E6950 Homework #4 Solution

Prob. 1.

a.
RF Bandwidth (or Transmission Bandwidth) = 1.25MHz
Baseband Bandwidth = RF Bandwidth / 2 = 625kHz < 871.5kHz Carrier Frequency
Let
B : Baseband Bandwidth
\( T_{\text{symbol}} \) : Symbol duration \( \quad R_{\text{symbol}} \) : Symbol rate
\( T_{\text{bit}} \) : Bit duration \( \quad R_{\text{bit}} \) : bit rate (or data rate)

Then,
\[
B = \frac{1}{2T_{\text{symbol}}(1+r)} = \frac{R_{\text{symbol}}}{2}(1+r)
\]
\[
R_{\text{symbol}} = \frac{2B}{1+r}
\]

Hence,
Data rate \( R_{\text{ba}} \) for QPSK is:
\[
R_{\text{bit}} = 2R_{\text{symbol}} = \frac{4B}{1+r}
\]

Data rate \( R_{\text{ba}} \) for PSK(or BPSK) is:
\[
R_{\text{bit}} = R_{\text{symbol}} = \frac{2B}{1+r}
\]

Therefore,

<table>
<thead>
<tr>
<th>r</th>
<th>QPSK data rate (kbps)</th>
<th>PSK(or BPSK) data rate (kbps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>2000</td>
<td>1000</td>
</tr>
<tr>
<td>0.5</td>
<td>1666.66</td>
<td>833.33</td>
</tr>
<tr>
<td>1</td>
<td>1250</td>
<td>625</td>
</tr>
</tbody>
</table>

b.
RF Bandwidth = 2B = 30kHz
Hence,
Baseband Bandwidth = B = 15kHz < 871.5kHz

<table>
<thead>
<tr>
<th>r</th>
<th>QPSK data rate (kbps)</th>
<th>PSK(or BPSK) data rate (kbps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>48</td>
<td>24</td>
</tr>
<tr>
<td>0.5</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>30</td>
<td>15</td>
</tr>
</tbody>
</table>
Prob. 2.

a.

\[ S_i(t) = a_i \cdot \text{COS}(\pi t/T) \cdot \text{COS}(W_0 t) + b_i \cdot \text{SIN}(\pi t/T) \cdot \text{SIN}(W_0 t) \]  \hfill (5-8)

If \( a_i = 1 \) and \( b_i = 1 \), then \( a_i \cdot b_i = 1 \), \( dW = \pi / T \)

\[ S_i(t) = \text{COS}(W_0 t - a_i \cdot b_i \cdot \pi t/T + 0) = \text{COS}(W_0 t - dW t + 0) \]

If \( a_i = 1 \) and \( b_i = -1 \), then \( a_i \cdot b_i = -1 \), \( dW = \pi / T \)

\[ S_i(t) = \text{COS}(W_0 t + a_i \cdot b_i \cdot \pi t/T + 0) = \text{COS}(W_0 t + dW t + 0) \]

If \( a_i = -1 \) and \( b_i = 1 \), then \( a_i \cdot b_i = -1 \), \( dW = \pi / T \)

\[ S_i(t) = \text{COS}(W_0 t + a_i \cdot b_i \cdot \pi t/T + p_i) = \text{COS}(W_0 t + dW t + p_i) \]

If \( a_i = -1 \) and \( b_i = -1 \), then \( a_i \cdot b_i = 1 \), \( dW = \pi / T \)

\[ S_i(t) = \text{COS}(W_0 t - a_i \cdot b_i \cdot \pi t/T + p_i) = \text{COS}(W_0 t - dW t + p_i) \]

Therefore, in general, \( S_i(t) = \text{COS}(W_0 t - a_i \cdot b_i \cdot \pi t/T + \text{theta}) \)

\( \text{theta} = 0 \), \( a_i = 1 \), \( \text{theta} = p_i \), \( a_i = -1 \).  \hfill (5-9)

as well as \( S_i(t) = \text{COS}(W_0 t + dW t + \text{theta}) \) \( dW = \pi / T \), \( a_i \cdot b_i = -1 \)  \hfill (5-10)

b.

1) First when \( a_i = 1 \) if \( b_i = 1 \), \( a_i \cdot b_i = +1 \), then \( S_i(t) = \text{COS}(W_1 t) \).

So the output of the first LPF (after integration) = \( 2 \cdot T \left[ \frac{1}{2} t + 1/(4 \cdot W_1) \cdot \text{SIN}(2 \cdot W_1 t) \right] t = T/2; t = 0 \)

\[ = 2 \cdot T \left[ \frac{1}{2} T/2 + 1/(4 \cdot W_1) \cdot \text{SIN}(W_1 t) \right] \]

\[ = 1/2 + 1/(2 \cdot W_1 t) \cdot \text{SIN}(W_1 t) \]

Thus when \( W_1 t >> 1 \), then \( = 1/2 \)

The output of the second LPF (after integration)

\[ = 1/(2 \cdot dW t) \cdot \text{SIN}(dW t) = 0 \] if \( dW t = p_i \), then \( df = R/4 \)

2) \( a_i = 1 \), if \( b_i = -1 \), then \( a_i \cdot b_i = -1 \), then \( S_i(t) = \text{COS}(W_2 t) \),

The output of the first LPF = \( 1/(2 \cdot dW t) \cdot \text{SIN}(dW t) = 0 \) if \( dW t = p_i \), then \( df = R/4 \)

Thus the output of the second LPF = \( 1/2 \)

3) \( a_i = -1 \), if \( b_i = +1 \), then \( a_i \cdot b_i = -1 \), then \( S_i(t) = \text{COS}(W_2 t) \)

The output of the first LPF = \( 1/(2 \cdot dW t) \cdot \text{SIN}(dW t) = 0 \) if \( dW t = p_i \), then \( df = R/4 \)

Thus the output of the second LPF = \( 1/2 \)

4) \( a_i = -1 \), if \( b_i = -1 \), then \( a_i \cdot b_i = +1 \), then \( S_i(t) = \text{COS}(W_1 t) \)

The output of the first LPF = \( 1/2 \)

Thus the output of the second LPF = \( 1/(2 \cdot dW t) \cdot \text{SIN}(dW t) = 0 \) if \( dW t = p_i \), then \( df = R/4 \)

So if \( dW t = p_i \), \( df = R/4 \), then the above result is expected, thus with the known value of \( a_i \), the value of \( b_i \) can be determined uniquely.

<table>
<thead>
<tr>
<th>( a_i )</th>
<th>( b_i )</th>
<th>( S_i(t) )</th>
<th>Top LPF</th>
<th>Bottom LPF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>( \text{COS}(W_1 t) )</td>
<td>( 1/2 )</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td>( \text{COS}(W_2 t) )</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
<td>( \text{COS}(W_2 t) )</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
<td>( \text{COS}(W_1 t) )</td>
<td>( 1/2 )</td>
<td>0</td>
</tr>
</tbody>
</table>
Prob. 3.

a. It is simply because the transmitted signal over bandwidth-limited wireless channel always has the potential problem of the random amplitude variations due to multipath-fading.

b. First, bandwidth is a limited resource so that the modulation scheme utilizing bandwidth more efficiently eventually will have more service channels. This will result the low blocking probability that gives better availability and the low cost that gives lower price to the users. Since the power resource of the mobile stations is based on the battery, it is important to design the system power-efficiently to make the service realistic and widely available.

c. QPSK has a problem of an abrupt 180 degree phase change occurring during the switch from one QPSK signal to another at interval of T sec when both of two coefficients change. But OQPSK reduces the maximum phase shift unlike the QPSK has by keeping the quasi-constant amplitude during the transmission.

Prob. 4.

<table>
<thead>
<tr>
<th>Successive Binary pairs</th>
<th>ai</th>
<th>bi</th>
<th>in sec</th>
<th>in Rad</th>
<th>Location of constellation in Rad</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0</td>
<td>-1</td>
<td>-1</td>
<td>1T</td>
<td>-3*\pi/4</td>
<td>-3*\pi/4</td>
</tr>
<tr>
<td>1 0</td>
<td>+1</td>
<td>-1</td>
<td>2T</td>
<td>\pi/4</td>
<td>-2*\pi/4</td>
</tr>
<tr>
<td>1 1</td>
<td>+1</td>
<td>+1</td>
<td>3T</td>
<td>3*\pi/4</td>
<td>\pi/4</td>
</tr>
</tbody>
</table>