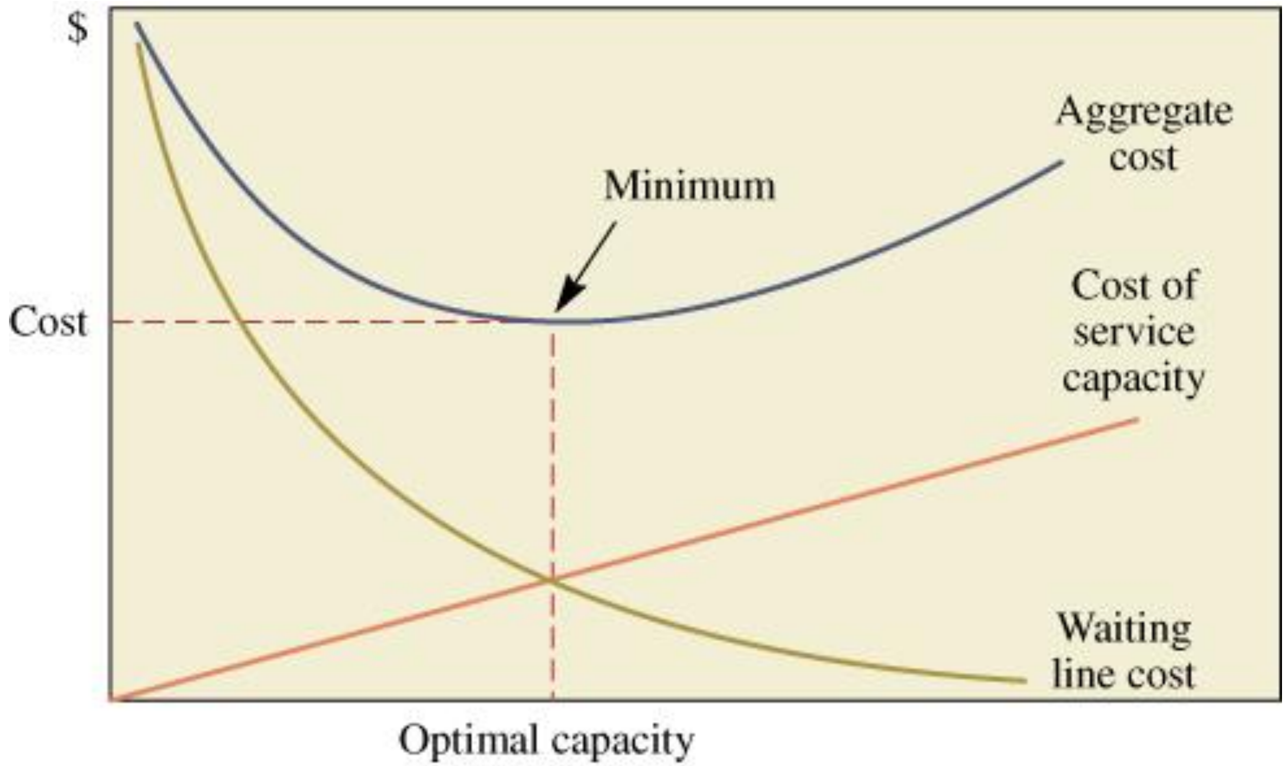
Q: How many waiter should the restaurant hires?

lambda	20
mu	3

s	Lq	Ls	Wq	Ws	P(0)	P(delay)	Utilization	Cw*Lq	Cs*S	Total
0										
1	infinity	infinity	infinity	infinity	0.0000	1.0000	1.0000			
2	infinity	infinity	infinity	infinity	0.0000	1.0000	1.0000			
3	infinity	infinity	infinity	infinity	0.0000	1.0000	1.0000			
4	infinity	infinity	infinity	infinity	0.0000	1.0000	1.0000			
5	infinity	infinity	infinity	infinity	0.0000	1.0000	1.0000			
6	infinity	infinity	infinity	infinity	0.0000	1.0000	1.0000			
7	17.2227	23.8894	0.8611	1.1945	0.0004	0.8611	0.9524	120.5591	70	190.5591
8	2.6634	9.3301	0.1332	0.4665	0.0009	0.5327	0.8333	18.64403	80	98.64403
9	0.8950	7.5617	0.0448	0.3781	0.0011	0.3133	0.7407	6.265113	90	96.26511
10	0.3492	7.0158	0.0175	0.3508	0.0012	0.1746	0.6667	2.444147	100	102.4441
11	0.1415	6.8082	0.0071	0.3404	0.0013	0.0920	0.6061	0.990771	110	110.9908
12	0.0572	6.7239	0.0029	0.3362	0.0013	0.0458	0.5556	0.40059	120	120.4006
13	0.0226	6.6893	0.0011	0.3345	0.0013	0.0215	0.5128	0.158456	130	130.1585
14	0.0087	6.6753	0.0004	0.3338	0.0013	0.0095	0.4762	0.0607	140	140.0607
15	0.0032	6.6699	0.0002	0.3335	0.0013	0.0040	0.4444	0.022396	150	150.0224
16	0.0011	6.6678	0.0001	0.3334	0.0013	0.0016	0.4167	0.007937	160	160.0079
17	0.0004	6.6671	0.0000	0.3334	0.0013	0.0006	0.3922	0.002698	170	170.0027
18	0.0001	6.6668	0.0000	0.3333	0.0013	0.0002	0.3704	0.00088	180	180.0009
19	0.0000	6.6667	0.0000	0.3333	0.0013	0.0001	0.3509	0.000275	190	190.0003
20	0.0000	6.6667	0.0000	0.3333	0.0013	0.0000	0.3333	8.26E-05	200	200.0001
21	0.0000	6.6667	0.0000	0.3333	0.0013	0.0000	0.3175	2.38E-05	210	210
22	0.0000	6.6667	0.0000	0.3333	0.0013	0.0000	0.3030	6.61E-06	220	220
23	0.0000	6.6667	0.0000	0.3333	0.0013	0.0000	0.2899	1.76E-06	230	230
24	0.0000	6.6667	0.0000	0.3333	0.0013	0.0000	0.2778	4.54E-07	240	240



Trade-off between service capacity and waiting line



Waiting cost is non-linear in # of servers: marginal gain decreases as number of servers increases

Example: Staff at ASO

Students arrive at the ASO at an average of one every 15 minutes and their requests take on average 10 minutes to be processed. The service counter is staffed by only one clerk, Judy, who works 8 hours per day. Assume Poisson arrivals and exponential service times.

a. What percentage of time is Judy idle?

b. How much time, on average, does a student spend waiting in line?

c. How long is the (waiting) line on average?

Use M/M/1 model:
Mean arrival rate $\lambda = 60 / 15 = 4$ per hour
Mean service rate $\mu = 60 / 10 = 6$ per hour

Example: Staff at ASO

a. What percentage of time is Judy idle?

Compute $\rho = \frac{\lambda}{\mu} = 0.667$

Judy is idle (1 – ρ) or 33.3% of the time since ρ is the proportion of time that she is busy.

b. How much time, on average, does a student spend waiting in line?

Compute $Wq = \frac{\rho}{\mu - \lambda} = \frac{0.667}{2} = 0.33 \text{ hour or } 20 \text{ mins}$

Student waits on average 20 minutes in line.

c. How long is the (waiting) line on average?

Compute $Lq = \frac{\rho^2}{1 - \rho} = \frac{0.667^2}{1 - 0.667} = 1.333$

There are on average 1.33 students waiting in line.

ASO Example: Interventions

The managers of the ASO estimate that the time a student spends waiting in line costs them (due to goodwill loss and so on) $10 per hour. To reduce the time a student spends waiting, they need to improve Judy's processing time. There are 2 options:

A. Install a computer system, with which Judy expects to be able to complete a student request in 6 minutes (original 10 minutes).

B. Hire another temporary clerk, who will work at the same rate as Judy.

The computer system costs $99.5 per day, while the temporary clerk gets paid $75 per day. Which option is preferred? Assume Poisson arrival rate and exponential service time.

ASO Example: Interventions

Option A: Maintain one staff at the counter with a new computer system

Use M/M/1 model with $\lambda = 4$ and $\mu = 60/6 = 10$: $\rho = 4/10 = 0.4$

Compute $L_q = \rho^2 / (1- \rho) = 0.267$

Total cost = Waiting cost + Additional service cost (i.e., installing a computer system)

Total cost = $0.267 \times \$10 \times 8$ hours + \$99.5 = \$120.83 per day

Option B: Have two staff at the counter without the use of a new computer system

Use M/M/s model with $s = 2$, $\rho = \lambda / (s \times \mu) = 4 / 12 = 0.333$

Obtain $L_q = 0.0833$ from Excel M/M/s sheet

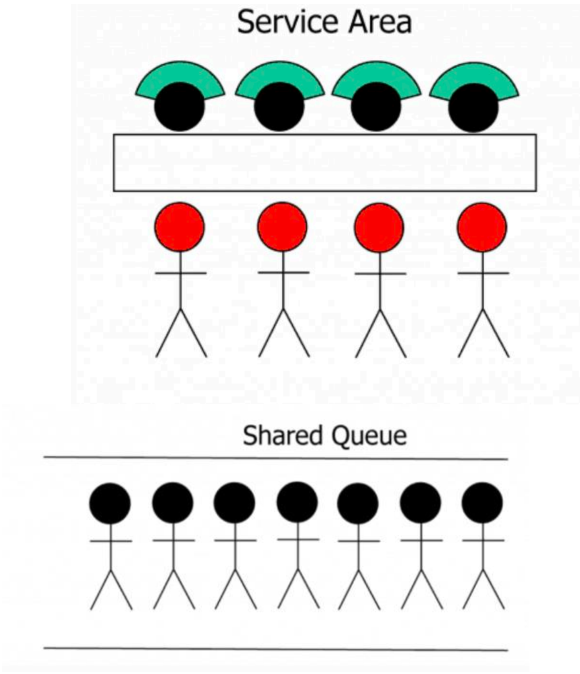
Total cost = Waiting cost + Additional service cost (i.e., hiring a temporary clerk)

Total cost = $0.0833 \times \$10 \times 8$ hours + \$75 = \$81.7 per day

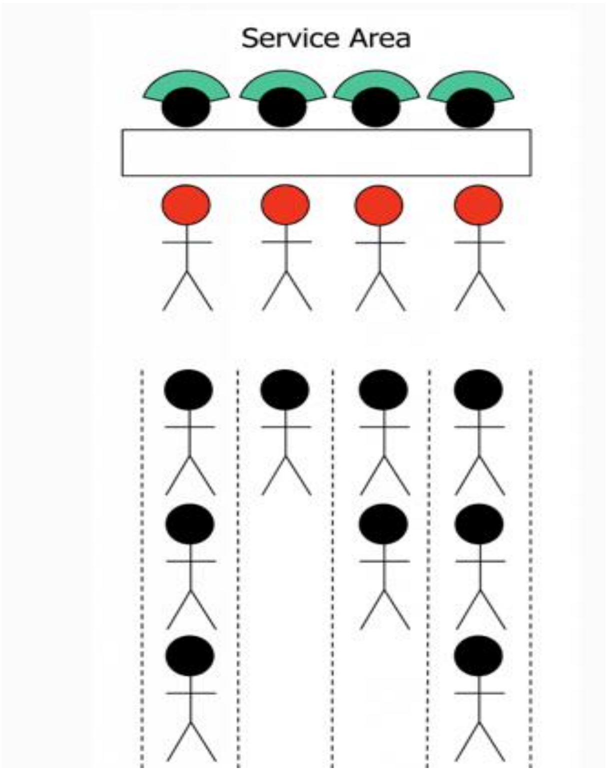
Conclusion: Option B is a better solution to minimize the total cost (both excluding Judy's existing cost, which is a constant)

Pooled vs. Separate Queues

Alternative 1: Pooled Queue



Alternative 2: Separate Queues



Examples …..

The comparison may be more common than you expect…

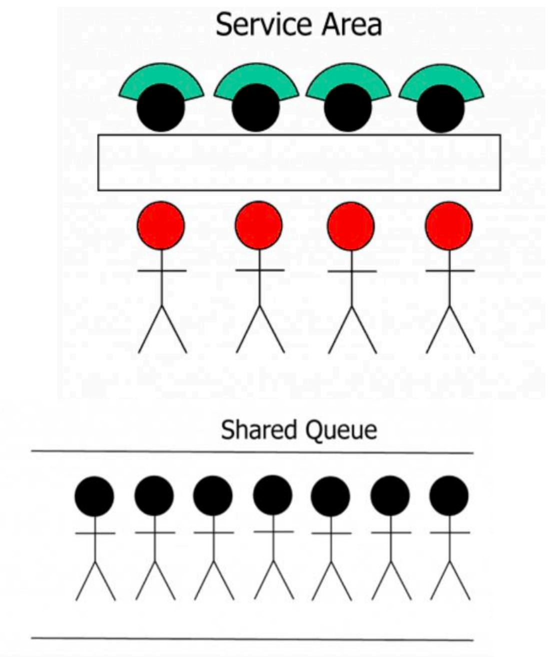
How to Solve the Problem?

Identify arrival rate and service rate

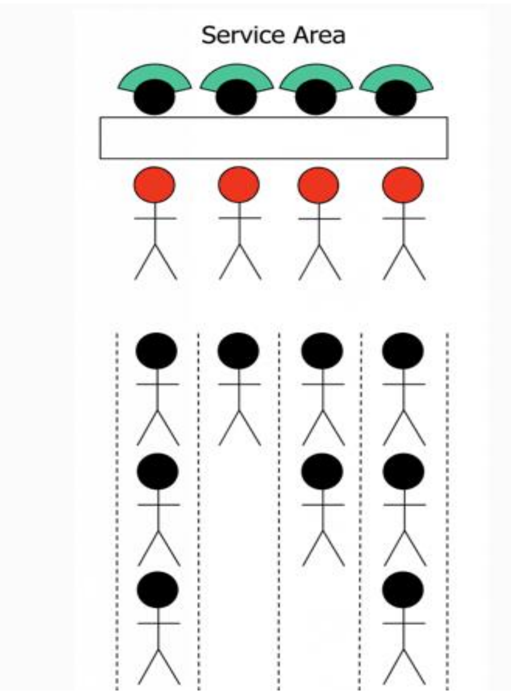
↓

Use M/M/s to analyze Alternative 1 and 2

Alternative 1: Pooled Queue



Alternative 2: Separate Queues



Use M/M/s to analyze Alternative 1 and 2

- λ : arrival rate to the system
- μ : service rate of each server
- s : number of servers

- $\lambda_1 = \lambda$
- $\mu_1 = \mu$
- $s_1 = s$

← Pooled queue

One M/M/s model

- $\lambda_2 = \lambda/s$
- $\mu_2 = \mu$
- $s_2 = 1$

← Separated queue

s M/M/1 model with smaller arrival rate

Pooled vs Separated Queue: Example

Arrival rate of customer to the store = **4.73** customers/min

Service rate of each server = **1.33** customers/min

Apply M/M/s for Alternative 1 and 2

Alternative 1: Common queue (four servers)

$\lambda =$

$\mu =$

$s =$

$\rho =$

$W_q =$

Alternative 2: four separate queues

$\lambda =$

$\mu =$

$s =$

$\rho =$

$W_q =$

Pooled Queue

Alternative 1: Common queue

$\lambda = 4.73$ customers/min

$\mu = 1.33$ customers/min

$s = 4$

$\rho = 0.8891$

$W_q = 1.30$ min

Inputs:

lambda	4.73
mu	1.33

Definitions of terms:

lambda = arrival rate

mu = service rate

s = number of servers

Lq = average number in the queue

Ls = average number in the system

Wq = average wait in the queue

Ws = average wait in the system

P(0) = probability of zero customers in the system

P(delay) = probability that an arriving customer has to wait

Outputs:

s	Lq	Ls	Wq	Ws	P(0)	P(delay)	Utilization
0							
1	infinity	infinity	infinity	infinity	0.0000	1.0000	1.0000
2	infinity	infinity	infinity	infinity	0.0000	1.0000	1.0000
3	infinity	infinity	infinity	infinity	0.0000	1.0000	1.0000
4	6.1396	9.6960	1.2980	2.0499	0.0127	0.7658	0.8891
5	0.9756	4.5320	0.2063	0.9581	0.0241	0.3960	0.7113
6	0.2737	3.8301	0.0579	0.8098	0.0273	0.1881	0.5927
7	0.0844	3.6408	0.0179	0.7697	0.0282	0.0818	0.5081
8	0.0260	3.5824	0.0055	0.7574	0.0284	0.0325	0.4445
9	0.0077	3.5641	0.0016	0.7535	0.0285	0.0118	0.3952
10	0.0022	3.5586	0.0005	0.7523	0.0285	0.0039	0.3556
11	0.0006	3.5570	0.0001	0.7520	0.0285	0.0012	0.3233

Separate Queue

Alternative 2: 4 separate queues

$\lambda = 4.73/4$ customers/min

$\mu = 1.33$ customers/min

$s = 1$

$\rho = 0.8891$

$W_q = 6.03\,\text{min}$

Inputs:

lambda	1.1825
mu	1.33

Definitions of terms:

lambda = arrival rate
mu = service rate
s = number of servers
Lq = average number in the queue
Ls = average number in the system
Wq = average wait in the queue
Ws = average wait in the system
P(0) = probability of zero customers in the system
P(delay) = probability that an arriving customer has to wait

Outputs:

s	Lq	Ls	Wq	Ws	P(0)	P(delay)	Utilization
0							
1	7.1279	8.0169	6.0278	6.7797	0.1109	0.8891	0.8891
2	0.2190	1.1081	0.1852	0.9371	0.3845	0.2736	0.4445
3	0.0286	0.9177	0.0242	0.7761	0.4080	0.0679	0.2964
4	0.0039	0.8930	0.0033	0.7552	0.4107	0.0137	0.2223
5	0.0005	0.8896	0.0004	0.7523	0.4110	0.0023	0.1778
6	0.0001	0.8892	0.0000	0.7519	0.4110	0.0003	0.1482
7	0.0000	0.8891	0.0000	0.7519	0.4110	0.0000	0.1270
8	0.0000	0.8891	0.0000	0.7519	0.4110	0.0000	0.1111

Comparison of Two Alternatives

Arrival rate of customer to the store = **4.73** customers/min

Service rate of each server = **1.33** customers/min

Alternative 1:
Common queue
(four servers)

$\lambda = 4.73$ customers/min

$\mu = 1.33$ customers/min

$s = 4$

$\rho = 0.8891$

$W_q = 1.30$ min

Alternative 2:
four separate queues

$\lambda = 4.73/4$ customers/min

$\mu = 1.33$ customers/min

$s = 1$

$\rho = 0.8891$

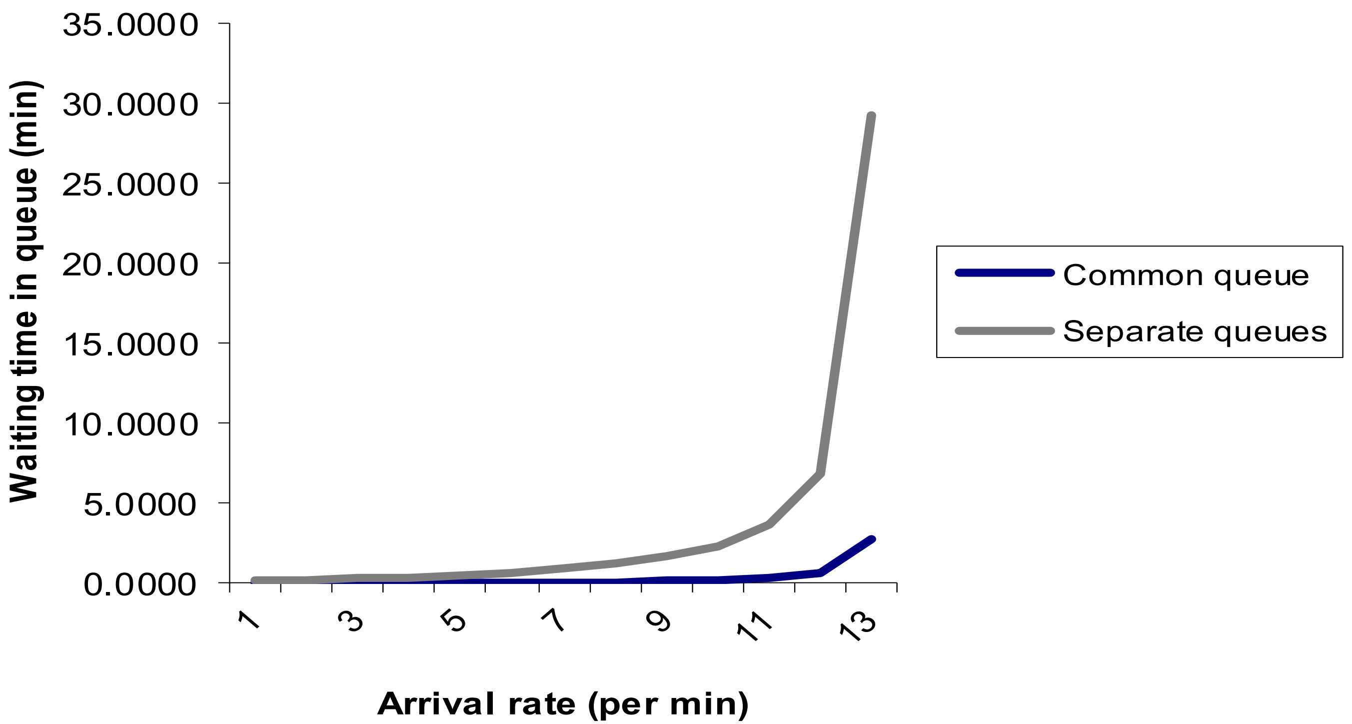
$W_q = 6.03$ min

Observations:

1. Same utilization
2. Less waiting time in common queue

Benefit of Pooled Queue

- **Input data:** **$\mu = 1.33$, $s = 10$**
- Pooled queue vs. separate queues: waiting time

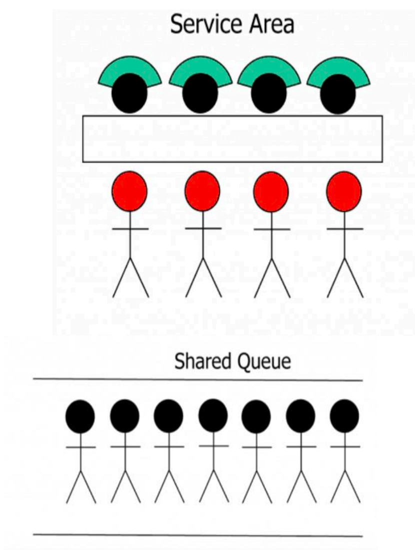


- Less waiting time in the pooled queue
- Difference is especially large under high utilization

Insight from Pooling

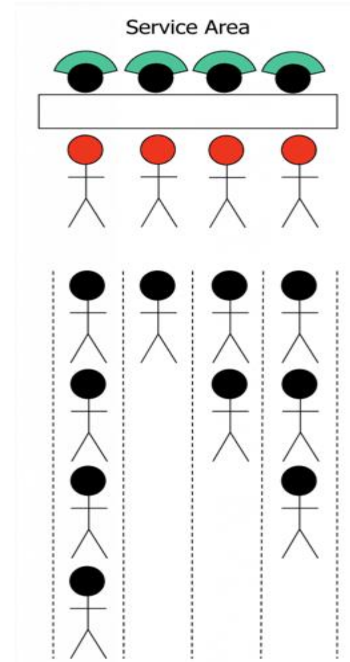
Some people are waiting in line, while some servers are idle?

Alternative 1: Pooled Queue



No, impossible

Alternative 2: Separate Queues



Yes, possible

Insights from two aspects:

- Pooling resources leads to better match between supply and demand for service
- Pooling resources reduces the variability in the system

Example: Benefit of Pooling

A super market has eight check-out counters and each of them has its own waiting line (eight separate queues). The total arrival rate to the check-out area is 20 cust/hour. The service rate of each counter is 4 cust/hour.

- What is the average time of waiting in the queue (Wq)?
- Suppose we pool the queues for each two check-out counters (i.e., now four separate queues), what is the new average waiting time?
- What if we further pool the queues: (1) use a common queue for four check-counters; (2) use a common queue for all the eight counters?

Example: Benefit of Pooling (2)

- With eight separate queues, we have eight M/M/1 queues
  - Arrival rate $\lambda = 20/8 = 2.5$ cust/hour and service rate $\mu=4$ cust/hour
  - Then, average waiting time is $\rho/(\mu - \lambda) = 0.42$ hours
- When we pool every two counters, we have 4 M/M/2 queues
  - Arrival rate $\lambda = 20/4 = 5$ cust/hour for each of them
  - The average waiting time can be checked from spreadsheet as 0.16 hours
- The reduction in waiting time is 0.42 – 0.16 = 0.26 hours (more than half!)

Example: Benefit of Pooling (3)

- When we pool every four counters, we have 2 M/M/4 queues
  - Arrival rate λ = 20/2 = 10 cust/hour for each of them
  - The average waiting time can be checked from spreadsheet as 0.0533 hours

- When we pool all the eight counters, we have one M/M/8 queue
  - Arrival rate λ = 20 cust/hour
  - The average waiting time is 0.0139 hours

- Pool 1, 2, 4, and 8 check-out counters: waiting time 0.42, 0.16, 0.053, and 0.0139 hours

- Intuition: Some initial degree of pooling can achieve majority of the benefit!

Pros and Cons of Pooled Queue

- Pros:
  - Less waiting time
  - Better match between demand and supply
- Cons:
  - A longer/crowded queue
  - Less ownership of customers
  - Psychological and behavioral effects
- In reality, we can operate somewhere in between of a fully separated queue and a fully pooled queue

Summary

- Use of M/M/s spreadsheet to obtain service system performance
- Cost analysis in queueing systems
  - Trade-off: waiting cost versus service cost
- Pooled queue versus separated queues:
  - Pooled queue is better in terms of reducing waiting time
  - Comparison of the metrics

ISOM 2700: Operations Management

Session 7. Basics of Simulation

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Fall 2025

Agenda

- Variability in Service System
- **Queueing Model**
  - M/M/1 model
  - M/M/s model
  - **G/G/1 model**
- Some Simulation Basics

M/M/1 and G/G/1 Model

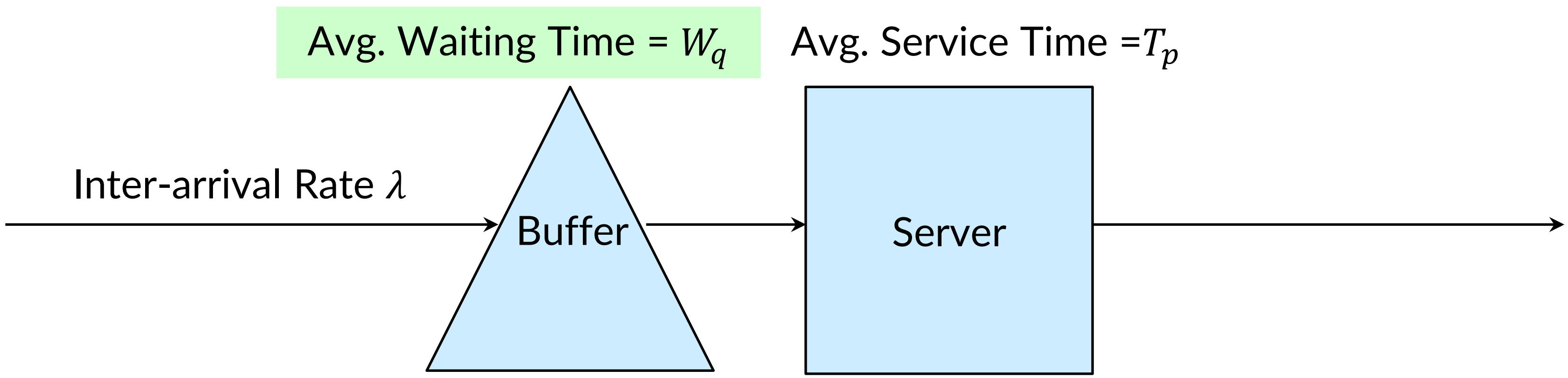
- In M/M/1 model, we assume both interarrival and service times follow an exponential distribution

- This brings mathematical convenience with explicit formulas, but may not hold in many circumstances

- If the interarrival and service times follow general probabilistic distributions, we call it a G/G/1 model
  - Still assuming a single server, FCFS

- M/M/1 model is a special case of a G/G/1 model

G/G/1 Model: Average Waiting Time



$$W_q = \underbrace{\left(\frac{CV_a^2 + CV_p^2}{2}\right)}_{\text{Variability factor}} \times \underbrace{\left(\frac{\text{Utilization}}{1-\text{Utilization}}\right)}_{\text{Utilization factor}} \times \underbrace{T_p}_{\text{Service time factor}}$$

CV_a and CV_p are coefficients of variations for inter-arrival and service time distributions, respectively

G/G/1 Model: Average Waiting Time

$$W_q = \left(\frac{CV_a^2 + CV_p^2}{2}\right) \times \left(\frac{\text{Utilization}}{1-\text{Utilization}}\right) \times T_p$$

Variability factor — Utilization factor — Service time factor

- $CV_a = \dfrac{\text{Standard deviationof interarrival time}}{\text{Mean of interarrival time}}$
- $CV_p = \dfrac{\text{Standard deviation of service time}}{\text{Mean of service time}}$
- $\text{Utilization} = \dfrac{\text{Arrival rate}}{\text{Service rate}}$

G/G/1 Model: Average Waiting Time

$$W_q = \left(\frac{CV_a^2 + CV_p^2}{2}\right) \times \left(\frac{\text{Utilization}}{1-\text{Utilization}}\right) \times T_p$$

Variability factor | Utilization factor | Service time factor

- Average arrival and service rates: affect the utilization factor and service time factor
  - Higher utilization and service time lead to longer waiting
- Variability in interarrival and service time: affect the variability factor
  - Higher variability leads to longer waiting
- Both average rates and variability matter in queueing system

G/G/1 Model: Example

Suppose a restaurant has one server. The interarrival time has a mean of 10 minutes and a standard deviation of 8 minutes. The service time has a mean of 8 minutes and a standard deviation of 10 minutes.
What is the average waiting time in queue?

- $\mathrm{CV_a} = \frac{8}{10} = 0.8; \mathrm{CV_p} = \frac{10}{8} = 1.25; \mathrm{Utilization} = \frac{60/10}{60/8} = 0.8$
- $\mathrm{W}_q = \left(\frac{0.8^2+1.25^2}{2}\right) \times \left(\frac{0.8}{1-0.8}\right) \times 8 = 35.2\ minutes$

Getting Other Measures in G/G/1

- Time in system: Time in queue + Service time

$$W_s = W_q + \frac{1}{\mu}$$

- Number of customers in queue: $L_q = \lambda \times W_q$

- Number of customers in system: Number in queue + Number in the server

$$L_s = L_q + \rho \quad \text{or} \quad L_s = \lambda \times W_s$$

- They yield the same results

Special Case: M/M/1 Model

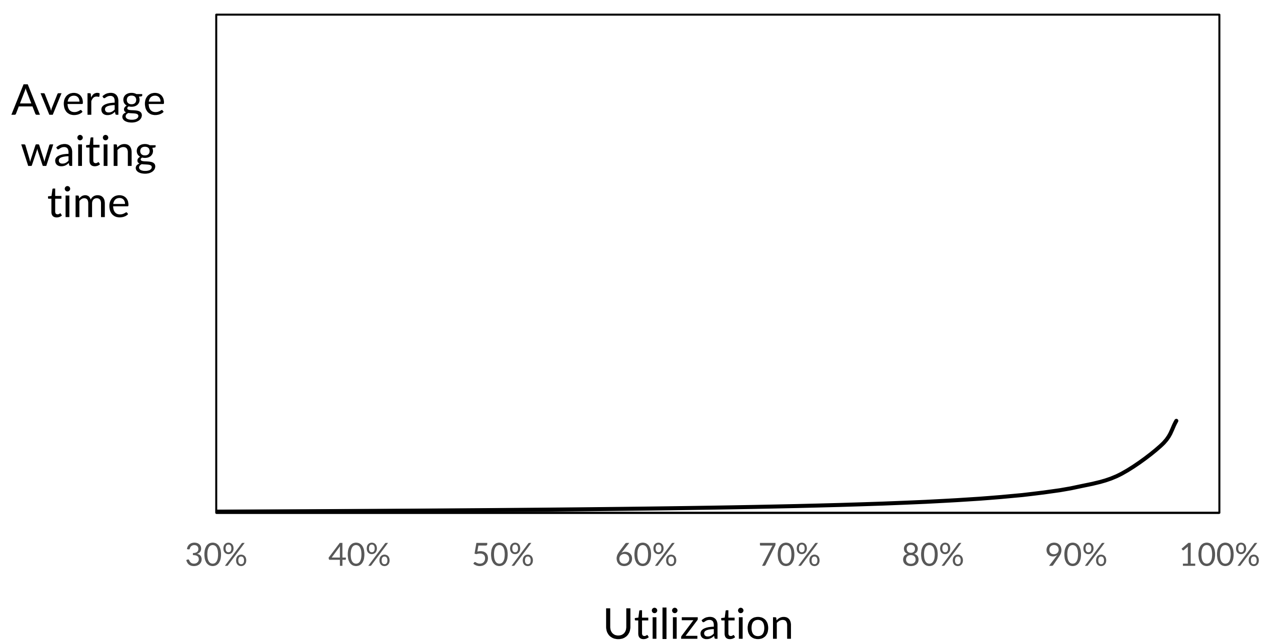
$$W_q = \underbrace{\left(\frac{CV_a^2 + CV_p^2}{2}\right)}_{\text{Variability factor}} \times \underbrace{\left(\frac{\text{Utilization}}{1-\text{Utilization}}\right)}_{\text{Utilization factor}} \times \underbrace{T_p}_{\text{Service time factor}}$$

- In M/M/1 model, the average service time is $T_p = \frac{1}{\mu}$
- As inter-arrival and service time follow exponential distributions, we have $CV_a = CV_p = 1$
- Thus, $W_q = \frac{\rho}{1-\rho} \times \frac{1}{\mu} = \frac{\rho}{\mu-\lambda}$, which coincides with the M/M/1 formula
- In M/M/1 model, if we have constant inter-arrival time or service time, the average waiting time decreases by a half
  - Benefit of reducing variability in service system

Effect of Utilization

$$W_q = \left(\frac{CV_a^2 + CV_p^2}{2}\right) \times \left(\frac{\text{Utilization}}{1-\text{Utilization}}\right) \times T_p$$

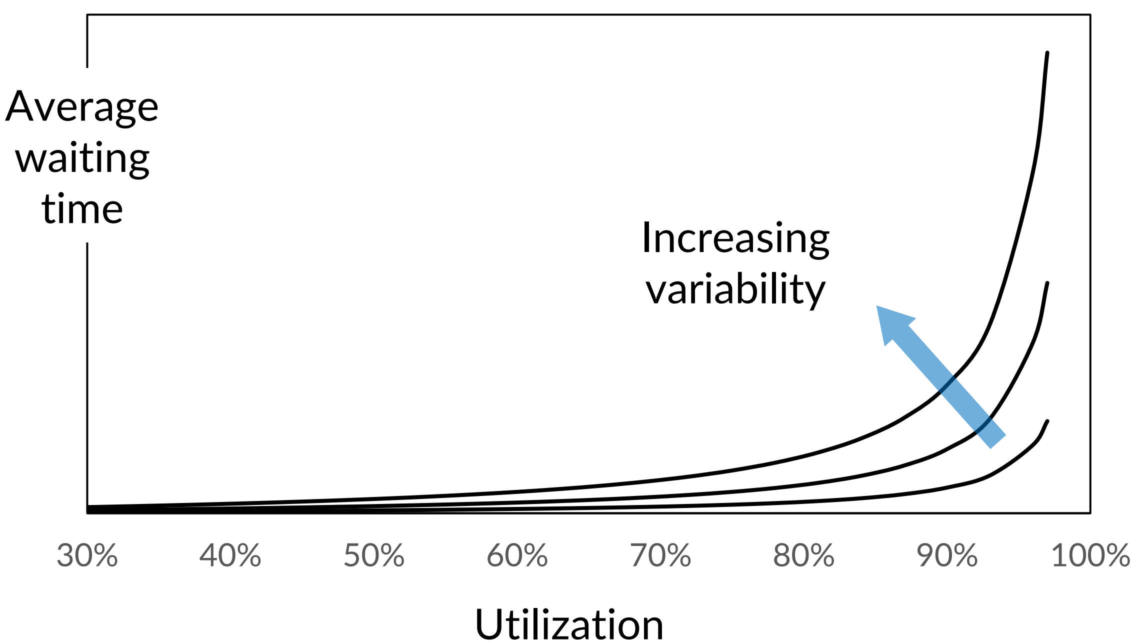
Variability factor | Utilization factor | Service time factor



Effect of Variability

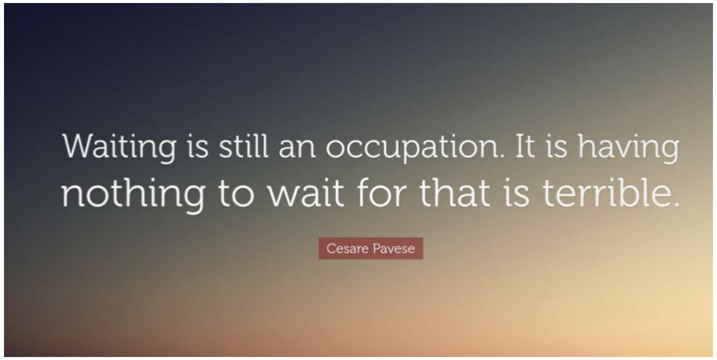
$$W_q = \left(\frac{CV_a^2 + CV_p^2}{2}\right) \times \left(\frac{\text{Utilization}}{1-\text{Utilization}}\right) \times T_p$$

Variability factor | Utilization factor | Service time factor



Psychology Effects for Queues (1)

- Unoccupied time feels longer than occupied time
  - Divert customer's attention when waiting
  - Entertain, enlighten, and engage (Katz et al., 1991)

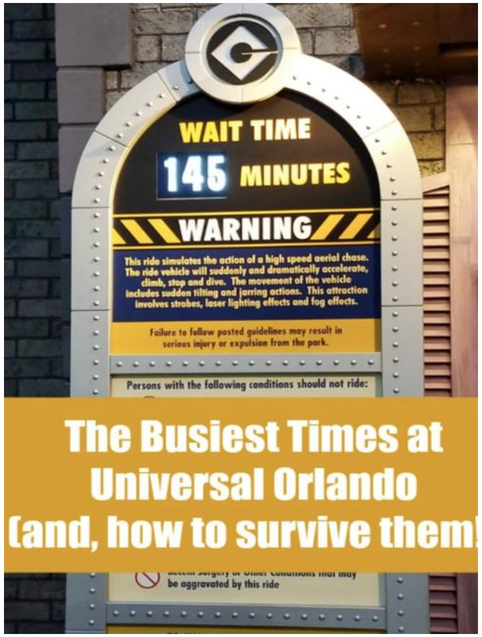


- Pre-process waiting feels longer than in-process waiting
  - Ensure a quick "a foot in the door" feeling to discourage balking

Psychology Effects for Queues (2)

- Anxiety makes the waiting seem longer
  - Ensure the waiting customers that they are not forgotten
  - Realize that new or infrequent customers feel they wait longer than frequent users

- Uncertain waiting feels longer than known, finite waiting
  - Inform the customers of what and how much longer to expect
  - Surprise the customer with shorter waiting time



Psychology Effects for Queues (3)

- Unexplained waiting feels longer than explained waiting
  - Provide real reasons for keeping customers waiting

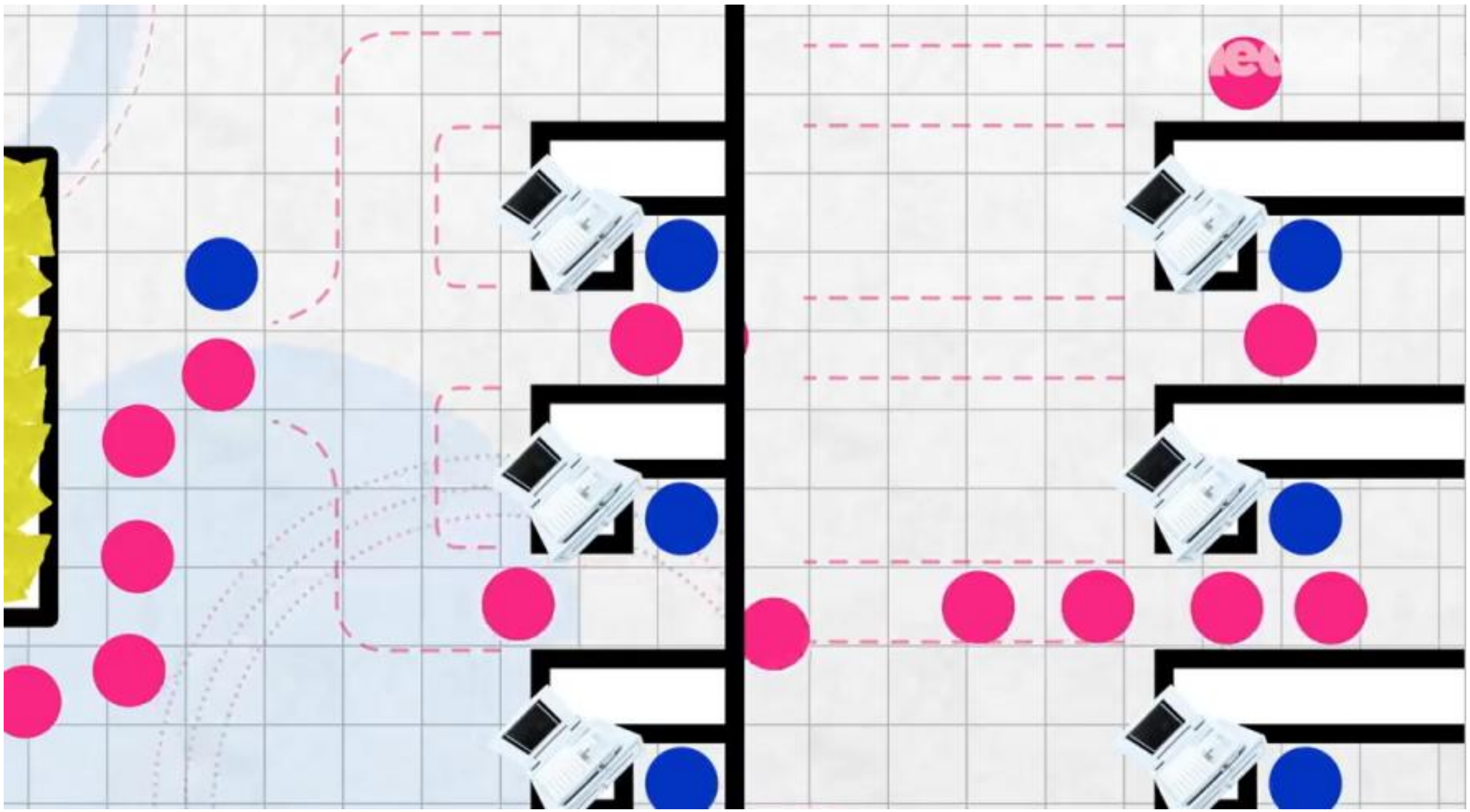
- Unfair waiting feels longer than equitable waiting
  - Redesign the waiting system to ensure fairness
  - Maintain FCFS, if possible

Psychology Effects for Queues (4)

- Uncomfortable waiting feels longer than comfortable waiting
  - Design a better waiting environment

- Solo waiting feels longer than group waiting
  - Create a waiting environment that encourages interaction

Video: Psychological Effect in Queueing



Video

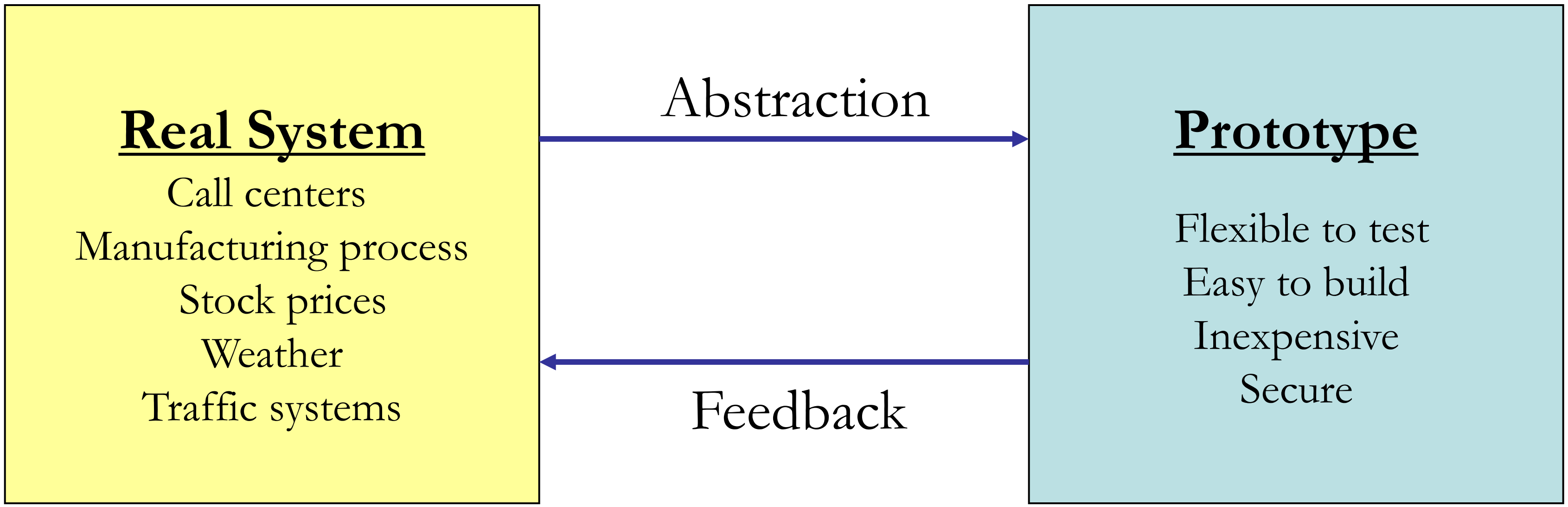
Agenda

- Variability in Service System
- Queueing Model
- **Some Simulation Basics**

Idea of Simulation

- When no explicit formula is available for the queueing process, what can we do?
  - E.g., multi-server or layers, abandonment, deterioration
- Idea: generate a model/process that mimics the real system dynamics
  - How the system works (decision-making)
  - The real variability in the system
- Estimate the system performance from the simulated model/process
  - Average from a large sample size/long enough periods

What is simulation?



- Attempts to duplicate a real-life system
- Numerical technique of experimentation
- Generates recommendations

Simulation: An Example

Suppose you are playing a game with your friend. You roll a dice with numbers from 1 to 6 with equal probability

- If the number is greater or equal to 4, you get $1, your friend loses $1
- If the number is smaller or equal to 2, you lose $1, your friend gets $1
- Otherwise, both your wealth do not change

You start with initial wealth $3; your friend starts with $5; If any of you goes bankruptcy (to $0), then the game ends

Question: what is your chance of winning the game?

Simulation: An Example

You don't know how to calculate the probability... But you have an efficient machine that can generate the dice roll…

You can mimic a path of the game and see who wins

Dice Roll	Your wealth	Your friend's wealth
0	3	5
1	2	6
6	3	5
2	2	6
2	1	7
1	0	8

To obtain the winning probability, you need to generate a lot of paths…

This is called "simulation" !

From 10,000 runs, we can find your winning probability is 73%

In this path, you lose the game!

Applications

- Waiting-line systems
  - AT&T Call Center
  - Fast-food restaurant: Burger King
- Financial investment
  - Simulate stock price movement over time and backtest trading strategies under different market scenarios
- Inventory ordering
  - How much to order in every time period?

Strength and Limitations of Simulation

Strength

- Can deal with large complex problems
- Allows "what if" type (counterfactual) questions
- Does not interfere with real system

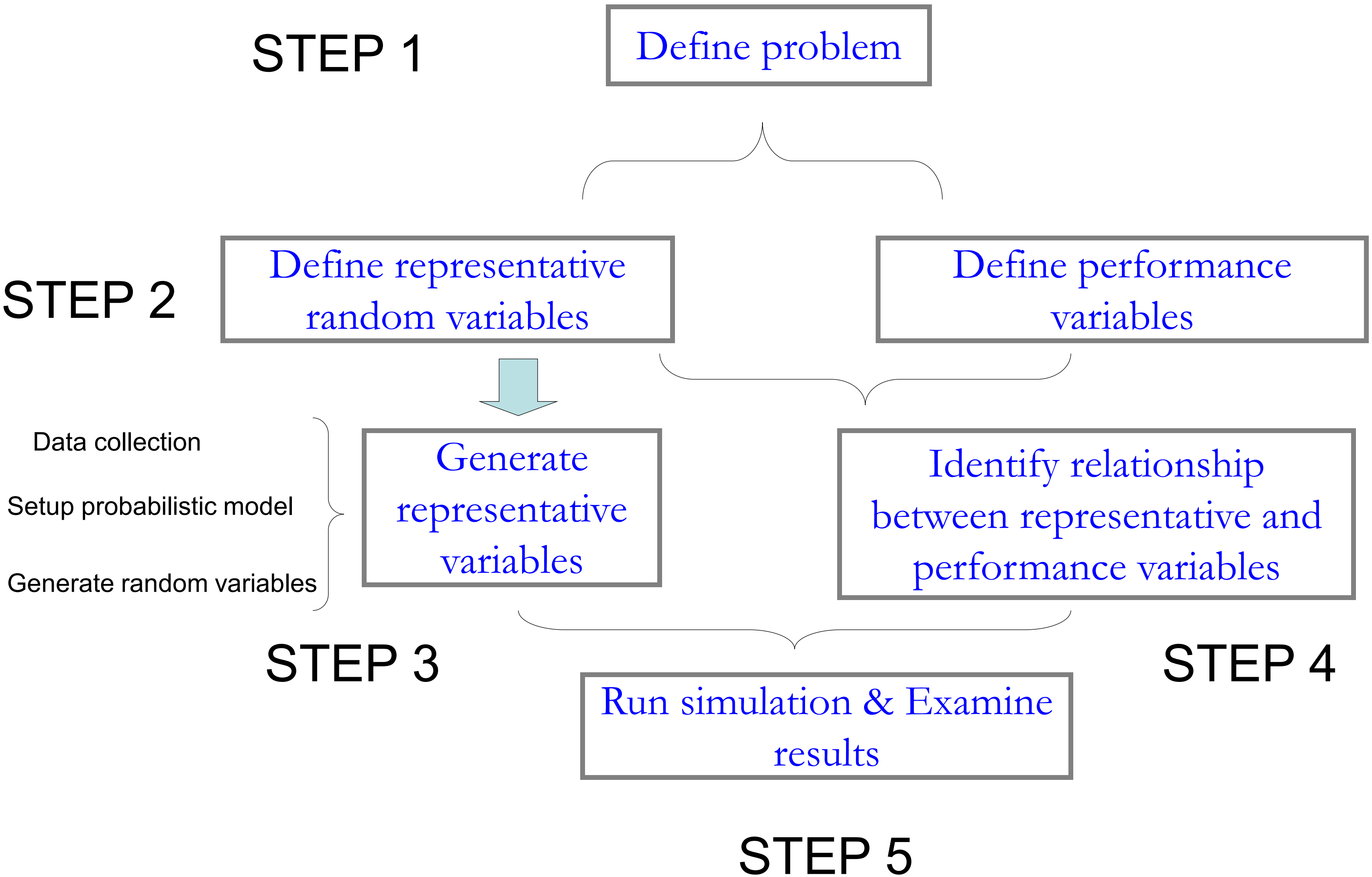
Limitation

- Does not yield optimal solution
- May not accurately reflect reality
- Requires good managerial input

Key Ingredients of Simulation

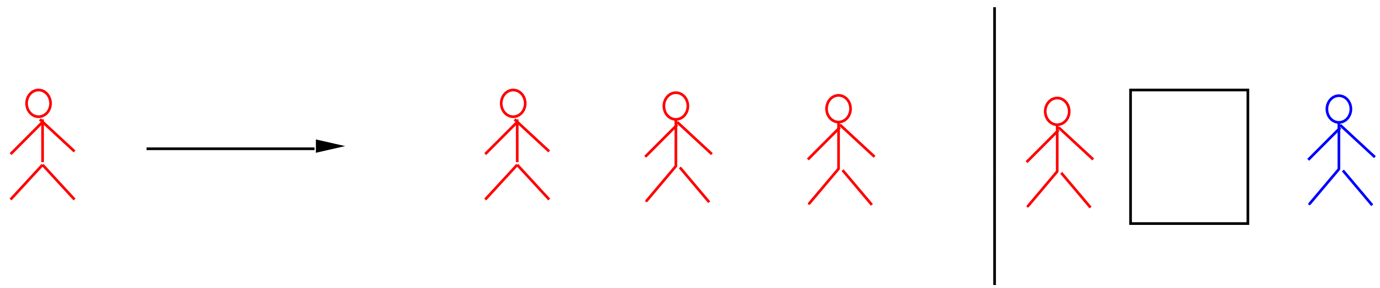
- How does the system evolve?
  - Dynamics with time, decision making at each step
- What is the key performance measure?
  - E.g., waiting time, inventory costs, portfolio return
- Where does the variability come from?
  - E.g., random arrivals/service times, market scenarios
- How to model the randomness?
  - Distribution of random variables

Monte-Carlo Simulation



Example: Single-Server Service System

- STEP 1: Define problem



- Estimate
  - Average waiting time in queue

STEP 2: Define Variables

Define representative random variables

Interarrival time: time between two consecutive customers

Service time: time to serve one customer

The representative variables are the exogenous random variables that determine the system dynamics

Define performance variables

Waiting time in queue

STEP 3: Generate Representative Variables

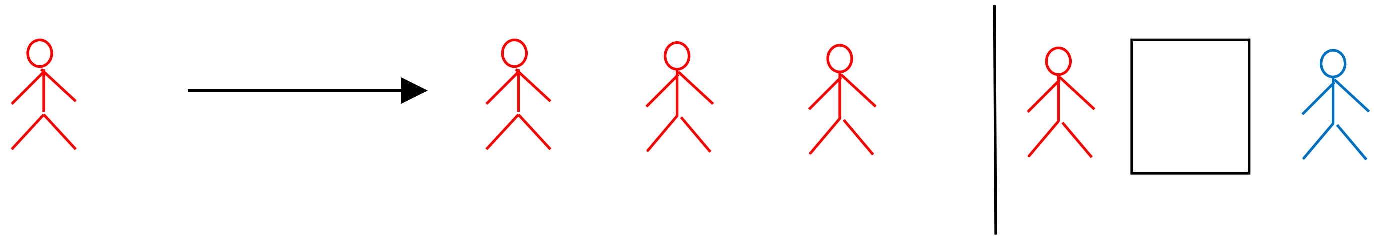
General steps:

1. Collect data from real-world scenarios

2. Set up the probabilistic model
   - Trade-off: analytical convenience vs. fitting power for reality

3. Generate random variable according to its distribution
   - An important topic in probability & statistics

STEP 4: Identify Relationship: Physics of Service System



Inter-arrival time	Arrival time	Waiting time in queue	Service start time	**Service time**	Service completion time
A1	B1 (=A1)	C1 (=0)	D1 (=B1+C1)	E1	F1 (=D1+E1)
A2	B2 (=B1+A2)	C2 (=max{F1-B2,0})	D2 (=B2+C2)	E2	F2 (=D2+E2)
A3	B3 (=B2+A3)	C3 (=max{F2-B3,0})	D3 (=B3+C3)	E3	F3 (=D3+E3)

STEP 5: Run simulation & examine results

Excel Demonstration for An Example

- Questions
  - What is the average waiting time in queue?
  - Can you use the M/M/1 formula to compute the average waiting time in this queue?

Summary

- Simulation attempts to mimic a real-life system
- Know how to perform Monte-Carlo simulation
  - Key steps and representative variable
  - System evolution and performance metrics
- Understand physics of the service system

ISOM 2700: Operations Management

Session 8. Quality Management: Capability Analysis

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Fall 2025

What's Quality?

- We hear and care about *quality* in many scenarios. But what is quality?
- Quality as good, excellence, or even perfection
  - Design quality
- Quality as consistency or conformance
  - Measurement of process variability
- Quality as a stakeholder-relative concept
  - Degree of satisfaction: expectation vs. perception
- Who defines quality:

  Customers, producers, competitors, regulators

Cost of Quality

- Cost of achieving good quality
  - Appraisal (inspection and testing)
  - Prevention (training and maintenance)
- Cost of poor quality
  - Internal failure costs (example: scrap and rework)
  - External failure costs (example: Warranty and recall)
  - Hidden costs of poor quality (example: customer dissatisfaction, loss of market share/brand reputation)

Cost of Poor Quality

Source: Wall Street Journal

Tesla Will Recall 135,000 Cars for Faulty Touch Screens.

A 2021 Tesla Model X sport-utility vehicle. The company said it would recall Model S vehicles from 2012 to 2018 and Model X vehicles from 2016 to 2018. David

CBS NEWS NEWS SHOWS LIVE Login

Quality control issues force Johnson & Johnson to scrap doses of COVID-19 vaccine

UPDATED ON: APRIL 1, 2021 / 7:04 PM / CBS/AP

Source: The New York Times

Video: Quality Problem of Boeing

Keeping good quality in complex setting can be important, and challenging!

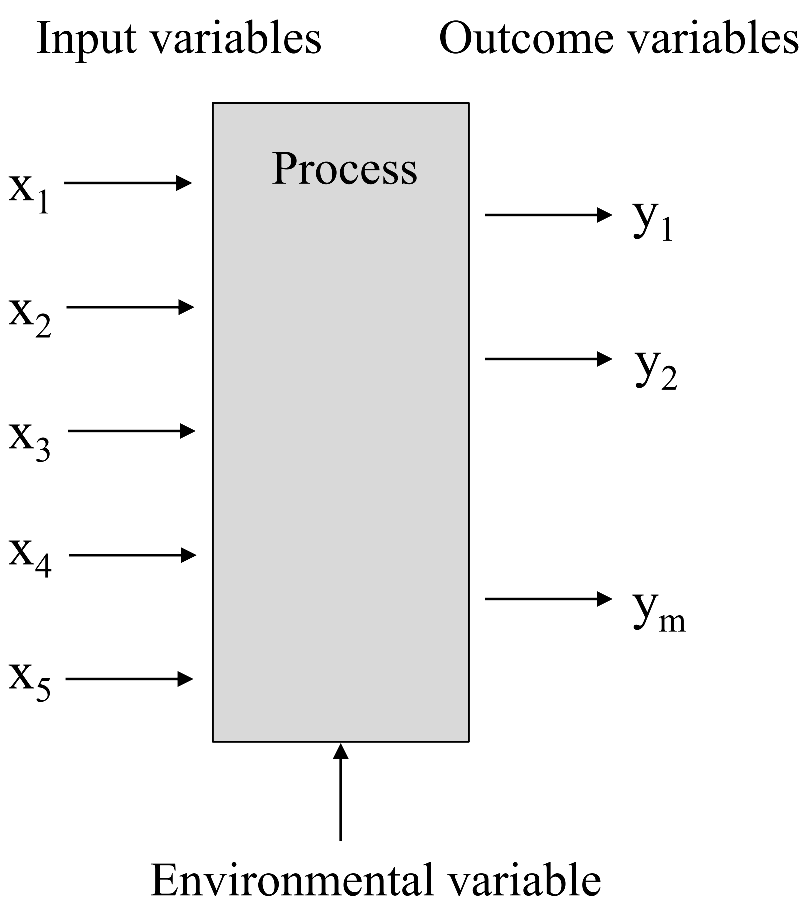
Video

Agenda

- **Types of Variations**
- Capability Analysis
- Conformance Analysis
- Acceptance Sampling

Root Cause: Variation

- From OM perspective, quality problem is caused by variation
  - If we can make exactly same products every time, we will have no quality issues

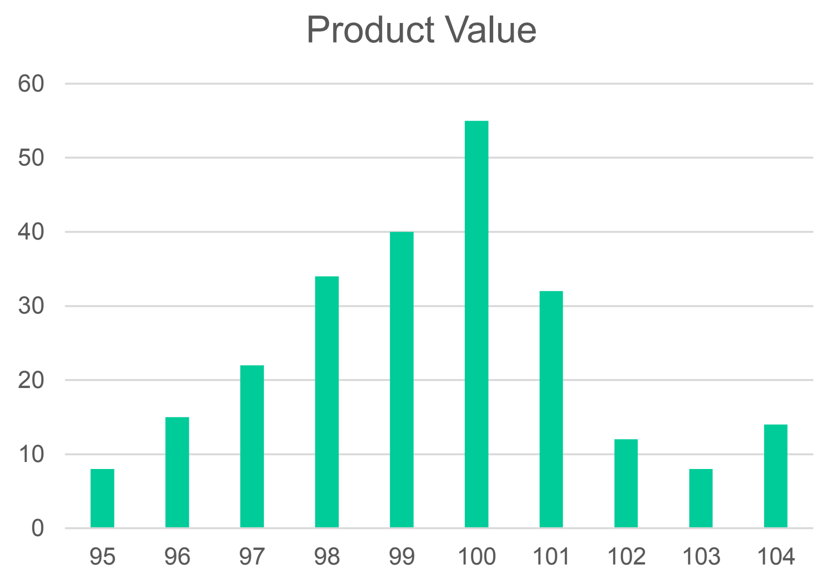


- What leads to variation:
  - Input variables, environmental variables
  - Process that transfers input to output

- Process variability is inevitable
  - Human variability, machine variability, etc

Probabilistic View of Quality

Product Value	Frequency (Out of 240)	Probability
95	8	3.3%
96	15	6.3%
97	22	9.2%
98	34	14.2%
99	40	16.7%
100	55	22.9%
101	32	13.3%
102	12	5.0%
103	8	3.3%
104	14	5.8%



In reality, the product value (e.g., length, weight) is usually random time to time, thus follows a probability distribution

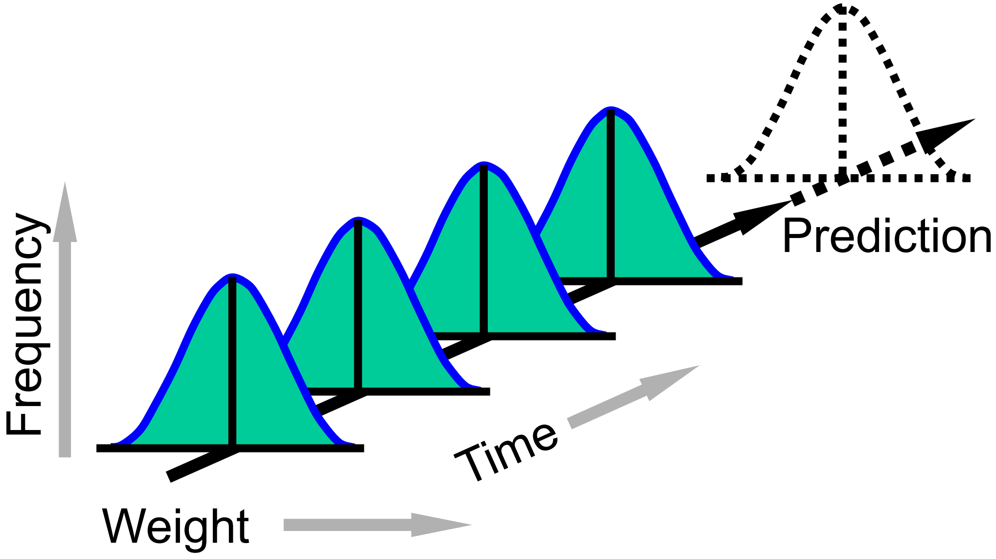
Type of Variations

- **Natural variations** (common cause variation, normal variability)
  - Random errors inherent to the process; impact all outputs
  - Cannot be eliminated without change in process itself
  - Form a pattern that can be described as a distribution

- **Assignable variations** (systematic errors, abnormal variability)
  - Can be traced to a specific reason
  - A "discoverable" cause
  - Examples: machine fault, unskilled workers, poor material
  - Can be eliminated by operator or management action

Processes with Only Natural Variations

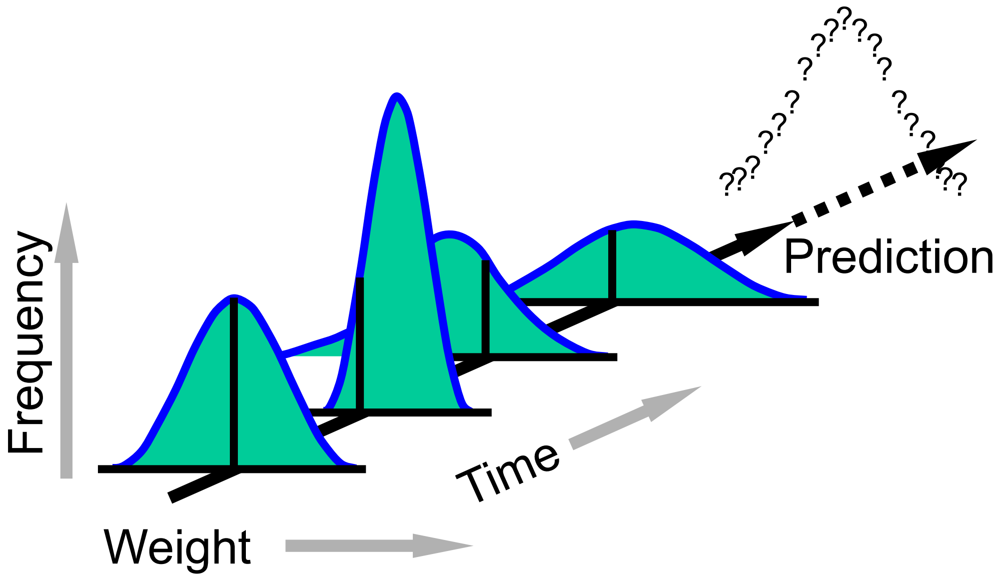
With only natural variations, the output forms a distribution that is stable over time and is predictable



Example: weight of boxes of cereal

Processes with Assignable Variations

If assignable variations are present, the process output is unstable over time and is not predicable



Example: weight of boxes of cereal

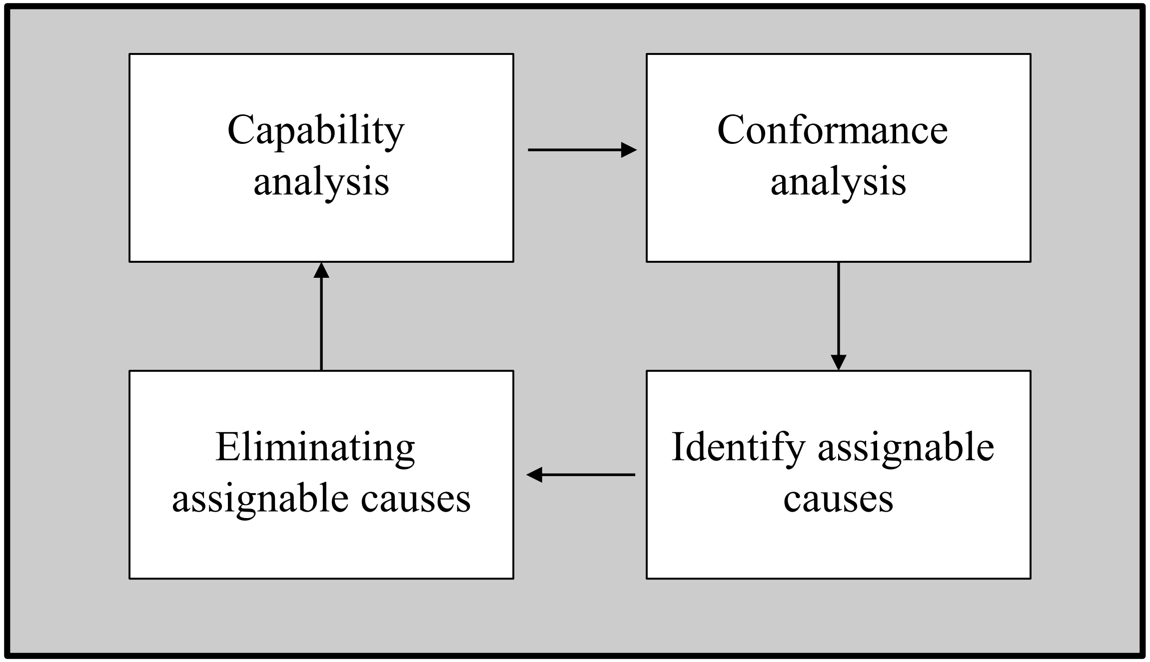
Key Questions

- How does the variation affect the process capacity?
  - Outcome specification, likelihood of defects
  - This measures the quality from a “static” aspect, i.e., taking sample at a given time point
  - Capability analysis: capability index, six-sigma criteria

- Is the production process consistent and stable over time?
  - Abnormal instances, system errors, actions to be taken
  - This measures the quality from a “dynamic” aspect, i.e., checking samples over different periods
  - Conformance analysis: control chart, p-chart

- How can we improve the system performance?

Statistical Process Control



Capable: the output of the process ***meets design specs*** with a sufficiently high probability

In-control: the process is ***consistent and stable*** over time

Agenda

- Types of Variations
- **Capability Analysis**
- Conformance Analysis
- Acceptance Sampling

Specification Limits

- Quality is defined by specification limits
  - Pre-specified by engineering designs
  - Lower and upper limits: LSL and USL
  - Quality is considered acceptable if it is within LSL and USL; otherwise it is considered a defect

Likelihood of Defect

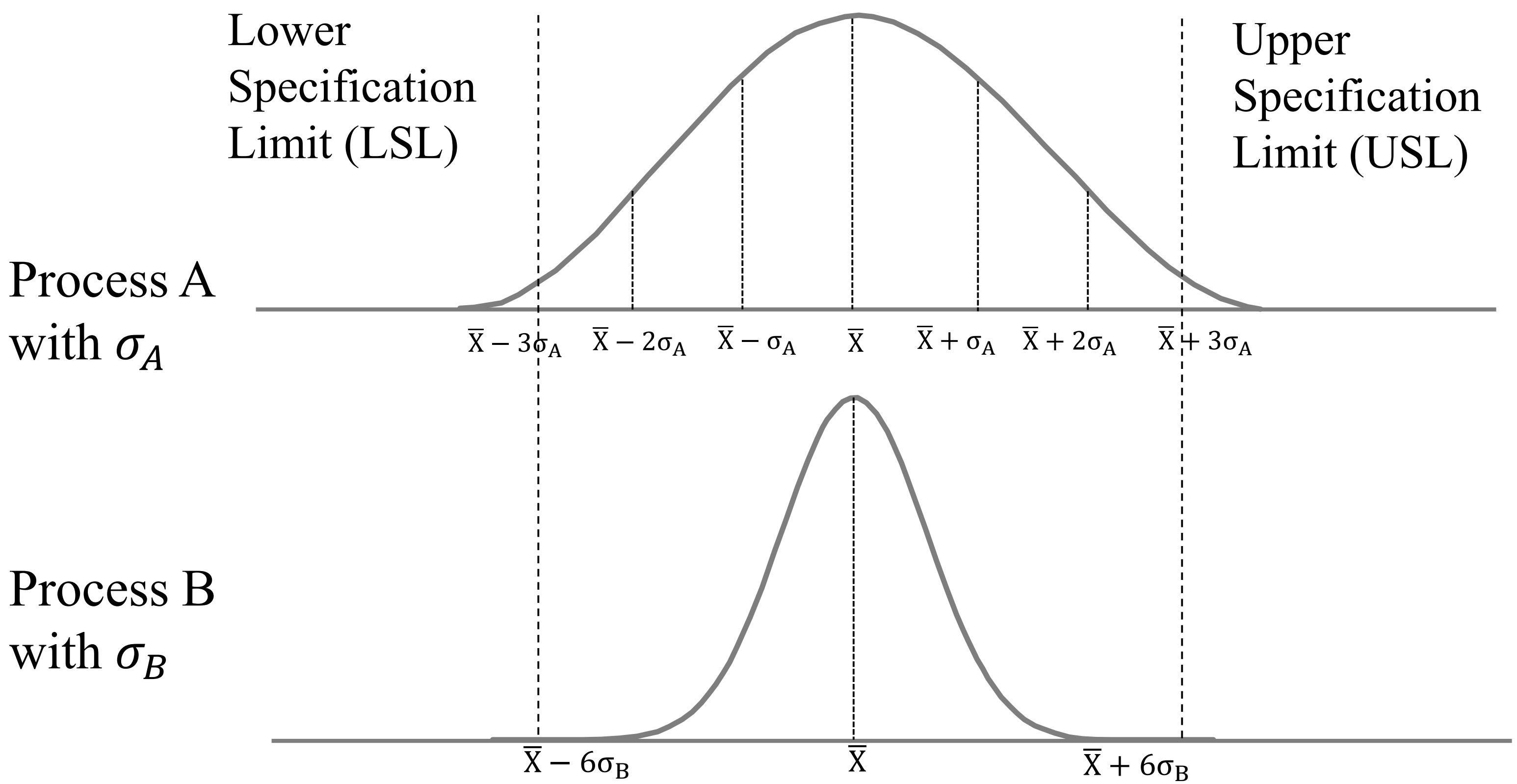
- What determines the likelihood that the process produces a defect?

- Tightness of specification limits: USL – LSL
  - Wider specification limits reduce the likelihood for defect

- Variability of actual products: how dispersed is the distribution of products' metrics
  - Larger variability increases the likelihood for defect

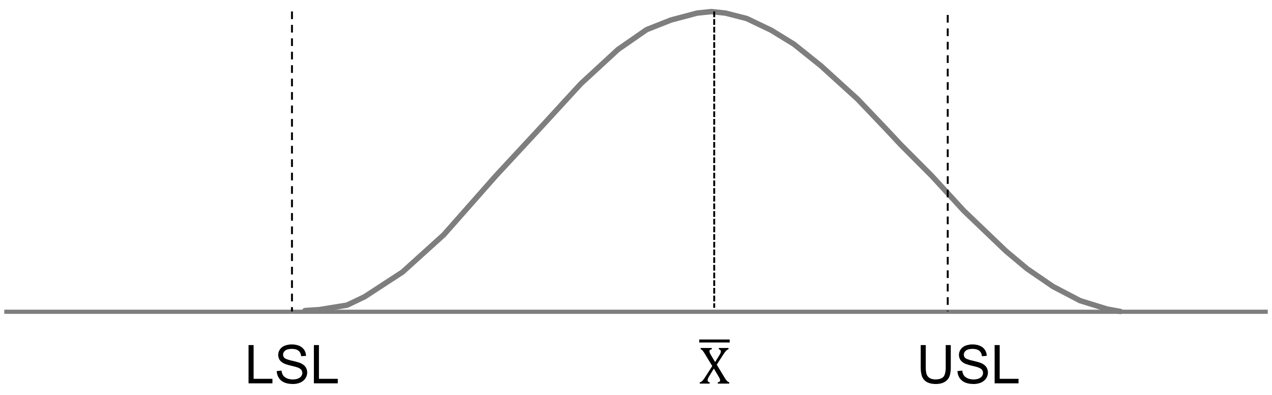
Impact of Variability

- We measure variability by the standard deviation of product metrics
- For example, case A has a larger standard deviation than case B in below



Asymmetric Case

- Sometimes the mean $\bar{X}$ does not lie in the middle of LSL and USL: asymmetric case



- In this case, we focus on the "narrow" side (here $USL - \bar{X}$) as it is more likely to violate the limit on this side
  - Be conservative in quality management

Capability Index

- Given a target defect likelihood, how to tell if current quality is good enough?
- There are three factors to consider:
  - Width of range between LSL and USL
  - Variability, as measured by the standard deviation
  - Asymmetric case: look at the narrower side
- Idea: measure the width of range on both sides relative to the standard deviation
  - how many standard deviations can we insert in on each side?

Capability Index

- Process Capability Index:

$$C_{pk} = \min\left[\frac{\bar{X} - LSL}{3\sigma}, \frac{USL - \bar{X}}{3\sigma}\right]$$

- $\bar{X}$ is the sample average, σ is the sample standard deviation

- A larger capability index means a lower likelihood of default, i.e., better quality

- A smaller standard deviation leads to a larger capability index

Capability Index and #-Sigma Quality

- N-sigma quality means that the smaller side of $\overline{X} - LSL$ and $USL - \overline{X}$ is N times of the standard deviation
- The larger N, the better

- Capability index is divided by three times of the standard deviation

$$C_{pk} = \min\left[\frac{\overline{X} - LSL}{3\sigma}, \frac{USL - \overline{X}}{3\sigma}\right]$$

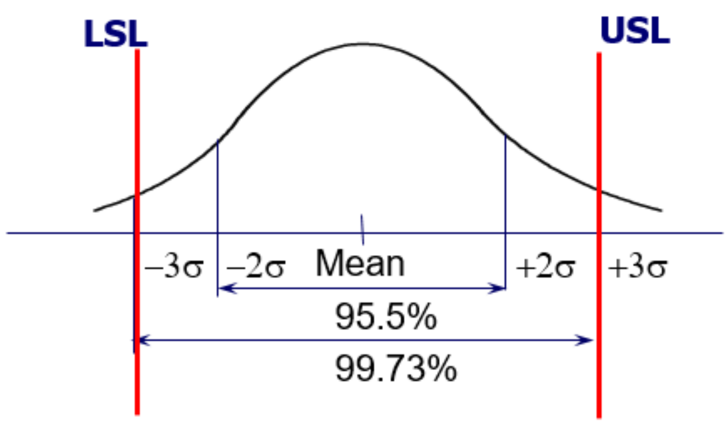
- Cpk = 1 means three-sigma quality
- Cpk = 2 means six-sigma quality

Three Sigma Quality

$$C_{pk} = \min\left[\frac{\bar{X} - LSL}{3\sigma}, \frac{USL - \bar{X}}{3\sigma}\right]$$

- $C_{pk} \geq 1$: 3-sigma quality
  - The tolerance distance on each side is at least 3 times of the standard deviation
  - Prob{within specification limits} greater than 99.73%
  - Defect probability smaller than 0.27%

The Normal Distribution



Six Sigma Condition

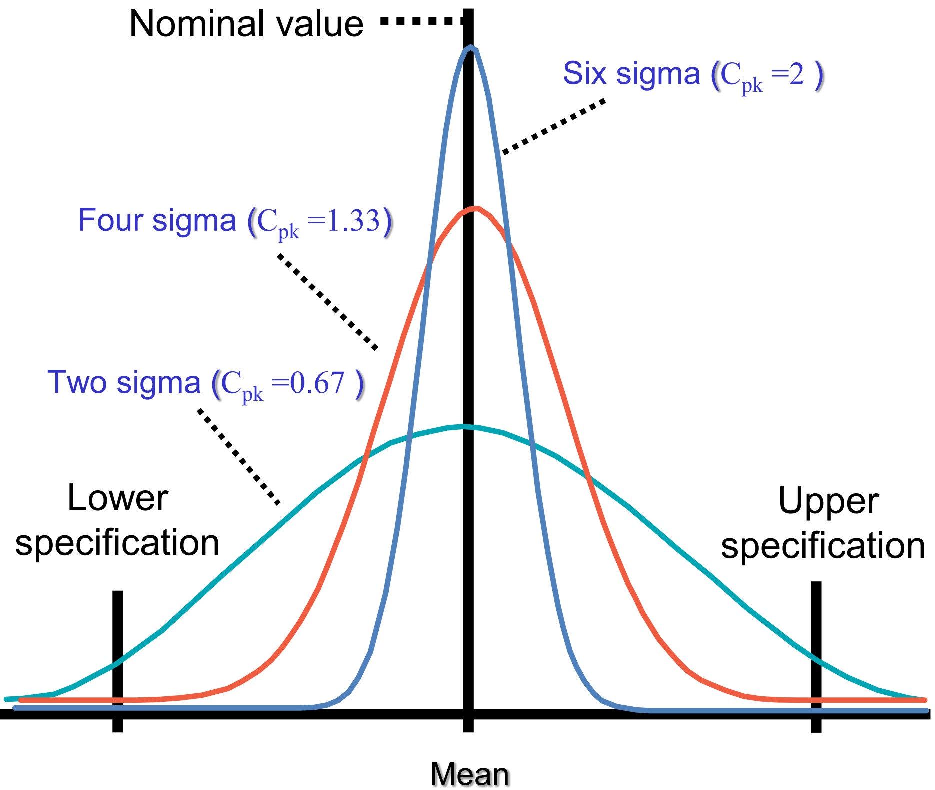
- Six-Sigma: tolerance distance on each side is at least six times of the standard deviation

$$C_{pk} = \min\left[\frac{\overline{X} - LSL}{3\sigma}, \frac{USL - \overline{X}}{3\sigma}\right]$$

- In terms of capability index, it requires $C_{pk} \geq 2$
  - This imposes a limit on the process variability

- Now, the defect probability decreases to 0.00000000197

Effect of Reducing Variability



#σ	C_{pk}	P{defect}	Defects
1σ	0.33	0.317	317 per thousand
2σ	0.67	0.0455	45 per thousand
3σ	1.00	0.0027	2.7 per thousand
4σ	1.33	0.000063	63 per million
5σ	1.67	0.0000006	574 per billion
6σ	2.00	$2x10^{-9}$	2 per billion

Six Sigma Quality

- Six-Sigma: An initiative and a set of techniques to improve manufacturing quality

- Historical development
  - First developed at Motorola in the 1980's
  - Adopted by two thirds of Fortune 500 organizations by late 1990

- Basic premises of six sigma
  - Variation is the main cause of many quality problems
  - Sources of variation can be identified, quantified, and controlled
  - Reducing variation is an effective way to improve quality

Six Sigma Quality

- Define: what you want to achieve with your project.
- Measure: how you will measure your goals and which statistical analysis and data collection method you will use.
- Analyze: analyze the process and discover potential influencing variables.
- Improve: make improvements based on the results of your analysis.
- Control: control the outcome by evaluating whether your changes have been successful or not.

Example Question

- Suppose you run a factory producing a computer table. The LSL and USL of the table length is 94cm and 106 cm respectively. Assume the sample average is 100cm, and the standard deviation is σ = 2cm.

- What is current capability index? How many sigma quality does the process have?

$$C_{pk} = \min\left[\frac{\bar{X}-LSL}{3\sigma}, \frac{USL-\bar{X}}{3\sigma}\right] = \min\left[\frac{100-94}{3\times 2}, \frac{106-100}{3\times 2}\right] = 1$$

- Thus, the capability index is one, meaning the process has a 3-sigma quality
  - we can insert 3 standard deviations on each side

Example Question

- If we want to achieve 6-sigma quality, what is the largest standard deviation we can have?

- To achieve 6-sigma quality, the capability index should be at least 2

- This means

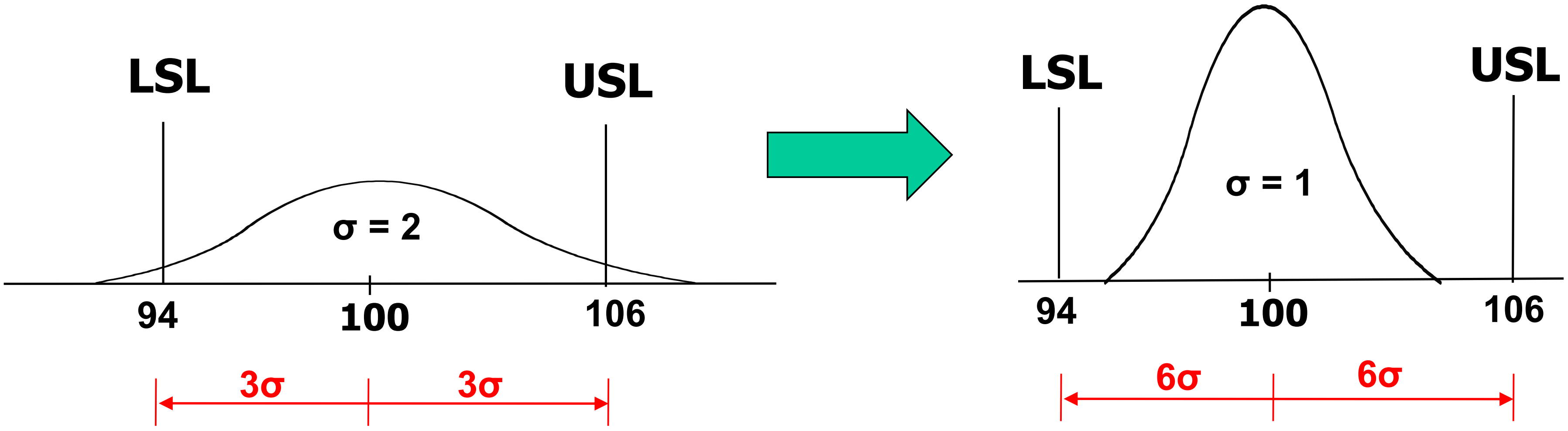
$$C_{pk} = \min\left[\frac{\bar{X}-LSL}{3\sigma}, \frac{USL-\bar{X}}{3\sigma}\right] = \min\left[\frac{100-94}{3\times\sigma}, \frac{106-100}{3\times\sigma}\right] \geq 2$$

- Then we know the standard deviation should be at most 1cm

Quality Improvement

- By decreasing the standard deviation from 2cm to 1cm, we improve from 3-sigma quality to 6-sigma quality

- This leads to a lower defect likelihood



Why 6-sigma is needed

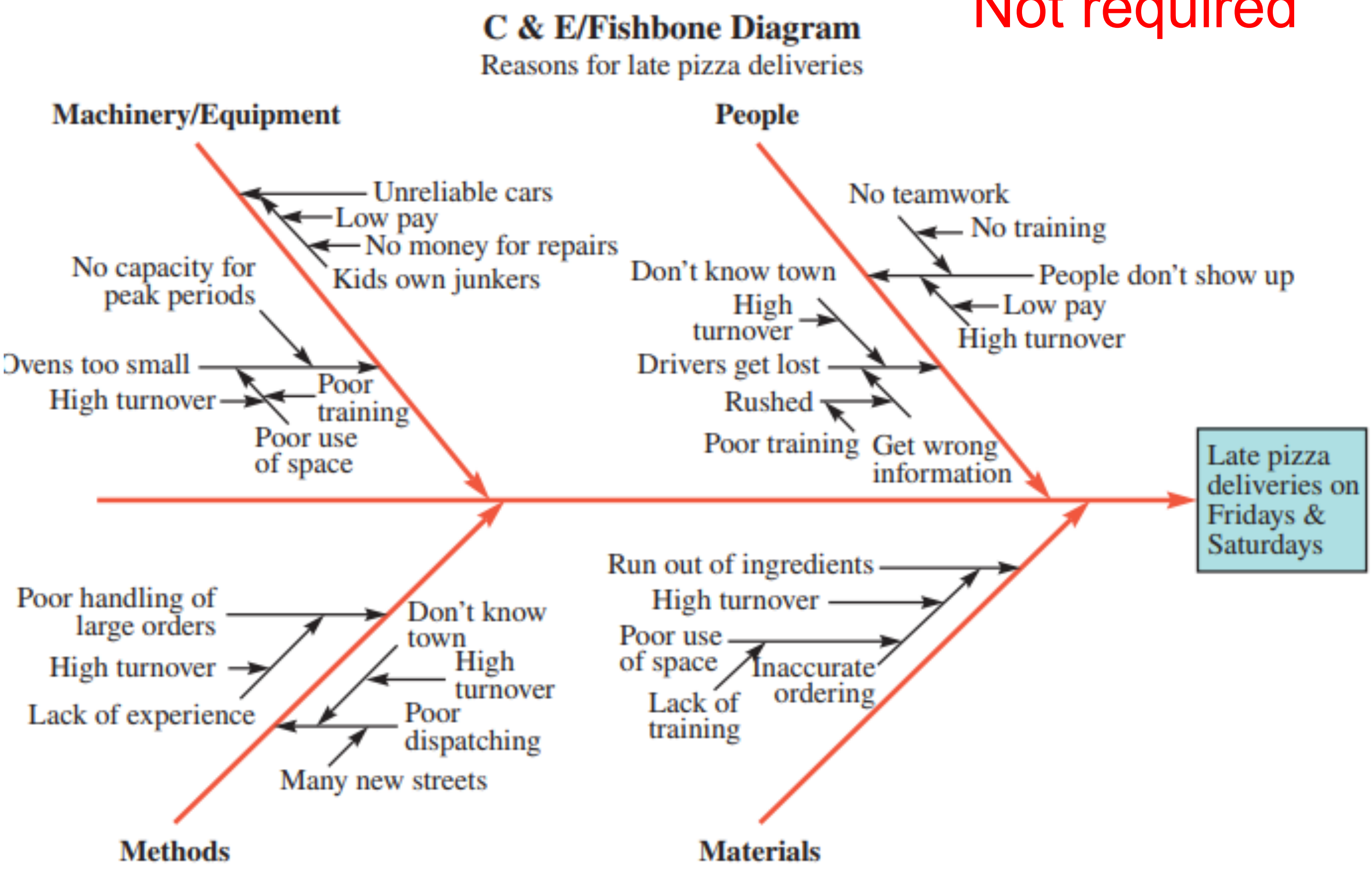
- Going through 3-sigma to 6-sigma decreases the defect probability from 0.0027 to 0.00000000197
- Change seems small. Why we need this?
- Industries with high quality requirements
  - Jet engines, cars, vaccines, medicines, etc
- Errors can accumulate: suppose a production chain with 100 components:
  - Defect probability with 3-sigma quality: 23.7%
  - Defect probability with 6-sigma quality: 0.00002%

Handling of Defects

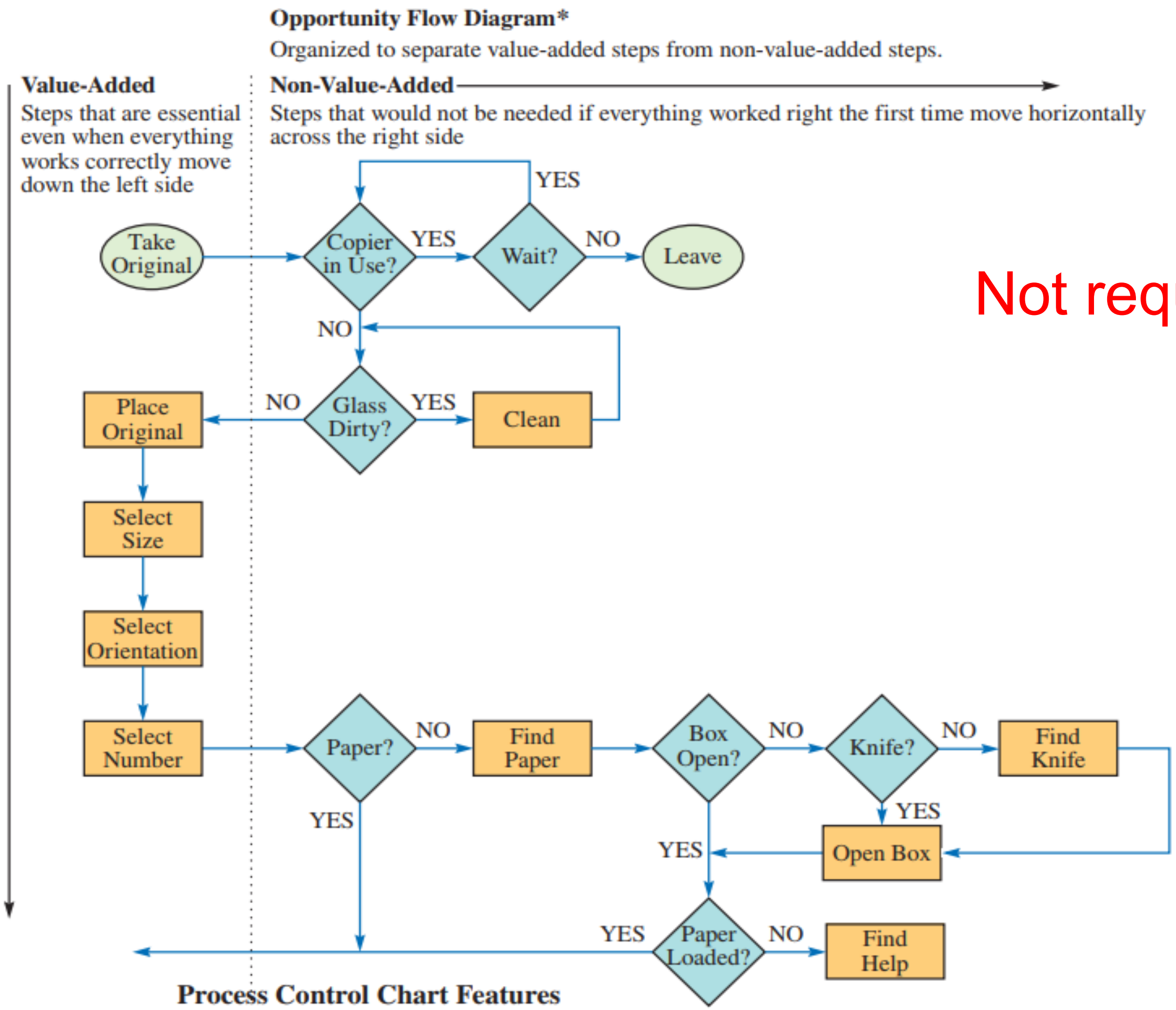
- Scrapping: eliminate from the process
  - Waste of resource, uncertainty in output
- Rework: bounce-backs
  - Requires additional resources, may increase system workload
  - Operational choice: save capacity vs. make things right at the first time
  - Example: early discharge of patients from hospital

6-Sigma Quality: Fishbone Diagram

Not required



6-Sigma Quality: Process Control Chart



Process Control Chart Features

Not required

ISOM 2700: Operations Management

Session 9. Quality Management: Control Charts and Acceptance Sampling

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Dept. of ISOM, HKUST

Fall 2025

Agenda

- Types of Variations
- Capability Analysis
- **Conformance Analysis**
- Acceptance Sampling

Conformance Analysis

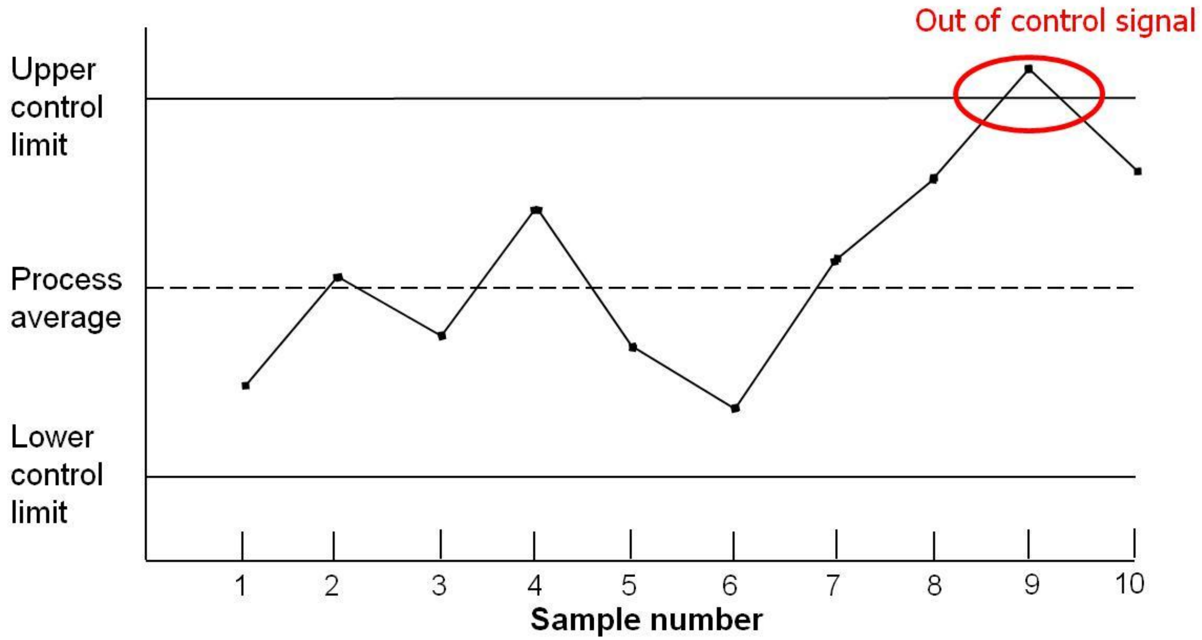
- A process is **in control** if it is operating without assignable cause variation.
- Otherwise, the process is **out of control**, and corrective action should be taken

Conformance Analysis

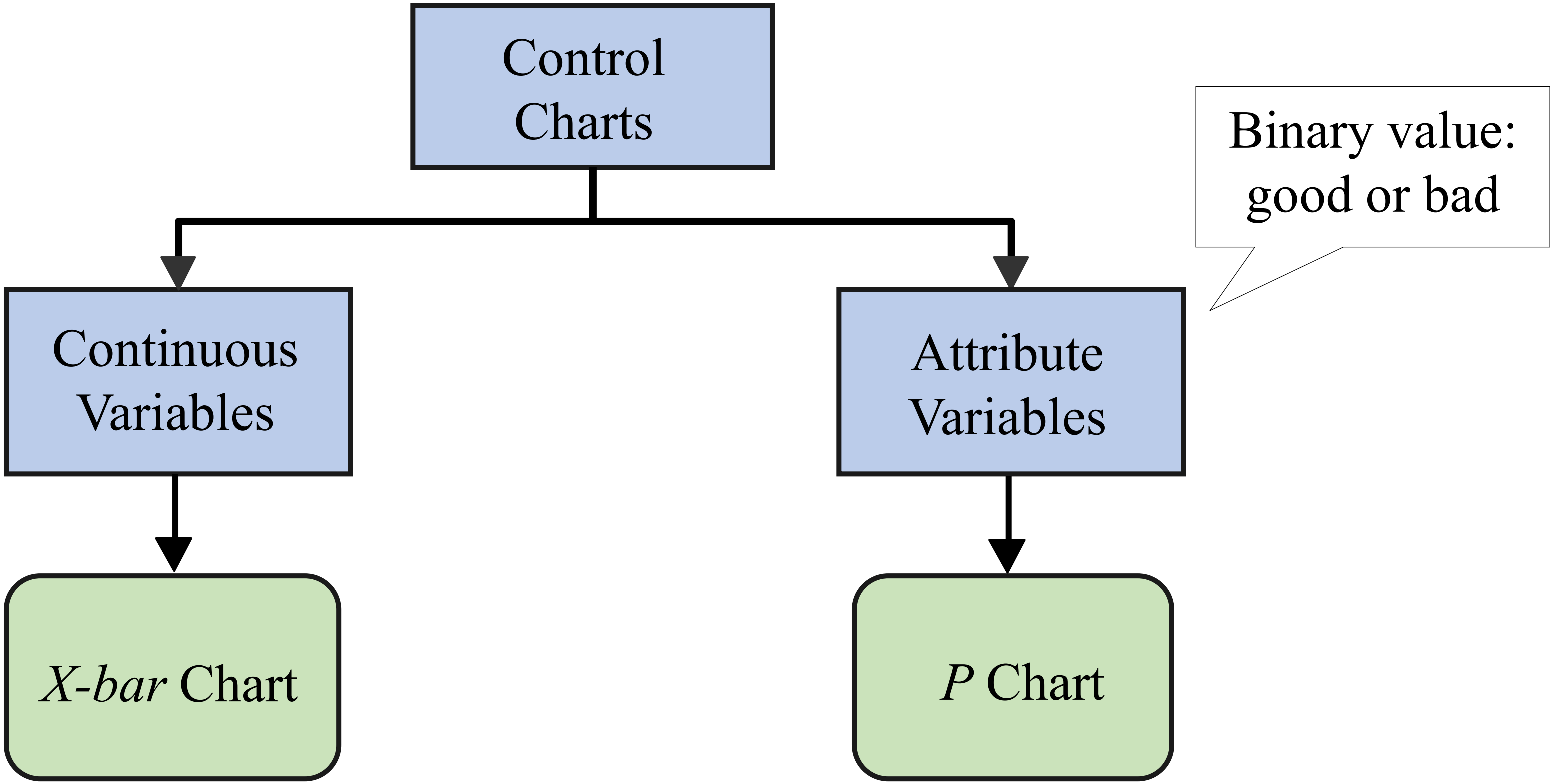
- Control the quality of the *output* by controlling the *process* that produces the output
- Uses statistics and control charts to tell when to adjust a process
- Involves
  - Creating standards (upper & lower limits)
  - Measuring sample output (e.g. mean and range)
  - Taking corrective action if necessary

Process Control: Control Charts

- Monitor the variation of outputs to assess how the process evolves over time
- Idea: If the process were stable, then the probability of violating the control limits is very small
- Thus if we do observe violation from data, it suggests the process is likely to be unstable



Types of Control Charts



Continuous Variables

- Product characteristic that can be measured continuously
  - Such as weight, length, width, temperature, speed…

- X-bar chart to control the values

- Type of data:
  - A series of samples over time
  - For each sample, measures taken on each unit

Method: x-bar chart

- Step 1. Identify size *n* for each sample

- Step 2. In each sample, compute:
  - $\overline{\mathrm{X}}$: mean value
  - R: range (max – min)

- Step 3. Compute mean of sample statistics (across all samples):
  - $\overline{\overline{\mathrm{X}}}$: mean of x-bar
  - $\bar{R}$: mean of R

- Step 4. Compute control limits

$$\mathrm{LCL}_{\overline{\mathrm{x}}} = \overline{\overline{\mathrm{X}}} - \mathrm{A}_2\overline{\mathrm{R}} \qquad \mathrm{UCL}_{\overline{\mathrm{x}}} = \overline{\overline{\mathrm{X}}} + \mathrm{A}_2\overline{\mathrm{R}}$$

Table for A_2

Sample size

N	A_2
2	1.880
3	1.023
4	0.729
5	0.577
6	0.483
10	0.308
15	0.223
20	0.180
25	0.153

- A_2 controls the width of lower and upper control limits
- Increased sample size is associated with smaller A_2, why?
- The mean of a larger sample should have less variation
- This table will be provided

Example of x-bar

- Data on circuit board thickness

  Sample size: $n = 3$

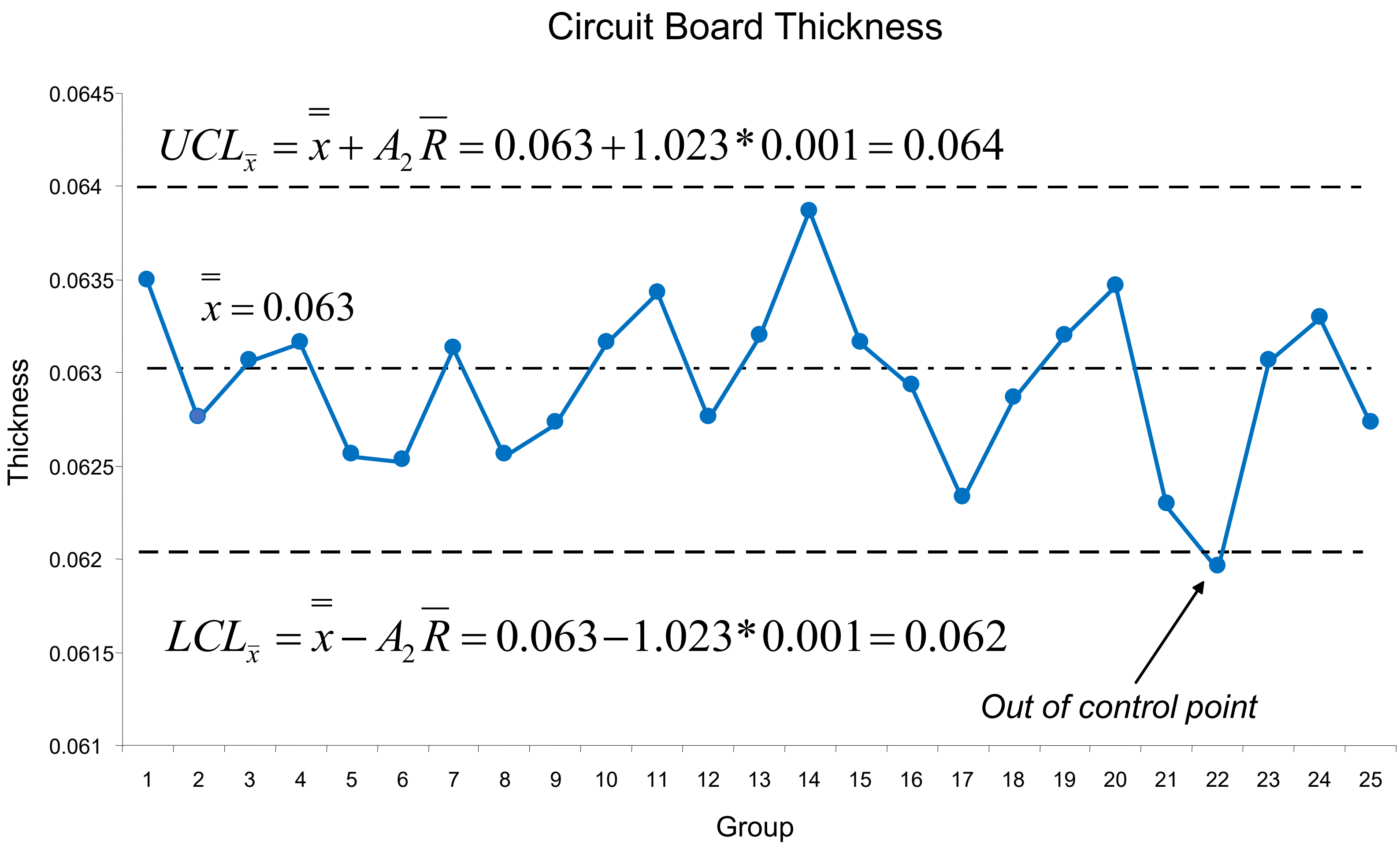
$$\bar{\bar{X}} = 0.0630$$

$$\bar{R} = 0.0009 \approx 0.001$$

Sample	x1	x2	x3
1	0.0629	0.0636	0.0640
2	0.0630	0.0631	0.0622
3	0.0628	0.0631	0.0633
4	0.0634	0.0630	0.0631
5	0.0619	0.0628	0.0630
6	0.0613	0.0629	0.0634
7	0.0630	0.0639	0.0625
8	0.0628	0.0627	0.0622
9	0.0623	0.0626	0.0633
10	0.0631	0.0631	0.0633
11	0.0635	0.0630	0.0638
12	0.0623	0.0630	0.0630
13	0.0635	0.0631	0.0630
14	0.0645	0.0640	0.0631
15	0.0619	0.0644	0.0632
16	0.0631	0.0627	0.0630
17	0.0616	0.0623	0.0631
18	0.0630	0.0630	0.0626
19	0.0636	0.0631	0.0629
20	0.0640	0.0635	0.0629
21	0.0628	0.0625	0.0616
22	0.0615	0.0625	0.0619
23	0.0630	0.0632	0.0630
24	0.0635	0.0629	0.0635
25	0.0623	0.0629	0.0630

	x-bar	R
	0.0635	0.0011
	0.0628	0.0009
	0.0631	0.0005
	0.0632	0.0004
	0.0626	0.0011
	0.0625	0.0021
	0.0631	0.0014
	0.0626	0.0006
	0.0627	0.0010
	0.0632	0.0002
	0.0634	0.0008
	0.0628	0.0007
	0.0632	0.0005
	0.0639	0.0014
	0.0632	0.0025
	0.0629	0.0004
	0.0623	0.0015
	0.0629	0.0004
	0.0632	0.0007
	0.0635	0.0011
	0.0623	0.0012
	0.0620	0.0010
	0.0631	0.0002
	0.0633	0.0006
	0.0627	0.0007
mean	0.0630	0.0009

Plotting x-bar Chart



Difference of LSL/USL and LCL/UCL

- We have learnt two types of limits: Specification limits and Control limits

- Specification limits: used in capability analysis
  - They determine what is a defect (if the product value falls out of the LSL and USL)
  - They are given exogenously by engineering designs, not depending on your sample

- Control limits: used in conformance analysis
  - They are used to tell if a process is in-control (consistent over time)
  - They are computed based on your series of sample

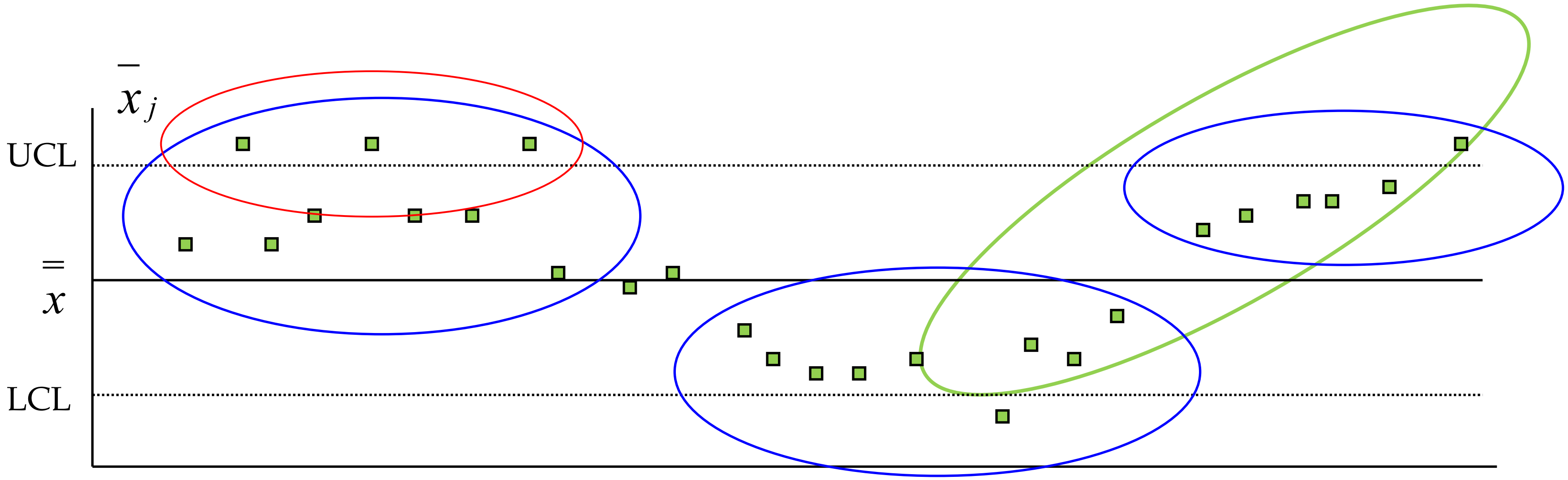
Identify Anomalies

- If the process were in-control, you would expect
  - The sample averages stay within the control limits
  - The sample averages fluctuate around the process average

- Thus, when the above is violated, there is high probability that some assignable cause variation (anomalies) happened
  - This implies that you should take corrective actions

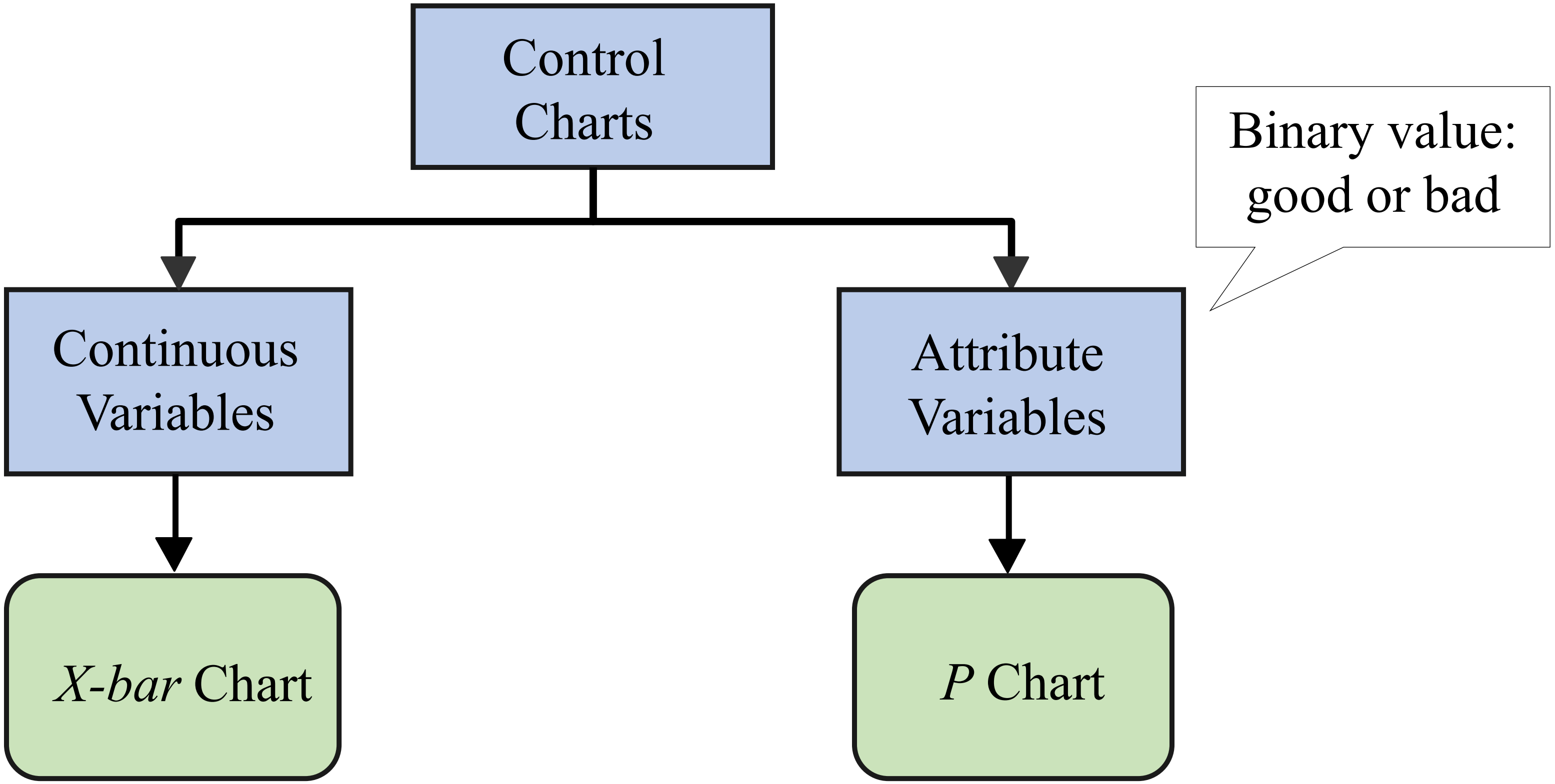
- Types of anomalies:
  - Point falls outside the control limits
  - Seven consecutive points in a row on one side of center line
  - Seven points in a row going up or going down

Identify Anomalies: Example



- Several points outside limits
- Long (>7) runs of points above/below mean
- Trends or drift over time

Types of Control Charts



Attribute (Binary) Variables

- Attribute variables: variables such as good or bad, defective or non-defective, yes or no

- P-chart
  - Controls the percentage of defective items
  - Same logic and steps as the x-chart

- Type of data:
  - A series of samples over time
  - For each sample, number of defective items

Method: P-chart

- Step 1. Identify sample size *n*
- Step 2. In each sample, compute:
  - p: fraction of defective items
- Step 3. Compute means of sample:
  - $\bar{p}$: mean of *p from each sample*
- Step 4. Compute standard deviation:

$$\sigma_p = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

- Step 5. Compute control limits

$$UCL_p = \bar{p} + 3\sigma_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$LCL_p = \max\{\bar{p} - 3\sigma_p, 0\} = \max\{\bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}, 0\}$$

Example

- Data on defective circuit boards
- Sample size: $n = 50$

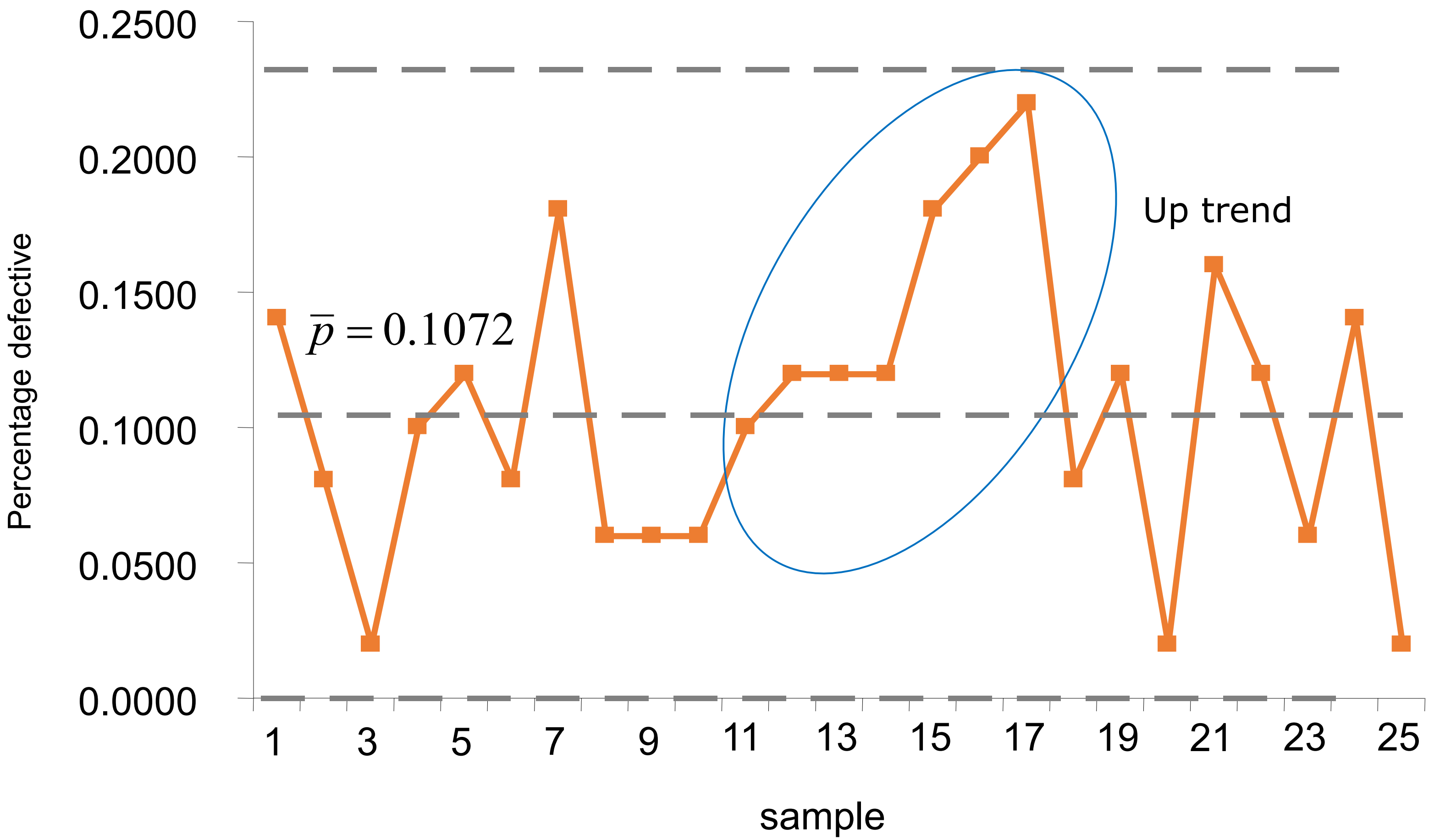
$$\bar{p} = 0.1072$$

$$\sigma_p = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0438$$

	Number of defective items	p
1	7	0.1400
2	4	0.0800
3	1	0.0200
4	5	0.1000
5	6	0.1200
6	4	0.0800
7	9	0.1800
8	3	0.0600
9	3	0.0600
10	3	0.0600
11	5	0.1000
12	6	0.1200
13	6	0.1200
14	6	0.1200
15	9	0.1800
16	10	0.2000
17	11	0.2200
18	4	0.0800
19	6	0.1200
20	1	0.0200
21	8	0.1600
22	6	0.1200
23	3	0.0600
24	7	0.1400
25	1	0.0200
	pbar	0.1072
	sigma p	0.0438

P-chart

$$\mathrm{UCL_p} = \bar{\mathrm{p}} + 3\sigma_\mathrm{p} = 0.1072 + 3 \times 0.0438 = 0.2384$$

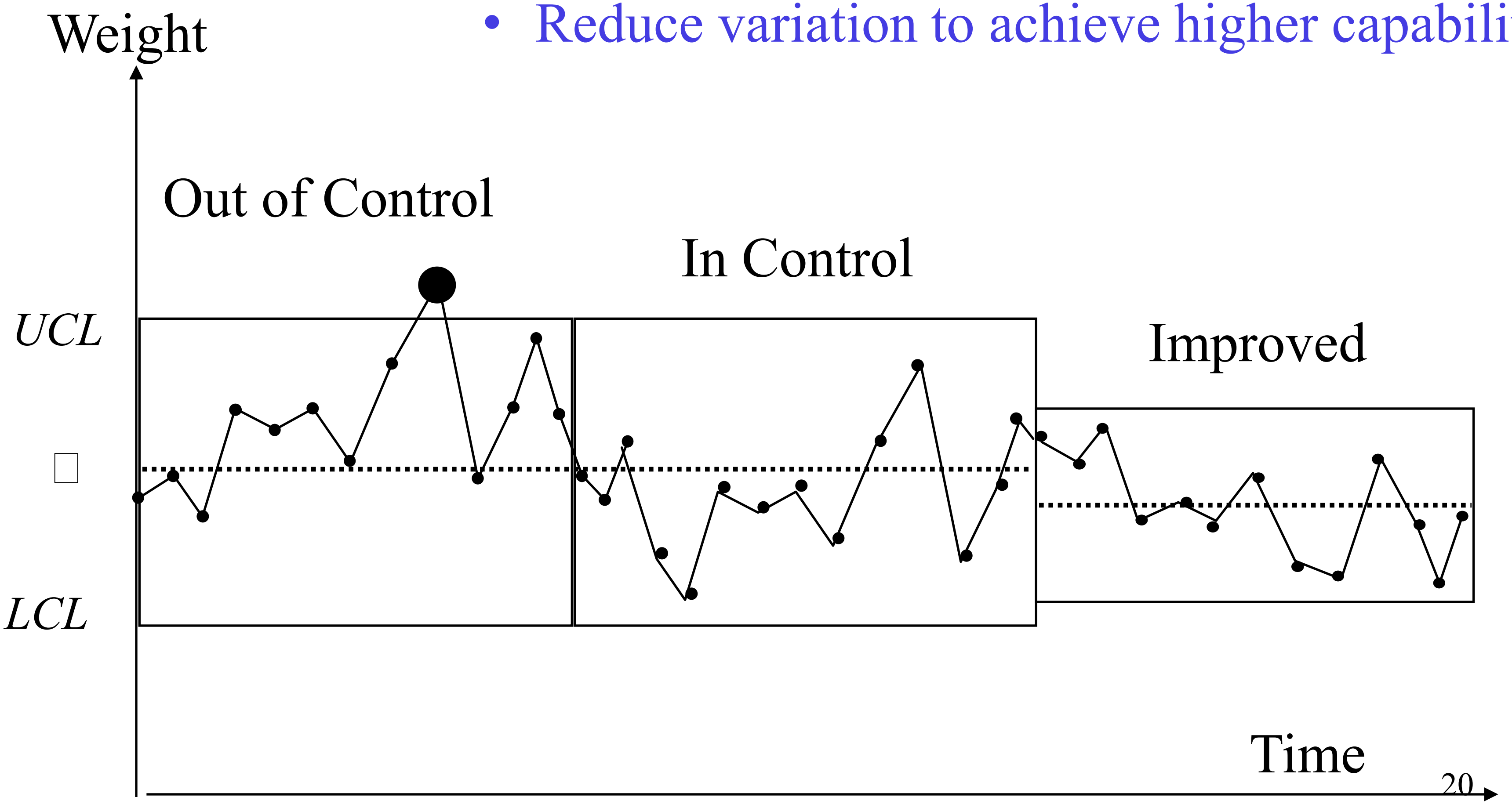


$$\mathrm{LCL_p} = \max\{\bar{\mathrm{p}} - 3\sigma_\mathrm{p}, 0\} = \max\{0.1072 - 3 \times 0.0438, 0\} = 0$$

From Control to Improvement

Quality = Process control + Improvement

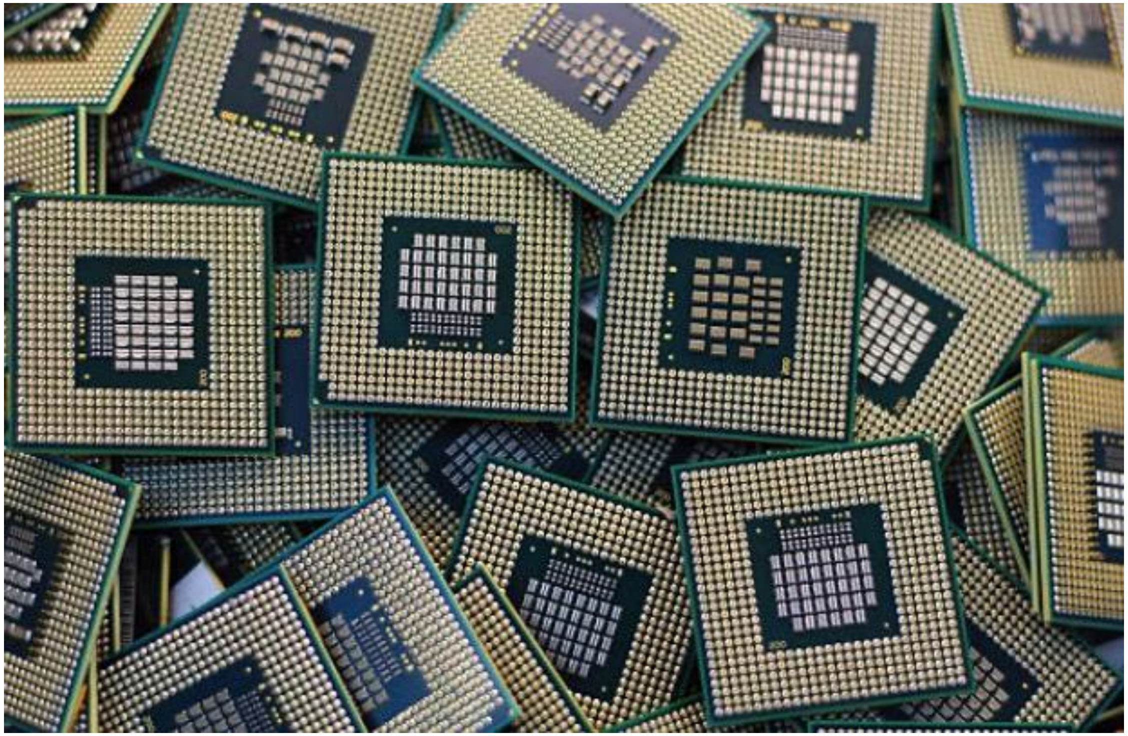
- Identify and eliminate anomalies
- Reduce variation to achieve higher capability



Agenda

- Types of Variations
- Capability Analysis
- Conformance Analysis
- **Acceptance Sampling**

Example: Computer Chips



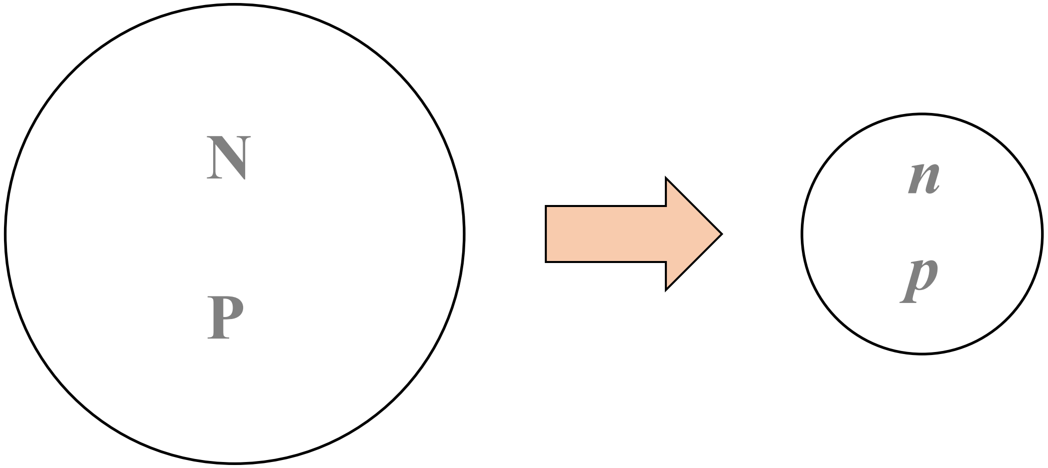
How can we know whether most of the chips (say more than 90%) are good?

Acceptance Sampling

- How many out of the 10,000 chips are good?
- Should we accept the lot?
- Test a sample of ***n*** chips: if # of malfunctioned chips is no more than ***c***, accept the lot
  - Otherwise, reject the lot
- Question: how should we decide the sampling plan, i.e., sample size ***n*** and acceptance number ***c***?

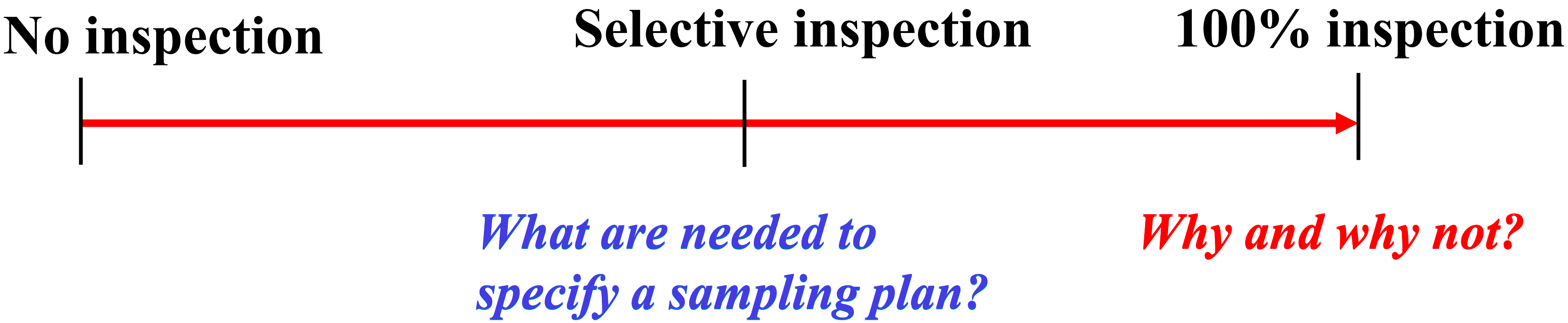
Acceptance Sampling Plan

- Measure quality by the percent defective
- Accept/reject entire lot based on sample results
  - Population size = N and true population defective rate = P
  - Sample size = n and estimated sample defective rate = p
- We are interested in the population defective rate P, but it is unknown unless we do a full inspection
  - So we use the sample defective rate p to approximate for P



where N >> *n* and *p* is used to estimate P

Why acceptance sampling plan?

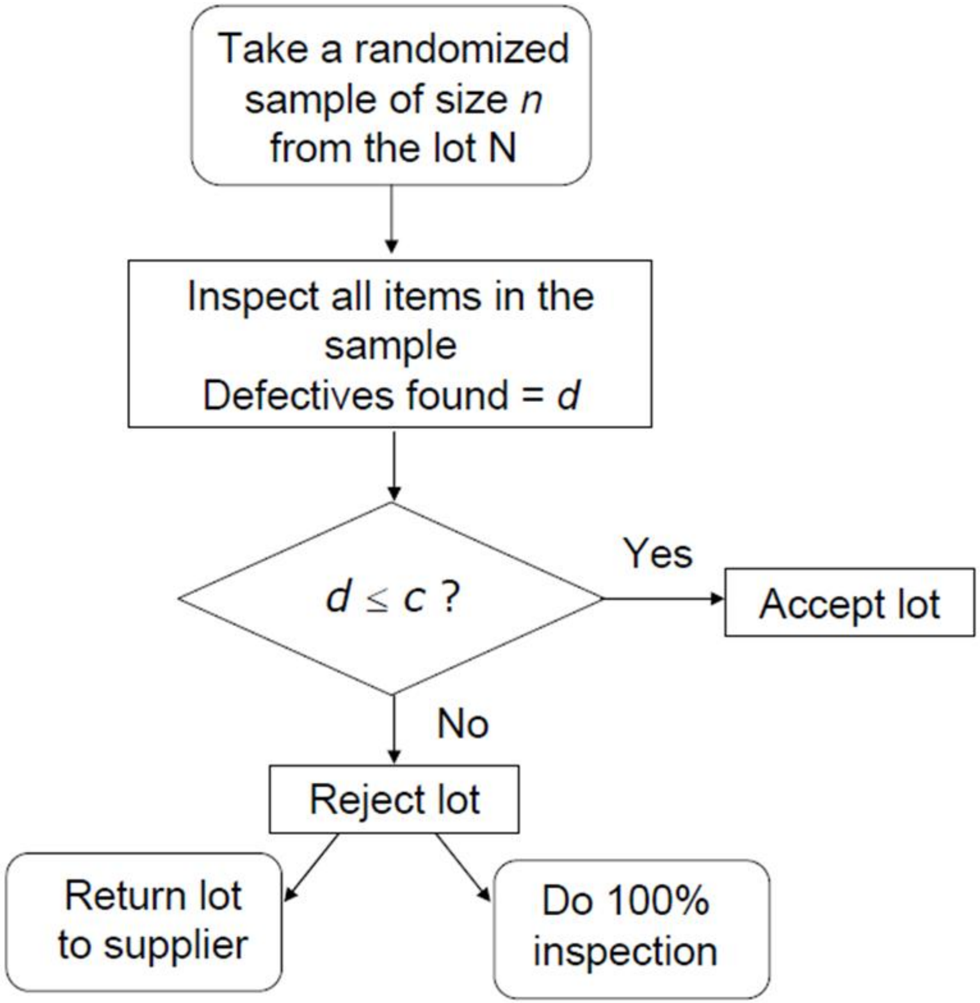


Trade-off: risk of wrong decision vs. cost of sampling

Acceptance Sampling Plan

- Decision variables: *(n,c)*
  - Sample size = ***n***
  - Acceptance number = ***c***
  - maximum defective number for the lot to be accepted
- Decision rule
  - Find the number of defective items in sample = d
  - If d ≤ c, accept the lot; else reject

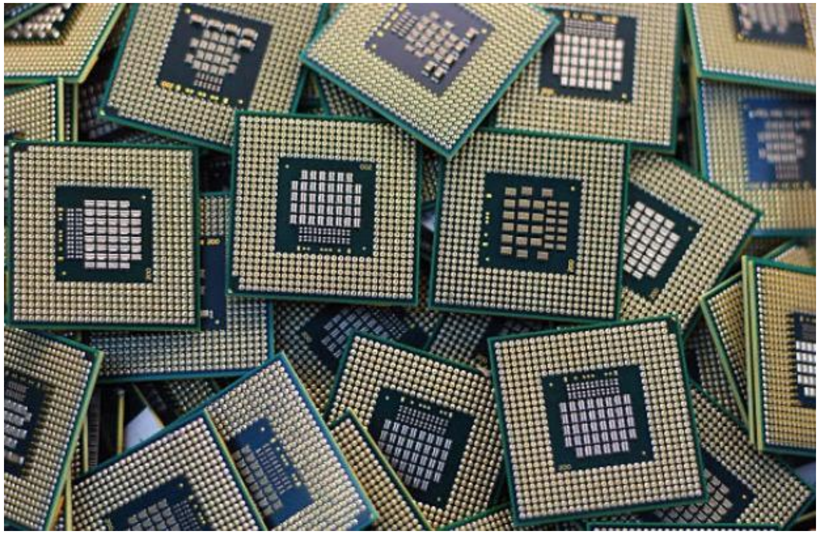
Acceptance Sampling Plan Procedure



Potential Errors

Sampling plan cannot be perfect:

- We never know true population defective rate from a subsample
- Errors are inevitable due to randomness in sampling
- Need to quantify the probability of errors



	Bad Lot: More than 20% bad chips	**Good Lot:** Less than 5% bad chips
Accept Lot	**Consumer's risk**	Correct decision
Reject Lot	Correct decision	**Producer's risk**

Two Types of Errors

- Producer's risk (Type I Error)
  - α = probability of rejecting a good lot (5% is common)
- Consumer's risk (Type II Error)
  - β = probability of accepting a bad lot (10% is typical)

	Bad Lot	Good Lot
Accept Lot	**Type II Error** with probability **β**	Correct decision
Reject Lot	Correct decision	**Type I Error** with probability **α**

Trade-off between Two Errors

- Suppose we sample n products and accept the lot if the number of defectives is no large than c

- If we increase the threshold c, we are more likely to accept the lot
  - This reduces the probability of Type I error, but increases that of Type II error --- a trade-off we face in acceptance sampling

- In practice, we need to choose the probabilities of the two types of errors depending on the concrete setting
  - If the penalty of a particular error is large, we want to have a small probability for making that error
  - E.g., we want to have a very small probability of Type II error for airplane engines, vaccines, medicines

Parameters for Sampling Plans

- To determine a sampling plan, what parameters we need?

- Probability of Type I error: rejecting a good lot
- Probability of Type II error: accepting a bad lot

- Next question: What is the definition for “good” lot and “bad” lot?
- Using the defective probability P of the underlying population

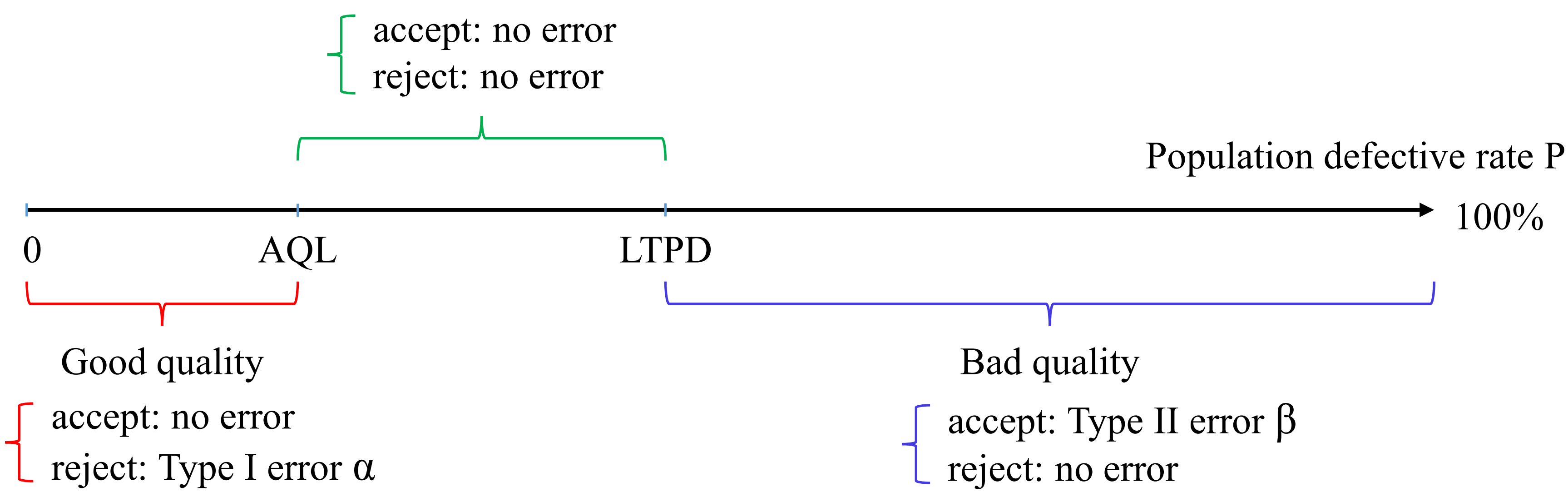
“Good” and “Bad” Lot

- Acceptance quality level (AQL)
  - Maximum defective rate such that the lot is acceptable
  - Quality is considered acceptable if the probability of defective is no more than AQL (“good” lot)
- Lot tolerance percent defective (LTPD)
  - Maximum defective rate consumer is willing to tolerate
  - Quality is considered unacceptable if the probability of defective is higher than LTPD (“bad” lot)
- Sampling plan: give α, β, AQL, and LTPD, we can solve for (***n*,*c***)

Clarification on AQL and LTPD

- The true population defective rate P is unknown
  - unless we do a 100% inspection
- AQL: used to define "good" or acceptable quality level
  - For defining Type I error (producer's risk)
  - If we reject a lot with P smaller or equal to AQL, we regard it as a Type I error
- LTPD: used to define "bad" or intolerable quality level
  - For defining Type II error (consumer's risk)
  - If we accept a lot with P greater or equal to LTPD, we regard it as a Type II error
- If true P lies between AQL and LTPD:
  - We may or may not accept the lot, depending on the random sampling result
  - If we reject it, we do NOT regard this as a Type I error
  - If we accept it, we do NOT regard it as a Type II error

Clarification on AQL and LTPD



Example: Sampling Plan

A manufacturer produces bulbs to an AQL of 1% defectives and is willing to run a 5% risk of having lots of this level or fewer defectives rejected. Lots of 6% or more defectives (LTPD) are low quality and should be accepted no more than 10% of the time.

What values of sample size (n) and acceptance number (c) should the manufacturer use to determine the quality of this lot of products?

Sampling parameters: AQL = 0.01, α = 0.05, LTPD = 0.06, β = 0.10

Example: Sampling Plan

Given AQL = 0.01, α = 0.05, LTPD = 0.06, and β = 0.10

We can solve the problem using the sampling plan table (will be provided in exam)

Excerpt from a sampling plan table for α = 0.05 and β = 0.10

C	LTPD÷AQL	n×AQL
0	44.890	0.052
1	10.946	0.355
2	6.509	0.818
3	4.890	1.366
4	4.057	1.970
5	3.549	2.613
6	3.206	3.286
7	...	

(1). Divide LTPD by AQL (0.06 ÷ 0.01 = 6).

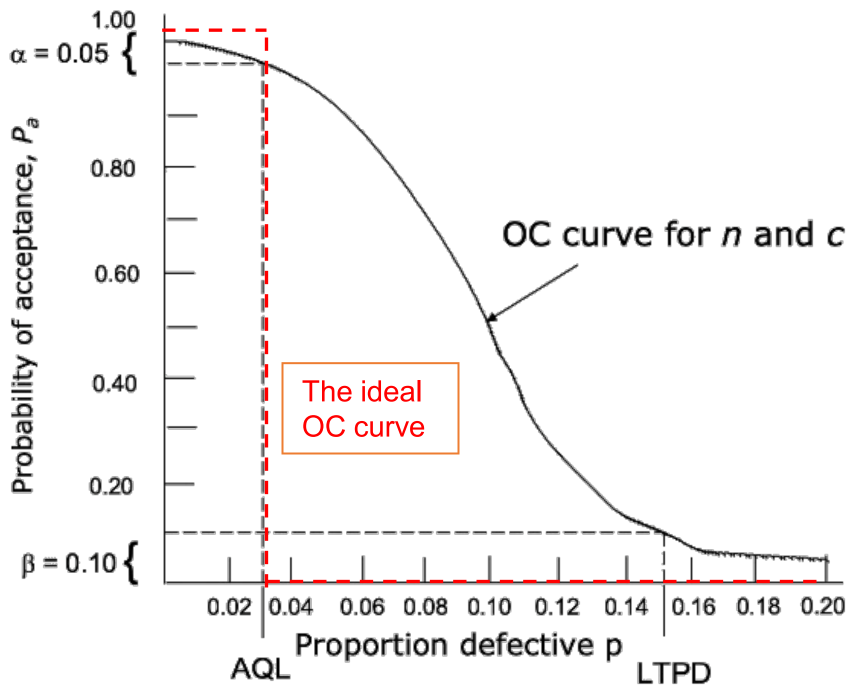
(2). Find the smallest ratio in column 2 that is equal to or greater than 6 (ratio found earlier), i.e., 6.509. That gives you c = 2.

(3). Find the value in column 3 that is in the same row as c = 2 and divide that quantity by AQL to find n, i.e., 0.818 ÷ 0.01 = 81.8 (round to 82)

The sampling plan is n = 82 and c = 2. (here we round up for n)

Operating Characteristic Curve

- The two types errors can be visualized on the operating characteristic (OC) curve (Not required)



- X-axis: population defective probability; y-axis: probability of acceptance given (n,c)

ISOM 2700: Operations Management

Session 10. Resource Allocation Decisions: Decision Trees

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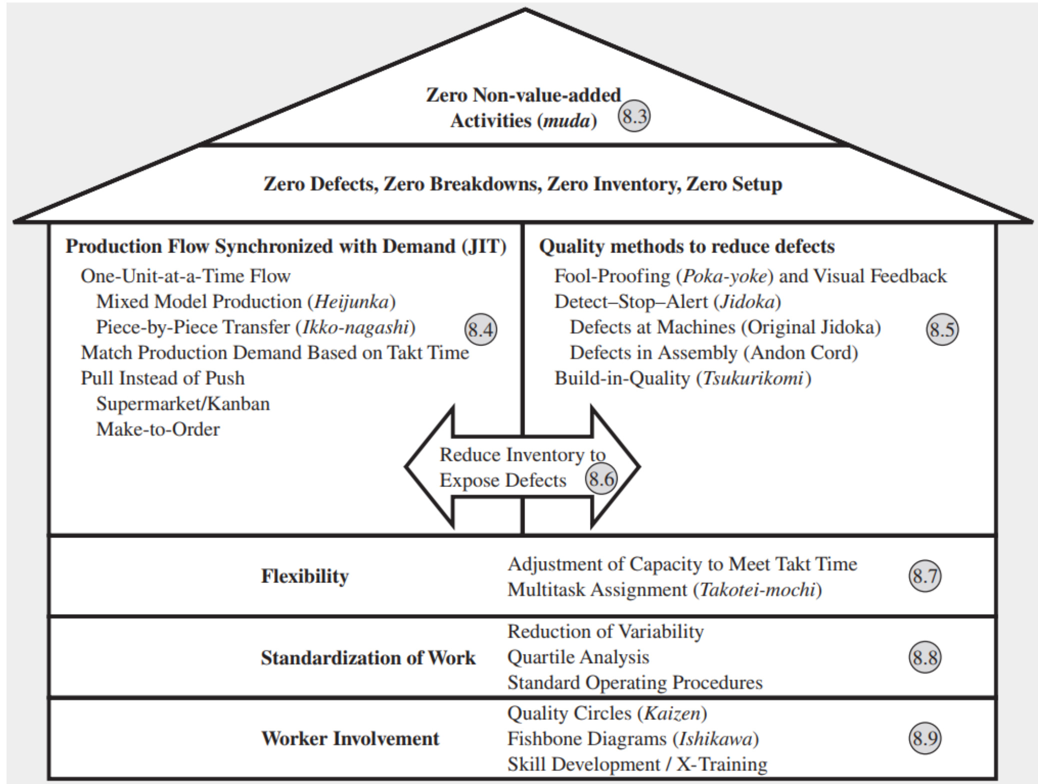
Fall 2025

Quality Management: JIT Manufacturing

(Not Required)

- Just-in-Time manufacturing (JIT, TPS)
  - Pioneered by Toyota in 1980s, adopted by many famous manufacturers globally
- Key: production flow management + quality improvement
- Goal: match supply with demand in a smooth and efficient way
- Cheaper, faster, and better

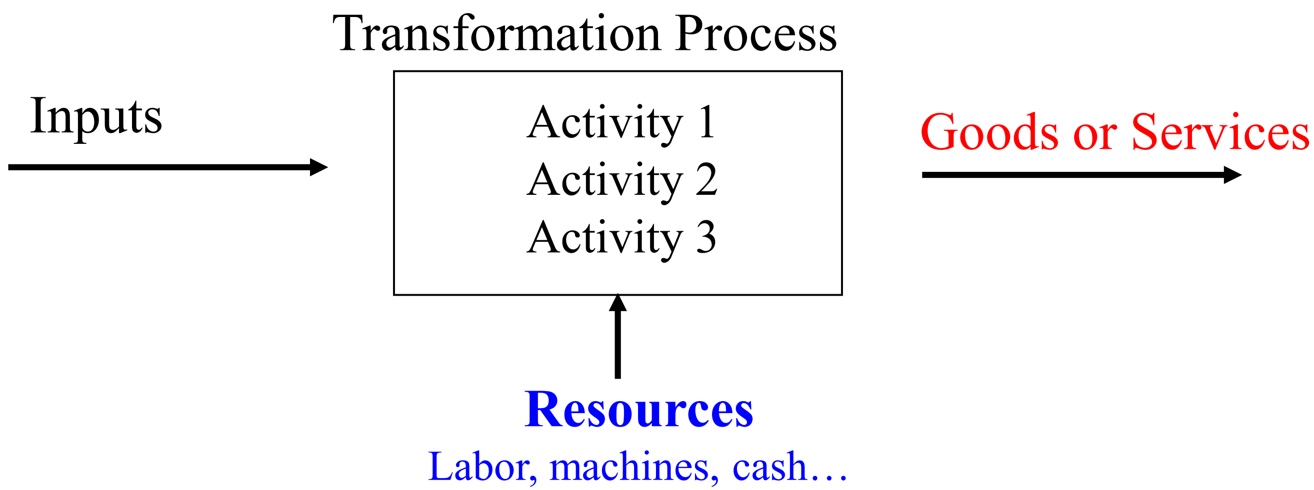


Video

Agenda

- **Capacity Planning: Overview**
- Decision Tree Method
- Linear Programming Method

Capacity Planning



If you plan to open a restaurant on campus, you must decide how many customers it should be able to serve

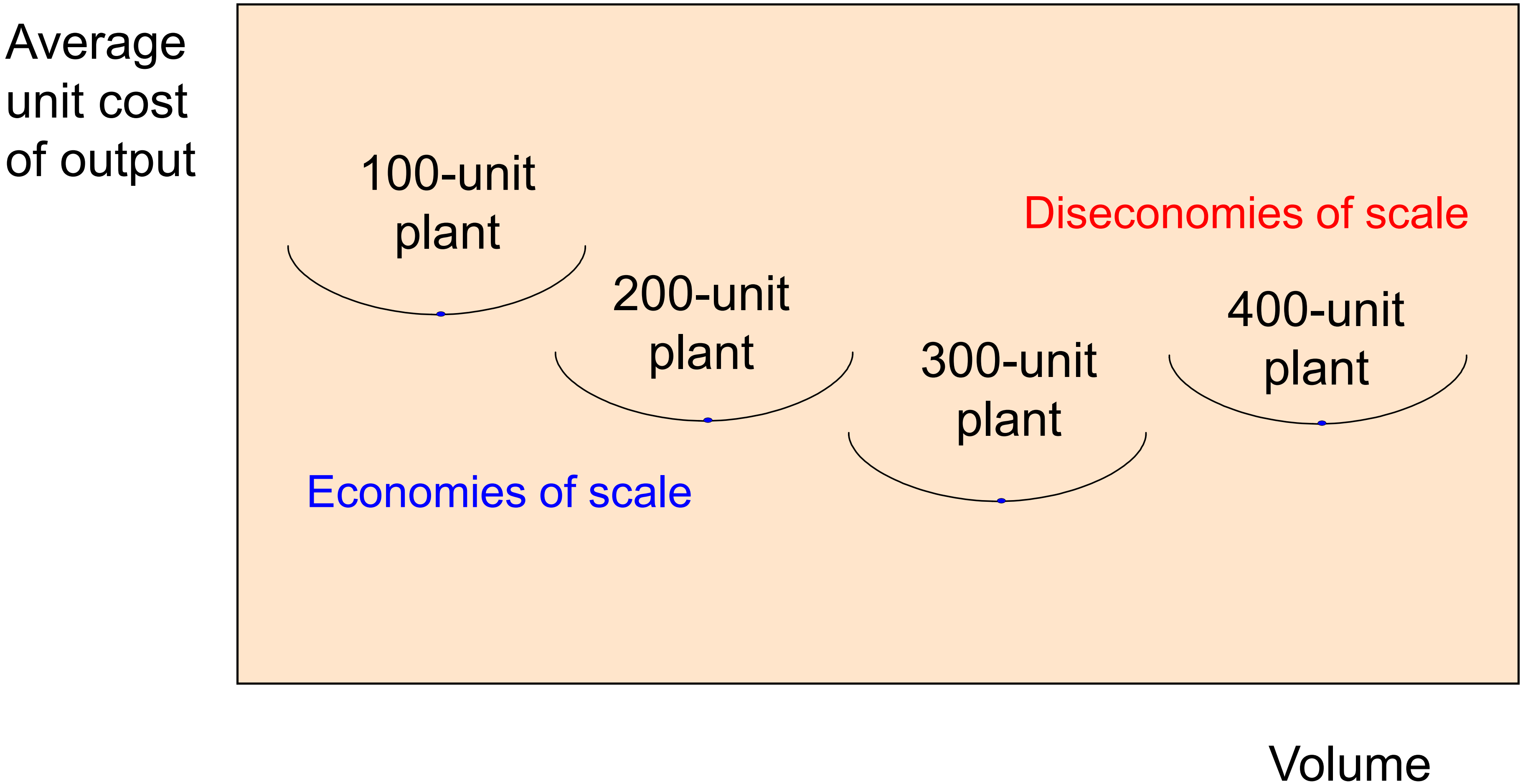
- Machines
- Tables
- Employees
- Capital
- …

Capacity Planning

- Capacity is the amount of output that a system is capable of achieving over a specific period of time.

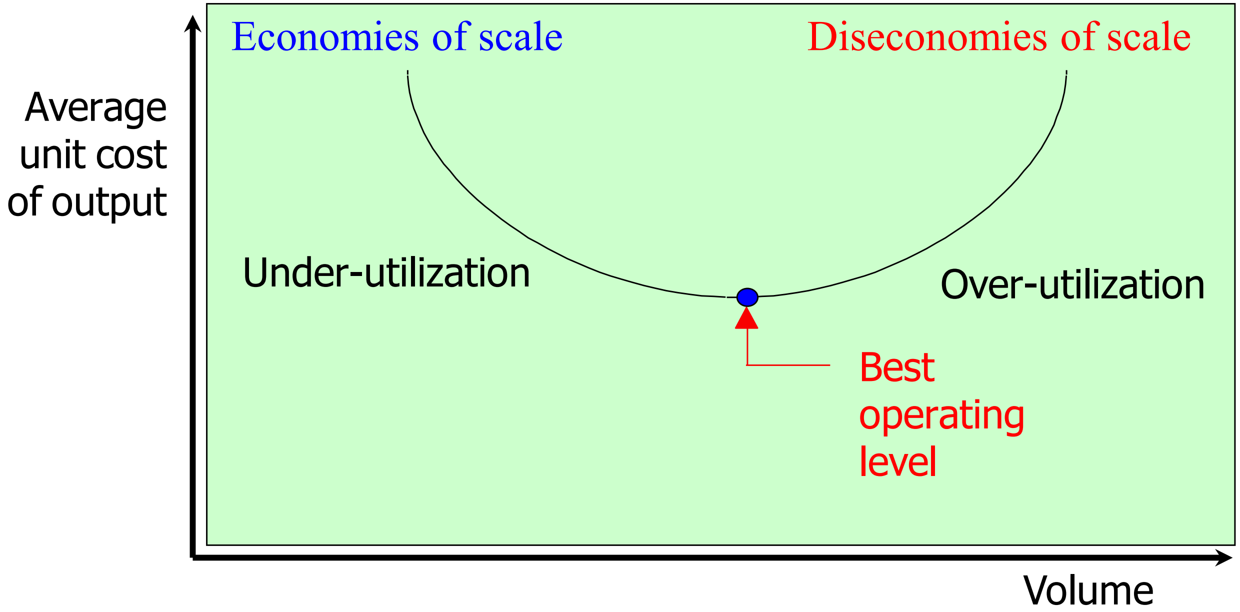
- Three levels of capacity planning and examples
  - Strategic: Plant and equipment investments, etc.
  - Tactical: Hiring, layoffs, new tools, minor equipment purchases, etc.
  - Operational: Detailed planning and control, job assignment, etc.

Economies & Diseconomies of Scale



Best Operating Level

- Economies of scale: average unit cost decreases with the scale
- Diseconomies of scale: average unit cost increases with the scale
- Best operating level: capacity level with lowest average unit cost (this is only one aspect in capacity planning)



Uncertainty in Capacity Planning

- In a real-world, uncertainty plays an important role in capacity planning

- Uncertainties in…
  - Supply: How many inputs you can get from supplier, at what price?
  - Demand: How many products you can sell to end customers? Will there be a powerful competitor by the time?
  - Production: Will there be any disruption in your manufacturing process?
  - Extreme events: Pandemic? Natural disasters? Wars?

- These uncertainties make capacity planning very challenging, especially in the long-term (strategic planning)

Agenda

- Capacity Planning: Overview
- **Decision Tree Method**
- Linear Programming Method

Decision Making under Uncertainty

- General steps:
  - Specify the objective for making a choice
  - Develop alternatives in each step
  - Analyze and compare alternatives
  - Select the best alternative
  - Implement and monitor the results

- How to evaluate alternatives under uncertainty: Decision Tree Method

Decision Tree Method

- Key requirements for using decision tree method
- You need to know the following before making decisions (sometime challenging)
  - possible scenarios (nature of state) and their probabilities
  - potential set of actions that can be taken in each step
  - payoff for each action under each scenario
- It requires uncertainty to be imposed in a certain way

Decision Tree Method: Example

A glass factory specializing in crystal is experiencing a substantial backlog, and the firm's management is considering three courses of action:

A) Subcontracting
B) Construct new facilities
C) Do nothing (no change)

The correct choice depends largely upon demand, which may be low, medium, or high. By consensus, management estimates the respective demand probabilities as 0.1, 0.5, and 0.4.

Glass Factory Example: Payoff Table

The management also estimates the profits when choosing from the three alternatives (A, B, and C) under the differing probable levels of demand. These profits, in thousands of dollars are presented in the table:

	0.1	**0.5**	**0.4**
	Low	Medium	High
Subcontracting	10	50	90
Construct new facility	-120	25	200
Do nothing	20	40	60

Glass Factory Example: EV

With the payoff table, we compute the expected value of each potential decision by summing up the profit under different scenario, multiplied by the probability.

	0.1 Low	0.5 Medium	0.4 High	EV
Subcontracting	10	50	90	62
New facility	-120	25	200	80.5
Do nothing	20	40	60	46

$$EV_B = 0.1\times(-120) + 0.5\times 25 + 0.4\times 200 = 80.5$$

Best Option by Expected Value

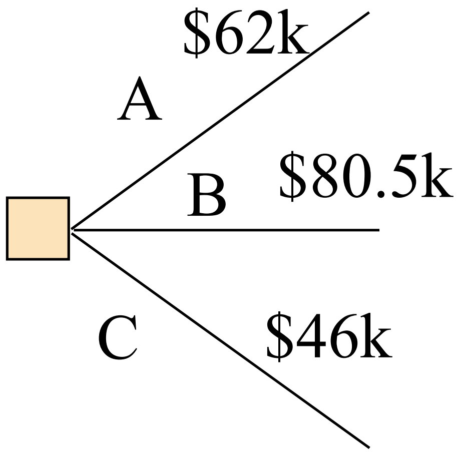
	0.1 Low	0.5 Medium	0.4 High	EV
Subcontracting	10	50	90	62
New facility	-120	25	200	80.5
Do nothing	20	40	60	46

$$\mathbf{EV_B = 0.1\times(-120) + 0.5\times25 + 0.4\times200 = 80.5}$$

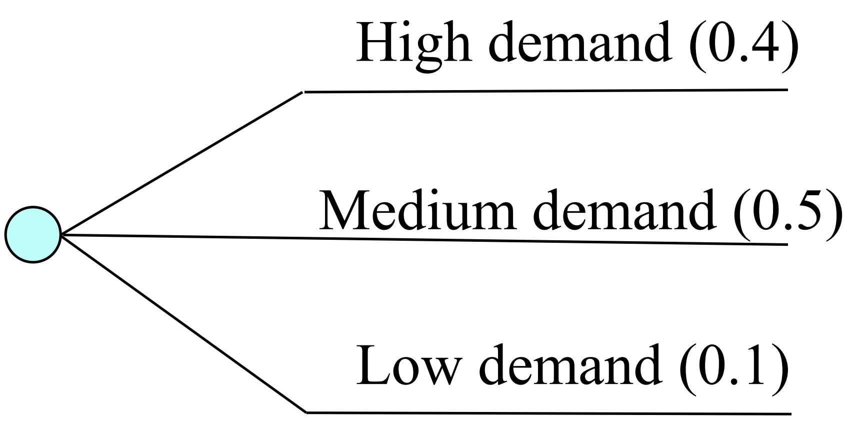
- We choose the "New facility" option as it delivers the highest expected value
- In this course, we always choose the options that leads to the highest expected value
- However, in reality, you also need to consider the risk related to the options. For example, the "New facility" option is quite risky.

Draw Decision Tree

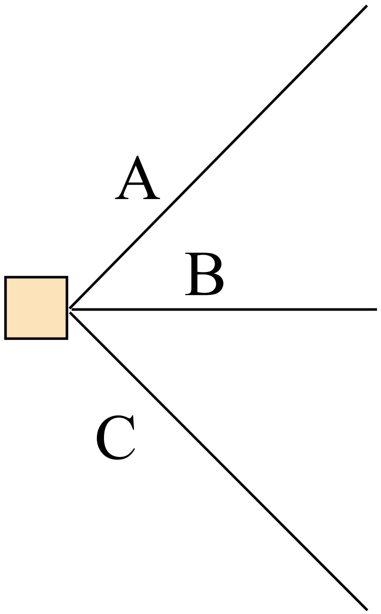
Decision point: followed by alternative decisions (and their EV computed backwardly)



Event point: followed by possible scenarios (and their probability)

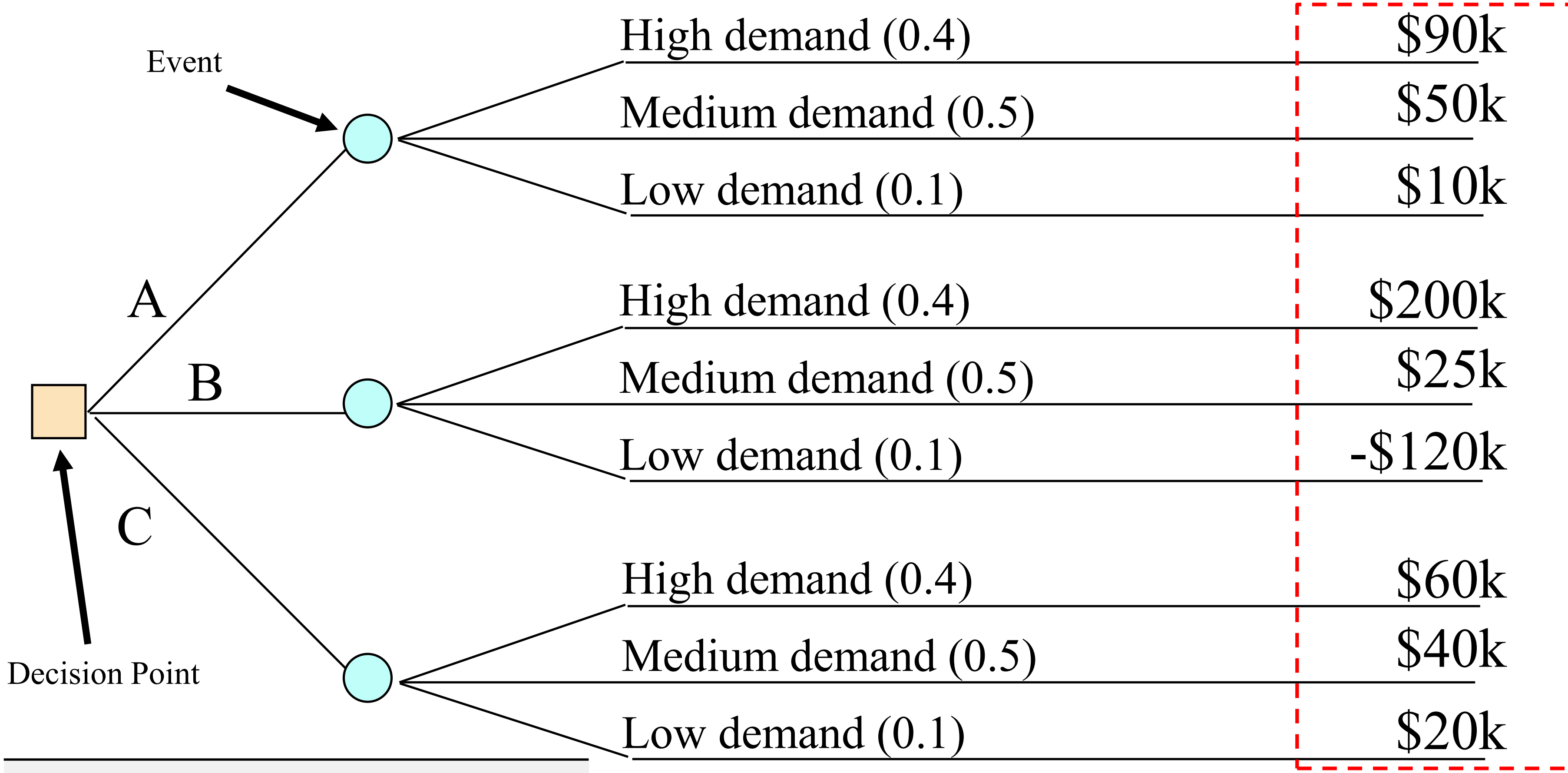


Step 1: 3 Decisions



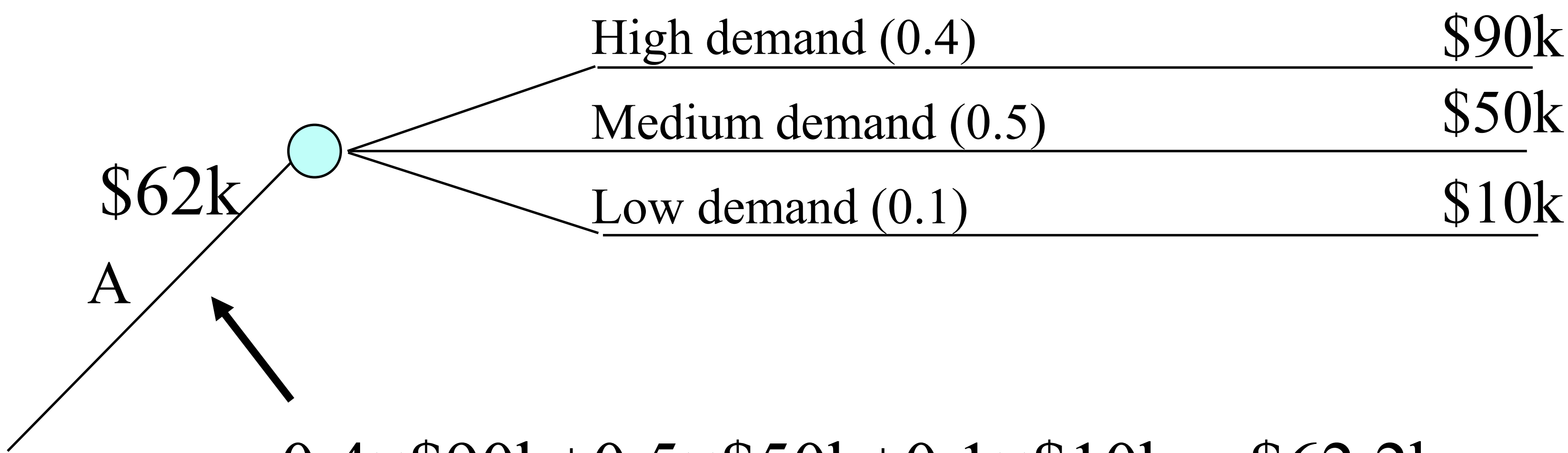
	0.1	0.5	0.4
	Low	Medium	High
A	10	50	90
B	-120	25	200
C	20	40	60

Step 2: States of Nature, Probabilities, and Payoffs



	0.1	**0.5**	**0.4**
	Low	Medium	High
A	10	50	90
B	-120	25	200
C	20	40	60

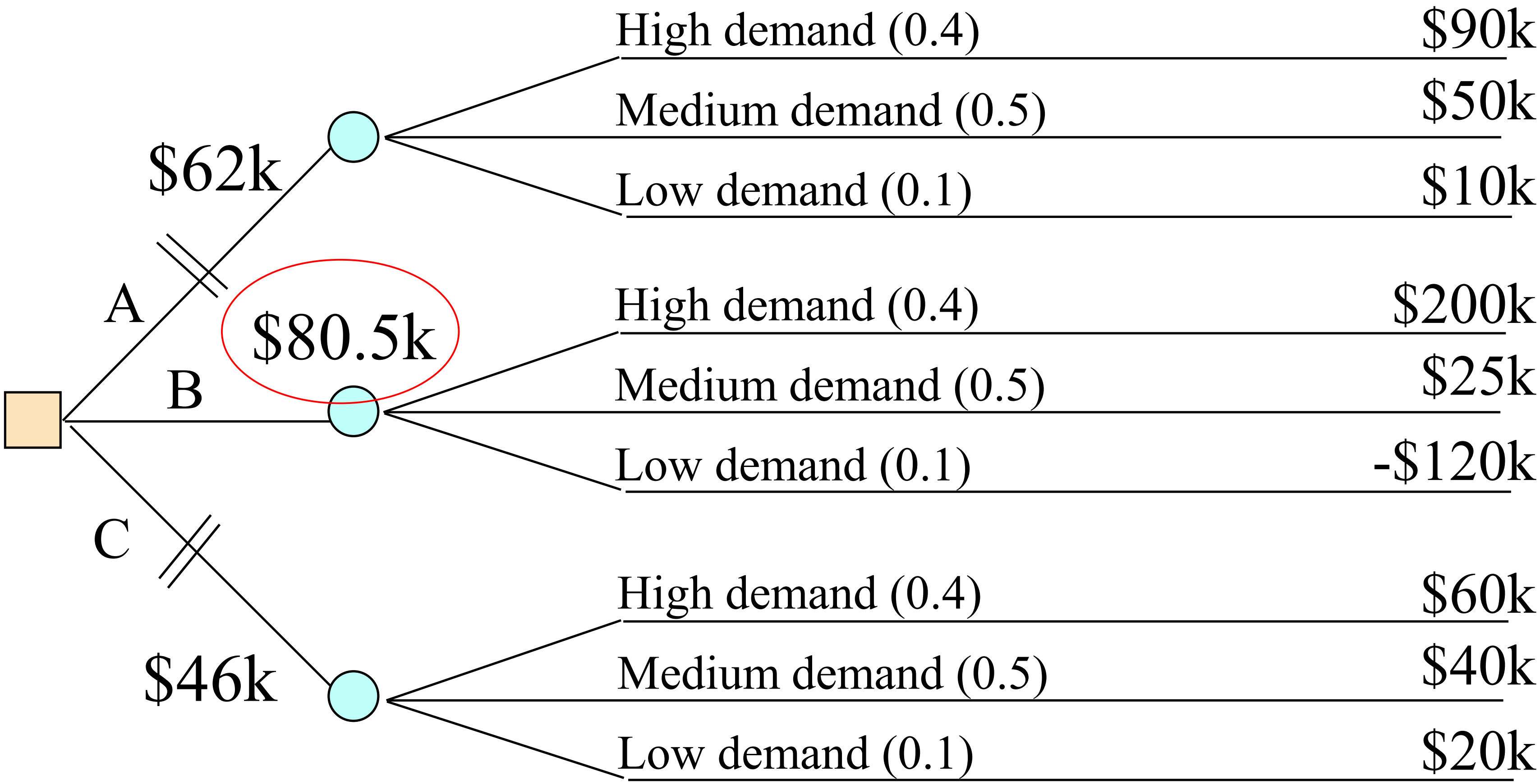
Step 3: Expected Value for each Decision



$$0.4 \times \$90k + 0.5 \times \$50k + 0.1 \times \$10k = \$62.2k$$

	0.1	**0.5**	**0.4**
	Low	Medium	High
A	10	50	90
B	-120	25	200
C	20	40	60

Step 4: Choose Decision



Value of Perfect Information

- When making the decision, you need to choose one action before the random demand scenario occurs
- This is undesirable as you cannot choose the best action for each demand scenario
- Question: How much are you willing to pay to pin down the demand scenario before making decisions?
- This reflects the (Expected) Value of Perfect Information

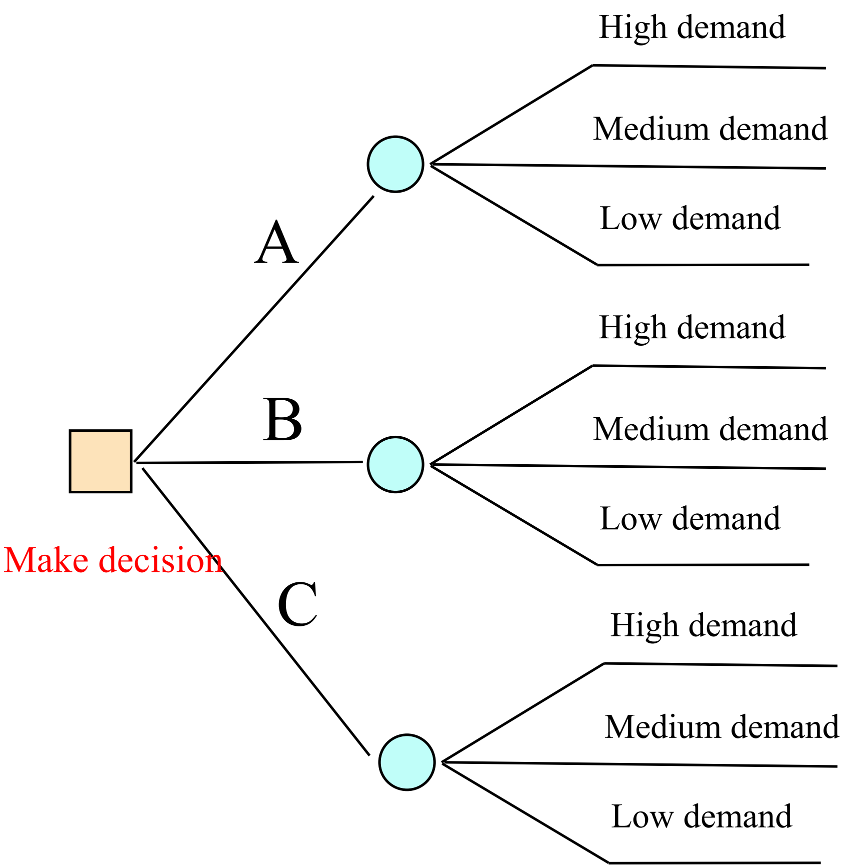
Value of Perfect Information

- Under Perfect information:
  - the scenarios are still random with given probabilities
  - but you can know it before making decisions
  - That is, you can choose different actions depending on the realized demand scenario

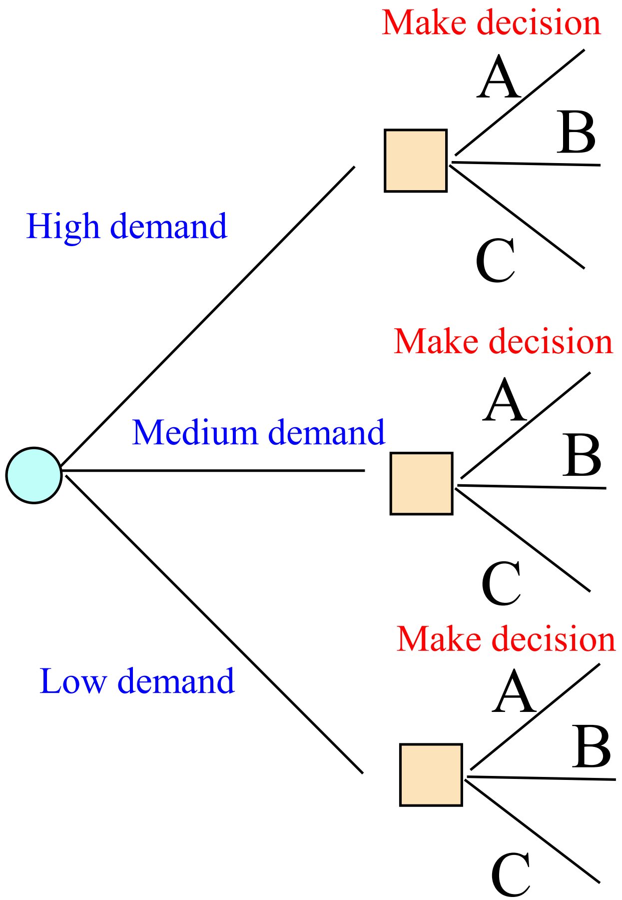
- (Expected) Value of Perfect Information:

$$\begin{pmatrix}\text{Value of}\\ \text{Perfect}\\ \text{Information}\end{pmatrix} = \begin{pmatrix}\text{Profit under}\\ \text{Perfect}\\ \text{Information}\end{pmatrix} - \begin{pmatrix}\text{Profit without}\\ \text{Perfect}\\ \text{Information}\end{pmatrix}$$

Perfect Information: Comparison



Original: make decision before knowing the demand scenario



Perfect information: make decision after knowing the demand scenario

Value of Perfect Information

- If you know the demand scenario before making decision, you can always choose the best action

	0.1 Low	0.5 Medium	0.4 High
Subcontracting	10	50	90
New facility	-120	25	200
Do nothing	20	40	60

- Low demand (0.1): do nothing, payoff = \$20k
- Medium demand (0.5): subcontracting, payoff = \$50k
- High demand (0.4): new facility, payoff = \$200k

Value of Perfect Information

- If you know the demand scenario before making decision, you can always choose the best action
- Low demand (0.1): do nothing, payoff = \$20k
- Medium demand (0.5): subcontracting, payoff = \$50k
- High demand (0.4): new facility, payoff = \$200k
- Expected value under perfect information = $0.1\times20+0.5\times50+0.4\times200$ = \$107k
- Increase from PI = \$107k – \$80.5k = \$26.5k
- This is the expected value of perfect information (EVPI)
  - The amount you are willing to pay to know the demand before making decision

Value of Perfect Information

- Note that even with perfect information (PI), your payoff is still random as different demand scenarios may occur
- The advantage is that you can choose your action after seeing demand scenario -- so you choose the best action accordingly
- The increase in the expected payoff is the EVPI
- E.g., you can invest in a better forecasting technology that can tell you the demand scenario in advance (e.g., big data tools)

Calculation of EVPI: Steps

1. Calculate the original expected value under the best alternative
   - Use decision tree to calculate backward
   - Only one alternative can be taken for each decision point

2. Calculate the new expected value with perfect information
   - For each scenario, always choose the best alternative
   - Different actions can be taken depending on the possible scenarios
   - There is still uncertainty in the scenario, but now we know what the scenario is before making decisions

3. EVPI as the difference of the two

Example: Hackers' Computer Store

Since the sales growth over the past couples of years has been good, the management is considering several options:

A. Move to new location

B. Expand store

C. Do nothing now, consider expansion one year later if demand is strong

Besides your decision, the cost and revenue also depends upon the future demand growth, which may be strong or weak. You estimate that

- Probability of strong demand = 0.55
- Probability of weak demand = 0.45

Example: Hackers' Computer Store

Objective: maximize total profit in next five years

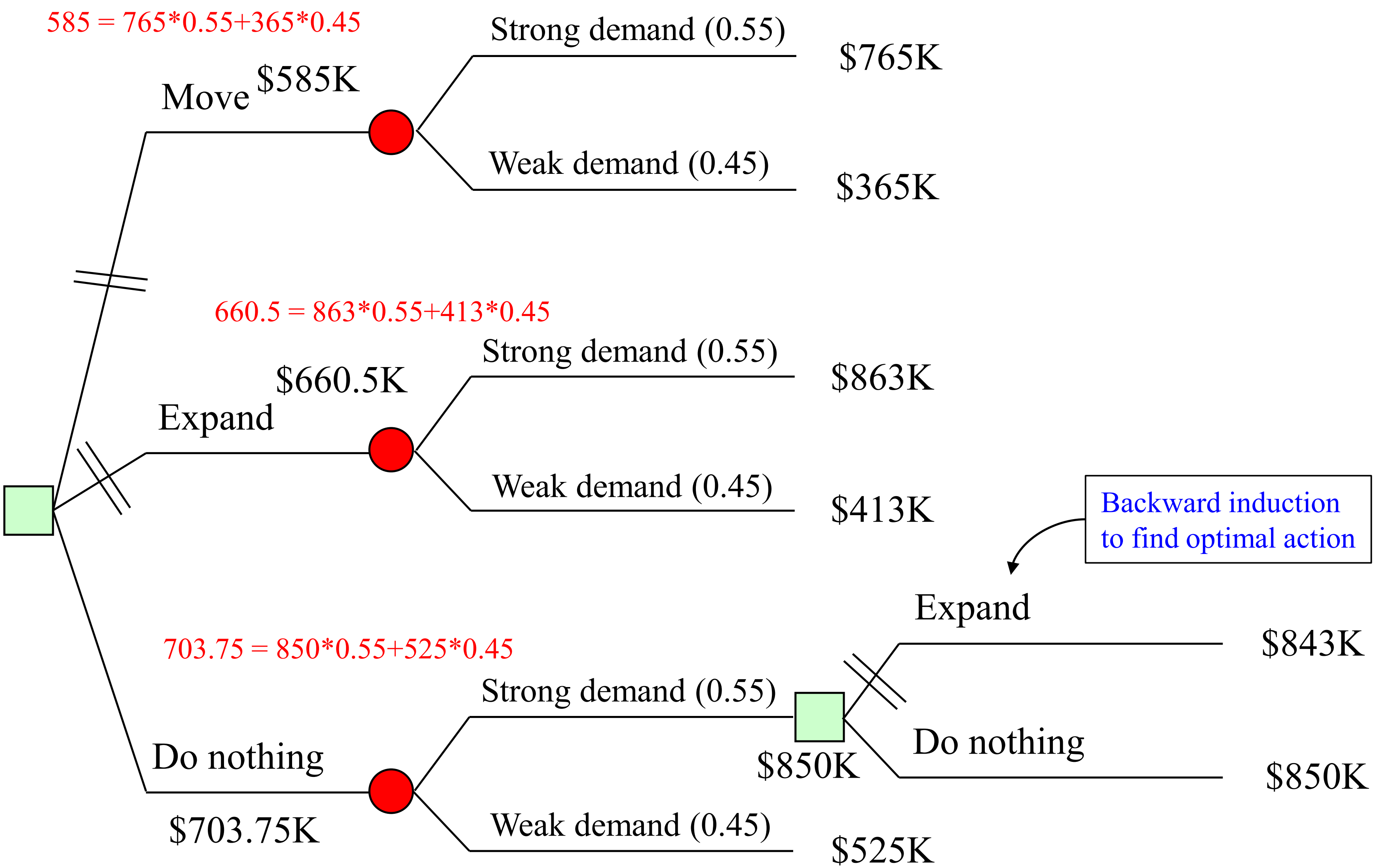
- Move to new location costs \$210k, generates \$195k per-year if demand is strong, \$115k if demand is weak
- Expand store costs \$87k, generates \$190k per-year if demand is strong, \$100k if demand is weak
- Do nothing has no cost, generates \$170k per-year if demand is strong, \$105k if demand is weak

Payoff table for next five years

Alternatives, Scenario	Revenue	Cost	Profit
Move to new location, strong	195,000 × 5 yrs	210,000	765,000
Move to new location, weak	115,000 × 5	210,000	365,000
Expand store, strong	190,000 × 5	87,000	863,000
Expand store, weak	100,000 × 5	87,000	413,000
Do nothing now, expand next year if strong	170,000 × 1+ 190,000 × 4	87,000	843,000
Do nothing now, do not expand next year if strong	170,000 × 5	0	850,000
Do nothing now, weak	105,000 × 5	0	525,000

We need to extend decision tree to multi-period setting

Decision Tree



Backward Induction in Decision Tree

- When we have multiple periods in decision tree, we use the backward induction to find optimal policy
- We start from the last period to find the optimal solution and expected value for it
- Then we go backwardly to previous periods using the expected value you have obtained for future periods

Perfect Information

- What happens if you can find out whether demand is weak or strong before you make the decision?

- If you know that the demand will definitely be strong
  - The best decision is: **Expand**
  - Your profit is: **$863000**

- If you know that the demand will definitely be weak
  - Your best decision is: **Do nothing**
  - Your profit is: **$525000**

Value of Perfect Information

- Expected profit under perfect information:

  $863,000×0.55 + $525,000×0.45 = $710900

- Increase from current profit:

  $710,900 – $703,750 = $7150

- This is the value of perfect information
  - The amount you are willing to pay to know the demand

Decision Tree: Summary

- Possible future scenarios or states of nature
- Probability of each future scenario
- Decision alternatives
- Payoff for each alternative under every future condition
- Decision criterion

- Information about future state is valuable
- Desirable to resolve uncertainty at early stages
- Approaches: feasibility study, market research, demand forecasting, data analytics

ISOM 2700: Operations Management

Session 11. Resource Allocation Decisions II
Linear Programming Method

Yiwen Shen
Dept. of ISOM, HKUST
Spring 2025

Agenda

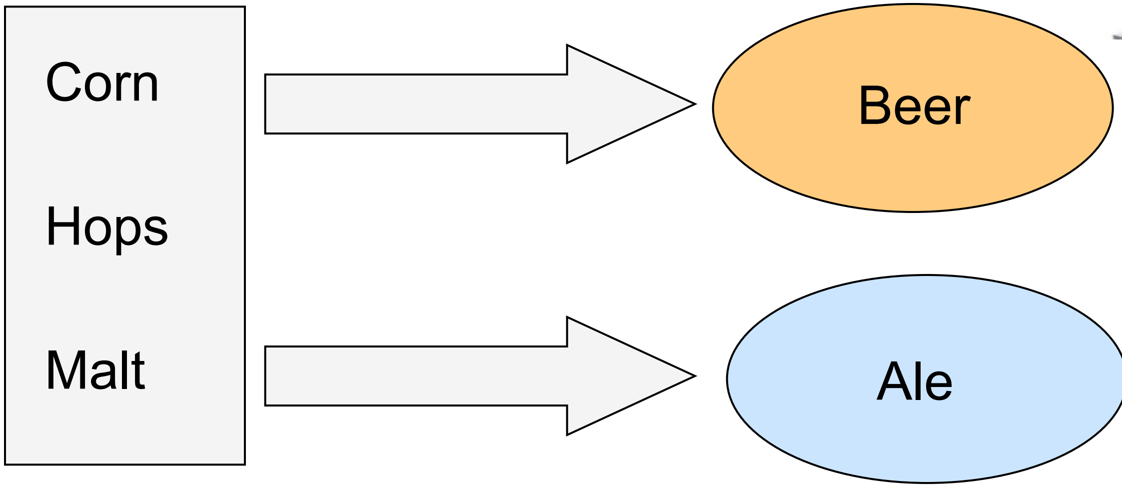
- **Formulate linear programming problems**

- Solve linear programming problems
  - Graphical Method (no more than 2 decision variables)
  - Excel Solver

- Sensitivity analysis

Resource Allocation Decision

Example: Bland Brewery Problem



Bland Brewery Problem (continued)

- Profitability

1 Barrel of Beer	1 Barrel of Ale
$23	$13

- Resource Requirement

	1 Barrel of Beer	1 Barrel of Ale
Corn	15 lbs	5 lbs
Hops	4 ozs	4 ozs
Malt	20 lbs	35 lbs

- Resource Availability

Corn	480 lbs
Hops	160 ozs
Malt	1,190 lbs

How much of beer and ale to produce, if we want to maximize profit?

Plan 1: Beer-only Production

- Profitability

1 Barrel of Beer
$23

- Resource Requirement

	1 Barrel of Beer
Corn	15 lbs
Hops	4 ozs
Malt	20 lbs

- Resource Availability

Corn	480 lbs
Hops	160 ozs
Malt	1,190 lbs

- We run out of **corn** after producing 480/15=**32** barrels of beer
- We run out of **hops** after producing 160/4=**40** barrels of beer
- We run out of **malt** after producing 1190/20=**59.5** barrels of beer

- We can produce at most 32 barrels of beer, making total profit of $\$23 \times 32 = \$736$

Plan 2: Ale-only Production

- Profitability

1 Barrel of Ale
$13

- Resource Requirement

	1 Barrel of Ale
Corn	5 lbs
Hops	4 ozs
Malt	35 lbs

- Resource Availability

Corn	480 lbs
Hops	160 ozs
Malt	1,190 lbs

- We run out of **corn** after producing 480/5=**96** barrels of ale
- We run out of **hops** after producing 160/4=**40** barrels of ale
- We run out of **malt** after producing 1190/35=**34** barrels of ale

- We can produce at most 34 barrels of ale, making total profit of $\$13 \times 34 = \$442$

How to best use our resources?

	Beer-only Production	Ale-only Production
Number of Barrels	32	34
Profit	$736	$442
Corn left **Malt left** **Hops left**	0 lbs 32 lbs 550 lbs	310 lbs 24 lbs 0 lbs

Can we have a chance of extracting more revenue if we "mix" beer and ale in the production plan?

Tradeoff: producing more beer vs. producing more ale

Bland Brewery Model: Standard Notation

- **Decision Variables**
  - A = # of barrels of ale to produce, and
  - B = # of barrels of beer to produce.

1 Barrel of Beer	1 Barrel of Ale
\$23	\$13

	1 Barrel of Beer	1 Barrel of Ale
Corn	15 lbs	5 lbs
Hops	4 ozs	4 ozs
Malt	20 lbs	35 lbs

- **Objective Function**
  Profit in \$ = $13A + 23B$
- **Availability Constraints**
  Corn: $5A + 15B \leq 480$
  Hops: $4A + 4B \leq 160$
  Malt: $35A + 20B \leq 1190$
- **Non-negativity Constraints**
  $A, B \geq 0$

Bland Brewery Linear Program

Objective Coefficients

Objective Function

max $13\,A + 23\,B$ (Profit)

subject to

Constraints

(corn) $5A + 15B \le 480$

(hops) $4A + 4B \le 160$

(malt) $35A + 20B \le 1190$

(non-negativity) $A, B \ge 0$

Right-hand-side Variables

Variables

Feasible/Infeasible Solution

- *Feasible and Infeasible Solutions*
  - A production plan (A,B) that satisfies all the constraints is called a *feasible solution*

@ For example, in the Bland Brewery LP, is solution (A=10, B=10) feasible?

@ **Check Constraints:**

Corn Availability: 5 x 10 + 15 x 10 = 200 $\leq$ 480

Hops Availability: 4 x 10 + 4 x 10 = 80 $\leq$ 160

Malt Availability: 35 x 10 + 20 x 10 = 550 $\leq$ 1190

Non-negativity: 10, 10 $\geq$ 0

@ The solution (A=10, B=10) is **feasible**.

Feasible/Infeasible Solution

- *Feasible and Infeasible Solutions*
  - Is production plan (A=40, B=10) feasible?

- **Check Constraints:**

  Corn Availability: $5 \times 40 + 15 \times 10 = 350 \leq 480$

  Hops Availability: $4 \times 40 + 4 \times 10 = 200 > 160$

  Malt Availability: $35 \times 40 + 20 \times 10 = 1600 > 1190$

  Non-negativity: $40, 10 \geq 0$

- The production plan (A=40, B=10) is not feasible, i.e. it is *infeasible* because the hops and malt constraints are *violated*

Optimal Solution

- For a maximization problem, an *optimal solution* is a feasible solution that has the largest objective function value among all feasible solutions

- For a minimization problem, we look for the smallest objective value

- The optimal solution for the Bland Brewery production model is (A=12, B=28). The optimal objective function value is $800.

Optimal Solution vs. Optimal Objective Value

- Recall we maximize the objective function = 13A + 23B
- Optimal solution: the decision variables that lead to the largest objective function (A,B) = (12,28)
- Optimal objective value: the value of the objective function at the optimal solution 13*12 + 23*28 = 800
- Make sure you differentiate between the two concepts!

Definition of LP Problem

- Both the objective function and the constraints are linear with respect to decision variables

- Linear functions: each variable appears in a separate term raised to the first power and is multiplied by a constant (which could be 0)
  - E.g., $f(x1,x2) = k1*x1+k2*x2$ for constants k1 and k2

- Linear constraints are linear functions that are restricted to be $\geq$, $\leq$, or $=$ to a constant.

Examples of Linear/Non-linear Functions

- **Linear** functions of *A* and *B*:
  - @13*A* + 23*B*
  - @0.5*A* + (2/3)*B*

- **Non-Linear** functions of *A* and *B*:
  - @$13A^2$ + 23AB
  - @1/A+B
  - @log(A) + cos(B)
  - @max(A,0)
  - @IF(A< 5,0,10)

Examples of Linear/Non-linear Constraints

- **Linear** Constraints of A, B, C
  - A-B ≤ 10;
  - A≥0;
  - A+B+3C=5;
  - A/B ≥ 2, A-2B ≥0 (given B ≥0);
  - A = B， A-B = 0;

- **Non-linear** Constraints of A, B, C
  - AB ≤ 2C;
  - log(AB) ≤ 100;
  - A^2+B ≤ 2C

Binding vs. Non-binding Constraints

- Binding constraints
  - equality reached in the constraint
  - limit the improvement in the objective function, e.g., use all resources available

- Non-binding constraints
  - equality not reached in the constraint
  - do not limit improvement, e.g., have “left over” resources

- Slack (Right-hand side minus Left-hand side): the amount of resources left over
  - positive (>0) for non-binding constraints
  - zero for binding constraints

Binding Constraints: Example

Example: (corn) $5A + 15B \leq 480$

(hops) $4A + 4B \leq 160$

(malt) $35A + 20B \leq 1190$

(non-negativity) $A, B \geq 0$

- For optimal solution A = 12, B = 28

- Corn: $5 \times 12 + 15 \times 28 = 480$, binding
- Hops: $4 \times 12 + 4 \times 28 = 160$, binding
- Malt: $35 \times 12 + 20 \times 28 = 980 < 1190$, non-binding

- Slack for Corn and Hops = 0
- Slack for Malt = 1190 – 980 = 210 > 0

Assumptions in Linear Program

- **Continuity:**
  - the decision variables are continuous, i.e., fractional values are allowed

- **Proportionality**
  - Each unit of output uses the same amount of resources
  - No economies/diseconomies of scale
  - Sometimes unrealistic

- **Additivity**
  - Each unit of output has the same market value
  - Profit is the *sum* of the profit contributions from each output
  - Sometimes unrealistic

Procedure of LP Formulation

- Step 1. Define the decision variables.
- Step 2. Write the objective in terms of the decision variables.
- Step 3. Write the constraints in terms of the decision variables (availability + non-negativity)

Make sure that objective function & constraints are linear

Applications of LP

Applications	Description
Production Planning	Determine the resource capacity needed to meet demand over a time period
Product Mix	Mix of different products to produce that will maximize profit given resource constraints
Transportation	Logistical flow of items from sources to destinations
Blend	Determine “recipe” requirements, i.e., how to blend different components to produce different products
Investment Strategies	Determine portfolio weights to invest in different assets, given return objectives and risk constraints

Example

A school board is investigating various ways of composing the faculty for a proposed new high school.

They can hire teachers and aides. The amount of money the school district will spend on salaries each year depends on how many teachers and how many aides are hired.

Objective

- The board finds that the annual teacher salary is \$15,000, and the average aide salary is \$10,000.

- Objective: minimizing the annual cost $=15t + 10a$

- Decision variables:
  - t = number of teachers hired
  - a = number of aides hired

School Board Requirements

- The building can accommodate no more than 50 faculty members. ⇨ $t + a \leq 50$
- A minimum of 20 faculty members is needed to staff the school. ⇨ $t + a \geq 20$
- The number of teachers must be at least half the number of aides. ⇨ $t/a \geq 1/2$, $\Rightarrow t - 1/2a \geq 0$
- It is impossible to hire a negative number of aides or teachers! ⇨ $t \geq 0$, $a \geq 0$

Linear Program (LP) Formulation

- Decision variables
  - t = number of teachers hired
  - a = number of aides hired

- LP problem

$$
\begin{aligned}
\text{Min} \quad & 15t + 10a \\
\text{s.t.} \quad & t + a \leq 50 \\
& t + a \geq 20 \\
& t - 1/2a \geq 0 \\
& t \geq 0 \\
& a \geq 0
\end{aligned}
$$

Objective function

"Subject to"

Constraints

Agenda

- Formulate linear programming problems
- **Solve linear programming problems**
  - **Graphical Method (no more than 2 decision variables)**
  - Excel Solver
- Sensitivity analysis

Solving LP by Graphical Method

- Formulate the problem in standard LP format
- Plot the constraints on a piece of graph paper and define the feasible solution space
  - x and y axis: the decision variables
- Find the optimal solution:
  - Examine all corner points of feasible region (vertex)
  - Use the iso-profit (or iso-cost) line method

(Note: graphical method is useful in understanding the LP concept but cannot solve problems with more than 2 decision variables.)

Example: Product Mix Problem

A manager must decide on the mix of products A and B for the coming week.

For each unit of Product A, it requires processing times of 1 minute for molding and 2 minutes for painting. For each unit of Product B, it requires 1 minute for molding, 1 minute for painting, and 1 minute for cutting.

Based on staff and machine schedules, there will be 300 minutes available for molding, 400 minutes for painting, and 250 minutes for cutting for the coming week.

The manager also estimates that the profit margins will be $3 per unit of A and $2 per unit of B.

(a) Formulate the problem and solve it graphically.

(b) What combination of the products will maximize the total profit?

(c) What is the maximum total profit?

(d) Is there any idle time in molding, painting, or cutting departments?

Example: Product Mix Problem

A manager must decide on the mix of products A and B for the coming week. For each unit of Product A, it requires processing times of 1 minute for molding and 2 minutes for painting. For each unit of Product B, it requires 1 minute for molding, 1 minute for painting, and 1 minute for cutting. Based on staff and machine schedules, there will be 300 minutes available for molding, 400 minutes for painting, and 250 minutes for cutting for the coming week. The manager also estimates that the profit margins will be $3 per unit of A and $2 per unit of B.

Let A = no. of units of product A to make in the coming week
B = no. of units of product B to make in the coming week

$$\text{Max } Z = 3A + 2B$$

$$\begin{aligned} \text{s.t.} \quad A + B &\le 300 \quad &\text{(1) Molding} \\ 2A + B &\le 400 \quad &\text{(2) Painting} \\ B &\le 250 \quad &\text{(3) Cutting} \\ A, B &\ge 0 \end{aligned}$$

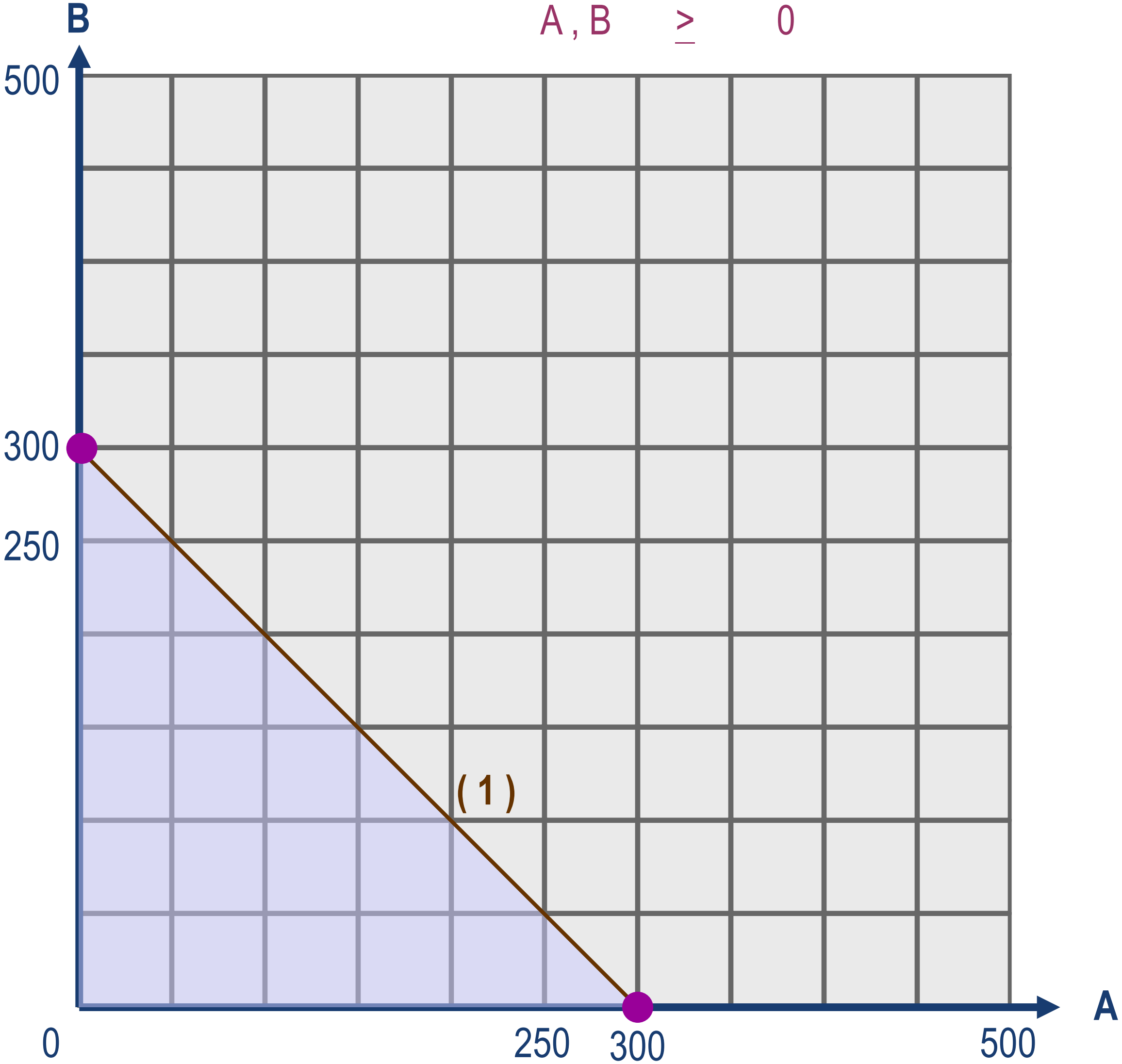
$$\text{Max } Z = 3A + 2B$$

$$\text{s.t.} \quad A + B \leq 300 \quad (1)$$

$$2A + B \leq 400 \quad (2)$$

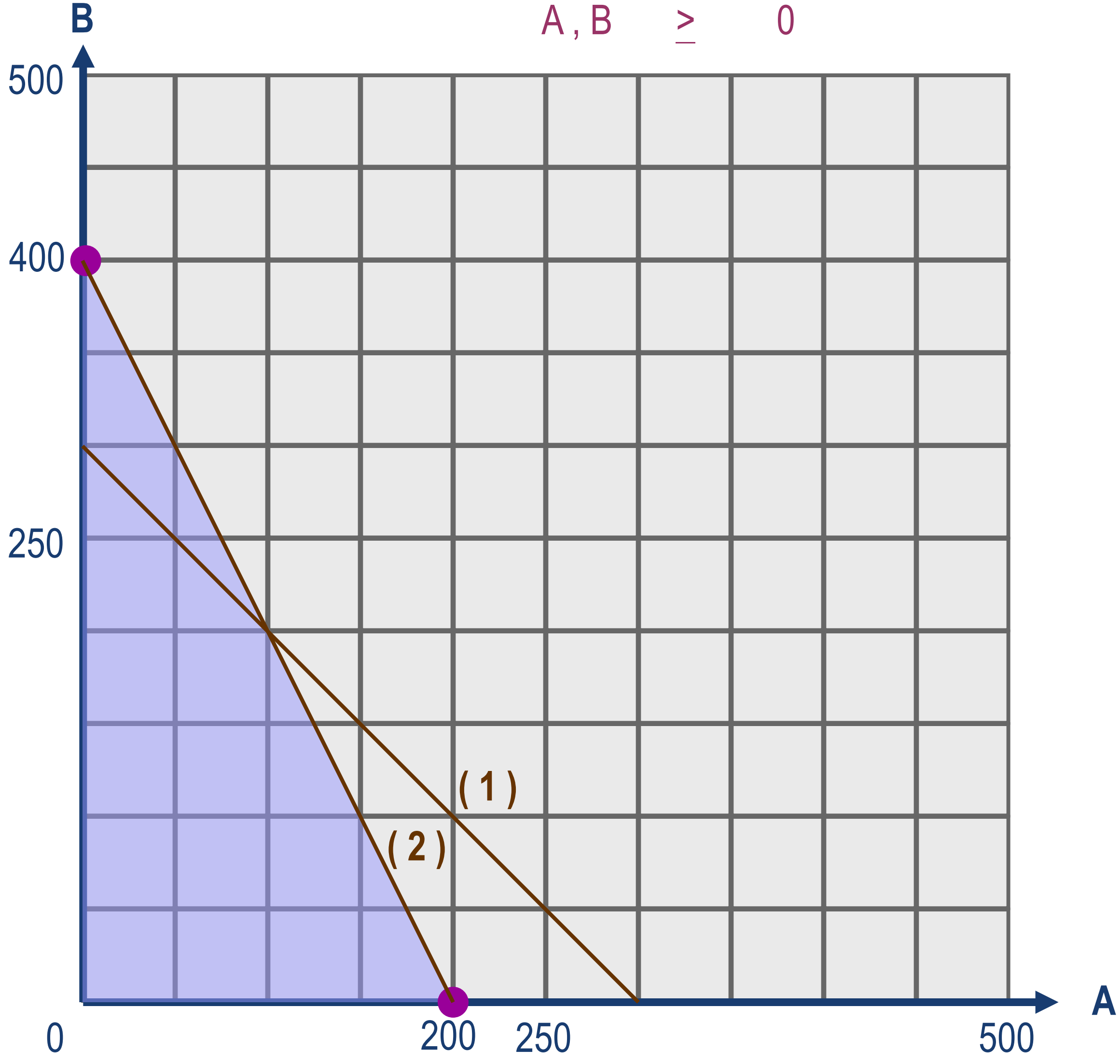
$$B \leq 250 \quad (3)$$

$$A, B \geq 0$$



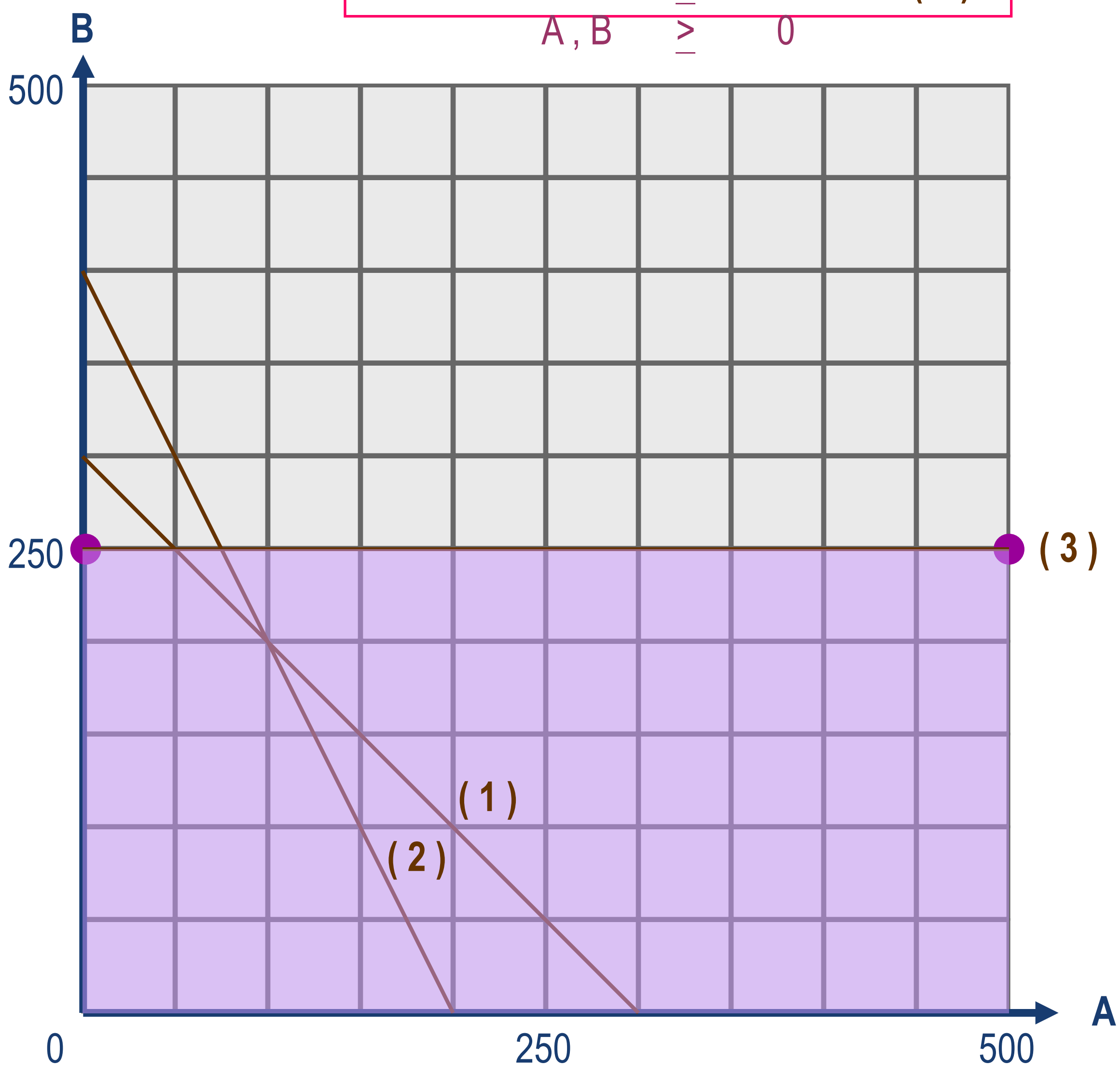
$$\text{Max } Z = 3A + 2B$$

$$\begin{aligned} \text{s.t.} \quad A + B &\le 300 \quad (1) \\ 2A + B &\le 400 \quad (2) \\ B &\le 250 \quad (3) \\ A, B &\ge 0 \end{aligned}$$

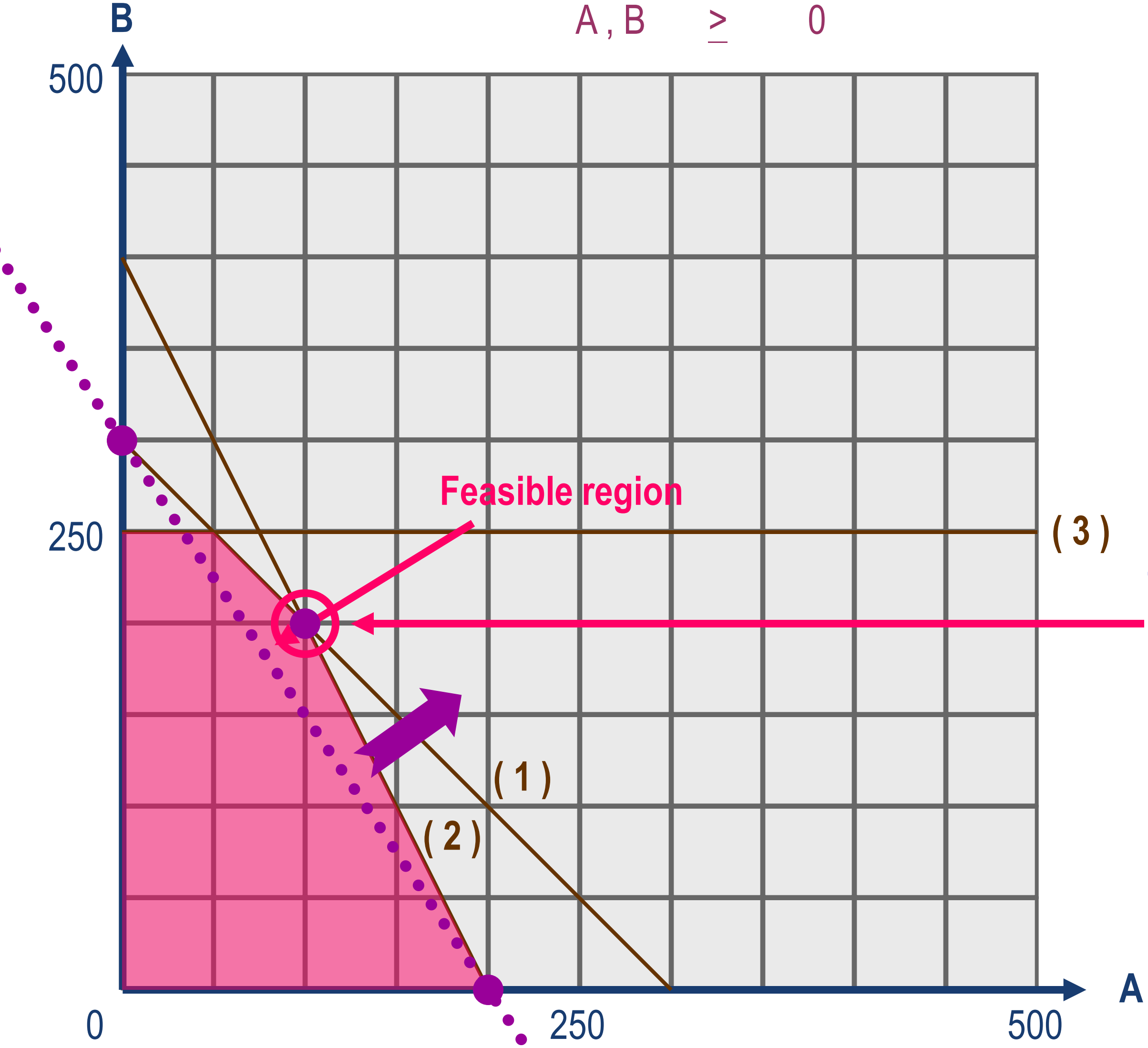


$$\text{Max } Z = 3A + 2B$$

$$\begin{aligned} \text{s.t.} \quad A + B &\leq 300 \quad (1) \\ 2A + B &\leq 400 \quad (2) \\ B &\leq 250 \quad (3) \\ A, B &\geq 0 \end{aligned}$$



$$
\begin{array}{llcllll}
\text{Max} & Z & = & 3A + 2B & & & \\
\text{s.t.} & A & + & B & \le & 300 & (1) \\
 & 2A & + & B & \le & 400 & (2) \\
 & & & B & \le & 250 & (3) \\
 & & & A, B & \ge & 0 &
\end{array}
$$



Iso-profit Line Method

The objective function line can be represented by a straight-line with 3A + 2B; its slope is -3/2

Iso-profit line: (1) points on the line leads to same profit; (2) lines on the upper right have larger profit

To maximize Z, we move the line to the right until it reaches the last corner point in the feasible region. (we can start anywhere for this line)

Optimal solution

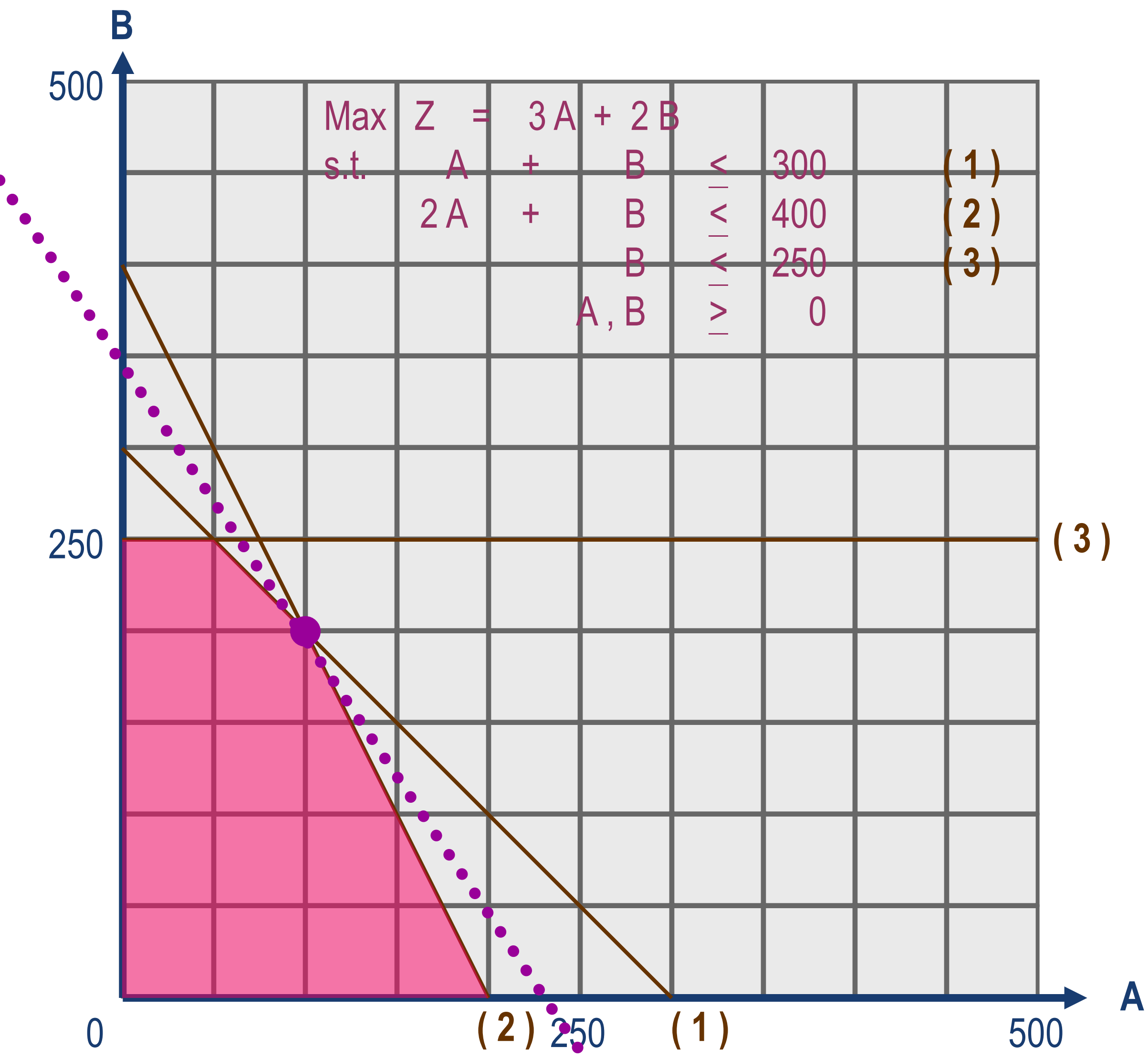
A = 100

B = 200

Max Z = 700

(b) What combination of the products will maximize the total profit?

100 A and 200 B



Optimal solution

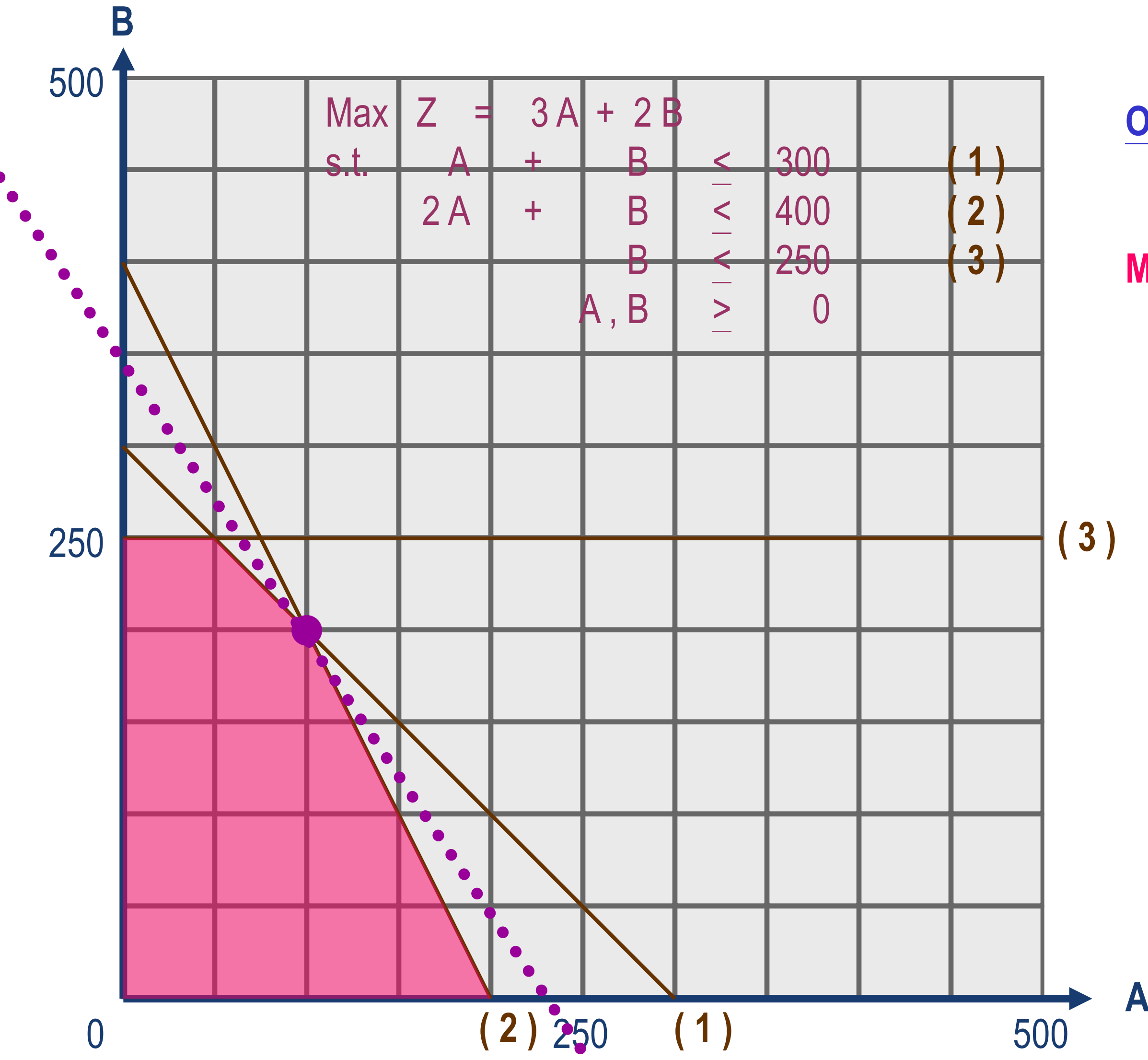
A = 100

B = 200

Max Z = 700

(c) What is the maximum total profit?

Total profit = 3(100) + 2(200) = $ 700



Optimal solution

A = 100

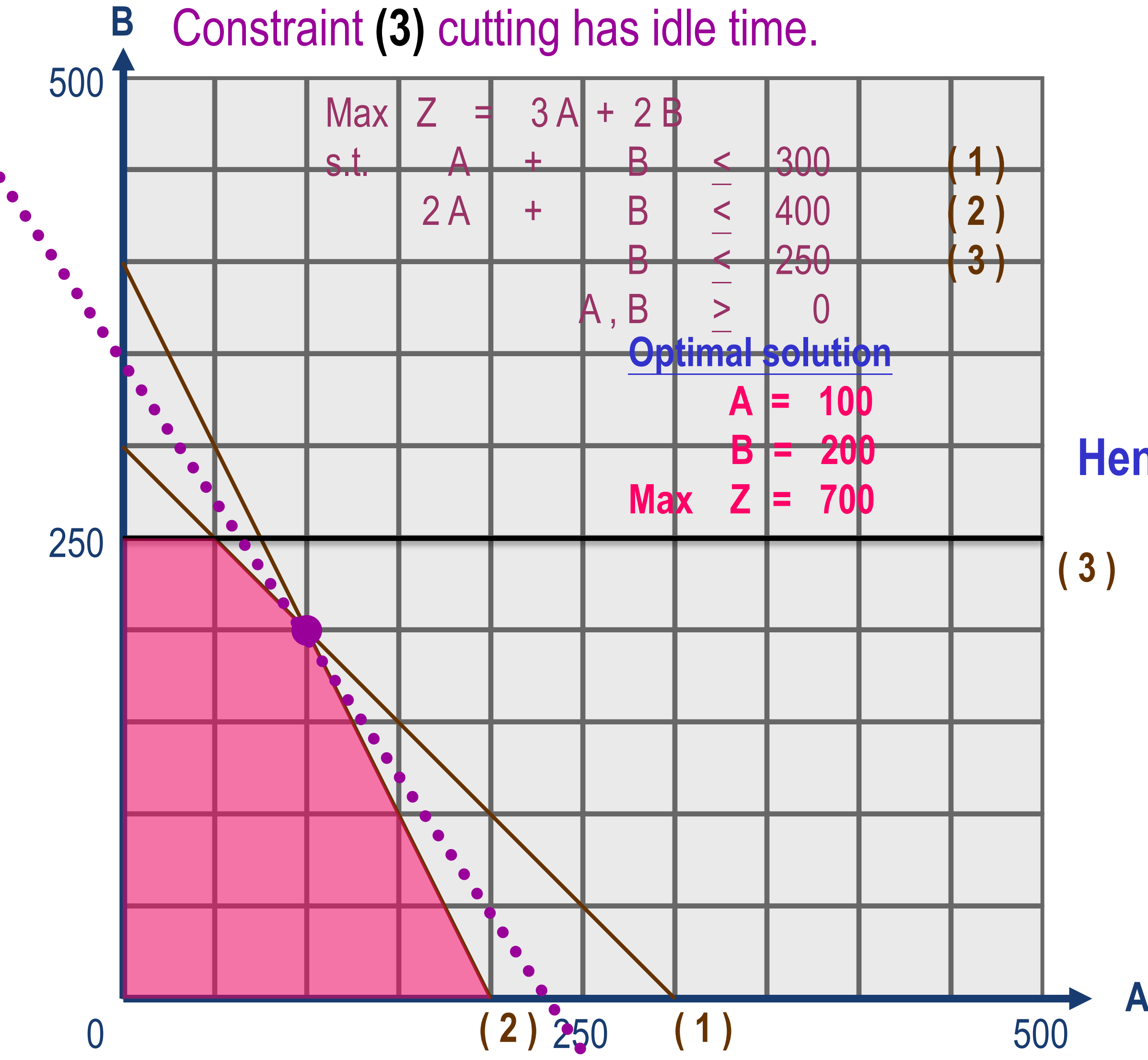
B = 200

Max Z = 700

(d) Is there any idle time in molding, painting, or cutting departments? If so, which department has idle time and how much?

Optimal solution is on the constraints **(1)** molding and **(2)** painting ⇒ No idle time

Constraint **(3)** cutting has idle time.



Idle time for (3):

$S_3 = 250 - B$

$= 250 - 200 = 50$

Hence, cutting has idle time = 50 min

If the solution falls on the boundary of a constraint, then the constraint is binding

ISOM 2700: Operations Management

Session 11. Resource Allocation Decisions II
Linear Programming Method

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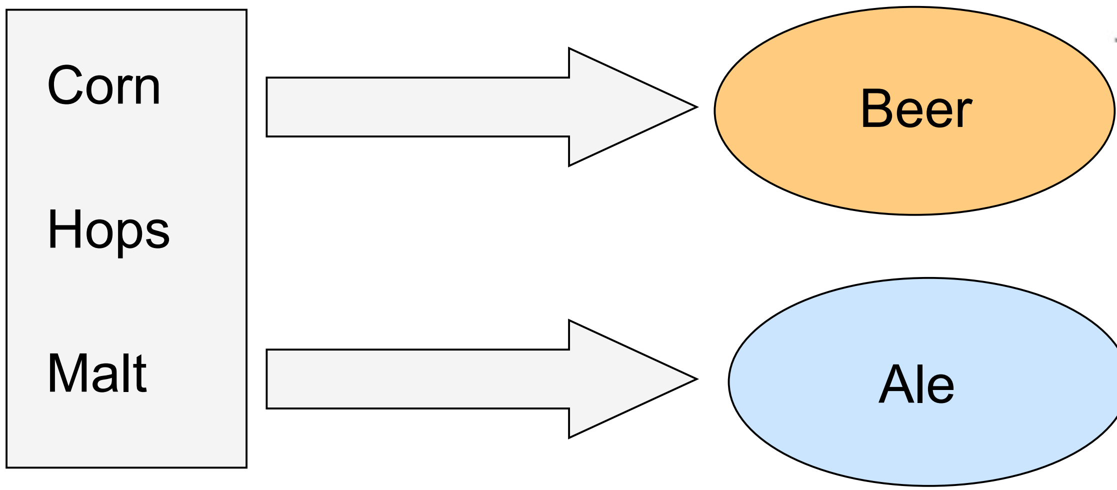
Fall 2025

Agenda

- **Formulate linear programming problems**
- Solve linear programming problems
  - Graphical Method (no more than 2 decision variables)
  - Excel Solver
- Sensitivity analysis

Resource Allocation Decision

Example: Bland Brewery Problem



Bland Brewery Problem (continued)

- Profitability

1 Barrel of Beer	1 Barrel of Ale
$23	$13

- Resource Requirement

	1 Barrel of Beer	1 Barrel of Ale
Corn	15 lbs	5 lbs
Hops	4 ozs	4 ozs
Malt	20 lbs	35 lbs

- Resource Availability

Corn	480 lbs
Hops	160 ozs
Malt	1,190 lbs

How much of beer and ale to produce, if we want to maximize profit?

Plan 1: Beer-only Production

- Profitability

1 Barrel of Beer
$23

- Resource Requirement

	1 Barrel of Beer
Corn	15 lbs
Hops	4 ozs
Malt	20 lbs

- Resource Availability

Corn	480 lbs
Hops	160 ozs
Malt	1,190 lbs

- We run out of **corn** after producing 480/15=**32** barrels of beer
- We run out of **hops** after producing 160/4=**40** barrels of beer
- We run out of **malt** after producing 1190/20=**59.5** barrels of beer

- We can produce at most 32 barrels of beer, making total profit of $\$23 \times 32 = \$736$

Plan 2: Ale-only Production

- Profitability

1 Barrel of Ale
$13

- Resource Requirement

	1 Barrel of Ale
Corn	5 lbs
Hops	4 ozs
Malt	35 lbs

- Resource Availability

Corn	480 lbs
Hops	160 ozs
Malt	1,190 lbs

- We run out of **corn** after producing 480/5=**96** barrels of ale
- We run out of **hops** after producing 160/4=**40** barrels of ale
- We run out of **malt** after producing 1190/35=**34** barrels of ale

- We can produce at most 34 barrels of ale, making total profit of $\$13 \times 34 = \$442$

How to best use our resources?

	Beer-only Production	Ale-only Production
Number of Barrels	32	34
Profit	$736	$442
Corn left **Malt left** **Hops left**	0 lbs 32 lbs 550 lbs	310 lbs 24 lbs 0 lbs

Can we have a chance of extracting more revenue if we "mix" beer and ale in the production plan?

Tradeoff: producing more beer vs. producing more ale

Bland Brewery Model: Standard Notation

- **Decision Variables**
  - A = # of barrels of ale to produce, and
  - B = # of barrels of beer to produce.

1 Barrel of Beer	1 Barrel of Ale
$23	$13

	1 Barrel of Beer	1 Barrel of Ale
Corn	15 lbs	5 lbs
Hops	4 ozs	4 ozs
Malt	20 lbs	35 lbs

- **Objective Function**

  Profit in $ = $13A + 23B$

- **Availability Constraints**

  Corn: $5A + 15B \leq 480$

  Hops: $4A + 4B \leq 160$

  Malt: $35A + 20B \leq 1190$

- **Non-negativity Constraints**

  $A, B \geq 0$

Bland Brewery Linear Program

Objective Function

Objective Coefficients

max 13 A + 23 B (Profit)

subject to

Constraints

(corn) $5A + 15B \leq 480$

(hops) $4A + 4B \leq 160$

(malt) $35A + 20B \leq 1190$

(non-negativity) $A, B \geq 0$

Right-hand-side Variables

Variables

Feasible/Infeasible Solution

- *Feasible and Infeasible Solutions*
  - A production plan (A,B) that satisfies all the constraints is called a *feasible solution (can be executed)*

@ For example, in the Bland Brewery LP, is solution (A=10, B=10) feasible?

@ **Check Constraints:**

Corn Availability: 5 x 10 + 15 x 10 = 200 $\leq$ 480

Hops Availability: 4 x 10 + 4 x 10 = 80 $\leq$ 160

Malt Availability: 35 x 10 + 20 x 10 = 550 $\leq$ 1190

Non-negativity: 10, 10 $\geq$ 0

@ The solution (A=10, B=10) is **feasible**.

Feasible/Infeasible Solution

- *Feasible and Infeasible Solutions*
  - Is production plan (A=40, B=10) feasible?

- **Check Constraints:**
  Corn Availability: $5 \times 40 + 15 \times 10 = 350 \leq 480$
  Hops Availability: $4 \times 40 + 4 \times 10 = 200 > 160$
  Malt Availability: $35 \times 40 + 20 \times 10 = 1600 > 1190$
  Non-negativity: $40, 10 \geq 0$

- The production plan (A=40, B=10) is not feasible, i.e. it is *infeasible* because the hops and malt constraints are *violated*

Optimal Solution

- For a maximization problem, an *optimal solution* is a feasible solution that has the largest objective function value among all feasible solution (best solution)

- For a minimization problem, we look for the smallest objective value

- The optimal solution for the Bland Brewery production model is (*A*=12, *B*=28). The optimal objective function value is $800.

Optimal Solution vs. Optimal Objective Value

- Recall we maximize the objective function = 13A + 23B
- Optimal solution: the decision variables that lead to the largest objective function (A,B) = (12,28)
- Optimal objective value: the value of the objective function at the optimal solution 13*12 + 23*28 = 800
- Make sure you differentiate between the two concepts!

Definition of LP Problem

- Both the objective function and the constraints are linear with respect to decision variables
- Linear functions: each variable appears in a separate term raised to the first power and is multiplied by a constant (which could be 0)
  - E.g., f(x1,x2) = k1*x1+k2*x2 for constants k1 and k2
- Linear constraints are linear functions that are restricted to be ≥, ≤, or = to a constant
  - Note that equality must be included in the constraint

Examples of Linear/Non-linear Functions

- **Linear** functions of *A* and *B*:
  - @13*A* + 23*B*
  - @0.5*A* + (2/3)*B*

- **Non-Linear** functions of *A* and *B*:
  - @13A^2 + 23AB
  - @1/A+B
  - @log(A) + cos(B)
  - @max(A,0)
  - @IF(A< 5,0,10)

Examples of Linear/Non-linear Constraints

- **Linear** Constraints of A, B, C
  - A-B ≤ 10;
  - A≥0;
  - A+B+3C=5;
  - A/B ≥ 2, A-2B ≥0 (given B ≥0);
  - A = B， A-B = 0;

- **Non-linear** Constraints of A, B, C
  - AB ≤ 2C;
  - log(AB) ≤ 100;
  - A^2+B ≤ 2C

Procedure of LP Formulation

- Step 1. Define the decision variables.
- Step 2. Write the objective in terms of the decision variables.
- Step 3. Write the constraints in terms of the decision variables (availability + non-negativity)

Make sure that objective function & constraints are linear

Assumptions in Linear Program

- **Continuity:**
  - the decision variables are continuous, i.e., fractional values are allowed
- **Proportionality**
  - Each unit of output uses the same amount of resources
  - No economies/diseconomies of scale
- **Additivity**
  - Each unit of output has the same market value
  - Profit is the *sum* of the profit contributions from each output

- These assumptions sometimes are unrealistic, but they bring good analytical convenience

Applications of LP

Applications	Description
Production Planning	Determine the resource capacity needed to meet demand over a time period
Product Mix	Mix of different products to produce that will maximize profit given resource constraints
Transportation	Logistical flow of items from sources to destinations
Blend	Determine "recipe" requirements, i.e., how to blend different components to produce different products
Investment Strategies	Determine portfolio weights to invest in different assets, given return objectives and risk constraints

Example

A school board is investigating various ways of composing the faculty for a proposed new high school.

They can hire teachers and aides. The amount of money the school district will spend on salaries each year depends on how many teachers and how many aides are hired.

Objective

- The board finds that the annual teacher salary is \$15,000, and the average aide salary is \$10,000.
- Objective: minimizing the annual cost $= 15t + 10a$
- Decision variables:
  - t = number of teachers hired
  - a = number of aides hired

School Board Requirements

- The building can accommodate no more than 50 faculty members. ⇨ $t + a \leq 50$
- A minimum of 20 faculty members is needed to staff the school. ⇨ $t + a \geq 20$
- The number of teachers must be at least half the number of aides. ⇨ $t/a \geq 1/2$, $\Rightarrow t - 1/2a \geq 0$
- It is impossible to hire a negative number of aides or teachers! ⇨ $t \geq 0$, $a \geq 0$

Linear Program (LP) Formulation

- Decision variables
  - t = number of teachers hired
  - a = number of aides hired

- LP problem

$$
\begin{aligned}
\text{Min} \quad & 15t + 10a \\
\text{s.t.} \quad & t + a \leq 50 \\
& t + a \geq 20 \\
& t - 1/2a \geq 0 \\
& t \geq 0 \\
& a \geq 0
\end{aligned}
$$

Objective function

Constraints

"Subject to"

Binding vs. Non-binding Constraints

- In each resource constraint, we require the amount used to be smaller or equal to the amount available

- Binding constraints
  - equality reached in the constraint
  - limit the improvement in the objective function, e.g., use all resources available

- Non-binding constraints
  - equality not reached in the constraint
  - do not limit improvement, e.g., have "left over" resources

Binding vs. Non-binding Constraints

- Slack (Right-hand side minus Left-hand side): the amount of resources left over
  - positive (>0) for non-binding constraints
  - zero for binding constraints

- What does it mean economically?

- If a resource has a positive slack, then we cannot improve our objective by further increasing this resource
  - because we already have leftovers and choose not to use them

- It is only valuable to increase the resource with zero slack

Binding Constraints: Example

Example: (corn) $5A + 15B \leq 480$

(hops) $4A + 4B \leq 160$

(malt) $35A + 20B \leq 1190$

(non-negativity) $A, B \geq 0$

- For optimal solution A = 12, B = 28
- Corn: $5 \times 12 + 15 \times 28 = 480$, binding
- Hops: $4 \times 12 + 4 \times 28 = 160$, binding
- Malt: $35 \times 12 + 20 \times 28 = 980 < 1190$, non-binding
- Slack for Corn and Hops = 0
- Slack for Malt = 1190 – 980 = 210 > 0

Agenda

- Formulate linear programming problems
- **Solve linear programming problems**
  - **Graphical Method (no more than 2 decision variables)**
  - Excel Solver
- Sensitivity analysis

Solving LP by Graphical Method

- Formulate the problem in standard LP format
- Plot the constraints on a piece of graph paper and define the feasible solution space
  - x and y axis: the decision variables
- Find the optimal solution:
  - Examine all corner points of feasible region (vertex)
  - Use the iso-profit (or iso-cost) line method

(Note: graphical method is useful in understanding the LP concept but cannot solve problems with more than 2 decision variables.)

Example: Product Mix Problem

A manager must decide on the mix of products A and B for the coming week.

For each unit of Product A, it requires processing times of 1 minute for molding and 2 minutes for painting. For each unit of Product B, it requires 1 minute for molding, 1 minute for painting, and 1 minute for cutting.

Based on staff and machine schedules, there will be 300 minutes available for molding, 400 minutes for painting, and 250 minutes for cutting for the coming week.

The manager also estimates that the profit margins will be \$3 per unit of A and \$2 per unit of B.

(a) Formulate the problem and solve it graphically.

(b) What combination of the products will maximize the total profit?

(c) What is the maximum total profit?

(d) Is there any idle time in molding, painting, or cutting departments?

Example: Product Mix Problem

A manager must decide on the mix of products A and B for the coming week. For each unit of Product A, it requires processing times of 1 minute for molding and 2 minutes for painting. For each unit of Product B, it requires 1 minute for molding, 1 minute for painting, and 1 minute for cutting. Based on staff and machine schedules, there will be 300 minutes available for molding, 400 minutes for painting, and 250 minutes for cutting for the coming week. The manager also estimates that the profit margins will be \$3 per unit of A and \$2 per unit of B.

Let A = no. of units of product A to make in the coming week
B = no. of units of product B to make in the coming week

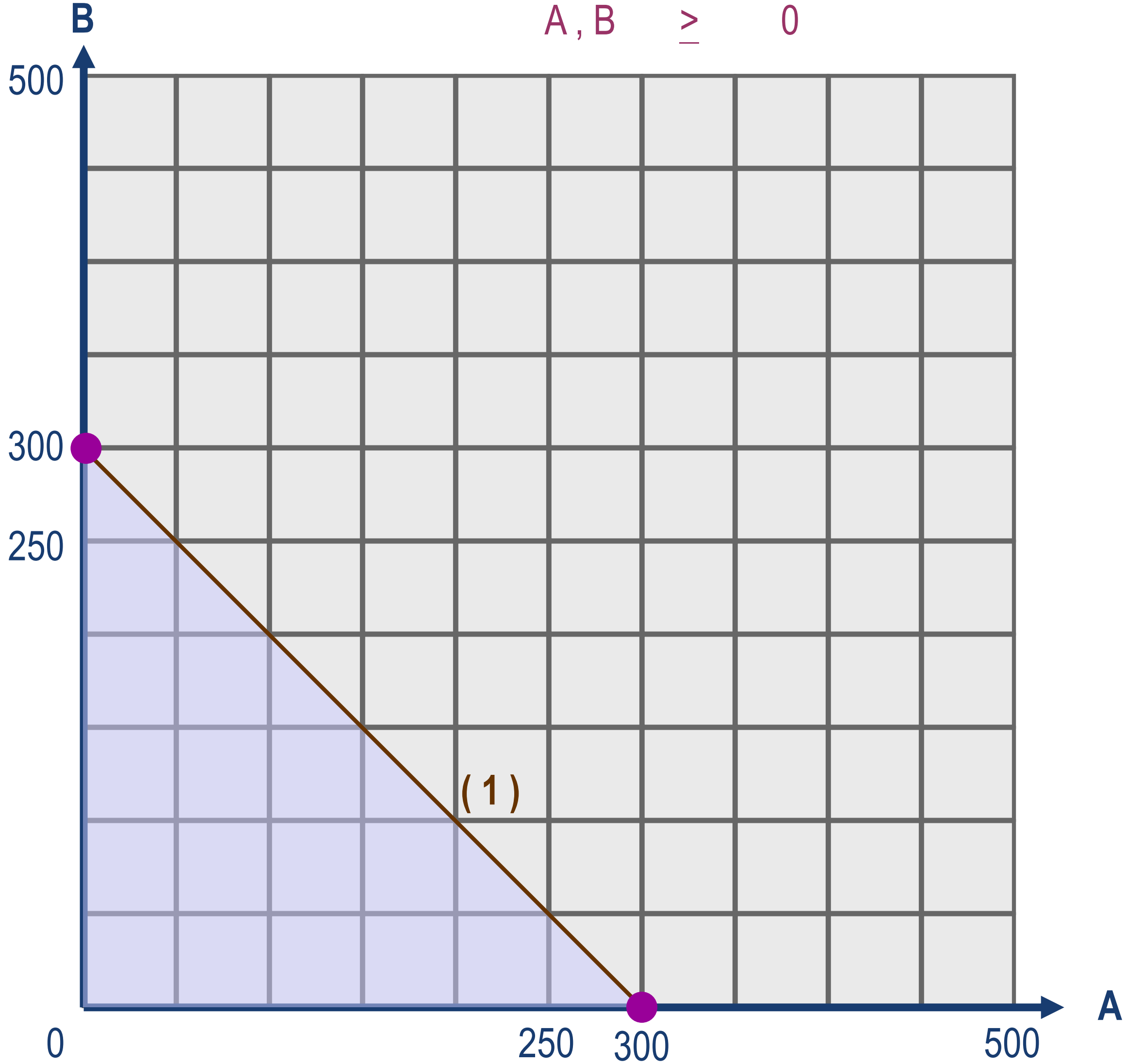
$$\text{Max } Z = 3A + 2B$$

$$\begin{aligned}
\text{s.t.} \quad A + B &\le 300 && \text{(1) Molding} \\
2A + B &\le 400 && \text{(2) Painting} \\
B &\le 250 && \text{(3) Cutting} \\
A, B &\ge 0
\end{aligned}$$

$$\text{Max } Z = 3A + 2B$$

s.t.

A	+	B	$\leq$	300	(1)
2 A	+	B	$\leq$	400	(2)
		B	$\leq$	250	(3)
		A , B	$\geq$	0	



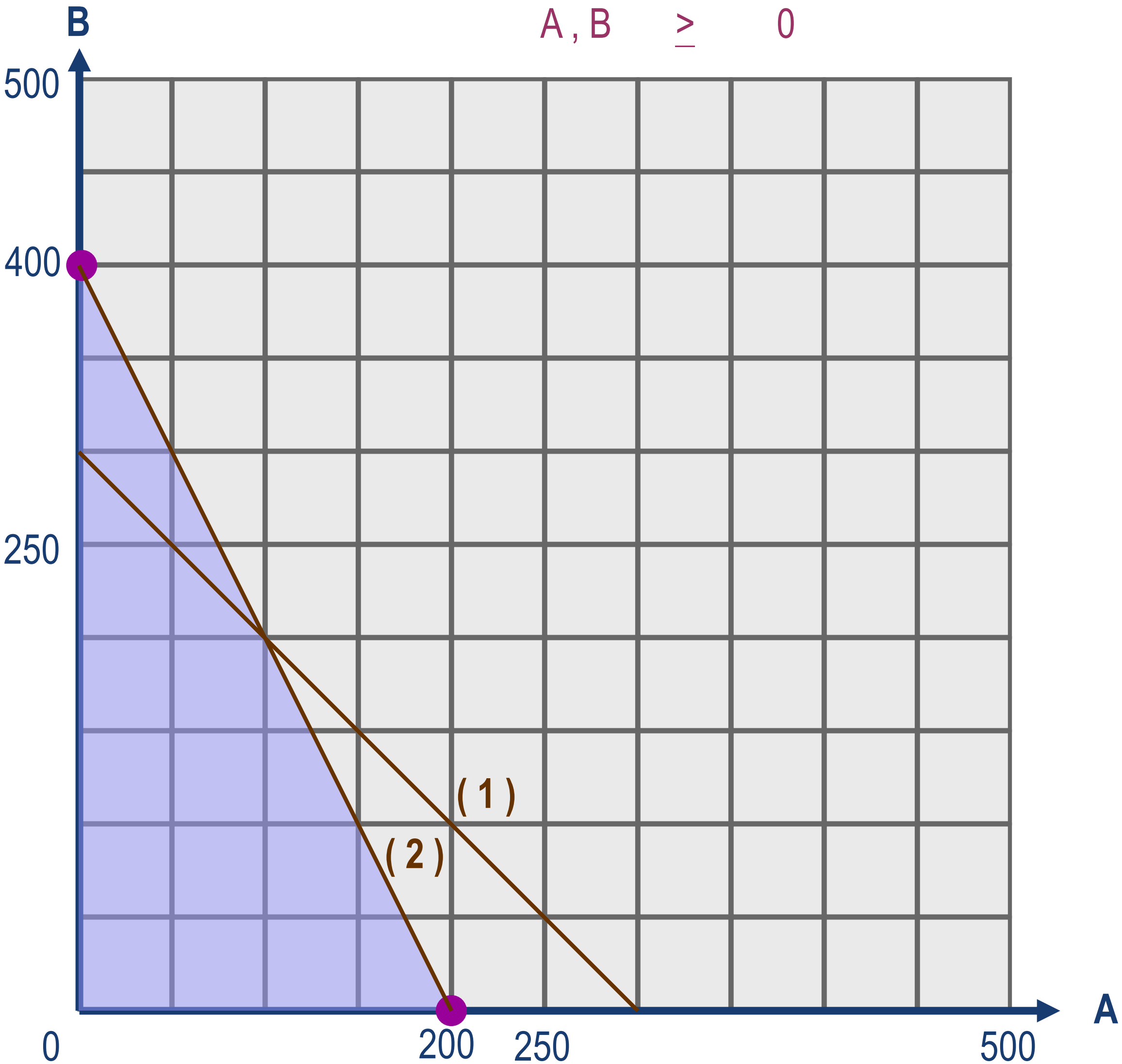
$$\text{Max } Z = 3A + 2B$$

$$\text{s.t.} \quad A + B \leq 300 \quad (1)$$

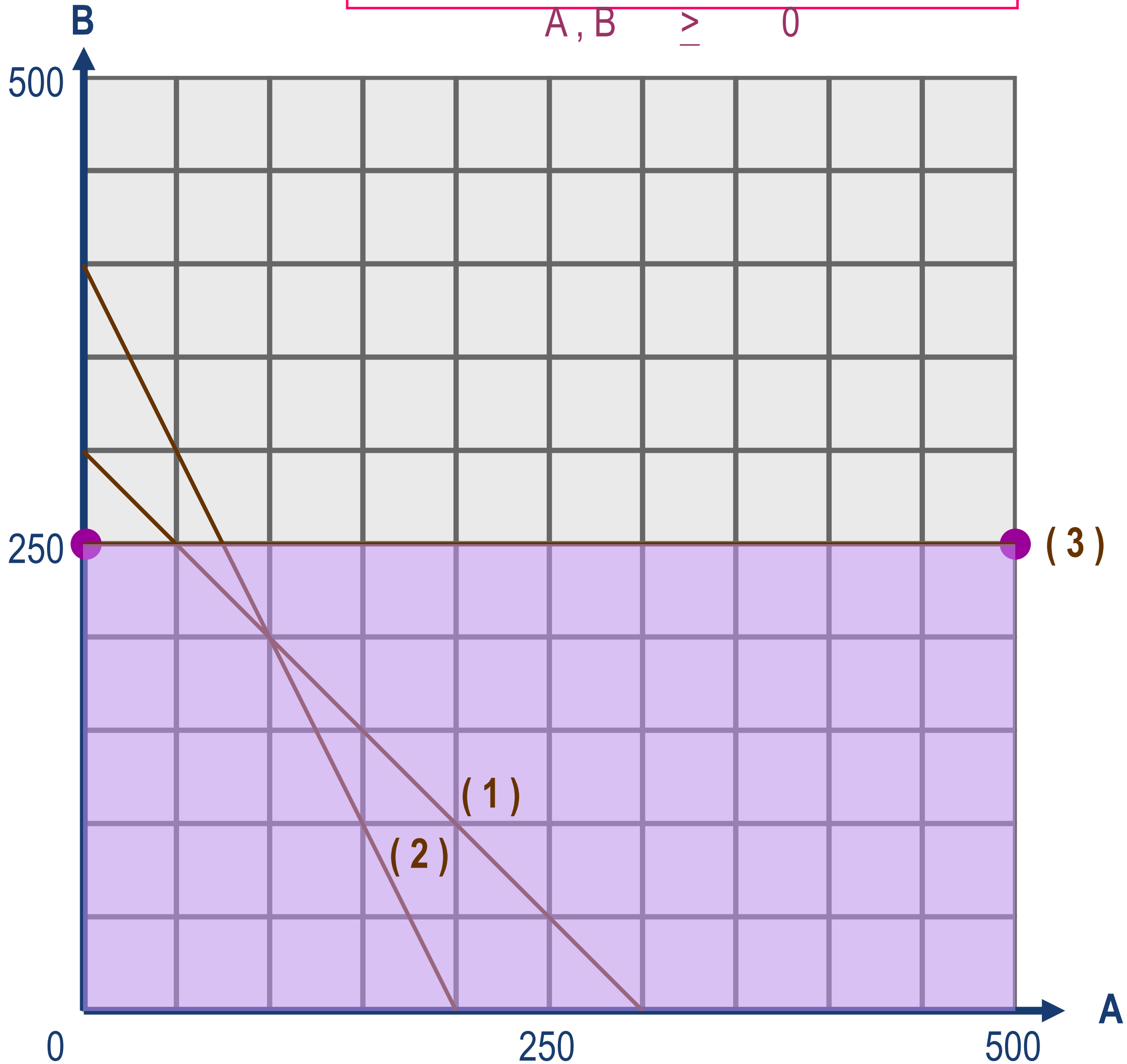
$$2A + B \leq 400 \quad (2)$$

$$B \leq 250 \quad (3)$$

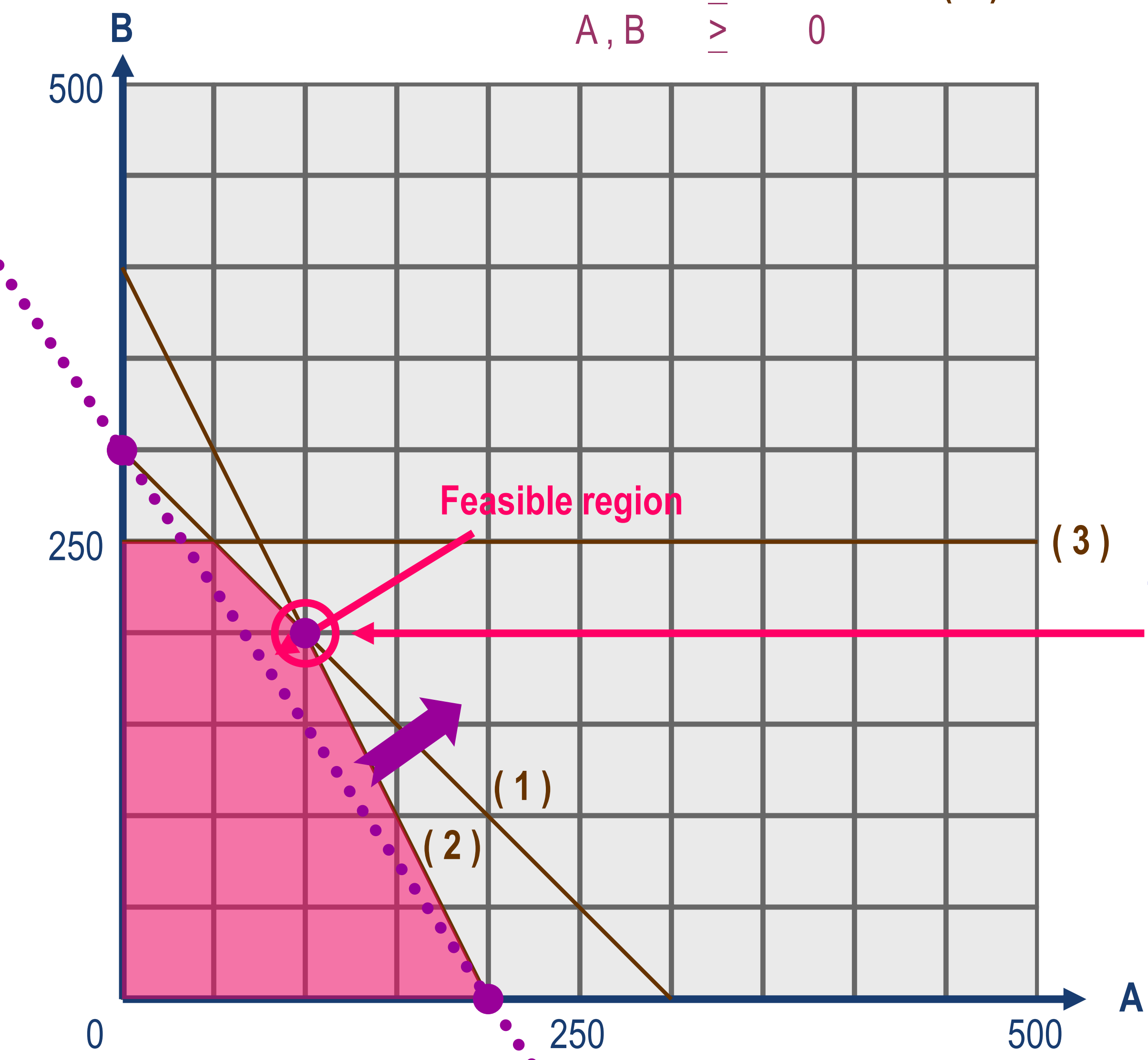
$$A, B \geq 0$$



$$
\begin{aligned}
\text{Max } Z &= 3A + 2B \\
\text{s.t. } A + B &\le 300 \quad (1) \\
2A + B &\le 400 \quad (2) \\
B &\le 250 \quad (3) \\
A, B &\ge 0
\end{aligned}
$$



$$
\begin{array}{llllllll}
\text{Max} & Z & = & 3A + 2B & & & & \\
\text{s.t.} & A & + & B & \leq & 300 & & (1) \\
 & 2A & + & B & \leq & 400 & & (2) \\
 & & & B & \leq & 250 & & (3) \\
 & & & A, B & \geq & 0 & &
\end{array}
$$



Iso-profit Line Method

The objective function line can be represented by a straight-line with 3A + 2B; its slope is -3/2

Iso-profit line: (1) points on the line leads to same profit; (2) lines on the upper right have larger profit

To maximize Z, we move the line to the right until it reaches the last corner point in the feasible region. (we can start anywhere for this line)

Optimal solution

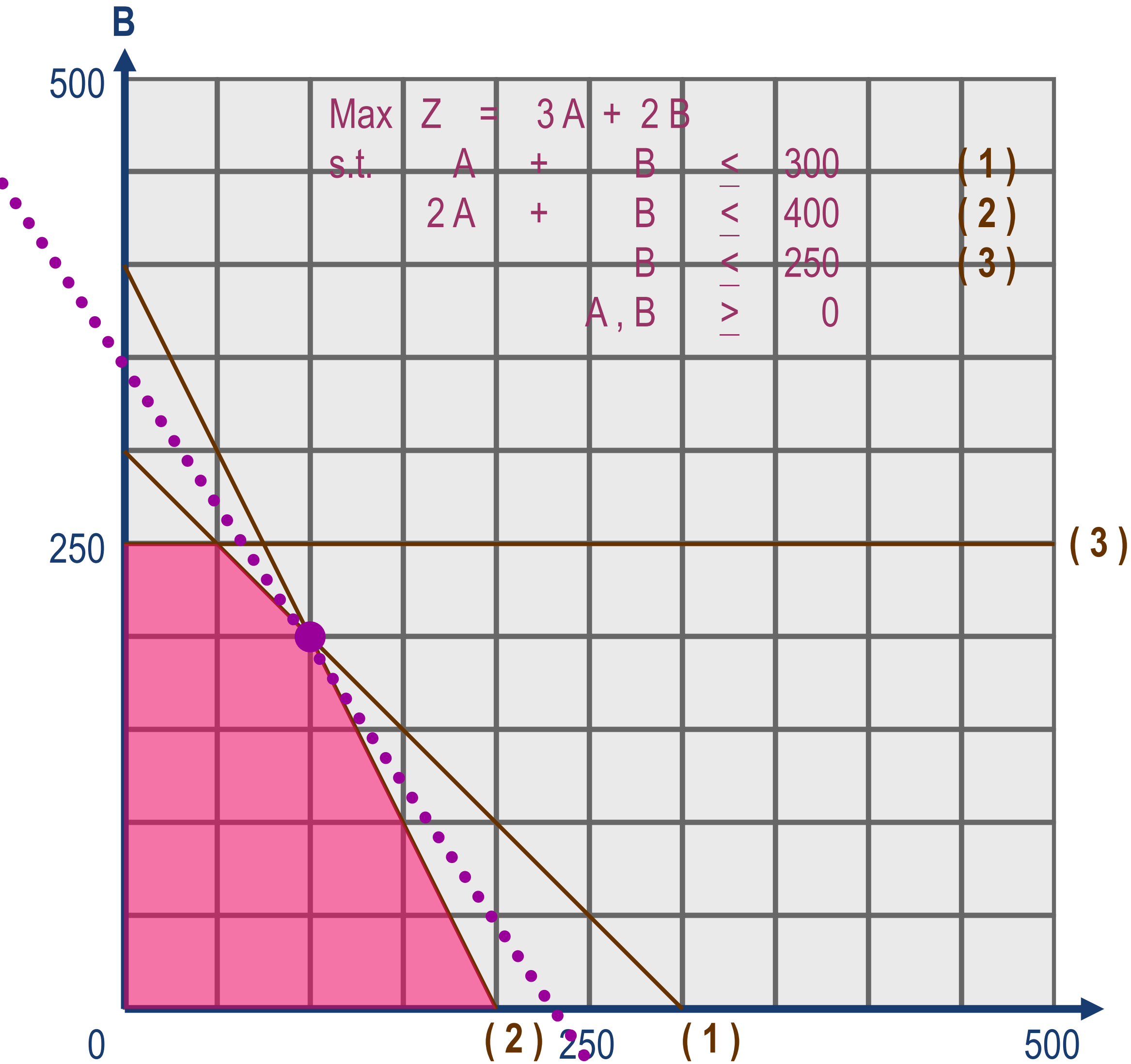
A = 100

B = 200

Max Z = 700

(b) What combination of the products will maximize the total profit?

100 A and 200 B



Optimal solution

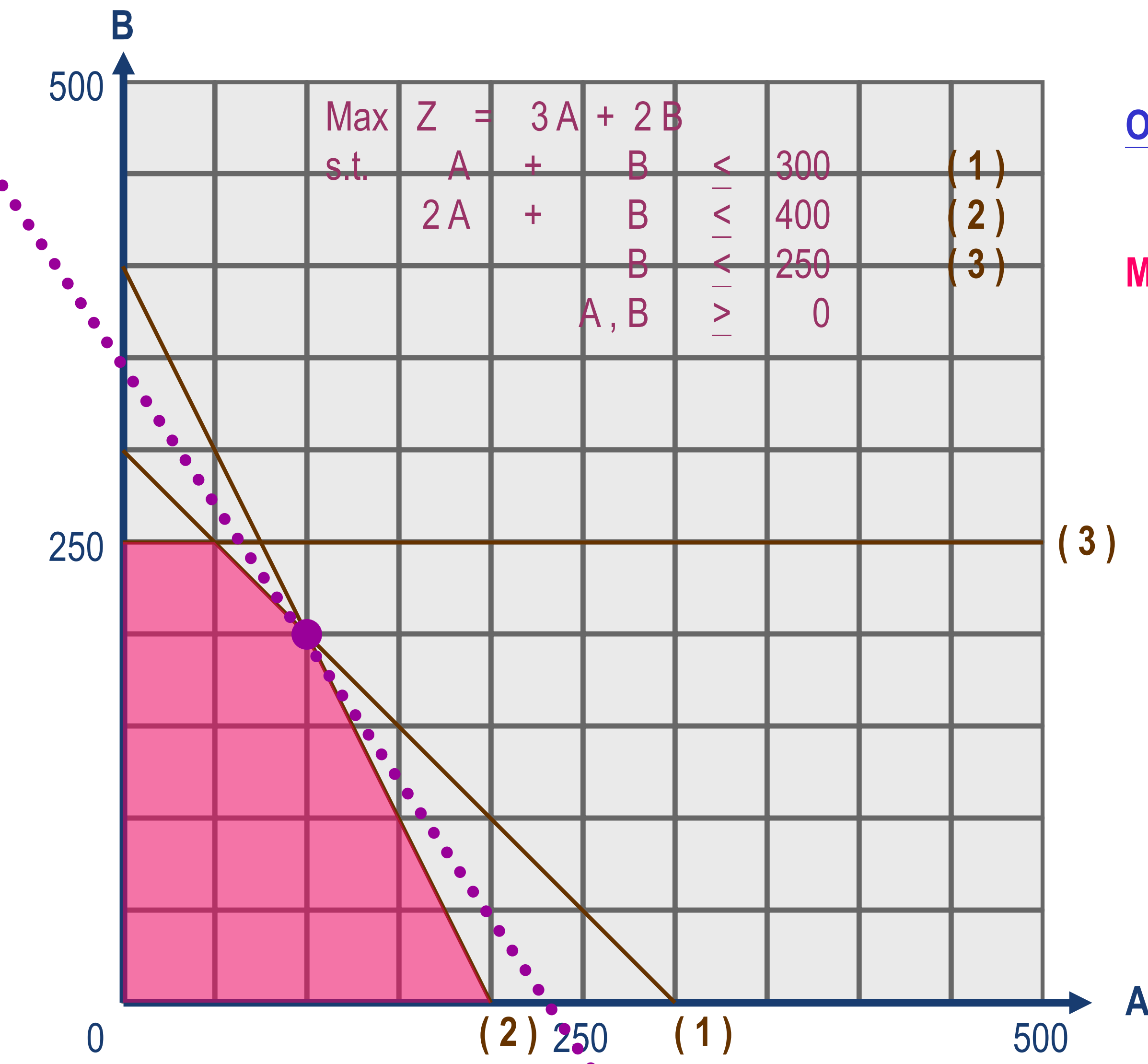
A = 100

B = 200

Max Z = 700

(c) What is the maximum total profit?

Total profit = 3(100) + 2(200) = $ 700



Optimal solution

A = 100

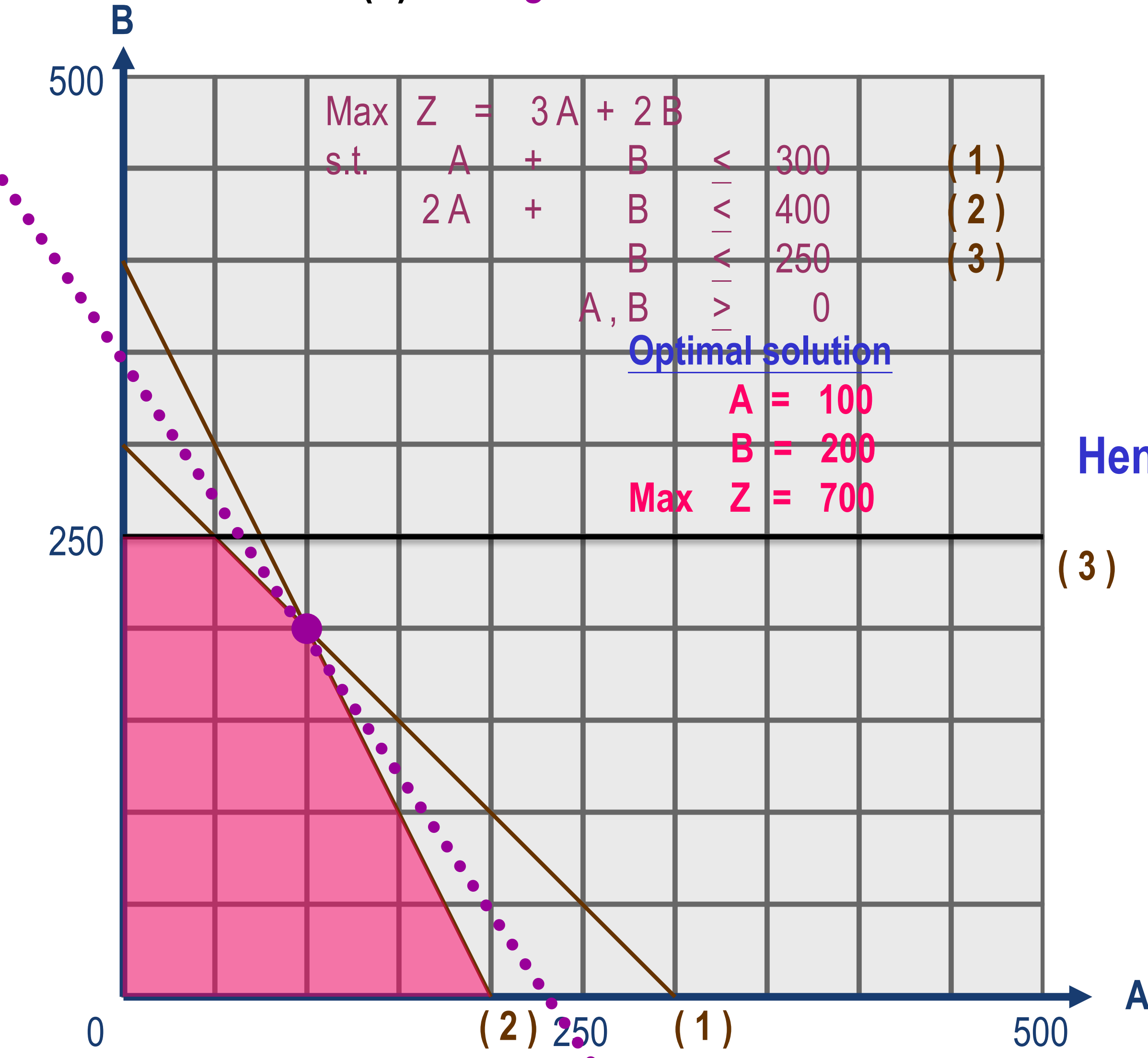
B = 200

Max Z = 700

(d) Is there any idle time in molding, painting, or cutting departments?
If so, which department has idle time and how much?

Optimal solution is on the constraints **(1)** molding and **(2)** painting $\Rightarrow$ No idle time

Constraint **(3)** cutting has idle time.



Idle time for (3):

$S_3 = 250 - B$
$= 250 - 200 = 50$

Hence, cutting has idle time = 50 min

If the solution falls on the boundary of a constraint, then the constraint is binding

ISOM 2700: Operations Management

Session 12. Solving LP using Excel Solver

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What we learnt so far...

- Overview of capacity planning
- Decision tree method
  - Alternative actions, uncertainty in demand scenario
  - Decision making under uncertainty in the future
- Linear programming method
  - Decision variables, linear objective and constraints
  - Decision making with constrained resources

Binding & Non-binding Constraint

- For a production plan, constraint for the availability of each resource:
  - Left-hand side (LHS): how many resources are needed
  - Right-hand side (RHS): how many resources are available
  - LHS ≤ RHS: availability constraint for each resource

- Binding vs Non-binding:
  - Binding: LHS = RHS, all of this resource is used up
  - Non-binding: LHS < RHS, some of this resource is left

- Slack: RHS – LHS, amount of resource left

Shadow Price

- Shadow price: defined for each resource (constraint)
  - Marginal revenue that can be generated by increasing one unit of the resource
  - How much more profit can we gain if we increase the resource availability by one unit

- Shadow price for resource i $=$ new optimal value after increasing i by one unit $-$ old optimal value

- In product mix problem, shadow price is non-negative
  - The more resource, the better
  - Equals to zero if optimal value remains unchanged

Shadow Price and Slack

- Shadow price and slack are related
- Slack > 0:
  - the resource is not used up (non-binding) ⇒ increasing by one unit does not affect optimal plan ⇒ shadow price must be zero
- Shadow price > 0:
  - Increasing the resource improves production plan ⇒ all the resource must be used up currently ⇒ Slack = 0 (binding)
- It is impossible to have shadow price and slack both greater than zero for a resource

Binding Constraints and Shadow Price

	Binding	Non-binding
Constraint	LHS = RHS	LHS < RHS
Slack	= 0	> 0
Improve the objective by increasing the resource by Δ	Yes	No
Shadow price	≥ 0	= 0

Complementary slackness: For each resource, its slack and shadow price cannot be both positive, i.e., Shadow price × Slack = 0

Agenda

- Formulate linear programming problems
- **Solve linear programming problems**
  - Graphical Method (no more than 2 decision variables)
  - **Excel Solver**
- Sensitivity analysis

Product Mix Problem at ST Powder

- ST. sells two products: **cologne** and **perfume**
- Cologne sells for \$3 per ounce; each ounce requires
  - 2 grams of fragrance
  - 6 grams of intensifier
- Perfume sells for \$8 per ounce; each ounce requires
  - 4 grams of fragrance
  - 2 grams of intensifier
  - 1 gram of stabilizer
- S. T. has limited supplies. In particular, it has
  - 1,600 grams of fragrance
  - 1,800 grams of intensifier
  - 350 grams of stabilizer

S. T. would like to use these ingredients immediately before they spoil. How can it maximize the revenue earned from these supplies?

LP Formulation of ST Powder's Problem

Decision Variables:

C: ounces of cologne produced

P: ounces of perfume produced

Objective Function:

Max $3C + 8P$

Subject to constraints:

$2C + 4P \leq 1600$ (fragrance)

$6C + 2P \leq 1800$ (intensifier)

$P \leq 350$ (stabilizer)

$C \geq 0$ (non-negativity)

$P \geq 0$ (non-negativity)

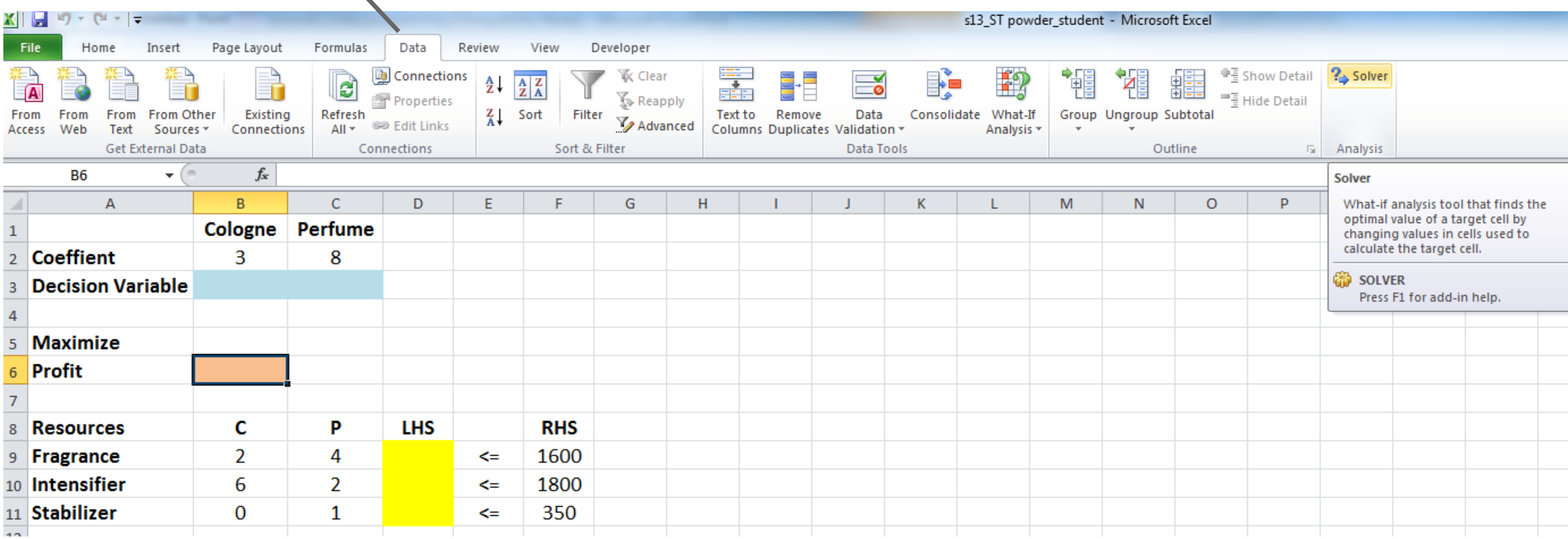
Solving LP by using Excel Solver

Step 1. Write LP in a spreadsheet as follows, and then click “Data”→”Solver”

(Remark: For the first time, you need to load the solver add-ins from Options.)

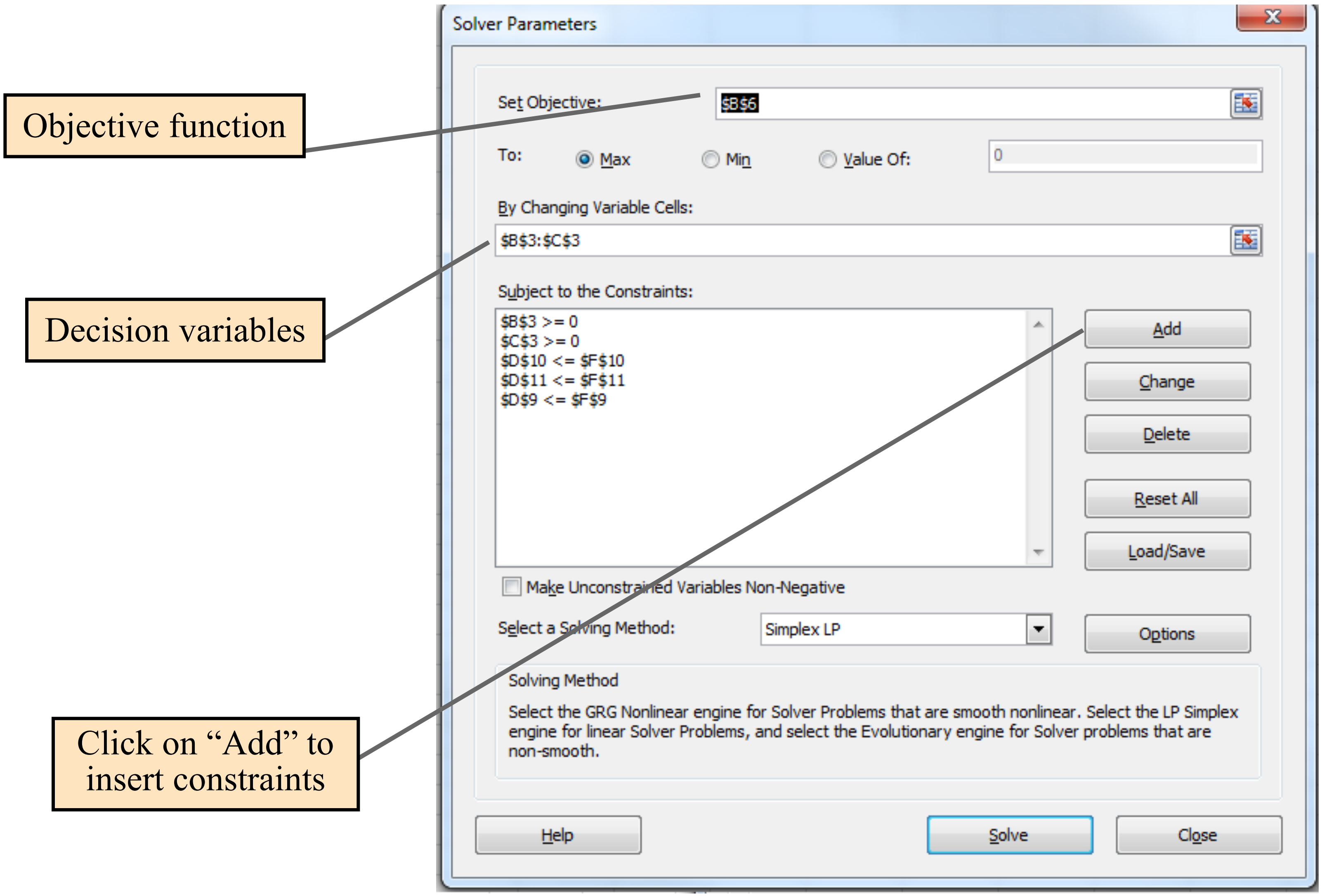
Click on “Data” to invoke “Solver.”

Load solver in Mac



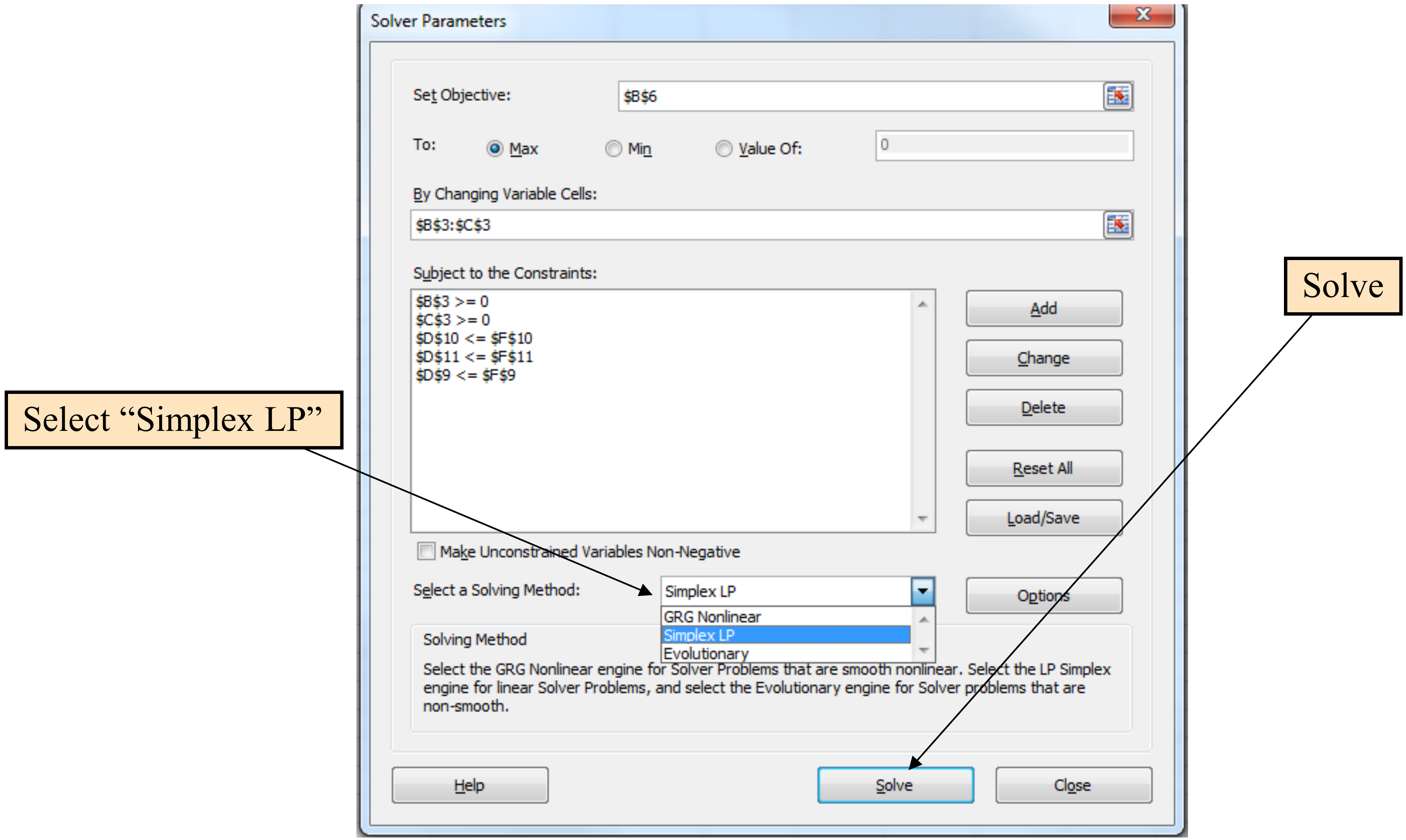
Solving LP by using Excel Solver

Step 2. Specify Solver parameters



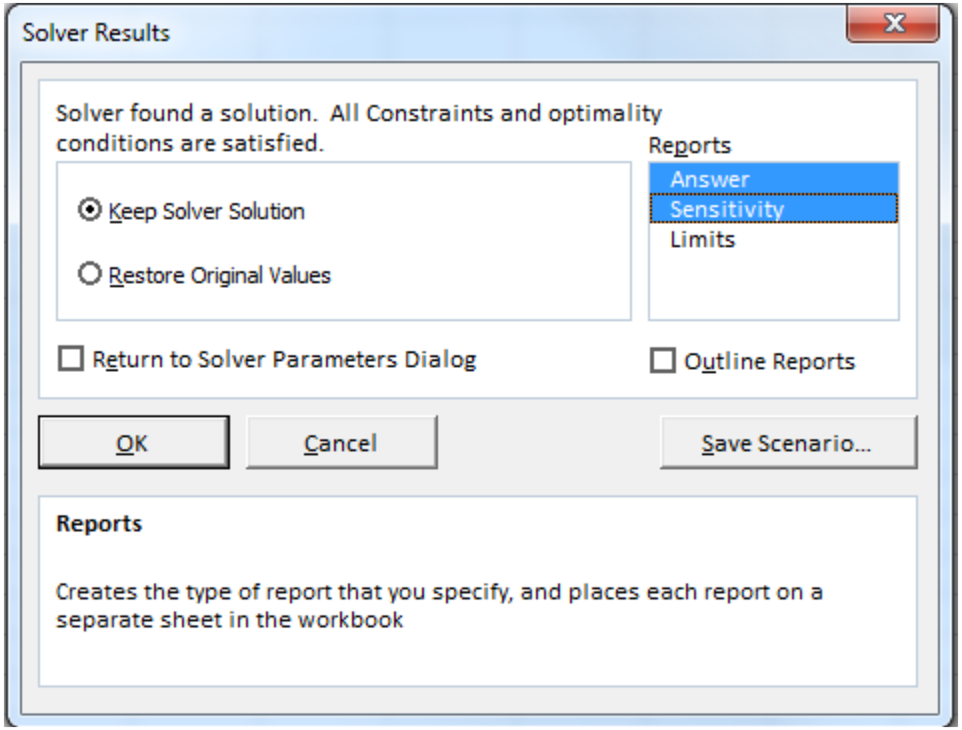
Solving LP by using Excel Solver

Step 3. Choose "Simplex LP" from the solving method menu;
Step 4. Click "Solve".



Solving LP by using Excel Solver

Step 5. Select “Answer” and “Sensitivity” to generate two extra spreadsheets, named as “Answer report” and “Sensitivity report”; Click OK.



Agenda

- Formulate linear programming problems
- Solve linear programming problems
- **Sensitivity analysis**

Sensitivity Analysis

- How do optimal objective value and solution react to changes in resource availability and product prices?

- Why do we care?
  - Impact of contract changes: e.g., should we increase a resource availability by 10 units at a price of $100?
  - True economic value of resources: e.g., what is the underlying marginal value of each type of resources?
  - Robustness of production plan: e.g., would the production plan change if the prices move by a little bit?

- These questions can be answered by sensitivity analysis

Price Coefficients

- If the change in the price coefficient is within the allowable range, then the optimal solution stays unchanged

- That is, a small change in price coefficients does not affect the optimal production plan

- However, the optimal objective value will change
  - It needs to be computed using the new price coefficients and original optimal production plan

Decision Variable Table

	Decision variables				For the objective coefficient	
Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B3	Decision Variable Cologne	100	0	3	1	3
C3	Decision Variable Perfume	350	0	8	1E+30	2
		Optimal production plan/soluion		(Price/Profit)		

If the price coefficient lies within the original value plus/minus the allowable increase/decrease

- the optimal solution (100,350) does NOT change
- the optimal objective value will change based on the new coefficient

- In the above question, the price for cologne can vary between 0 (3-3) and 4 (3+1)
- The price for perfume can vary between 6 (8-2) and +∞

Change in Right Hand Side (RHS)

- How will the optimal objective value change if the right-hand side (RHS) of some constraint changes?
- If the change in the RHS is within the allowable range
  - then the change in the optimal objective value can be directly computed by shadow price × change in RHS
- If the RHS increases by Δ (≤ allowable increase) then the optimal objective value increases by shadow price × Δ
- If the RHS decreases by Δ (≤ allowable decrease) then the optimal objective value decreases by shadow price × Δ

Need to first confirm the change of RHS is within the allowable range!

Constraint Table

	Resource/Constraint				For the RHS availability	
Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D10	Intensifier LHS	1300	0	1800	1E+30	500
D11	Stabilizer LHS	350	2	350	50	50
D9	Fragrance LHS	1600	1.5	1600	166.6666667	200
		resource used in optimal plan (LHS)		availability (RHS)		

If the RHS lies within the original value plus/minus allowable increase/decrease

- the shadow price will NOT change
- we can compute the change in optimal objective value by shadow price times the change in the RHS

- In the above question, the amount of stabilizer can vary between 300 (350 - 50) to 400 (350 +50), and its shadow price (2) does not change

Sensitivity Analysis of ST Powder

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B3	Decision Variable Cologne	100	0	3	1	3
C3	Decision Variable Perfume	350	0	8	1E+30	2

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D10	Intensifier LHS	1300	0	1800	1E+30	500
D11	Stabilizer LHS	350	2	350	50	50
D9	Fragrance LHS	1600	1.5	1600	166.6666667	200

- How much should ST be willing to pay for 100 grams more of fragrance?

  100 grams is less than the allowable increase of 166.67 grams for fragrance. So, we can apply the shadow price of fragrance. The increase in revenue with 100 grams more of fragrance is \$1.5x100 = \$150. Thus, ST should pay at most \$150 for 100 grams more of fragrance.

- If 400 grams of intensifier is wasted, can ST still meet its revenue target?

  Yes. Intensifier has a slack of 500 grams (and the shadow price is zero). Thus, a waste of 400 grams (which is smaller than allowable decrease 500) of intensifier will have no effect on the production plan and the revenues.

Sensitivity Analysis of ST Powder

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B3	Decision Variable Cologne	100	0	3	1	3
C3	Decision Variable Perfume	350	0	8	1E+30	2

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D10	Intensifier LHS	1300	0	1800	1E+30	500
D11	Stabilizer LHS	350	2	350	50	50
D9	Fragrance LHS	1600	1.5	1600	166.6666667	200

- How much should ST pay for 60 grams more of stabilizer?

  60 grams is more than the allowable increase of 50 grams for stabilizer. This means the current solution is no longer optimal. Therefore, we cannot apply the shadow price of stabilizer.

- The price of perfume falls to $7. Should ST still produce any perfume?

  The decrease from $8 to $7 is less than the allowable decrease of $2. Thus, the current solution is still optimal – produce 350 ounces of perfume.

Shelby Shelving Case

- A small company that produces two types of shelves for grocery stores: Model S and Model LX

- Three steps in manufacturing process:
  - Stamping, forming and assembly
  - Capacity of Stamping and Forming machines: **800** hours a month
  - Capacity of model S assembly department: **1900** units/month
  - Capacity of model LX assembly department: **1400** units/month

- Machine Requirements (Hours/per unit):
  - Stamping: 0.3h/u for model S and 0.3h/u for model LX
  - Forming: 0.25h/u for model S and 0.5h/u for model LX
- Profit: **Model S $260/unit**; **Model LX $245/unit**

Shelby Shelving: LP Formulation

$$\max \quad 260\,S + 245\,LX \qquad \text{(Net Profit)}$$

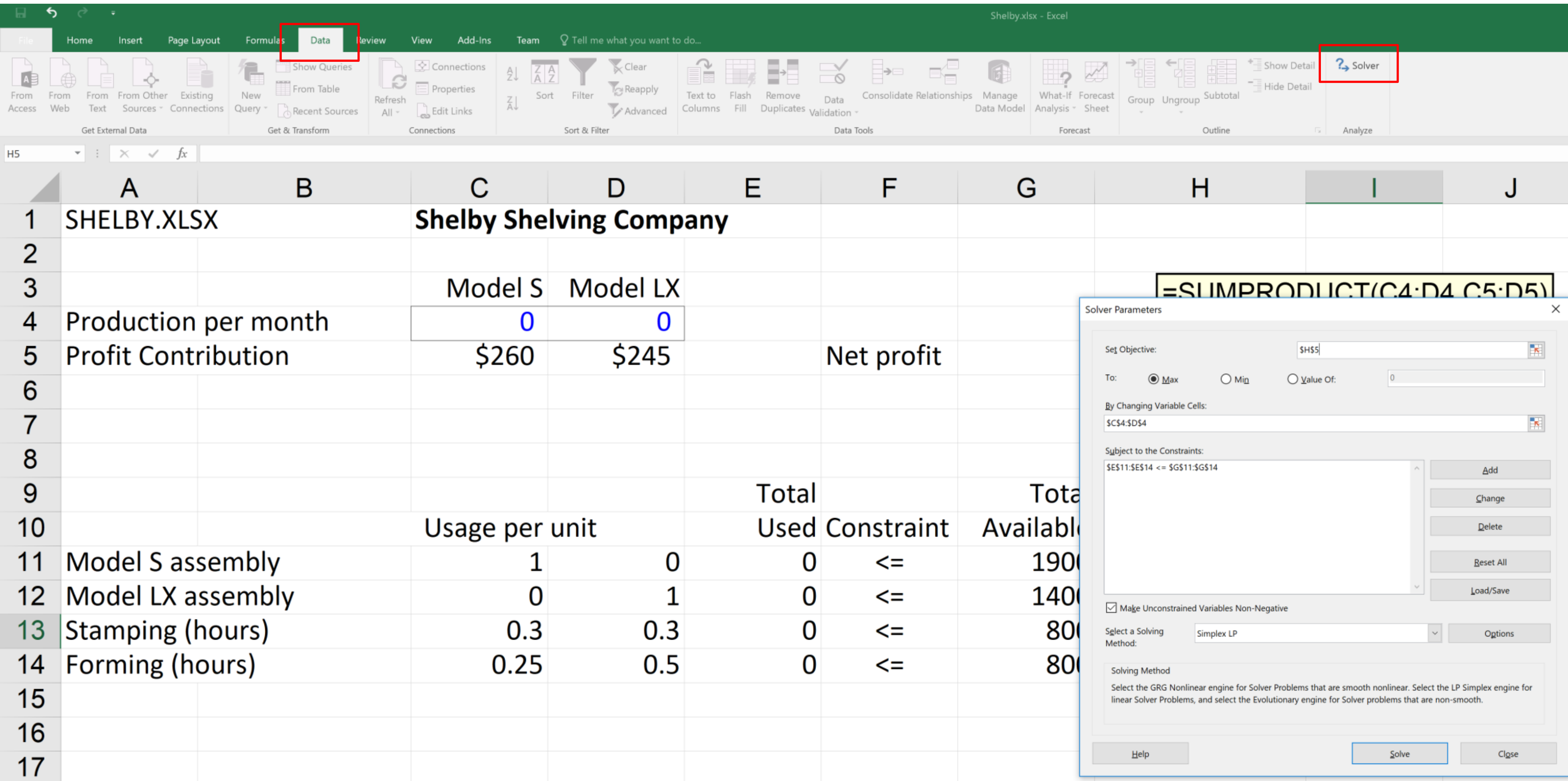
subject to:

(S assembly)	$S \leq \mathbf{1900}$
(LX assembly)	$LX \leq \mathbf{1400}$
(Stamping)	$0.3\,S + 0.3\,LX \leq \mathbf{800}$
(Forming)	$0.25\,S + 0.5\,LX \leq \mathbf{800}$
(Non-negativity)	$S, LX \geq 0$

Optimal solution: ***S* =1900, *LX* = 650**, Net Profit = $653,250.

The Solver Dialog Box

Load solver in Mac



The Spreadsheet after Optimizing

	A	B	C	D	E	F	G	H
1	SHELBY.XLSX		**Shelby Shelving Company**					
2								
3			Model S	Model LX				=SUMPRODUCT(C4:D4,C5:D5)
4	Production per month		1900	650				
5	Profit Contribution		$260	$245		Net profit		$653,250.00
6								
7								
8								
9					Total		Total	
10			Usage per unit		Used	Constraint	Available	
11	Model S assembly		1	0	1900	<=	1900	
12	Model LX assembly		0	1	650	<=	1400	
13	Stamping (hours)		0.3	0.3	765	<=	800	
14	Forming (hours)		0.25	0.5	800	<=	800	
15								
16								
17								

Shelby Shelving: Sensitivity Report

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B3	Decision Variable S	1900	0	260	1E+30	137.5
C3	Decision Variable LX	650	0	245	275	245

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D9	S assembly LHS	1900	137.5	1900	233.3333333	1500
D10	LX assembly LHS	650	0	1400	1E+30	750
D11	Stamping LHS	765	0	800	1E+30	35
D12	Forming LHS	800	490	800	58.33333333	325

Change in profit = Shadow Price × Change in Resources

- An increase in Model *S* assembly capacity from 1900 to 1902 would increase profit by:

$$137.5 \times (1902-1900) = 275$$

- A decrease in Model S assembly capacity from 1900 to 1897 would reduce profit by:

$$137.5 \times (1900-1897) = 412.5$$

Right-hand side Range

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B3	Decision Variable S	1900	0	260	1E+30	137.5
C3	Decision Variable LX	650	0	245	275	245

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D9	S assembly LHS	1900	137.5	1900	233.3333333	1500
D10	LX assembly LHS	650	0	1400	1E+30	750
D11	Stamping LHS	765	0	800	1E+30	35
D12	Forming LHS	800	490	800	58.33333333	325

- The sensitivity report indicates that the shadow price for Model *S* assembly, $137.5, is valid for Model S assembly capacity ranging from:

400 (= 1900-1500) to 2133.33 (= 1900+233.33)

Shelby Shelving: Sensitivity Report

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B3	Decision Variable S	1900	0	260	1E+30	137.5
C3	Decision Variable LX	650	0	245	275	245

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
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D10	LX assembly LHS	650	0	1400	1E+30	750
D11	Stamping LHS	765	0	800	1E+30	35
D12	Forming LHS	800	490	800	58.33333333	325

- Would Shelby be willing to pay $259 for an extra unit of capacity of the Model *S* assembly department?

Shadow price = 137.5 < 259 => should not pay

- How much would Shelby be willing to pay to increase the capacity of the Model LX assembly by 1 unit?

Shadow price = 0 => should not pay

Objective Coefficient Range

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B3	Decision Variable S	1900	0	260	1E+30	137.5
C3	Decision Variable LX	650	0	245	275	245

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D9	S assembly LHS	1900	137.5	1900	233.3333333	1500
D10	LX assembly LHS	650	0	1400	1E+30	750
D11	Stamping LHS	765	0	800	1E+30	35
D12	Forming LHS	800	490	800	58.33333333	325

- Optimal production plan will not change if
  - profit contribution of model S ranges from:

    122.5 (=260-137.5) to infinity (1E+30 means infinity)

  - profit contribution of model LX ranges from:

    0 (=245-245) to 520 (=245+275)

Objective Coefficient Range

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B3	Decision Variable S	1900	0	260	1E+30	137.5
C3	Decision Variable LX	650	0	245	275	245

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D9	S assembly LHS	1900	137.5	1900	233.3333333	1500
D10	LX assembly LHS	650	0	1400	1E+30	750
D11	Stamping LHS	765	0	800	1E+30	35
D12	Forming LHS	800	490	800	58.33333333	325

- If the profit of model S increase to 280, what is the increase in total profit
- The increase in profit = 280 – 260 = 20, which is within the allowable increase
- Thus, the optimal plan stays the same, thus the profit increase = (280 – 260)*1900 = 38000

Shadow Price and Slack

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B3	Decision Variable S	1900	0	260	1E+30	137.5
C3	Decision Variable LX	650	0	245	275	245

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D9	S assembly LHS	1900	137.5	1900	233.3333333	1500
D10	LX assembly LHS	650	0	1400	1E+30	750
D11	Stamping LHS	765	0	800	1E+30	35
D12	Forming LHS	800	490	800	58.33333333	325

Complementary Slackness

Slack > 0 (Non-binding)
$\Rightarrow$ Shadow Price = 0

Shadow Price > 0
$\Rightarrow$ Slack = 0 (Binding)

	Capacity	Used	Slack	Shadow Price
S Assembly	1900	1900	**0**	**137.5**
LX Assembly	1400	650	**750**	**0**
Stamping	800	765	**35**	**0**
Forming	800	800	**0**	**490**

“Pricing” a New Product

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D9	S assembly LHS	1900	137.5	1900	233.3333333	1500
D10	LX assembly LHS	650	0	1400	1E+30	750
D11	Stamping LHS	765	0	800	1E+30	35
D12	Forming LHS	800	490	800	58.33333333	325

A new product, GX, is proposed with the following characteristics:

Revenue:	**$1000**
Variable Cost :	**$ 850**
Hrs. Stamping :	**2.0**
Hrs. Forming :	**0.5**
No assembly :	**0**

Profit = 1000 – 850 = $150

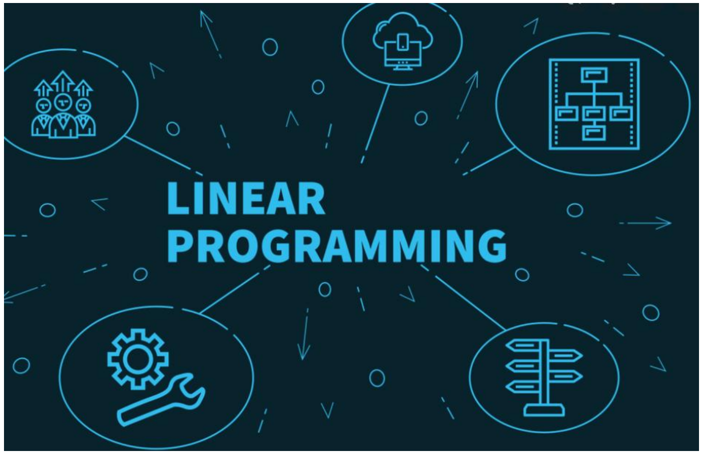
It would decrease the current profit by:
$2\times0+0.5\times490$ = $245 > $150

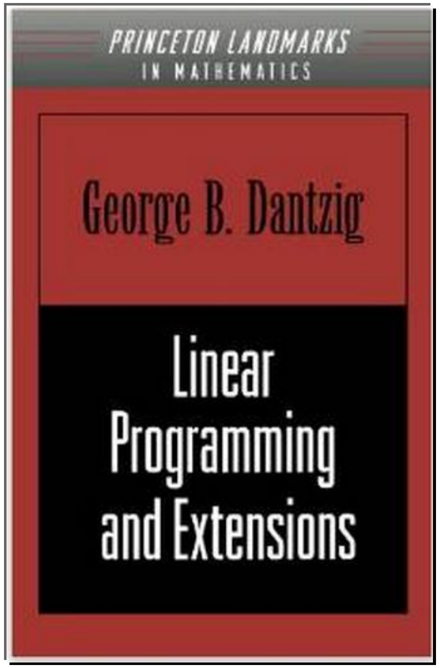
Thus, should not produce GX

Q: Is it worth producing?

History of Linear Programming

- A discipline from 1940's
- Simplex method (1947)
- Interior method (1984)





Linear Programming

- Powerful modeling device
  - Simple
  - Wide variety of applications
  - Flexible (sensitivity analysis)
- Solver for many programming languages:
  - Excel solver, Python, MATLAB,…

Summary

- Capacity planning and resource allocation can be a complex multidimensional problem
  - Decision tree method and linear programming
- Formulate linear programming problems
  - Decision variables, Objective function, Constraints
- Solve LP: Graphical Method or Excel Solver
- Sensitivity analysis
  - marginal value of a resource (i.e., shadow price), effect of a changing environment on operational decisions