E3101: Problem set 2

Okay... hope the book comes in soon... In the meantime here is a pdf of the second problem set. Have fun.

2.2.12 Apply elimination (circle the pivots) and back substitution to solve

\[
\begin{align*}
2x - 3y &= 3 \\
4x - 5y + z &= 7 \\
2x - y - 3z &= 5
\end{align*}
\]

2.2.13 Which number \(d\) forces a row exchange, and what is the triangular system (not singular) for that \(d\)? Which \(d\) makes this system singular (no third pivot)?

\[
\begin{align*}
2x + 5y + z &= 0 \\
4x + dy + z &= 2 \\
y - z &= 3
\end{align*}
\]

2.2.26 For which three numbers \(a\) will elimination fail to give three pivots?

\[
A = \begin{bmatrix}
a & 2 & 3 \\ a & a & 4 \\ a & a & a
\end{bmatrix}
\]

2.3.3 Which three matrices \(E_{21}, E_{31}, E_{32}\) put \(A\) into triangular form \(U\)?

\[
A = \begin{bmatrix}
1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0
\end{bmatrix}
\]

and \(E_{32}E_{31}E_{21}A = U\)

2.3.17 The parabola \(y = a + bx + cx^2\) goes through the points \((x, y) = (1, 4)\) and \((2, 8)\) and \((3, 14)\). Find and solve a matrix equation for the unknowns \((a, b, c)\).

2.4.5 Compute \(A^2\) and \(A^3\). Make a prediction for \(A^5\) and \(A^n\)

\[
A = \begin{bmatrix}
1 & b \\ 0 & 1
\end{bmatrix}
\]

and \(A = \begin{bmatrix}
2 & 2 \\ 0 & 0
\end{bmatrix}\)

2.4.15 If \(A\) is \(m\) by \(n\), how many separate multiplications are involved when

(a) \(A\) multiplies a vector \(x\) with \(n\) components?

(b) \(A\) multiplies an \(n\) by \(p\) matrix \(B\)

(c) \(A\) multiplies itself to produce \(A^2\)? Here \(m = n\).