Midterm Solutions:

1: (a) F
(b) T
(c) F
(d) F
(e) F
(f) T
(g) F
(h) F
(i) T
(j) T

2: (a) $N = N(\Omega) = \binom{15}{3}$ $\quad A = \{$all 3 bulbs are rated 18-watt$\}$

$N(A) = \binom{6}{3}$

$\Rightarrow \quad P(A) = \frac{N(A)}{N} = \frac{\binom{6}{3}}{\binom{15}{3}}$

(b) $B = \{$one bulb of each (type) is selected$\}$.

$\Rightarrow \quad N(B) = \binom{5}{1}\binom{6}{1}\binom{4}{1}$ (product rule)

$\Rightarrow \quad P(B) = \frac{N(B)}{N} = \frac{\binom{5}{1}\binom{6}{1}\binom{4}{1}}{\binom{15}{3}}$

(c) $C = \{$2 rated 23-watt$\} = \{$2 rated 23-watt, 1 rated 13-watt$\}$
$\cup \{$2 rated 23-watt, 1 rated 18-watt$\}$

$\downarrow$ mutually exclusive

$\Rightarrow \quad N(C) = \binom{4}{2}\binom{5}{1} + \binom{4}{2}\binom{6}{1}$

$\Rightarrow \quad P(C) = \frac{N(C)}{N} = \frac{\binom{4}{2}\binom{5}{1} + \binom{4}{2}\binom{6}{1}}{\binom{15}{3}}$. □.

3. From the problem: $P(\text{disease}) = 0.02$, $P(\text{positive} \mid \text{disease}) = 0.9$
$P(\text{positive} \mid \text{disease}^c) = 0.01$.

(a) $P(\text{positive}) = P(\text{positive} \mid \text{disease}) \times P(\text{disease}) + P(\text{positive} \mid \text{disease}^c) \times P(\text{disease}^c)$
$= 0.9 \times 0.02 + 0.01 \times (1 - 0.02)$
$= 0.0278$ (rule of total probability)

(b) $\mathbb{P}(\text{disease} \mid \text{positive}) = \dfrac{\mathbb{P}(\text{positive} \mid \text{disease})\,\mathbb{P}(\text{disease})}{\mathbb{P}(\text{positive})}$

$= \dfrac{0.9 \times 0.02}{0.0278}$

$= 0.6475.$ (Bayes' Theorem) □.

4. (a) Let $\sum_{y=1}^{5} p(y) = 1$, i.e. $a \cdot \sum_{y=1}^{5} y = 1 \Rightarrow a = \frac{1}{15}$

(b) $\mathbb{P}(Y \leq 3) = p(1) + p(2) + p(3) = \frac{1}{15} + \frac{2}{15} + \frac{3}{15} = 0.4$

(c) $\mathbb{P}(2 \leq Y \leq 4) = p(2) + p(3) + p(4) = \frac{2}{15} + \frac{3}{15} + \frac{4}{15} = 0.6$ □.

5. (a) $IQR = Q3 - Q1 = 86 - 76 = 10.$

Range = max − min = 100 − 58 = 42.

(b) Left-skewed, because median > mean. (or any other reasonable answers).

(c) Maximum lower whisker reach = $Q1 - 1.5 \times IQR = 34 <$ Min

Maximum upper whisker reach = $Q3 + 1.5 \times IQR = 128 >$ Max

$\Rightarrow$ No outliers. □.