Time-varying Risk of Nominal Bonds: How Important Are Macroeconomic Shocks?

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Abstract

I analyze the risk of nominal assets within an external habit model supplemented with novel macroeconomic dynamics. Despite featuring flexible non-Gaussian fundamental processes, the model can be solved in closed-form. The estimation identifies time-varying "demand-like" and "supply-like" macroeconomic shocks directly linked to the risk of nominal assets. In addition to matching standard properties of US stock and bond returns, I examine how the model reproduces the time-variation in the stock and bond return correlation. I find that macroeconomic shocks generate sizeable positive and negative correlations, although negative correlations occur less frequently and are smaller than in data. Historically, macroeconomic shocks are most important in explaining high correlations from the late 70’s until the early 90’s and low correlations pre- and during the Great Recession.

Keywords: equity, fixed income, stock and bond return correlation, macroeconomic volatility

JEL codes: E43, E44, G11, G12, G17

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1 Introduction

The correlation between aggregate stock market and government bond returns in the United States has varied significantly over time. As illustrated in Figure 1, while the unconditional correlation is essentially 0, the correlation has been as low as -0.78 in the first quarter of 2009 and as high as 0.75 in the third quarter of 1994. Furthermore, while correlations were predominantly positive before 2000, they switched to being negative thereafter. The primary importance of stocks and bonds in portfolios of institutional and individual investors makes understanding the economic sources of this correlation and its time variation an important input for asset allocation.

Explaining stock and bond return correlations has been a challenge for asset pricing models. Shiller and Beltratti (1992) argue that it is difficult to reproduce these correlations in a stylized present value framework. More recently, Bekaert, Engstrom, and Grenadier (2010) show that a consumption-based asset pricing model with habit utility also does not replicate substantial variation in stock-bond return correlations. Even an empirical dynamic factor model (Baele, Bekaert, and Inghelbrecht, 2010) has difficulties to match stock and bond return correlations. Particularly difficult to generate, both theoretically and empirically, are the sign changes in the correlation.

In this article, I propose and estimate a structural model to evaluate the impact of macroeconomic shocks (consumption growth and inflation) on the time variation in stock and bond return correlations. The model is an external habit model (see, e.g., Campbell and Cochrane, 1999) augmented with novel macroeconomic dynamics. The habit utility delivers realistic asset pricing moments, in particular, a realistic term structure. However, it is the novel macroeconomic dynamics that, at least in theory, generate both positive and negative stock and bond return correlations.

In particular, I model consumption growth and inflation to depend on two shocks directly related to the risk of nominal assets: the first shock moves consumption growth and inflation in the same direction (in real terms, such shocks are often referred to as demand shocks), while the second shock moves them in the opposite directions (in real terms, such shocks are often referred to as supply shocks). It is natural to think that in a demand-shock environment nominal bonds are a better hedge against stock market fluctuations than in a supply-shock environment, because in a demand-shock environment inflation is procyclical.
implying higher bond returns in recessions.¹

To identify the model econometrically and provide a realistic and economically intuitive description of macroeconomic shocks, I use the Bad Environment-Good Environment (BEGE) dynamics of Bekaert and Engstrom (2009).² Using that framework, I model both demand and supply shocks as a combination of a good component generating positive skewness and a bad component generating negative skewness. The components are modeled with demeaned gamma distributions which, as Bekaert and Engstrom (2009) show, deliver empirically flexible and theoretically tractable distributions. In my model, the BEGE structure has further economic appeal as, for instance, for supply shocks, it is intuitive to think that commodity crises mainly come from the bad component and technological progress predominantly from the good component.

My results are twofold. First, the novel macroeconomic structure allows characterizing the history of the US consumption growth and inflation in terms of demand and supply shocks. I find that the demand shock consists of a persistent Gaussian component and a rare-disaster type negative shock, which was most pronounced during the Great Recession. The supply shock consists, roughly speaking, of a good component generating positive skewness during periods of low commodity prices and technological innovation and, on average a more important, bad component generating negative skewness during periods of high commodity prices.

Second, using the model, I establish a direct link between time-variation in macroeconomic risks and time-variation in stock and bond return correlations. To the best of my knowledge, this article is the first showing that macroeconomic shocks alone (shocks extracted from macro data only) generate large positive and negative stock and bond return correlations. However, negative correlations are less extreme and frequent than historically observed in the

¹My definition of demand and supply shocks follows Blanchard (1989), who also provides empirical support for the terminology. There are several other meanings associated with demand and supply shocks. For example, Blanchard and Quah (1989) define supply shocks as shocks which permanently affect output and demand shocks as shocks which do not. In a New Keynesian setting (e.g., Woodford, 2003), the demand shock is a shock to output in the aggregate demand equation, while the supply shock is a shock to inflation in the aggregate supply equation.

²In a concurrent complementary paper, Bekaert, Engstrom, and Ermolov (2014c) use the same dynamics to build a term structure model using GDP and inflation data. However, their analysis is reduced-form and they do not consider equity and the time-varying stock and bond return correlations, the main topic of this paper.
US, although still very economically significant during, for instance, the Great Recession. Historically, macroeconomic shocks are the most important in explaining high stock and bond return correlations from the late 1970’s until the early 1990’s, which is identified as a supply environment, and low correlations pre- and during the Great Recession, which is identified as a demand environment. The role of macroeconomic shocks in explaining stock and bond return correlations is time-varying and other effects dominate during certain periods.\(^3\)

I contribute to the recent literature on the importance of macroeconomic shocks for time-varying stock and bond return correlations, which has delivered very disparate results regarding the importance of such shocks.\(^4\) In a reduced-form model, Baele, Bekaert, and Inghelbrecht (2010) find very few links between macroeconomic variables and time-varying stock and bond return correlations. However, Campbell, Pflueger, and Viceira (2014), using a habit model, argue that macroeconomic shocks are important during certain periods, mostly in the 1970’s. Finally, the long-run risk models of Burkhardt and Hasseltoft (2012) and Song (2014) and a reduced-form model of Campbell, Sunderam, and Viceira (2013) suggest that macroeconomic shocks might be the main driver of the time variation in stock and bond return correlations.

The difference in results between my work and the extant literature is not surprising as I take a substantially different approach to model macroeconomic shocks and stock and bond return correlations. First, my approach combines flexible non-Gaussian dynamics with exact closed-form asset pricing solutions. Most of the previous literature (Baele, Bekaert, and Inghelbrecht, 2010; Campbell, Sunderam, and Viceira (2013); Campbell, Pflueger and Viceira, 2014) assumes conditional Gaussian dynamics for the macroeconomic series.\(^5\) While Burkhardt and Hasseltoft (2012) and Song (2014) employ regime-switching models of the Hamilton (1989)-type and therefore embed non-Gaussian aspects of the data, they obtain pricing solutions using the Bansal and Zhou (2002) approximation and do not provide an

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\(^3\)Other proposed explanations include flight-to-quality (Connolly, Stivers, and Sun, 2005; Bansal, Connolly, and Stivers, 2014), liquidity (Baele, Bekaert, and Inghelbrecht, 2010), preference shocks (Bekaert, Engstrom, and Grenadier, 2010), learning and money illusion (David and Veronesi, 2013), and monetary policy (Campbell, Pflueger, and Viceira, 2014). My work is complementary to these studies.

\(^4\)Although the idea that macroeconomic shocks can be related to stock and bond return correlations has been proposed by Fama (1981), its formal evaluations have not been conducted until very recently.

economic interpretation of the shocks. Importantly, Burkhardt and Hasseltoft (2012) and Campbell, Pflueger, and Viceira (2014) identify macroeconomic regimes by explicitly setting breakpoints in data using different auxiliary assumptions. I estimate the dynamic macroeconomic environment directly from macroeconomic data. Furthermore, in Campbell, Pflueger, and Viceira (2014), the agent is surprised about any regime change, whereas regime changes are likely to be anticipated and such anticipated changes might contribute significantly to asset prices (Veronesi, 1999). My model does not have this restriction.

Second, my model is based on the habit utility which reproduces a realistic term structure. In particular, the model reproduces on average an upward-sloping real yield curve, an important feature of two most studied financial markets, the US and the UK. The long-run risk models (Burkhardt and Hasseltoft, 2012; Song, 2014) imply on average a downward-sloping real yield curve.

Third, I estimate macroeconomic shocks from consumption and inflation data only. Thus, my model minimizes the reliance on unobservable fundamentals dynamics (see the Chen, Dou, and Kogan (2014) critique on the use of such unobservable factors) and provides a conservative assessment of the role of macroeconomic shocks. In contrast, Song (2014) jointly estimates the macroeconomic dynamics and asset prices, allowing for measurement errors in consumption and inflation data. This might lead to situations where large fluctuations in asset prices, which are in reality unrelated to macroeconomic shocks, are accounted for by large unobserved fluctuations in consumption hidden behind observational errors, thereby overstating the importance of macroeconomic shocks. Burkhardt and Hasseltoft (2012) are also subject to this critique as they explicitly calibrate macroeconomic parameters to match stock and bond return correlations. Additionally, although Campbell, Pflueger and Viceira (2014) formulate their model as a consumption-based model, it is estimated from GDP, not consumption, data, which is theoretically questionable.

Finally, my model is structural with an economically motivated utility function, allowing to analyze economic effects driving time-variation in stock and bond return correlations. The

6For example, Ang and Bekaert (2002) show this empirically for US Treasuries.
7The US real yield curve is directly observed from 2004 to 2014 and has on average a positive slope (Gürkaynak, Sack, and Wright, 2010b). Chernov and Mueller (2012) and Ang and Ulrich (2012) argue that the average slope of the real yield curve has been positive starting from the 1970's. For the UK, Heyerdahl-Larsen (2014) shows that from 1985 to 2011, the average real yield curve slope is positive.
8Campbell, Sunderam, and Viceira (2013) and David and Veronesi (2013) pursue similar approaches.
reduced-form models of Baele, Bekaert, and Inghelbrecht (2010) and Campbell, Sunderam and Viceira (2013) are more flexible but are silent with respect to the underlying mechanisms.

2 Model

The economy consists of a representative agent with external habit formation preferences as in Campbell and Cochrane (1999). The key innovation is introducing macroeconomic dynamics from Bekaert, Engstrom, and Ermolov (2014c), which allows for a tractable analysis of risk transmission mechanisms from the macroeconomy to asset prices, with a particular focus on nominal asset prices.

2.1 Consumption and Inflation

The logarithmic consumption growth, \( g_{t+1} = \ln \frac{C_{t+1}}{C_t} \), is a constant mean process:

\[
g_{t+1} = \bar{g} + \epsilon^g_{t+1},
\]

where \( \epsilon^g_{t+1} \) is a heteroskedastic shock specified in the next subsection. Unlike in the long-run literature started by Bansal and Yaron (2004), there is no persistent component in the consumption growth, because in habit models it is usually of secondary importance (refer e.g., Wachter (2002) versus Wachter (2006)). Heteroskedasticity of the consumption growth is documented by numerous empirical studies starting with Ferson and Merrick (1987).

Because there is substantial evidence about inflation being persistent, logarithmic inflation is modeled with a persistent component:

\[
\pi_{t+1} = \bar{\pi} + x^\pi_t + \epsilon^\pi_{t+1},
\]

where \( x^\pi_t \) is a zero-mean persistent component following the dynamics specified in subsection 2.3 and \( \epsilon^\pi_{t+1} \) is a heteroskedastic shock explained in the next subsection. The persistence and heteroskedasticity of inflation are motivated by voluminous empirical work starting with Engle (1983).
2.2 Preferences

The representative agent maximizes expected utility as in Campbell and Cochrane (1999):

\[ E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t - H_t)^{1-\gamma} - 1}{1-\gamma}, \]

where \( C_t \) is the consumption at time \( t \) and \( H_t \) is a slow moving aggregate of past consumption, satisfying \( H_t < C_t \). The motivation behind the habit preferences is that the agent becomes sensitive to relatively mild fluctuations in consumption, because, due to the slow moving nature of the habit, fluctuations in the surplus \( C_t - H_t \) are percentually larger than fluctuations in absolute consumption.\(^9\)

The main reason for using the habit utility instead of the Epstein-Zin utilty, popular in the previous literature on the topic (Burkhardt and Hasseltoft, 2012; Song, 2014), is that, as shown by several papers starting with Wachter (2006), unlike most models with Epstein-Zin utility, habit models produce a more realistic term structure: in particular, they generate on average an upward-sloping real yield curve, as discussed above, an important feature of the US and the UK term structure.

Instead of modeling habit, \( H_t \), following most of the external habit literature starting from Campbell and Cochrane (1999), in order to ensure that the condition \( C_t > H_t \) always holds, the inverse surplus ratio \( Q_t = \frac{C_t}{C_t - H_t} \) is modeled. In this setting, the local coefficient of relative risk-aversion is equal to \( \gamma Q_t \). Following Campbell and Cochrane (1999) and Bekaert and Engstrom (2014), the inverse surplus ratio is an autoregressive process with its innovations being the same as innovations to the consumption growth:

\[ q_{t+1} = \bar{q} + \rho_q (q_t - \bar{q}) + \gamma_q \epsilon_{t+1}, \]

where \( \bar{q} \), \( \rho_q \), and \( \gamma_q \) are constants.

\(^9\)Following most of the habit literature after Campbell and Cochrane (1999), the difference habit (that is the utility is determined by \( C_t - H_t \)) is used instead of the ratio habit (that is the utility determined by \( \frac{C_t}{H_t} \)) used, for instance, in Abel (1999), although its economic interpretation is less obvious: it allows to interpret the habit as a weighted average of past consumption shocks with weights declining over time only as an approximation. This is because, as Chan and Kogan (2002) point out, replicating modern asset pricing moments using the ratio habit utility is difficult and this paper deals with the relatively recent data starting from 1970.
The specification in (4) differs from the Campbell and Cochrane (1999)’s. In (4) the sensitivity of the inverse surplus ratio to the consumption shocks, $\gamma_q$, is constant, and the action comes from the time-varying volatility of the consumption growth. This specification can be interpreted as a ”time-varying amount of risk” specification. Instead in Campbell and Cochrane (1999), the volatility of the consumption growth is constant, and the sensitivity of the inverse-surplus ratio to consumption growth shocks is time-varying. This specification can be interpreted as a ”time-varying prices of risk” specification. Ermolov (2014) shows that the ”time-varying amount of risk” specification has important advantages in terms of the term structure modeling. Economically, positive consumption shocks are expected to decrease the inverse surplus ratio, and thus the coefficient $\gamma_q$ is expected to be negative. I also experimented with a component unrelated to consumption in (4), as in Bekaert, Engstrom, and Grenadier (2010), but it was empirically insignificant.

2.3 Macroeconomic Shocks

The consumption and inflation shocks are modeled as a linear combination of two independent shocks:

$$
\epsilon_{t+1}^{g} = \sigma_d^{d} u_{t+1}^{d} + \sigma_s^{s} u_{t+1}^{s},
$$

$$
\epsilon_{t+1}^{\pi} = \sigma_d^{d} u_{t+1}^{d} - \sigma_s^{s} u_{t+1}^{s},
$$

$$
Var(u_{t+1}^{d}) = Var(u_{t+1}^{s}) = 1, Cov(u_{t+1}^{d}, u_{t+1}^{s}) = 0,
$$

(5)

where $\sigma_d^{d}$, $\sigma_s^{s}$, $\sigma_d^{d}$, and $\sigma_s^{s}$ are positive constants. $u_{t+1}^{d}$ is a shock which moves consumption and inflation in the same direction. In real terms, it is intuitive to refer to such shocks as ”demand shocks”. $u_{t+1}^{s}$ is a shock which moves consumption and inflation in the opposite directions and thus makes nominal bonds a riskier asset. In real terms, it is intuitive to refer to such shocks as ”supply shocks”. $Var(u_{t+1}^{d}) = Var(u_{t+1}^{s}) = 1$ is just a scaling assumption necessary for the empirical identification.

Note that, if $u_{t+1}^{d}$ and $u_{t+1}^{s}$ are heteroskedastic, specification (5) will imply a time-varying conditional covariance between consumption growth and inflation shocks, as is observed in the data (Burkhardt and Hasseltoft, 2012):

$$
Cov_t(\epsilon_{t+1}^{g}, \epsilon_{t+1}^{\pi}) = \sigma_d^{d} \sigma_d^{d} Var_t(u_{t+1}^{d}) - \sigma_s^{s} \sigma_s^{s} Var_t(u_{t+1}^{s}).
$$

(6)

From (6), in the demand shock environment the conditional covariance between consumption growth
growth and inflation is relatively high (relatively large $\text{Var}_t(u_{t+1}^d)$), while in the supply shock environment (relatively large $\text{Var}_t(u_{t+1}^s)$) the covariance is relatively low, making nominal bonds riskier. Depending on the estimated model parameters, the covariance could also switch the sign.

The key innovation of this paper compared to the previous structural literature investigating the macroeconomic risk of nominal assets (Burkhardt and Hasseltoft, 2012; Campbell, Pflueger, and Viceira, 2014; Song, 2014) is twofold. First, demand and supply shocks are modeled directly instead of modeling consumption growth and inflation, which, as will be shown in the next session, clarifies the economic intuition behind macroeconomic risk of nominal bonds. Second, macroeconomic shocks are modeled using the BEGE dynamics of Bekaert and Engstrom (2009). Under this framework, both demand and supply shocks consist of two zero-mean shocks: the first ”good environment” shock is bounded from the left and generates positive skewness, while the second ”bad environment” shock is bounded from the right and generates negative skewness. These shocks have time-varying higher order moments which, as shown by Bekaert and Engstrom (2009), help to generate rich fundamentals dynamics capturing important conditional non-Gaussian features in the data.\(^\text{10}\)

Formally, the demand and supply shocks are modeled as a component model of demeaned gamma shocks:

\[
\begin{align*}
    u_{t+1}^d &= \sigma_p^d \omega_{p,t+1}^d - \sigma_n^d \omega_{n,t+1}^d, \\
    u_{t+1}^s &= \sigma_p^s \omega_{p,t+1}^s - \sigma_n^s \omega_{n,t+1}^s, \\
    \omega_{p,t+1}^d &\sim \Gamma(p_t^d, 1) - p_t^d, \\
    \omega_{n,t+1}^d &\sim \Gamma(n_t^d, 1) - n_t^d, \\
    \omega_{p,t+1}^s &\sim \Gamma(p_t^s, 1) - p_t^s, \\
    \omega_{n,t+1}^s &\sim \Gamma(n_t^s, 1) - n_t^s,
\end{align*}
\]

(7)

where $\Gamma(x, y)$ denotes a gamma-distributed random variable with the shape parameter $x$ and the scale parameter $y$. Figure 2 illustrates the construction of the BEGE distribution from its good and bad components. Top panel of Figure 2 plots the probability density function of the good component corresponding to $\sigma_p \omega_{p,t+1}$ in equation (7). The distribution is cut

\(^{10}\)Bekaert, Engstrom, and Ermolov (2014a,b) find empirical evidence in favor of BEGE dynamics in the time series of US aggregate stock returns. This supports the fundamental dynamics, because in the model, as will be shown in the next section, stock returns under a first-order approximation of the equilibrium price-dividend ratio expression are a component model of gamma shocks.
from the left and has a right tail generating positive skewness. The middle panel of Figure 2 plots the probability density function of the bad component corresponding to $\sigma_n \omega_{n,t}$ in (7). The distribution is cut from the right and has a left tail generating negative skewness. The bottom panel of Figure 2 plots the probability density function of the component model of the good and bad components from top and middle panels. The distribution has now both tails and, as will be shown below, the skewness can be both positive and negative and will be determined by scale and shape parameters of the two demeaned gamma distributions.

The heteroskedasticity of the shocks is modeled through the time-varying shape parameters of the gamma distributions\textsuperscript{11}, which follow an autoregressive square-root-type volatility processes, as in Gourieroux and Jasiak (2006):

\begin{align*}
    p_{t+1}^d &= \bar{p}^d + \rho_p^d (p_t^d - \bar{p}^d) + \sigma_{pp}^d \omega_{p,t+1}^d, \\
    n_{t+1}^d &= \bar{n}^d + \rho_n^d (n_t^d - \bar{n}^d) + \sigma_{nn}^d \omega_{n,t+1}^d, \\
    p_{t+1}^s &= \bar{p}^s + \rho_p^s (p_t^s - \bar{p}^s) + \sigma_{ps}^s \omega_{p,t+1}^s, \\
    n_{t+1}^s &= \bar{n}^s + \rho_n^s (n_t^s - \bar{n}^s) + \sigma_{ns}^s \omega_{n,t+1}^s. 
\end{align*}

(8)

In (8) the volatility shocks are the same as the shocks to realizations in (7) capturing volatility clustering in data. Also note that in (8), assuming $\rho_{p/n} > \sigma_{pp/nn}$ ensures strict positivity of the shape parameters. This is one of the advantages of the BEGE dynamics over the traditional discrete square-root processes. Figure 3 illustrates the volatility dynamics in the model. The top panel shows the BEGE probability density function for the case where the good component is relatively more important (large $p_t$): this corresponds to the case where the good component, as graphed in the top panel of Figure 2, is more important and the distribution is positively skewed. The bottom panel plots the BEGE probability density function for the case where the bad component is relatively more important (large $n_t$): this corresponds to the case where the bad component, shown in the middle panel of Figure 2, is more important and the distribution is negatively skewed.

Using the properties of gamma distributions, under the BEGE dynamics, the covariance between consumption growth and inflation innovations is:

\begin{align*}
    Cov_t(\epsilon_{t+1}^g, \epsilon_{t+1}^\pi) &= \sigma_d^g \sigma_d^\pi (\sigma_n^d)^2 p_t^d + \sigma_g^d \sigma_n^d (\sigma_n^d)^2 n_t^d - \sigma_g^s \sigma_n^s (\sigma_n^s)^2 p_t^s - \sigma_g^s \sigma_n^s (\sigma_n^s)^2 n_t^s. 
\end{align*}

(9)

\textsuperscript{11}Alternatively, heteroskedasticity of the shocks can be modeled through the time-varying scale parameters, but in this case the empirical fit tends to be worse and the theoretical expressions more complicated.
The intuition in (9) is the same as in (6): demand shocks push consumption growth and inflation in the same direction, and, thus, higher volatility of these shocks (which in the model consists of a good and a bad component proportional to \( p_t^d \) and \( n_t^d \), respectively) increases the conditional covariance between the consumption growth and inflation. Supply shocks push consumption growth and inflation in opposite directions, and, thus, higher volatility of these shocks (which in the model again consists of a good and a bad component proportional to \( p_t^s \) and \( n_t^s \), respectively) decreases the conditional covariance between consumption growth and inflation.

The BEGE framework has several attractive features. First, economically having a good and bad component is intuitive in characterizing demand and supply shocks (think, e.g., of high commodity prices as a bad supply environment and low commodity prices as a good supply environment). Second, compared to the regime-switching literature, which predominantly relies on Bansal and Zhou (2002)’s approximation to compute asset prices, BEGE provides intuitive closed-form solutions. Third, as will be shown in section 4, BEGE dynamics can be estimated in a relatively efficient manner providing, unlike Burkhardt and Hasseltoft (2012) and Campbell, Pflueger, and Viceira (2014), an endogenous identification and estimation of the macroeconomic environment.

In addition to driving consumption growth and inflation shocks, demand and supply shocks also determine the dynamics of expected inflation in (2):

\[
x_{t+1}^\pi = \rho_{x\pi} x_t^\pi + \gamma_{x\pi} \left( \sigma_{u_{t+1}}^{d} - \sigma_{u_{t+1}}^{s} \right) + \gamma_{x\pi} \sigma_{u_{t+1}}^{d} + \sigma_{x\pi} \epsilon_{t+1}^{x\pi},
\]

where \( \gamma_{x\pi} \) and \( \gamma_{x\pi}^{d} \) are constants. In (10) demand and supply shocks can affect the expected inflation differently. I also introduce \( \epsilon_{t+1}^{x\pi} \sim N(0,1) \), a shock unrelated to demand and supply shocks.

### 2.4 Financial Assets

The two main financial assets considered are nominal bonds and an aggregate equity claim. The bonds are risk-free zero-coupon securities with a prespecified nominal payment of 1$ at maturity.

Aggregate equity is modeled as a claim to aggregate dividends. The log aggregate real
dividend growth follows a constant mean heteroskedastic process:

\[ \Delta d_{t+1} = \bar{g} + \gamma_d (\sigma_d^d u_{t+1}^d + \sigma_d^s u_{t+1}^s) + \gamma_d \sigma_d^d u_{t+1}^d + \sigma_d \epsilon_{t+1}^d, \]  

(11)

where \( \gamma_d, \gamma_d^s, \) and \( \sigma_d \) are constants and \( \epsilon_{t+1}^d \sim N(0,1) \).

There are four dividend modeling assumptions in (11). First, following the most of the habit literature, starting from Campbell and Cochrane (1999), the mean of the dividend growth is the same as the mean of consumption growth. Second, following numerous papers starting with Campbell (1986), the dividend claim is priced instead of the consumption claim to compare the model predictions with equity data. This implicitly corresponds to the fact that the agent has additional sources of income, such as labour income. Third, demand and supply shocks can affect the dividend growth differently. Finally, \( \epsilon_{t+1}^d \), a shock unrelated to demand and supply shocks, is also included.

3 Asset Pricing Implications

This section derives bond and equity prices and shows how the model can generate time-varying stock and bond return correlations. The stochastic discount factor is standard for an external habit model: the agent does not take into account the impact of her consumption choice on the habit stock. Thus, the stochastic discount factor, \( M_{t+1} \) follows from the first order conditions of maximizing the utility in (3) with respect to consumption at times \( t \) and \( t+1 \):

\[ M_{t+1} = \beta e^{-\gamma_{t+1} + q_{t+1} q_t}. \]  

(12)

The economic interpretation of (12) is that the marginal utility is low in times of high consumption growth and a low inverse surplus ratio.

Risk premia on all assets are determined by the covariance of their returns with the innovations to the stochastic discount factor. In the model, the innovations to the real log
stochastic discount factor are:
\[ m_{t+1} - E_t m_{t+1} = a_{dp} \omega^d_{p,t+1} + a_{dn} \omega^d_{n,t+1} + a_{sp} \omega^s_{p,t+1} + a_{sn} \omega^s_{n,t+1}, \]
\[ a_{dp} = \sigma^d \sigma^g \gamma (-1 + \gamma_q) < 0, \]
\[ a_{dn} = -\sigma^d \sigma^g \gamma (-1 + \gamma_q) > 0, \]
\[ a_{sp} = \sigma^s \sigma^g \gamma (-1 + \gamma_q) < 0, \]
\[ a_{sn} = -\sigma^s \sigma^g \gamma (-1 + \gamma_q) > 0, \]
where the signs are for the calibrated region. The interpretation of equation (13) is that shocks which increase consumption or the surplus ratio decrease marginal utility and shocks which decrease consumption increase marginal utility.

As usual, the nominal stochastic discount factor is a linear transformation of the real stochastic discount factor:
\[ m_t^\$ = m_{t+1} - \pi_{t+1}. \]

### 3.1 Bonds

In the model, the logarithmic price of a bond with maturity \( n \) at time \( t \) can be written as an affine function of the state variables:
\[ \ln P^s_{n,t} = C^s_n + X^\pi_n x^\pi_t + Q^s_n q_t + P^d_{n} p^d_t + N^d_{n} n^d_t + P^s_{n} p^s_t + N^s_{n} n^s_t, \]
where \( C^s_n, X^\pi_n, Q^s_n, P^d_{n}, N^d_{n}, P^s_{n}, \) and \( N^s_{n} \) are constants defined in Appendix A.

The price of a bond in equation (15) reflects three economic effects. First, the loading on expected inflation, \( X^\pi_n \), is negative, because higher inflation reduces the value of the nominal payout. Second, the loading on the inverse surplus ratio, \( Q^s_n \), is also negative. This is an intertemporal smoothing effect: when the inverse surplus ratio is high, it is expected to decrease in future so that marginal utility is relatively high today, therefore the agent prefers to consume now instead of saving for the future and thus bond prices are low. Finally, the volatility loadings \( P^d_{n}, N^d_{n}, P^s_{n}, \) and \( N^s_{n} \) are positive corresponding to the standard precautionary savings effect: in times of high uncertainty, the agent is willing to hedge uncertainty through investing in bonds.

For convenience, from now on I focus on continuously compounded yields:
\[ y^s_{n,t} = -\frac{1}{n} \ln P^s_{n,t}. \]
3.2 Equity

In the model, the price-dividend ratio can be written as:

\[
P_t \div D_t = \sum_{n=1}^{\infty} \exp(\left(C_n^e + Q_n^e q_t + P_n^{de} p_t^d + N_n^{de} n_t^d + P_n^{se} p_t^s + N_n^{se} n_t^s\right) > 0),
\]

where \(Q_n^e, P_n^{de}, N_n^{de}, P_n^{se}, \) and \(N_n^{se}\) are constants defined in Appendix A.

Equity pricing reflects two economic effects. First, the loading on the inverse-surplus ratio, \(Q_n^e\), is negative. This is the same intertemporal smoothing effect as for bonds: when the inverse-surplus ratio is high, the agent wants to consume now instead of saving for the future and thus equity prices are low. Second, the \(P_n^{de}, N_n^{de}, P_n^{se},\) and \(N_n^{se}\) coefficients are positive corresponding to the precautionary savings effect. In the model, increased volatility (higher \(p_t^d, n_t^d, p_t^s, \) and \(n_t^s\)) has two effects on equity prices. First, it decreases risk-free rates increasing equity prices. Second, it increases the risk-premium decreasing equity prices. The intuition is that as consumption growth volatility increases, the covariance between the consumption growth and dividends also increases, making the dividend stream covary more negatively with the stochastic discount factor and thus riskier. This implicitly assumes that in equation (11) the loading \(\gamma_d\) is greater than a certain threshold and the loading \(\gamma_d^t > 0\). This is later shown to be true in the data. The first effect is the dominant effect in the model.\(^{12}\)

3.3 Stock and Bond Return Correlation

The main focus of this paper is the time-varying conditional correlation between stock and bond returns:

\[
Corr_t(r_{n,t+1}^b, r_{t+1}^e) = \frac{Cov_t(r_{n,t+1}^b, r_{t+1}^e)}{\sqrt{Var_t(r_{n,t+1}^b)Var_t(r_{t+1}^e)}},
\]

where \(r_{n,t+1}^b\) is the logarithmic return on holding an \(n\) period bond from time \(t\) to time \(t + 1\),

\(^{12}\)There is no conclusive evidence on the role of volatility for equity prices. Theoretically, Wu (2001) and Bansal, Kiku, and Yaron (2010) argue in favor of a negative relationship, while Pastor and Veronesi (2006) or Bekaert, Engstrom, and Xing (2009) suggest a positive relationship. Empirically, results in Poterba and Summers (1986) imply an insignificant relationship, while Segal, Shaliastovich, and Yaron (2014) find that both relationships are possible.
\[ \ln \frac{P_{n,t+1}^s}{P_{n,t}^s}, \quad \text{and} \quad r_{t+1}^e \] is the logarithmic return on holding the aggregate equity claim from time \( t \) to time \( t + 1 \), \( \Delta d_{t+1} + \ln(1 + \frac{P_{t+1}}{D_{t+1}}) - \ln \frac{P_t}{D_t} \).

The term of the primary interest in (18) is the time-varying conditional covariance, \( \text{Cov}_t(r_{n,t+1}^b, r_{t+1}^e) \), as it determines the sign of the stock and bond return correlation:

\[ \text{Cov}_t(r_{n,t+1}^b, r_{t+1}^e) = \text{Cov}_t(r_{n,t+1}^b - E_t(r_{n,t+1}^b), r_{t+1}^e - E_t(r_{t+1}^e)). \tag{19} \]

The unexpected return on holding an \( n \) period bond from time \( t \) to time \( t + 1 \) in the model, \( r_{n,t+1}^b - E_t r_{n,t+1}^b \), consists of four terms corresponding to the basic macroeconomic shocks driving the BEGE dynamics in (7), \( \omega_{p,t+1}^d, \omega_{n,t+1}^d, \omega_{p,t+1}^s, \omega_{n,t+1}^s \), and the expected inflation shock, \( \epsilon_{t+1}^x \):

\[
\begin{align*}
\omega_{p,t+1}^d - E_t \omega_{p,t+1}^d &= \left( X_{n-1}^\pi (\gamma_n + \gamma_{n^d}) \sigma_n^d \sigma_p^d + Q_{n-1}^\sigma \gamma_n \sigma_p^d \sigma_n^d + P_{n-1} \gamma_n \sigma_n^d \right) \omega_n^d + \\
&\quad \left( -X_{n-1}^\pi (\gamma_n + \gamma_{n^d}) \sigma_n^d \sigma_n^d - Q_{n-1}^\sigma \gamma_n \sigma_p^d \sigma_n^d + N_{n-1} \gamma_n \sigma_n^d \right) \omega_n^d + \\
&\quad \left( -X_{n-1}^\pi \gamma_n \sigma_p^d \sigma_n^d + Q_{n-1}^\sigma \gamma_n \sigma_p^d \sigma_n^d + P_{n-1} \sigma_p^d \right) \omega_p^d + \\
&\quad \left( X_{n-1}^\pi \gamma_n \sigma_p^d \sigma_n^d - Q_{n-1}^\sigma \gamma_n \sigma_p^d \sigma_p^d + N_{n-1} \sigma_n^d \right) \omega_p^d + \left( X_{n-1}^\pi \sigma_n^d - \epsilon_{t+1}^x \right), \tag{20}
\end{align*}
\]

where the signs are for the region estimated later in the paper.

As can be seen from the first line of (20), the impact of a good demand shock (\( \omega_{p,t+1}^d \)) on the unexpected bond return consists of three components. The first component, \( X_{n-1}^\pi (\gamma_n + \gamma_{n^d}) \sigma_n^d \sigma_p^d \), corresponds to the increased expected inflation decreasing nominal bond prices and thus returns on nominal bonds. The second component, \( Q_{n-1}^\sigma \gamma_n \sigma_p^d \sigma_n^d \), corresponds to the increased intertemporal smoothing motive (note from (4) that a positive good demand shock decreases the inverse surplus ratio) increasing bond prices. The final component, \( P_{n-1} \sigma_p^d \), corresponds to the increased precautionary savings motive (see equation (8), where \( \omega_{p,t+1}^d \) increases future volatility) increasing bond prices. The impact of a bad demand shock (\( \omega_{n,t+1}^d \)) on the second line of (20) follows the same logic.

The impact of a good supply shock (\( \omega_{p,t+1}^s \)) on the unexpected bond return also consists of
three components illustrated on the third line of (20). The first component, \(-X_{n-1}^{s} \gamma_{s} \sigma_{s}^{2} \), corresponds to the decreased expected inflation increasing bond prices. The second component, \(Q_{n-1}^{s} \gamma_{s} \sigma_{s}^{2} \), corresponds to the increased intertemporal smoothing motive increasing bond prices. The final component, \(P_{n-1}^{s} \sigma_{p}^{2} \), corresponds to the increased precautionary savings motive increasing bond prices. The impact of a bad supply shock \((\omega_{n,t+1}^{s})\) on the fourth line of (20) follows the same logic. The independent expected inflation shock on the last line of (20), \(\epsilon_{t+1}^{x} \), is negatively related to the unexpected bond return through the higher inflation decreasing the bond prices.

The logic above allows to sign the impact of macroeconomic shocks on the unexpected bond returns. The unexpected bond return loads positively on the bad demand and good supply shocks and negatively on the good demand and bad supply shocks. Thus, nominal bonds are a good hedge against demand shocks and a bad hedge against supply shocks. Panel A of Table 1 summarizes the effects.

The unexpected return on holding the aggregate equity claim from time \(t\) to time \(t+1\) in the model, \(r_{t+1}^{e} - E_{t}r_{t+1}^{e}\), consists of four terms corresponding to the basic shocks driving the BEGE dynamics in (7), \(\omega_{p,t+1}^{d}, \omega_{n,t+1}^{d}, \omega_{p,t+1}^{s}, \omega_{n,t+1}^{s}\), and the dividend shock, \(\epsilon_{t+1}^{d}\):

\[
r_{t+1}^{e} - E_{t}r_{t+1}^{e} \approx \left( (\gamma_{d} + \gamma_{d}^{d}) \sigma_{d}^{2} \sigma_{p}^{2} + K_{2}^{q} \gamma_{q} \sigma_{g}^{2} \sigma_{p}^{2} + K_{2}^{d} \sigma_{p}^{2} \right) \omega_{p,t+1}^{d} +
\left( -(-\gamma_{d} + \gamma_{d}^{d}) \sigma_{d}^{2} \sigma_{p}^{2} - K_{2}^{q} \gamma_{q} \sigma_{g}^{2} \sigma_{p}^{2} + K_{2}^{d} \sigma_{p}^{2} \right) \omega_{n,t+1}^{d} +
\left( \gamma_{d} \sigma_{g}^{2} \sigma_{p}^{2} + K_{2}^{q} \gamma_{q} \sigma_{g}^{2} \sigma_{p}^{2} + K_{2}^{d} \sigma_{p}^{2} \right) \omega_{p,t+1}^{s} +
\left( -(-\gamma_{d} \sigma_{g}^{2} \sigma_{p}^{2} - K_{2}^{q} \gamma_{q} \sigma_{g}^{2} \sigma_{p}^{2} + K_{2}^{d} \sigma_{p}^{2} \right) \omega_{n,t+1}^{s} + \sigma_{d} \epsilon_{t+1}^{d},
\]

where \(K_{2}^{q}, K_{2}^{d}, K_{2}^{n}, K_{2}^{p}, K_{2}^{s}\) are constants specified in Appendix A and where the signs are for the region estimated later in the paper. Note that, in order to clarify the intuition,
(21) is a log-linearization of the equilibrium solution.  

As can be seen from the first line of (21), the impact of a good demand shock ($\omega_{p,t+1}^d$) on the unexpected equity return consists of three components. The first component, $(\gamma_d + \gamma_{de})\sigma_d^g\sigma_p^d$, corresponds to the increased dividend payments increasing equity returns. The second component, $K^q_{d}e^d\sigma_d^g\sigma_p^d$, corresponds to the increased intertemporal smoothing motive (note from (4) that a positive good demand shock decreases the inverse surplus ratio) increasing equity prices. The final component, $K^p_{d}e^d\sigma_p^d$, corresponds to the increased precautionary savings motive (see equation (8), where $\omega_{p,t+1}^d$ increases future volatility) increasing equity prices. The impact of a bad demand shock ($\omega_{n,t+1}^d$) on the second line of (21) follows the same logic.

The impact of a good supply shock ($\omega_{s,t+1}^d$) on the unexpected equity return also consists of three components illustrated on the third line of equation (21). The first component, $\gamma_d\sigma_s^d\sigma_p^s$, corresponds to the increased dividend payments increasing equity returns. The second component, $K^q_{s}e^s\sigma_s^g\sigma_p^s$, corresponds to the increased intertemporal smoothing motive increasing equity prices. The final component, $K^p_{s}e^s\sigma_p^s$, corresponds to the increased precautionary savings motive increasing equity prices. The impact of a bad supply shock ($\omega_{n,t+1}^s$) on the fourth line of equation (21) follows the same logic. The independent dividend shock on the last line of (21), $\epsilon_{t+1}^d$ is positively related to the unexpected equity return through the higher dividend payoff increasing the equity return.

The logic above allows to sign the impact of macroeconomic shocks on the unexpected equity returns. The unexpected equity return loads positively on the good demand and supply shocks and negatively on the bad demand and supply shocks. Thus, equity is a bad hedge against both demand and supply shocks. Panel B of Table 1 summarizes the effects.

By combining the conclusions of (20) and (21), it can be seen that demand shocks decrease and supply shocks increase the conditional covariance between stock and bond returns:

$$\text{Cov}_t(r_{n,t+1}^b, r_{t+1}^e) \approx \alpha_{p^d} p_t^d + \alpha_{n^d} n_t^d + \alpha_{p^s} p_t^s + \alpha_{n^s} n_t^s. \quad (22)$$

where $\alpha_{p^d}$, $\alpha_{n^d}$, $\alpha_{p^s}$, and $\alpha_{n^s}$ are constants, which can be computed from the coefficients

---

13It is important to point out that (21) is a log-linearization of the exact equilibrium solution taken in order to clarify the economic intuition and in that sense is very different from the vast majority of the literature following Campbell and Shiller (1988), where the return definition is log-linearized BEFORE computing the equilibrium.
in (20) and (21) and, as all coefficients in this section, are signed for the economically sensible region. Note from (22) that the time-variation in covariance is driven purely by macroeconomic state variables \((p_t^d, n_t^d, p_t^s, n_t^s)\), while the preference parameters affect the sensitivity of the covariance to these fluctuations through constant \(\alpha\)-coefficients. Panel C of Table 1 summarizes the effects.

4 Estimation

The estimation is conducted in two stages. First, the macroeconomic dynamics is estimated purely from macroeconomic data (no financial data). Second, given the dynamics of macroeconomics shocks, preference parameters are estimated using the generalized method of moments (GMM) in order to fit asset prices.

4.1 Data

The data is standard US macroeconomic and financial data. The Working (1960)-autocorrelation adjusted per-capita consumption growth of non-durables and services is from the National Income and Product Accounts (NIPA) website. Inflation is the change in the seasonally adjusted consumer price index for all urban customers from the Federal Reserve Bank of St.Louis website. The inflation forecasts are from the Survey of Professional Forecasters (SPF) obtained from the Federal Reserve Bank of Philadelphia website. The bond prices are from Gürkaynak, Sack, and Wright (2010a). Excess stock returns are from the Kenneth French’s data library. The sample period is quarterly from 1970Q1 to 2012Q4, because the inflation forecasts are not available before that.

Following Longstaff and Piazzesi (2004), the unsmoothed dividends are used. The economic motivation for using the unsmoothed dividends is, as is discussed in details in Longstaff and Piazzesi (2004), that firms artificially smooth dividends for reasons such as signaling content or resolving agency conflicts. Because dividends only affect equities, modeling their dynamics accurately is important for producing realistically low stock and bond return correlations. The motivation for this is that dividends are a very volatile cash flow channel which affects only equities (not bonds) and thus modeling the dividend dynamics accurately might be important for reproducing imperfect stock and bond return correlation.
Quarterly unsmoothed after-tax seasonally adjusted per-capita dividends are constructed in three stages. First, the average tax rate for each year is computed as $\tau = 1 - \frac{\text{Corporate Profits After Tax}}{\text{Corporate Profits Before Tax}}$, where seasonally adjusted corporate profits before and after tax are from NIPA. Second, quarterly corporate profits before tax from NIPA are adjusted using the corporate tax rate for this year and the population for the corresponding quarter (also from NIPA). Third, following Lee, Myers, and Swaminathan (1999), Bakshi and Chen (2005), and Longstaff and Piazzesi (2004), the unsmoothed dividend growth is computed from the after-tax per-capita corporate profits assuming a constant profit payout ratio of 50%, which is close to the historical average. Real dividend growth is computed by adjusting nominal dividend growth for inflation.

4.2 Macroeconomic Dynamics

The macroeconomic dynamics in the model consists of the consumption growth, inflation, expected inflation, and dividend growth processes. While all 4 time series are driven by the same fundamental shocks ($\omega_{\text{p,t+1}}, \omega_{\text{n,t+1}}, \omega_{\text{p,n,t+1}}, \omega_{\text{n,n,t+1}}$), computational complexity prevents joint estimation and the dynamics in equations (1), (2), (5), (7), (8), (10), and (11) are estimated in two stages. First, the dynamics of demand and supply shocks are estimated from consumption growth and inflation time series: this includes the estimation of the system parameters, values of demand and supply shocks, and filtering the expected values of the macroeconomic state variables ($p^d_t, n^d_t, p^s_t, n^s_t$). Then the demand and supply shock time series are used to estimate the dividend growth and expected inflation processes.

4.2.1 Consumption Growth and Inflation

The consumption growth and inflation dynamics in equations (1), (2), (5), (7), and (8) are governed by 18 parameters: $\bar{g}, \bar{\pi}, \sigma^g, \sigma^s, \sigma^d, \sigma^s, \bar{p}, \bar{d}, \rho^d, \rho^s, \bar{\rho}, \bar{\sigma}, \bar{\rho}, \sigma^s, \bar{n}, \sigma^s, \bar{\rho}, \sigma^s$. Due to the computational complexity of the system (observations being component models of 4 gamma distributions), the parameters are estimated in three stages: estimating consumption growth and inflation shocks ($\epsilon^g_t$ and $\epsilon^s_t$), obtaining demand and supply shocks ($u^d_t$ and $u^s_t$) from consumption growth and inflation shocks, and estimating the parameters of demand and supply shock dynamics.

First, consumption growth shocks ($\epsilon^g_{t+1}$) are obtained, following equation (1), by regressing
the observed consumption growth on a constant. This directly gives the estimate of the consumption growth mean, $\bar{g}$. Inflation shocks, $\epsilon_{\pi t+1}$, are filtered, following equation (2), by regressing the observed inflation on a constant and the demeaned expected inflation. This also produces the estimate of the inflation mean, $\bar{\pi}$. Expected inflation is constructed as the linear ordinary least squares (OLS) projection of quarterly inflation realizations on the previous quarter inflation forecasts for that quarter from the SPF and the realized inflation value from the previous quarter. This set of predictors minimizes the Bayesian information criterion in a forecasting exercise described in Appendix B.

Second, the demand and supply shocks are extracted from the consumption growth and inflation shocks by matching unconditional moments of consumption growth and inflation using GMM. The match is performed based on the unconditional second and third order moments (including the cross-moments) of consumption growth and inflation. Given the structure in (5), the values of the moments are:

$$E(u_{g t}^2) = (\sigma_d^g)^2 + (\sigma_s^g)^2,$$
$$E(u_{\pi t}^2) = (\sigma_d^\pi)^2 + (\sigma_s^\pi)^2,$$
$$E(u_{g t} u_{\pi t}^3) = \sigma_d^g (\sigma_d^\pi)^3 Uskw(u_{d t}^d) + (\sigma_s^g)^3 Uskw(u_{s t}^s),$$
$$E(u_{g t} u_{\pi t}^2) = \sigma_d^g (\sigma_d^\pi)^2 Uskw(u_{d t}^d) - (\sigma_s^g)^2 \sigma_s^\pi Uskw(u_{s t}^s),$$
$$E(u_{g t}^2 u_{\pi t}^3) = (\sigma_d^g)^2 (\sigma_d^\pi)^2 Uskw(u_{d t}^d) - (\sigma_s^g)^2 \sigma_s^\pi Uskw(u_{s t}^s),$$
$$E(u_{g t}^2 u_{\pi t}^2) = \sigma_d^g (\sigma_d^\pi)^2 Uskw(u_{d t}^d) + (\sigma_s^g)^2 \sigma_s^\pi Uskw(u_{s t}^s),$$
$$E(u_{g t}^2) = \sigma_d^g (\sigma_d^\pi)^2 Uskw(u_{d t}^d) - (\sigma_s^g)^2 \sigma_s^\pi Uskw(u_{s t}^s),$$

where $Uskw$ is the unscaled (by variance to the power of three over two) skewness. The weighting matrix for the GMM optimization is computed by bootstrapping 10,000 time series of historical length consisting of $u_{g t}^d$ and $u_{\pi t}^s$ and computing the moments in (23) for each of the time series. The inverse of the obtained covariance matrix is then used as a weighting matrix. The GMM is performed using the full covariance matrix. Because there are 7 moments in (23) and only 6 parameters to match ($\sigma_d^g, \sigma_s^g, \sigma_d^\pi, \sigma_s^\pi, Uskw(u_{d t}^d), Uskw(u_{s t}^s)$), an overidentification test can be performed. Given the estimated parameters $\sigma_d^g, \sigma_s^g, \sigma_d^\pi,$ and $\sigma_s^\pi,$ at each time point $t$ demand and supply shocks are extracted from consumption growth.

14The use of SPF forecasts is motivated by Ang, Bekaert, and Wei (2007).
and inflation shocks:

\[
\begin{bmatrix}
    u_t^d \\
    u_t^s
\end{bmatrix} = \begin{bmatrix}
    \sigma_d^d & \sigma_s^d \\
    \sigma_d^s & -\sigma_s^s
\end{bmatrix}^{-1} \begin{bmatrix}
    g_t \\
    \pi_t
\end{bmatrix}.
\]

Note that if demand and supply shocks would be modeled using static Gaussian distributions, demand and supply shocks would not be identified from consumption growth and inflation shocks, because there would be only three moments \(E(u_t^d 2), E(u_t^s 2), E(u_t^d u_t^s)\) to use and four parameters \((\sigma_d^d, \sigma_s^d, \sigma_d^s, \sigma_s^s)\) to estimate.

The third and the final stage is estimating the parameters of demand and supply shocks: \(\bar{p}^d, \rho_p^d, \sigma_{pp}^d, \bar{n}^d, \rho_n^d, \sigma_{nn}^d, \bar{p}^s, \rho_p^s, \sigma_{pp}^s, \bar{n}^s, \rho_n^s, \sigma_{nn}^s\). This is done for demand and supply time series separately (as they are independent) using maximum likelihood estimation (MLE).

The maximum likelihood estimation is used instead of a moment matching technique, such as GMM, because in order to compute conditional covariances and correlations in equations (18) and (22), the values of the state variables at each time point are needed, and moments matching techniques do not directly provide them.

Each of the demand and supply time series follows a stochastic volatility model, where each observation consists of two gamma distributed components. Because the exact likelihood function is not available in closed form, the characteristic function domain approximate maximum likelihood (AML) estimation of Bates (2006) is used. This is a fast but still a relatively accurate numerical method.

The details of the estimation are in Appendix C and here only the informal intuition is presented. Only the demand shock estimation is considered, as the supply shock estimation is identical. The system to estimate is:

\[
\begin{align*}
    u_{t+1}^d &= \sigma_p^d \omega_{p,t+1}^d - \sigma_n^d \omega_{n,t+1}^d, \\
    \omega_{p,t+1}^d &\sim \Gamma(p_t^d, 1) - p_t^d, \\
    \omega_{n,t+1}^d &\sim \Gamma(n_t^d, 1) - n_t^d, \\
    p_{t+1}^d &= \bar{p}^d + \rho_p^d (p_t^d - \bar{p}^d) + \sigma_{pp}^d \omega_{p,t+1}^d, \\
    n_{t+1}^d &= \bar{n}^d + \rho_n^d (n_t^d - \bar{n}^d) + \sigma_{nn}^d \omega_{n,t+1}^d.
\end{align*}
\]

Note that this econometric specification is different from Bekker, Engstrom, and Ermolov (2014a), who use the GARCH-version of the model. The advantages of the stochastic volatility model over the GARCH model include the better data fit, the more straightforward economic interpretation and the closed-form asset pricing solutions. The disadvantage is the more time-consuming estimation.
where only the time series of demand shock realizations, \( \{ u_t \}_{t=1}^T \) is observed. The estimation consists of three stages:

**Stage 0. Initialization.** At time 0, the distributions of \( p_0^d \) and \( n_0^d \) are initialized with the unconditional distributions of \( p_t^d \) and \( n_t^d \) (not in the initial parameters).

**Stage 1. Computing the likelihood.** The likelihood of the observation \( u_1^d \) given the distributions of \( p_0^d \) and \( n_0^d \) is computed as in lines 1-3 of (24).

**Stage 2. Bayesian updating of the \( p_0^d \) and \( n_0^d \) distributions given the value of \( u_1^d \).** Note from lines 1-3 of (24), that for some values of \( p_0^d \) and \( n_0^d \) the likelihood of observing \( u_1^d \) will be higher than for others. The prior distributions of \( p_0^d \) and \( n_0^d \) are updated so that these values yielding higher likelihoods get more probability weight. For instance, if \( u_1^d \) is very negative, the expected value of \( n_0^d \) is likely to go up and the expected value of \( p_0^d \) is likely to go down.

**Stage 3. Computing the conditional distributions of \( p_1^d \) and \( n_1^d \) given the value of \( u_1^d \).** This is done using the evolution processes in lines 4 and 5 of (24). The distributions of \( n_0^d \) and \( p_0^d \) are available from the previous stage, but also the distributions of \( \omega_{p,1}^d \) and \( \omega_{n,1}^d \) are needed. To compute these distributions, note from the first line of (24) that \( u_{t+1}^d \) is a linear combination of \( \omega_{p,1}^d \) and \( \omega_{n,1}^d \). From lines 2 and 3 of (24), the distributions of \( \omega_{p,1}^d \) and \( \omega_{n,1}^d \) depend on \( p_0^d \) and \( n_0^d \), respectively. Thus, given the distributions of \( p_0^d \) and \( n_0^d \), some of the \( \omega_{p,1}^d \) and \( \omega_{n,1}^d \) combinations will be more likely than others, yielding the distributions of \( \omega_{p,1}^d \) and \( \omega_{n,1}^d \).

The stages 1-3 are then repeated for all the following time points. The total likelihood is computed as the sum of individual likelihoods from stage 1. Note that from stage 2 the estimation also yields the expected values of the state variables, \( p_t^d \) and \( n_t^d \). The estimation is conducted under the restrictions that \( \sigma_{pp}^d < \rho_{p}^d, \, \sigma_{pp}^s < \rho_{s}^d, \, \sigma_{nn}^d < \rho_{n}^d, \, \sigma_{nn}^s < \rho_{n}^s \), preventing shape parameters in equation (8) from going negative, which ensures the accuracy of the closed form solutions for the asset pricing model. Standard errors are computed using a parametric bootstrap, where 250 time series of historical length are simulated using the estimated model parameters and parameters are reestimated for each time series.

Because Bates (2006) is an approximate likelihood technique, the accuracy of the likelihood function and parameter values in the economically sensible region is verified by computing the likelihood on the grid of the state variables, as in Section 13.3.3 of Zucchini and Mac-
Donald (2009). Accurately computing the likelihood function on the grid requires a very high grid precision and thus optimization using that method is practically infeasible.

### 4.2.2 Dividend Growth and Expected Inflation

Given the dynamics of demand and supply shocks estimated in the previous subsection, the dividend growth loadings on supply and demand shocks ($\gamma_d$ and $\gamma_{dd}$) are computed following equation (11) by regressing the real dividend growth on a constant, $\sigma_d^u u_t^d + \sigma_s^u u_t^s$, and $\sigma_d^d u_t^d$. The volatility of the dividend growth shock unrelated to supply and demand shocks, $\sigma_d$, is computed as the standard deviation of the residuals from this regression.

The expected inflation loadings on supply and demand shocks ($\gamma_\pi$ and $\gamma_{d\pi}$) are computed following equation (10) by regressing the demeaned inflation expectation on its lagged value, $\sigma_\pi^d u_t^d - \sigma_\pi^s u_t^s$, and $\sigma_\pi^d u_t^d$. The volatility of the expected inflation shock unrelated to supply and demand shocks, $\sigma_\pi^x$, is computed as the standard deviation of the residuals from this regression. The standard errors for the dividend growth and expected inflation processes parameters are computed using a bootstrap, where 10,000 time series of historical length are simulated.

### 4.3 Preferences

Given the macroeconomic fundamentals parameters, the preference parameters are estimated using the GMM to match asset prices. The preference parameters to estimate are: $\gamma$ (the coefficient of local risk-aversion), $\rho_q$ (inverse surplus ratio persistence), and $\gamma_q$ (sensitivity of inverse surplus ratio to the consumption shocks). The discount rate $\beta$ is fixed to an economically sensible value used in the literature, because, if estimated, it obtains unrealistically low values (around 0.97 quarterly). The value of $\bar{q}$ can be without loss of generality fixed to an arbitrary positive value because it not identified in the model. Note that this does not restrict the level of risk-aversion, as $\gamma$ is not restricted.

The GMM moments used for the estimation can be divided into 3 groups: bond moments, equity moments, and joint equity and bond moments. The bond moments are the quarterly nominal risk-free rate and its variance, the 5 year bond expected excess holding return and the variance of this return. The equity moments are the price-dividend ratio and its variance,
the equity premium mean and the variance of excess equity returns. The cross-moment is the unconditional covariance between 5 year bond and stock returns. The weighting matrix for the GMM optimization is computed by bootstrapping 10,000 time series of historical length and computing the covariance matrix of the moments across these bootstrap samples. The inverse of this covariance matrix is then used as a weighting matrix. I use a diagonal weighting matrix, because it leads to a more balanced moments fit: using the full matrix the $p$-value of the overidentification test is even slightly higher, but some moments tend to be further away from the data counterparts while others are essentially at their historical values.

5 Results

First, macroeconomic shocks (demand and supply) are characterized. Then I discuss the general fit with respect to asset prices and explore model-implied stock and bond return correlations.

5.1 Properties of Macroeconomic Shocks

A major advantage of the methodology in this paper is that, unlike regime-switching models used in the prior literature, it allows to characterize demand and supply shocks in an economically intuitive way. In the first stage of the estimation, demand and supply shocks are extracted from consumption growth and inflation shocks. Table 2 shows the loadings of consumption growth and inflation shocks on demand and supply shocks suggesting that, in line with the real business cycle theory, consumption growth shocks are supply driven ($\sigma_g^s > \sigma_g^d$). Inflation shocks tend to be more demand driven. The overidentification test fails to reject ($p$-value=0.87).

Figure 4 illustrates the model implied variances of demand and supply shocks over time indicating that supply shocks are most pronounced during commodity crises (such as the oil crises in the 1970’s, early 1990’s, and late 2000’s to present). At the same time, the importance of demand shocks has increased in early and mid 2000’s.

$17$Recall that $Var(u_{t+1}^d) = Var(u_{t+1}^s) = 1$. Thus, the coefficients in Table 2 directly reflect the relative importance of the two types of shocks.
The properties of demand and supply shocks are now analyzed in more detail. Panel A of Table 3 shows that the demand shock consists of a Gaussian good component and a strongly non-Gaussian bad component: the good component has a large shape parameter ($\bar{p}^d$) and a small scale parameter ($\sigma_p^d$) and the bad component has a small shape parameter ($\bar{n}^d$) and a large scale parameter ($\sigma_n^d$), which statistically generates a rare-disaster type left tail.\(^{18}\) The bad component is also clearly less persistent than the good component ($\rho_p^d$ is 0.97 and $\rho_n^d$ is 0.76). This is in line with the common wisdom that disaster-type events are relatively short-lived.

Panel B of Table 3 shows that both good and bad supply shock components are relatively Gaussian (large $\bar{p}^s$ and $\bar{n}^s$, small $\sigma_p^s$ and $\sigma_n^s$). Also both good and bad components are relatively persistent, although the bad component is more so ($\rho_p^s=0.90$ and $\rho_n^s=0.99$).

Figure 5 shows the extracted demand shocks and the "good" and "bad" demand variances. The bottom panel illustrates that the good demand variance has been high in 1970’s and then again during early and mid-2000’s. The bad demand variance is characterized by a pronounced peak during the Great Recession, but also by smaller peaks during some other recessions (such as the early 1990’s recession) and periods of weaker economic growth such as in 1986 (e.g., Cacy, Miller, and Robert, 1986). Most of the time, however, good demand variance is relatively more important (has a larger variance) than the bad variance.

Figure 6 repeats the exercise for the supply shocks. It illustrates that the bad supply component is most pronounced during the commodity shocks of 1970’s, early 1990’s and late 2000’s. The good supply component, although on average less important, is dominant in the periods of low commodity prices (late 1980’s, early 1990’s, early 2000’s) and technological progress (late 1990’s and early 2000’s).

In order to gain further economic intuition about the nature of macroeconomic shocks, Figure 7 plots the correlation between filtered shocks and excess returns on 10 Fama-French industry portfolios.\(^{19}\) Correlations with the bad supply shock are negative for all industries. This is not surprising because, as the results in Tables 2 and 3 indicate, consumption growth is

\(^{18}\)This ability to incorporate both a Gaussian (through the combination of large shape and small scale parameters) and a rare disaster-type (through the combination of small shape and large scale parameters) dynamics is one of the strengths of the BEGE methodology.

\(^{19}\)Others-portfolio is excluded, because it contains stocks from many industries not related to each other, making the interpretation of the results difficult. Results with a higher number of industry portfolios (17 and 49) are economically and statistically similar.
supply shock driven and the supply shock is bad supply shock driven. Thus, large realizations of the bad supply shock are likely to correspond to economic recessions, when most of the firms do poorly. Energy sector returns have the highest (the least negative) correlation with the bad supply shock: this is consistent with the previous observation that bad supply shocks correspond to commodity crises and energy companies may benefit from high commodity prices (for example, oil companies benefit from high oil prices). The returns on utilities stocks are also relatively highly correlated with the bad supply shock. This might be explained by the fact that many of the prices in the sector (for instance, wholesale electricity prices) are linked to commodity prices and thus firms suffer less from high commodity prices. The shops (wholesale and retail) industry does relatively poorly when a bad supply shock realizes, which could be attributed to its heavy dependence on transportation, which is hurt by high commodity prices. Correlations with the good supply shock are positive for all industries: as the consumption growth shock is supply driven, large good supply shocks are likely to correspond to economic expansions, where most of the industries perform well. The shops industry has the highest and utilities and energy industries the lowest correlations with the shock, which, in line with the discussion above, could be related to the dependence of these industries on commodity prices. Overall, the correlations of industry portfolio returns with supply shocks provide evidence in favor of a commodity driven nature of the shocks.

Return correlations with the bad demand shock are negative for all industries. This can be explained by the fact, that, as Figure 5 suggests, the bad demand variance tends to increase (implying large $\omega_{n,t,s}$) during recessions, when the economy performs poorly as a whole. Non-durables, shops and healthcare industries have the highest (closest to 0) excess return correlations with the bad demand shock. These industries provide everyday goods for which it is plausible to assume a relatively inelastic demand. At the same time, durables and manufacturing have the most negative correlations with the bad demand shock. This makes economic sense as durables are on average more expensive and longer-term purchases and thus are likely to be more affected by demand fluctuations. Manufacturing might suffer because it is likely to be a supplier for durable good producers. Correlations of industry portfolio returns with the good demand shock are puzzling. In particular, the energy industry, which is commonly thought of as a supply industry, has the highest correlation. This might be linked to the fact that, as can be seen from Figure 5, the good demand variance

\[20\] This is indirectly supported by the fact that if 49 instead of 10 industry portfolios are used, the transportation industry has the lowest correlation with the bad supply shock.
had increased (implying large $\omega_{p,t}$s) during the 1970s and then during the mid-2000s which also coincided with the period of increasing commodity prices, benefitting the energy sector. Industries with the next highest correlations with the good demand shock (manufacturing, high tech, durables) are more easily interpretable.

Figure 8 plots implied correlations between consumption growth and inflation innovations (see equation (9)). The proportion of positive correlations was historically high between 2000 and 2010. According to the logic of the model in this paper, nominal bonds were less risky securities during this period.

For completeness, Table 4 reports the rest of the macroeconomic parameter estimates. Consumption (and thus dividend) growth has a quarterly mean of 42 basis points. The $\gamma_d$ parameter roughly (as there is an additional loading on the demand shock, $\gamma_{dd'}$) measures the dividend growth "leverage" coefficient, which is noisily estimated to be 1.35. This is in between values used in the literature: e.g., Campbell and Cochrane (1999) and Wachter (2006) use the value of 1, Burkhardt and Hasseltoft (2012) the value of 2, and Bansal and Yaron (2004) and Song (2014) the values of 2.5-3.5. Dividend growth shocks are economically strongly more sensitive to demand shocks ($\gamma_{dd'}$) but this coefficient is also estimated without much precision. Expected inflation is not surprisingly quite persistent with an autocorrelation coefficient of 0.93. This coefficient governs the decay in the autocorrelogram of inflation itself. Expected inflation shocks load significantly on inflation shocks ($\gamma_{\pi}=-0.22$) but load relatively more on demand shocks than does actual inflation ($\gamma_{\pi d'}=0.09$).

Because I use maximum likelihood estimation instead of a moments matching technique (as, for instance, in Burkhardt and Hasseltoft, 2012, or Campbell, Pflueger, and Viceira, 2014) to estimate the fundamentals dynamics, Table 5 verifies that the model replicates unconditional moments. To do so, I simulate a 100,000 quarters long time series at the estimated parameters described in Tables 2-4. The data standard errors are computed via the bootstrap. The vast majority of the model implied moments are inside one standard deviation of the data counterparts. The only moment missed by more than two standard deviations is the probability of a 4 standard deviations positive consumption shock, which is 0 in data with the standard error of 0 and in the model is 0.03% (but statistically insignificant with the standard error of 0.05%).
5.2 General Asset Pricing Fit

Table 6 presents preference parameters estimated via GMM. They turn out to be fairly similar to those usually reported for heteroskedastic habit models (Bekaert and Engstrom, 2009; Ermolov, 2014). The inverse surplus ratio process is highly persistent with an autoregressive parameter of 0.9890, which is very similar to the parameter in Campbell and Cochrane (1999). The risk aversion coefficient is 4.12 but of course does not represent risk aversion at all. Table 7 shows properties of the unconditional distribution of local risk aversion, \( \gamma Q_t \). The distribution is skewed to the right and has the median is only 10.58. The median local risk aversion smaller than in most of the Gaussian habit literature (e.g., Wachter, 2006; Bekaert, Engstrom, and Grenadier, 2010). This is because gamma distributions have fatter tails than Gaussian distributions with equal variance, making the agent more sensitive to economic fluctuations due to her concave utility function.

Table 8 shows that the model fits GMM moments very well: 7 out of 9 moments are within 1 standard deviation of their data counterparts (and all moments are inside 1.5 standard deviations of their data counterparts), and the model is not rejected at any conventional confidence level. Therefore, the model is consistent with standard salient facts of asset returns: a high equity premium, a low nominal risk free rate, and the variability of bond returns, price-dividend ratios and equity returns.

Table 9 shows that the model also reproduces many static and dynamic features of bond and equity markets, such as autocorrelation and predictability, not included in the estimation reasonably well. Importantly, the model replicates on average the upward-sloping real yield curve, as observed in the US and the UK, which is difficult to reproduce in long-run risk models (such as Burkhardt and Hasseltoft, 2012, or Song, 2014). The model also reproduces a positive nominal term spread. To assess the bond predictability, I ran a Fama-Bliss (1987) regression of the log excess holding return on the excess log forward rate. As in the historical sample, the model coefficient is negative. However, its magnitude is underestimated. Similarly for equities, a univariate regression of the log equity excess return on the log price dividend ratio yields a slope which is negative but has a smaller magnitude than in data.
5.3 Time-varying stock and bond return correlations

Table 10 shows that macroeconomic shocks are able to generate large positive and negative stock and bond return correlations. Indeed, the maximum correlations of 0.60 in the data and 0.55 in the model are very close to each other. The minimum correlation in the model is -0.48, which is mathematically larger than the minimum correlation of -0.71 in data, but is economically significantly negative. The distribution of conditional correlations suggests that the model reproduces large positive correlations well (97.5\textsuperscript{th} and 99\textsuperscript{th} percentile of the model and data distribution are very similar) and is able to generate negative stock and bond return correlations (1\textsuperscript{st} and 2.5\textsuperscript{th} percentiles of the model implied conditional correlation distribution are negative) which, however, are larger than in data. The unconditional correlation in the model is higher than historically, but within 2 standard deviations of the data counterpart. Possible reasons for this will be discussed later in the section.

Figure 9 plots the model implied stock and bond return correlations over time suggesting that macroeconomic shocks are able to replicate the general low frequency temporal patterns in stock and bond return correlations but their explanatory power is time-varying. During the 1970’s, the 1980’s, and the 1990’s, in line with the data, the model predicts on average positive correlations. In the model, as can be seen from Figures 4 and 8, supply shocks are dominant during these periods naturally implying positive stock and bond return correlations, as discussed in the asset pricing implications section. In terms of the time pattern, the fit of the model is very accurate in replicating the increase in the correlations from the late 1970’s until the mid-1980’s and then the drop from the mid 1980’s to the early 1990’s. During the early and mid-1970’s the macroeconomic shocks implied correlations are somewhat too high and during the mid-1990’s they are too low. However, the positive sign of the macroeconomic shocks implied correlation is correct during both of these intervals.

The problematic periods for the model start to occur in late 1990’s. During 1998-2000 the model implied correlations are clearly too high. This might be partially attributed to the fact that in this period, characterized by the LTCM and Asian crises, there were significant capital inflows into the US bond market and outflows from the US stock market due to flights to safety considerations (which are not in the model), but in terms of consumption barely anything happened. Another possible explanation is that during this period there were significant discretionary monetary policy shocks (setting the interest rate differently
from what would be usually implied by the macroeconomic fundamentals) by the Federal Reserve (as shown in Baele et.al., 2014a), which significantly affected the bond returns but are also not in the model.

Similarly to the data, after 2000, Figure 9 indicates that the model implied correlations decrease, although the model misses the strong negative correlations during 2001-2004. Again this period (2001-2004) is characterized by the fears that the Dot-com bubble could spread to other sectors of economy, motivated by a number of large corporate frauds (e.g., Enron). These considerations might have leaded to purely liquidity driven flights to safety phenomena, which is not visible in the consumption growth data and is thus outside the scope of the model. Also, as shown in Baele et.al. (2014a), there were large discretionary monetary policy shocks during this period, affecting bond returns.

The model matches well the low stock and bond return correlations between 2004 and 2010 including the strong negative correlations during the Great Recession. Indeed, as illustrated in Figures 4 and 8, this period is characterized by the growing importance of first good and then bad demand variance (Figure 5), which in the model implies low stock and bond return correlations. While the stock and bond correlations in data stayed low after the Great Recession, the model implied correlations have increased considerably due to the increase in the supply variance (which might be related to high commodity prices) as in Figure 4. This discrepancy between the macroeconomic shocks implied correlations and the data might be attributed to the non-conventional policy of quantitative easing, which is a textbook example of a discretionary monetary policy. Also, given the high general uncertainty during this period, flights to safety might have played a role.

Table 11 shows that the macroeconomic risk driven model implies, as in data, that the stock and bond return correlations have been statistically significantly smaller after 2000 than before 2000. However, the difference between correlations is less pronounced than in data. The model is able to closely replicate the correlation during 1970-2000, but is not able to generate low enough correlation during the early 2000’s and 2010’s, which are likely to be related to non-macroeconomic factors.

In order to quantify these seemingly macroeconomy unrelated effects driving stock and bond return correlations in 1998-2004 and 2011-2012, a binary flights to safety proxy as in
Baele et.al. (2014b) show that their proxy identified from high-frequency data is correlated with periods of extreme stress in financial markets and argue that it is unlikely to be related to macroeconomic events. Thus, it does not have a counterpart in this paper’s lower frequency macroeconomic model. Because the proxy in Baele et.al. (2014b) is at daily frequency and my analysis is at quarterly frequency, I compute the quarterly value of the proxy by counting the number of the days inside the quarter for which the flights to safety dummy is on and divide it by the number of days in the quarter.

Top panel of Figure 10 plots the flights to safety variable indicating that flights to safety episodes are relatively infrequent (consistently with Baele et.al., 2014b) and coincide with the episodes of poor performance by the macroeconomic stock and bond return correlations model in Figure 9 (1998-2004, 2011-2012). Indeed before 1998, there are on only two non-zero flights to safety values, corresponding to the episodes such as Black Monday of 1987. However, during 1998-2004 and from the beginning of the Great Recession onwards several large values of the flights to safety variable are observed. Bottom panel of Figure 10 plots the results of the regression of the stock and bond return correlation not explained by macroeconomic shocks on a constant and the flights to safety variable. This formally shows that flights to safety episodes indeed explain the episodes most problematic for the macroeconomic shocks (1998-2004, 2011-2012) fairly well.

Excluding the flights to safety periods (1998-2004, 2011-2012), the correlation between the model implied and data stock and bond return correlations in Figure 9 goes from 0.39 to 0.58 and the unconditional stock and bond return correlation in data becomes 0.27, which is very close to the model implied unconditional correlation of 0.30 in Table 10. Generally, the results in this section suggest that, although many of the negative stock and bond return correlation episodes are not macroeconomic in their nature, purely macroeconomic shocks are able to generate strong negative stock and bond return correlations and are important.

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21 Ghysels, Plazzi, and Valkanov (2013) use a similar proxy. I thank Lieven Baele for providing the data. The proxy is available only starting in 1980. By reconstructing the first component of the proxy, I show that flights to safety were historically rare during the 1970s and, thus, set the value of the flights to safety variable during this period to 0. Using any values greater than 0 only improves the results below.

22 A disadvantage of the used proxy is that it cannot directly disentangle the economic sources of the flights to safety phenomena, as it is likely to capture both fear and sentiment driven episodes, such as during the Enron case, and unconventional monetary policy regimes, such as quantitative easing.
in doing so during important historical periods such as the Great Recession.

6 Conclusion

I provide an empirically flexible and theoretically tractable framework for analyzing the macroeconomic risk of nominal assets. Estimation of the model allows to characterize the macroeconomic shocks in an economically intuitive way. The demand shock has a persistent Gaussian component supplemented with a rare-disaster type component. This component becomes most pronounced during the Great Recession. The supply shock consists of a good component emerging during times of low commodity prices and technological innovation and a bad component dominating during periods of high commodity prices.

Macroeconomic shocks produce large positive and negative stock and bond return correlations, comparable to the historically observed values. However, the model implied negative correlations are smaller and less frequent than in data, although the model identifies macroeconomic shocks as an important driver of the large negative correlations during periods such as the Great Recession.

There are at least two promising directions for future research. First, given the tractability of the framework, it can be applied to study the role of macroeconomic risk in other important nominal assets. For instance, Ermolov (2014) shows that a similar model can be easily extended to an international setting, allowing to analyze differences between term structures of different countries. An interesting extension is applying the framework to study the co-movement of government bond returns with the stocks of different firms (Baker and Wurgler, 2012): correlations between macroeconomic shocks and industry portfolio returns analyzed in this paper are the first step in this direction. As the model features loglinear preferences and fundamental dynamics, pricing under both physical and risk-neutral measure is straightforward, enabling the analysis of inflation options (e.g., Fleckenstein, Longstaff, and Lustig, 2013). The impact of macroeconomic conditions on the risk of corporate bonds and capital structure decisions (Bhamra, Kuehn, and Strebulaev, 2010; Bhamra, Fisher, and Kuehn, 2011; Kang and Pflueger, 2014) is another important application area. As unexpectedly low inflation increases real liabilities and thus default risk, in a demand shock environment the default risk becomes more countercyclical making the corporate bonds riskier.
Second, the results have implications for the research on the time-varying stock and bond return correlation. In particular, my findings indicate that although macroeconomic shocks explain some important episodes of negative return correlations, such as the Great Recession, they generate much less negative correlations than observed in the data. Thus, the future theoretical and empirical research should pay particular attention to these episodes of the low stock and bond return correlations.
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Appendix A - Asset prices

Properties of gamma distributions

Suppose $X \sim \Gamma(a, 1)$, where $a$ is the shape parameter. Then all consumption and asset pricing formulas below can be obtained using these formulas:

$$E(X) = a,$$
$$Var(X) = a,$$
$$Skw(X) = \frac{2}{\sqrt{a}},$$
$$\text{Excess kurtosis}(X) = \frac{6}{a},$$
$$E(e^{bX}) = e^{-a \ln(1-b)}.$$

In order to simplify the notation, define the following function: $f(x) = x + \ln(1-x)$.

Real Bonds

The time $t$ price of an $n$ period zero-coupon bond paying one unit of consumption at maturity can be obtained by recursively computing the expectations of the real stochastic discount factor $P_{n,t} = E_t(M_{t+n})$:

$$P_{n,t} = \exp(C_n + Q_n q_n + P^d_n p^d_n + N^d_n n^d_n + P^s_n p^s_n + N^s_n n^s_n),$$
$$C_n = \ln \beta - \gamma \bar{g} + \bar{g}(1 - \rho_q),$$
$$Q_n = -\gamma(1 - \rho_q),$$
$$P^d_1 = -f(a_{dp}),$$
$$N^d_1 = -f(a_{dn}),$$
$$P^s_1 = -f(a_{sp}),$$
$$N^s_1 = -f(a_{sn}),$$
$$C_n = C_{n-1} + C_1 + Q_{n-1}(1 - \rho_q)q + P^d_{n-1}(1 - \rho_p^d)p^d + N^d_{n-1}(1 - \rho_n^d)n^d + P^s_{n-1}(1 - \rho_p^s)p^s + N^s_{n-1}(1 - \rho_n^s)n^s,$$
$$Q_n = -\gamma(1 - \rho_q) + \rho_q Q_{n-1},$$
$$P^d_n = \rho_p^d P^d_{n-1} - f(a_{dp} + Q_{n-1} \gamma q^d \sigma_{dp} + P^d_{n-1} \sigma_{pp}),$$
$$N^d_n = \rho_n^d N^d_{n-1} - f(a_{dn} - Q_{n-1} \gamma q^d \sigma_{dn} + N^d_{n-1} \sigma_{nn}),$$
$$P^s_n = \rho_p^s P^s_{n-1} - f(a_{sp} + Q_{n-1} \gamma q^s \sigma_{sp} + P^s_{n-1} \sigma_{pp}),$$
$$N^s_n = \rho_n^s N^s_{n-1} - f(a_{sn} - Q_{n-1} \gamma q^s \sigma_{sn} + N^s_{n-1} \sigma_{nn}).$$
Nominal Bonds

The time $t$ price of an $n$ period zero-coupon bond paying one nominal unit at maturity can be obtained by recursively computing the expectations of the nominal stochastic discount factor $P^S_{n,t} = E_t(M^S_{t,n})$:

$$P^S_{n,t} = \exp(C^S_n + Q^S_n q_t + X^u_{n,t} + P^d^S_p \rho_t^d + N^d_{n,t} + P^o_{n,t} + N^o_{n,t})$$

$$C_1 = \ln \beta - \gamma \bar{q} + \gamma q (1 - \rho_t) - \pi,$$

$$Q_1 = -\gamma (1 - \rho_t),$$

$$a^S_n = -1.$$

$$P^S_{1,t} = -f(a^S_{dp}),$$

$$N^d_{1,t} = -f(a^S_{dp}),$$

$$P^S_{1,t} = -f(a^S_{dp}),$$

$$N^d_{1,t} = -f(a^S_{dp}).$$

$$C^S_n = C^S_{n-1} + C^S_{n-1} (1 - \rho_t) q_t + P^d_{n-1} (1 - \rho_t^d) p^d_t + N^d_{n-1} (1 - \rho_t^d) n^d_t + P^o_{n-1} (1 - \rho_t^o) n^o_t + N^o_{n-1} (1 - \rho_t^o) n^o_t + \frac{1}{2} X_{n-1}^\pi \sigma^2,$$

$$Q^S_n = -\gamma (1 - \rho_t) + \rho_t Q^S_{n-1},$$

$$X^\pi_n = \rho_t \pi X^\pi_{n-1} - 1,$$

$$P^S_{n,t} = \rho_t P^S_{n-1,t} - f(a^S_{dp} + Q^S_{n-1} \gamma \sigma^d \sigma^d_p + X^\pi_{n-1} \sigma^d \sigma^d_p (\gamma n + \gamma^d + P^d_{n-1} \sigma^d_p),$$

$$N^d_{n,t} = \rho_t N^d_{n-1,t} - f(a^S_{dp} - Q^S_{n-1} \gamma \sigma^d \sigma^d_p - X^\pi_{n-1} \sigma^d \sigma^d_p (\gamma n + \gamma^d + N^d_{n-1} \sigma^d_p),$$

$$P^S_{n,t} = \rho_t P^S_{n-1,t} - f(a^S_{dp} + Q^S_{n-1} \gamma \sigma^d \sigma^d_p - X^\pi_{n-1} \sigma^d \sigma^d_p \gamma n + P^o_{n-1} \sigma^o_p),$$

$$N^d_{n,t} = \rho_t N^d_{n-1,t} - f(a^S_{dp} - Q^S_{n-1} \gamma \sigma^d \sigma^d_p - X^\pi_{n-1} \sigma^d \sigma^d_p \gamma n + N^o_{n-1} \sigma^o_p),$$

$$a^S_{dp} = a_{dp} - \sigma_x \sigma^d_p,$$

$$a^S_{dp} = a_{dp} - \sigma_x \sigma^d_p,$$

$$a^S_{dp} = a_{dp} - \sigma_x \sigma^d_p,$$

$$a^S_{dp} = a_{dp} - \sigma_x \sigma^d_p,$$

$$a^S_{dp} = a_{dp} - \sigma_x \sigma^d_p.$$
**Equity**

The price-dividend ratio can be computed by recursively pricing the dividend claim in each period of time and summing these prices over time:

\[
\frac{P_t}{D_t} = \sum_{n=1}^{\infty} \exp(C_n + Q_n \dot{q} + P_n \dot{d} + N_n \dot{a} + P^e_n \dot{p} + N^e_n \dot{r}),
\]

\[
C_t = \ln \beta + (1 - \gamma)\bar{g} + \gamma \bar{q}(1 - \rho_q) + \frac{1}{2}\sigma_d^2,
\]

\[
Q_t = -\gamma(1 - \rho_q),
\]

\[
P_{de}^t = -f(a_{dp}^t),
\]

\[
N_1^{de} = -f(a_{dn}^t),
\]

\[
P_{se}^t = -f(a_{sp}^t),
\]

\[
N_1^{se} = -f(a_{sn}^t),
\]

\[
C_t = C_{t-1} + C_t + Q_{t-1}(1 - \rho_q) + P_{de}^{t-1} \dot{d}(1 - \rho_d^p) + N_{de-1} \dot{a}(1 - \rho_a^p) + P_{se}^{t-1} \dot{p}(1 - \rho_p^s) + N_{se-1} \dot{r}(1 - \rho_r^s),
\]

\[
Q_t = -\gamma(1 - \rho_q) + \rho_q C_{t-1},
\]

\[
P_{de}^t = \rho_d^p P_{de}^{t-1} - f(a_{dp}^t) + Q_{t-1} \gamma \sigma_d^p \sigma_d^p + P_{de}^{t-1} \sigma_d^p,
\]

\[
N_{de}^t = \rho_d^p N_{de-1} - f(a_{dn}^t) - Q_{t-1} \gamma \sigma_d^p \sigma_d^p + N_{de-1} \sigma_d^p,
\]

\[
P_{se}^t = \rho_p^s P_{se}^{t-1} - f(a_{sp}^t) + Q_{t-1} \gamma \sigma_d^p \sigma_d^p + P_{se}^{t-1} \sigma_d^p,
\]

\[
N_{se}^t = \rho_p^s N_{se-1} - f(a_{sn}^t) - Q_{t-1} \gamma \sigma_d^p \sigma_d^p + N_{se-1} \sigma_d^p,
\]

\[
a_{dp}^t = a_{dp} + \sigma_d^p (\gamma_d + \gamma_q) \sigma_d^p,
\]

\[
a_{dn}^t = a_{dn} - \sigma_d^p (\gamma_d + \gamma_q) \sigma_d^p,
\]

\[
a_{sp}^t = a_{sp} + \sigma_d^s (\gamma_d + \gamma_q) \sigma_d^s.
\]

The logarithmic return on equity, \(r^e_{t+1} = \ln(1 + \frac{P_{t+1}}{D_{t+1}}) - \ln(\frac{P_t}{D_t}) + d_{t+1} \), can be computed as a linear function of state variables and fundamental shocks by log-linearizing the equilibrium solutions for \(\ln(\frac{P_t}{D_t})\) and \(\ln(1 + \frac{P_{t+1}}{D_{t+1}})\) around the steady state values of the state variables \(q_t, p_d^t, n_a^t, p_e^t, n_r^t\):

\[
p_{de} \approx \ln \sum_{n=1}^{\infty} \exp(C_n + Q_n \dot{q} + P_n \dot{d} + N_n \dot{a} + P^e_n \dot{p} + N^e_n \dot{r}) + \\
\sum_{n=1}^{\infty} \exp(C_n + Q_n \dot{q} + P_n \dot{d} + N_n \dot{a} + P^e_n \dot{p} + N^e_n \dot{r}) (q_t - \bar{q}) + \\
\sum_{n=1}^{\infty} \exp(C_n + Q_n \dot{q} + P_n \dot{d} + N_n \dot{a} + P^e_n \dot{p} + N^e_n \dot{r}) (p_t - \bar{p}) + \\
\sum_{n=1}^{\infty} \exp(C_n + Q_n \dot{q} + P_n \dot{d} + N_n \dot{a} + P^e_n \dot{p} + N^e_n \dot{r}) (n_t - \bar{n}) + \\
\sum_{n=1}^{\infty} \exp(C_n + Q_n \dot{q} + P_n \dot{d} + N_n \dot{a} + P^e_n \dot{p} + N^e_n \dot{r}) (p_{t+1} - \bar{p}) + \\
\sum_{n=1}^{\infty} \exp(C_n + Q_n \dot{q} + P_n \dot{d} + N_n \dot{a} + P^e_n \dot{p} + N^e_n \dot{r}) (n_{t+1} - \bar{n}) = \\
K_1 + K_1^d q_t + K_1^a p_d^t + K_1^e n_a^t + K_1^s p_e^t + K_1^r n_r^t,
\]
where $K_1$, $K_1^q$, $K_1^{p^d}$, $K_1^{n^d}$, $K_1^{p^n}$, and $K_1^{n^n}$ are implicitly defined. Similarly:

$$\ln(1 + \frac{P_t}{D_t}) \approx \ln(1 + \sum_{n=1}^{\infty} \exp(C_n^e + Q_n^e \tilde{q} + P_n^{p^e} p^e + N_n^{p^e} \tilde{p}^e + P_n^{n^e} n^e_d + P_n^{P^e} P^e) + \sum_{n=1}^{\infty} \exp(C_n^e + Q_n^e \tilde{q} + P_n^{p^e} p^e + N_n^{p^e} \tilde{p}^e + P_n^{n^e} n^e_d + P_n^{P^e} P^e) + N_n^{p^e} n^e_d)) +$$

$$\sum_{n=1}^{\infty} \frac{Q_n^e}{p^e} \exp(C_n^e + Q_n^e \tilde{q} + P_n^{p^e} p^e + N_n^{p^e} \tilde{p}^e + P_n^{n^e} n^e_d + P_n^{P^e} P^e) + N_n^{p^e} n^e_d) +$$

where $K_2$, $K_2^q$, $K_2^{p^d}$, $K_2^{n^d}$, $K_2^{p^n}$, and $K_2^{n^n}$ are implicitly defined. The excess one period holding return on equity, $r_{t+1}^{ex}$, can now be expressed as:

$$r_{t+1}^{ex} = \ln(1 + \frac{P_{t+1}}{D_{t+1}}) - \ln(\frac{P_t}{D_t}) + d_{t+1} - y_{1,t} +$$

$$\sum_{n=1}^{\infty} n^e_q + K_2^{p^d} p^d + K_2^{n^d} n^d + K_2^{p^n} p^n + K_2^{n^n} n^n.$$
Appendix B - Constructing inflation expectations

The inflation expectation for quarter $t + 1$ at quarter $t$ are modeled as the fitted values from the regression:

$$\pi_{t+1} = \alpha_0 + \alpha_1 \pi_{t,t+1}^e + \alpha_2 \pi_t + \epsilon_{t+1}^{\pi_{OLS}},$$

where $\pi_t$ is the quarter $t$ inflation, $\pi_{t,t+1}^e$ is the expectation of quarter $t + 1$ inflation in quarter $t$ from the Survey of Professional Forecasters, $\alpha_0$, $\alpha_1$, and $\alpha_2$ are constant coefficients and $\epsilon_{t+1}^{\pi_{OLS}}$ is a zero-mean noise term independent from $\pi_t$ and $\pi_{t,t+1}^e$. The estimated coefficients are below.

<table>
<thead>
<tr>
<th>OLS coefficient</th>
<th>Newey-West standard error (4 lags)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>0.0000 (0.0010)</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.8856 (0.1758)</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.2477 (0.1181)</td>
</tr>
</tbody>
</table>

The set of predictors is optimal in terms of minimizing Bayesian information criterion. The summary of considered predictor specifications with corresponding Bayesian information criteria is below.

<table>
<thead>
<tr>
<th>Predictors (in addition to a constant)</th>
<th>Bayesian information criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_{t,t+1}^e$, $\pi_t$</td>
<td>-1723</td>
</tr>
<tr>
<td>$\pi_{t,t+1}^e$, $\pi_t$, $\pi_{t-2}$</td>
<td>-1722</td>
</tr>
<tr>
<td>$\pi_{t,t+1}^e$, $\pi_{t-1,t+1}^e$, $\pi_t$</td>
<td>-1722</td>
</tr>
<tr>
<td>$\pi_{t,t+1}^e$, $\pi_{t-2}$</td>
<td>-1720</td>
</tr>
<tr>
<td>$\pi_{t-1,t+1}^e$, $\pi_t$, $\pi_{t-2}$</td>
<td>-1720</td>
</tr>
<tr>
<td>$\pi_{t,t+1}^e$</td>
<td>-1719</td>
</tr>
<tr>
<td>$\pi_{t,t+1}^e$, $\pi_t$, $\pi_{t-1}$</td>
<td>-1719</td>
</tr>
<tr>
<td>$\pi_{t,t+1}^e$, $\pi_t$, $\pi_{t-1}$, $\pi_{t-2}$</td>
<td>-1719</td>
</tr>
<tr>
<td>$\pi_{t,t+1}^e$, $\pi_{t-1,t+1}^e$</td>
<td>-1719</td>
</tr>
<tr>
<td>$\pi_t$, $\pi_{t-2}$</td>
<td>-1718</td>
</tr>
<tr>
<td>$\pi_{t-1,t+1}^e$, $\pi_t$, $\pi_{t-1}$, $\pi_{t-2}$</td>
<td>-1717</td>
</tr>
<tr>
<td>$\pi_t$, $\pi_{t-2}$</td>
<td>-1716</td>
</tr>
</tbody>
</table>
\begin{align*}
&\pi_{t-1,t+1}, \pi_t \quad -1716 \\
&\pi_{t,t+1}, \pi_{t-1} \quad -1715 \\
&\pi_{t-1,t+1}, \pi_{t-2} \quad -1711 \\
&\pi_{t-1,t+1}, \pi_t, \pi_{t-1} \quad -1707 \\
&\pi_{t-1,t+1} \quad -1705 \\
&\pi_{t-1,t+1}, \pi_{t-1} \quad -1704 \\
&\pi_t, \pi_{t-1} \quad -1702 \\
&\pi_t \quad -1695 \\
&\pi_{t-2} \quad -1693 \\
&\pi_{t-1} \quad -1677
\end{align*}
Appendix C - Maximum likelihood estimation of demand and supply shocks parameters

The estimation procedure is a modification of Bates (2006) algorithm for the component model of two gamma distributed variables. Below the step-by-step estimation strategy for the demand shock is described. The estimation for the supply shock is identical.

The methodology below is an approximation, because, in order to facilitate the computation, at each time point the conditional distribution of state variables \( p_{d,t} \) and \( n_{d,t} \) is assumed to be gamma, although the distribution does not have a closed form solution. The choice of the approximating distributions is discussed in details in section 1.3 of Bates (2006). Here the gamma distributions are used, because they are bounded from the left at 0, which ensures that the shape parameters of the gamma distribution in the model (\( p_{d,t} \) and \( n_{d,t} \)) will always stay positive, like they should.

The system to estimate is:

\[
\begin{align*}
\omega_{p,t+1}^d &\sim \Gamma(p_{d,t}^d,1) - p_{d,t}^d, \\
\omega_{n,t+1}^d &\sim \Gamma(n_{d,t}^d,1) - n_{d,t}^d, \\
p_{t+1}^d &\sim \tilde{p}^d + \rho_p^d(p_t^d - \tilde{p}^d) + \sigma_{p,p}^d\omega_{p,t+1}^d, \\
n_{t+1}^d &\sim \tilde{n}^d + \rho_n^d(n_t^d - \tilde{n}^d) + \sigma_{n,n}^d\omega_{n,t+1}^d.
\end{align*}
\]

The following notation is defined:

\( U_t^d \equiv \{u_1^d, \ldots, u_t^d\} \) is the sequence of observations up to time \( t \).

\( F(i\phi, i\psi_1, i\psi_2|U_t^d) \equiv E(e^{i\phi u_{t+1}^d+i\psi_1 p_{t+1}^d+i\psi_2 n_{t+1}^d}|U_t^d) \) is the next period’s joint conditional characteristic function of the observation and the state variables.

\( G_{t|s}(i\psi_1, i\psi_2) \equiv E(e^{i\psi_1 p_s^d+i\psi_2 n_s^d}|U_s^d) \) is the characteristic function of the time \( t \) state variables conditioned on observing data up to time \( s \).

At time 0, the characteristic function of the state variables \( G_{0|0}(i\psi_1, i\psi_2) \) is initialized. As mentioned above, the distribution of \( p_0^d \) and \( n_0^d \) is approximated with gamma distributions. Note that the unconditional mean and variance of \( p_t^d \) are \( E(p_t^d) = \tilde{p}^d \) and \( \text{Var}(p_t^d) = \frac{\sigma_{p,p}^d}{1-\rho_p^2} \tilde{p}^d \), respectively. The approximation by the gamma distribution with the shape parameter \( k_0 \) and the scale parameter \( \sigma_0^p \) is done by matching the first two unconditional moments.
Using the properties of the gamma distribution, \( k_0^p = \frac{E^2 p_0^d}{\text{Var}(p_0^d)} \) and \( \theta_0^p = \frac{\text{Var}(p_0^d)}{E(p_0^d)} \). Thus, \( p_0^d \) is assumed to follow \( \Gamma(k_0^p, \theta_0^p) \) and \( n_0^d \) is assumed to follow \( \Gamma(k_0^n, \theta_0^n) \), where \( k_0^n \) and \( \theta_0^n \) are computed in the same way. Using the properties of the expectations of the gamma variables, 
\[
G_{0|0}(i\psi^1, i\psi^2) = e^{-k_0^n \ln(1-\theta_0^n i\psi^1) - k_0^n \ln(1-\theta_0^n i\psi^2)}.
\]
Given \( G_{0|0}(i\psi^1, i\psi^2) \), computing the likelihood of \( U_t^d \) is performed by repeating the steps 1-3 below for all subsequent values of \( t \).

**Step 1.** Computing the next period’s joint conditional characteristic function of the observation and the state variables:

\[
F(i\Phi, i\psi^1, i\psi^2 | U_t^d) = E(E(e^{(i\Phi\sigma_p^d + i\psi^1 \sigma_p^2 + i\psi^2 \sigma_{pp}^d + \frac{1}{2}i\psi^1 \psi^2 \sigma_{pp}^d + \frac{1}{2}i\psi^1 \psi^2 \sigma_{pp}^d)} | U_t^d) e^{-i\Phi u_{t+1}^d}) d\Phi,
\]
where the function \( F \) is defined in step 1 and the integral is evaluated numerically.

**Step 2.** Evaluating the conditional likelihood of the time \( t + 1 \) observation:

\[
p(u_{t+1}^d | U_t^d) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(i\Phi, 0, 0 | U_t^d) e^{-i\Phi u_{t+1}^d}) d\Phi,
\]

**Step 3.** Computing the conditional characteristic function for the next period, \( G_{t+1|t+1}(i\psi^1, i\psi^2) \):

\[
G_{t+1|t+1}(i\psi^1, i\psi^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(i\Phi, i\psi^1, i\psi^2 | U_t^d) e^{-i\Phi u_{t+1}^d} d\Phi.
\]

As above, the function \( G_{t+1|t+1}(i\psi^1, i\psi^2) \) is also approximated with the gamma distribution via matching the first two moments of the distribution. The moments are obtained by taking the first and second partial derivatives of the joint characteristic function:

\[
E_{t+1} p_{t+1}^d = \frac{1}{2\pi p(u_{t+1}^d | U_t^d)} \int_{-\infty}^{\infty} F_{\psi^1}(i\Phi, 0, 0 | U_t^d) e^{-i\Phi u_{t+1}^d} d\Phi,
\]

\[
\text{Var}_{t+1} p_{t+1}^d = \frac{1}{2\pi p(u_{t+1}^d | U_t^d)} \int_{-\infty}^{\infty} F_{\psi^1 \psi^1}(i\Phi, 0, 0 | U_t^d) e^{-i\Phi u_{t+1}^d} d\Phi - E_{t+1} p_{t+1}^d,
\]

\[
E_{t+1} n_{t+1}^d = \frac{1}{2\pi p(u_{t+1}^d | U_t^d)} \int_{-\infty}^{\infty} F_{\psi^2}(i\Phi, 0, 0 | U_t^d) e^{-i\Phi u_{t+1}^d} d\Phi,
\]

\[
\text{Var}_{t+1} n_{t+1}^d = \frac{1}{2\pi p(u_{t+1}^d | U_t^d)} \int_{-\infty}^{\infty} F_{\psi^2 \psi^2}(i\Phi, 0, 0 | U_t^d) e^{-i\Phi u_{t+1}^d} d\Phi - E_{t+1} n_{t+1}^d,
\]

where \( F_{\psi^i} \) denotes the derivative of \( F \) with respect to \( \psi^i \). The expressions inside the integral are obtained in closed form by deriving the function \( F(i\Phi, i\psi^1, i\psi^2 | U_t^d) \) in step 1, and integrals are evaluated numerically. Using the properties of the gamma distribution, the values of the shape and the scale parameters are \( k_{t+1}^p = \frac{E_{t+1} p_{t+1}^d}{\text{Var}_{t+1} p_{t+1}^d} \) and \( \theta_{t+1}^p = \frac{\text{Var}_{t+1} p_{t+1}^d}{E_{t+1} p_{t+1}^d} \), respectively. The expressions for \( k_{t+1}^n \) and \( \theta_{t+1}^n \) are similar.
The total likelihood of the time series is the sum of individual likelihoods from step 2:

\[ L(Y_T) = \ln p(u^d_1 | k_0^p, \theta_0^p) + \sum_{t=2}^{T} \ln p(u^d_t | U^d_t). \]
Figure 1: Correlation Between Returns on the US 5 Year Treasury Bond and the Aggregate Stock Market. Correlations are computed quarterly from daily data. Expectations are the long-run (quarterly) conditional correlations from the MIDAS-dynamic conditional correlation model of Colacito, Engle, and Ghysels (2011). The unconditional correlation between returns is 0.05.
Figure 2: Components of the Bad Environment-Good Environment (BEGE) probability density function.

- Probability density function: Good component $\sigma_p(I(p,1)-p)$
- Probability density function: Bad component $-\sigma_n(I(n,1)-n)$
- Probability density function: Both Components $\sigma_p(I(p,1)-p)-\sigma_n(I(n,1)-n)$
Figure 3: Bad Environment-Good Environment (BEGE): time-varying volatility.
Figure 4: Properties of the Macroeconomic Shocks. The graph is quarterly. The demand shock is defined as a shock which moves the consumption growth and inflation in the same direction. The supply shock is defined as a shock which moves the consumption growth and inflation in the opposite directions. The shocks are filtered from the US consumption growth of non-durables and services and the changes in the CPI index for all urban customers. Good and bad demand and supply variances are conditional model implied variances filtered using the characteristic domain approximate maximum likelihood methodology of Bates (2006).
Figure 5: Properties of the Demand Shock. The graph is quarterly. The demand shock is defined as a shock which moves the consumption growth and inflation in the same direction. The demand shock is filtered from the US consumption growth of non-durables and services and the changes in the CPI index for all urban customers. Good and bad demand variances are conditional model implied variances filtered using the characteristic domain approximate maximum likelihood methodology of Bates (2006).
Figure 6: Properties of the Supply Shock. The graph is quarterly. The supply shock is defined as a shock which moves the consumption growth and inflation in the opposite directions. The supply shock is filtered from the US consumption growth of non-durables and services and the changes in the CPI index for all urban customers. Good and bad supply variances are conditional model implied variances filtered using the characteristic domain approximate maximum likelihood methodology of Bates (2006).
Figure 7: Correlation between Macroeconomic Shocks and Industry Portfolio Returns. Data are quarterly. Industry portfolio returns are Fama-French 9 industry (10 industry excluding "Others"-industry) excess returns from Kenneth French’s data library. Macroeconomic shocks and portfolio returns are contemporaneous.
Figure 8: Model Implied Conditional Consumption Growth and Inflation Correlation. The correlation is the model implied correlation based on the filtered values of conditional macroeconomic state variables.
Figure 9: Macroeconomic Shocks Implied Stock and Bond Return Correlations. The graph is quarterly. Realized correlations are computed quarterly from daily data. Data expectations are the long-run (quarterly) conditional correlations from the MIDAS-dynamic conditional correlation model of Colacito, Engle, and Ghysels (2011). Model correlations are conditional correlations computed using macroeconomic state variables.
Figure 10: Explaining Macroeconomy Unrelated Stock and Bond Return Correlation with Flights to Safety. Graph is quarterly. Top panel plots the flights to safety variable from Baele et.al. (2014b). Bottom panel plots residual stock and bond return correlation defined as the difference between macroeconomic shocks implied conditional and data expected correlations. Data expectations are the long-run (quarterly) conditional correlations from the MIDAS-dynamic conditional correlation model of Colacito, Engle, and Ghysels (2011). Residual correlation implied by flights to safety in the bottom panel is computed by regressing the residual correlation on a constant and the flights to safety variable.
Table 1: Loadings of Unexpected Stock and Bond Returns on Macroeconomic Shocks. $\omega_{d,t}$ and $\omega_{n,t}$ are good and bad demand shocks, respectively. $\omega_{p,t}$ and $\omega_{s,t}$ are good and bad supply shocks, respectively. $\epsilon_t$ is the shock to expected inflation unrelated to demand and supply shocks. $\epsilon_{t+1}^d$ is the shock to dividend growth unrelated to demand and supply shocks.

<table>
<thead>
<tr>
<th>Shock</th>
<th>Sign</th>
<th>Dominant effect(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_{d,t}$</td>
<td>-</td>
<td>expected inflation</td>
</tr>
<tr>
<td>$\omega_{n,t}$</td>
<td>+</td>
<td>expected inflation, precautionary savings</td>
</tr>
<tr>
<td>$\omega_{p,t}$</td>
<td>+</td>
<td>expected inflation, intertemporal smoothing, precautionary savings</td>
</tr>
<tr>
<td>$\omega_{s,t}$</td>
<td>-</td>
<td>expected inflation, intertemporal smoothing</td>
</tr>
<tr>
<td>$\epsilon_t$</td>
<td>-</td>
<td>expected inflation</td>
</tr>
<tr>
<td>$\epsilon_{t+1}^d$</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Panel A: Unexpected Bond Return Loadings

<table>
<thead>
<tr>
<th>Shock</th>
<th>Sign</th>
<th>Dominant effect(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_{d,t}$</td>
<td>+</td>
<td>dividend growth, intertemporal smoothing, precautionary savings</td>
</tr>
<tr>
<td>$\omega_{n,t}$</td>
<td>-</td>
<td>dividend growth, intertemporal smoothing</td>
</tr>
<tr>
<td>$\omega_{p,t}$</td>
<td>+</td>
<td>dividend growth, intertemporal smoothing, precautionary savings</td>
</tr>
<tr>
<td>$\omega_{s,t}$</td>
<td>-</td>
<td>dividend growth, intertemporal smoothing</td>
</tr>
<tr>
<td>$\epsilon_t$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$\epsilon_{t+1}^d$</td>
<td>+</td>
<td>dividend growth</td>
</tr>
</tbody>
</table>

Panel B: Unexpected Equity Return Loadings

<table>
<thead>
<tr>
<th>Shock</th>
<th>Sign</th>
<th>Dominant effect(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_{d,t}$</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>$\omega_{n,t}$</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>$\omega_{p,t}$</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>$\omega_{s,t}$</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>$\epsilon_t$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$\epsilon_{t+1}^d$</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Panel C: Contribution to Conditional Stock and Bond Return Covariance
Table 2: Consumption Growth and Inflation Loadings on Demand and Supply Shocks. Data are the US quarterly observations from 1970Q1 to 2012Q4. The parameters are estimated via GMM. The standard errors are in parentheses.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{gd}$</td>
<td>0.0015</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>$\sigma_{gs}$</td>
<td>0.0037</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>$\sigma_{\pi d}$</td>
<td>0.0055</td>
<td>(0.0010)</td>
</tr>
<tr>
<td>$\sigma_{\pi s}$</td>
<td>0.0032</td>
<td>(0.0006)</td>
</tr>
</tbody>
</table>

$p$-value of the overidentification test 0.8964

The dynamics are the following:

1. $\epsilon_{gt} = \sigma_{gd} u_{dt} + \sigma_{gs} u_{st}$,
2. $\epsilon_{\pi t+1} = \sigma_{\pi d} u_{dt} - \sigma_{\pi s} u_{st}$,
3. $Cov(u_{dt}, u_{st}) = 0$, $Var(u_{dt}) = Var(u_{st}) = 1$. 

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Table 3: Demand and Supply Shock Parameter Estimates. Data are the US quarterly observations from 1970Q1 to 2012Q4. The shocks are inverted from the US consumption growth of non-durables and services and the changes in the CPI index for all urban customers. The parameters are estimated using the characteristic domain approximate maximum likelihood methodology of Bates (2006). The log-likelihood for the demand time series is -214.5901 and the log-likelihood of the supply time series is -235.9547. The standard errors in parentheses are parameteric bootstrap errors computed by simulating the 250 time series of historical length using the estimated model parameters.

<table>
<thead>
<tr>
<th>Panel A: Demand shock</th>
<th>Panel B: Supply shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good variance</td>
<td>Bad variance</td>
</tr>
<tr>
<td>( \sigma^d_p ) 0.0675</td>
<td>( \sigma^d_n ) 5.3873</td>
</tr>
<tr>
<td>(0.0344)</td>
<td>(1.3268)</td>
</tr>
<tr>
<td>( \tilde{p}^d ) 139.8355</td>
<td>0.0077</td>
</tr>
<tr>
<td>(7.1719)</td>
<td>(0.0060)</td>
</tr>
<tr>
<td>( \rho^d_p ) 0.9649</td>
<td>0.7594</td>
</tr>
<tr>
<td>(0.0308)</td>
<td>(0.2008)</td>
</tr>
<tr>
<td>( \sigma^d_{pp} ) 0.9649</td>
<td>0.0830</td>
</tr>
<tr>
<td>(0.1418)</td>
<td>(0.0404)</td>
</tr>
</tbody>
</table>

The dynamics are the following:

\[
\begin{align*}
    u^d_{t+1} &= \sigma^d_p \omega^d_{p,t+1} - \sigma^d_n \omega^d_{n,t+1}, \\
    u^s_{t+1} &= \sigma^s_p \omega^s_{p,t+1} - \sigma^s_n \omega^s_{n,t+1}, \\
    \omega^d_{p,t+1} &= \Gamma(p^d_t, 1) - p^d_t, \\
    \omega^d_{n,t+1} &= \Gamma(n^d_t, 1) - n^d_t, \\
    \omega^s_{p,t+1} &= \Gamma(p^s_t, 1) - p^s_t, \\
    \omega^s_{n,t+1} &= \Gamma(n^s_t, 1) - n^s_t, \\
    p^d_{t+1} &= \tilde{p}^d + \rho^d_p (p^d_t - \tilde{p}^d) + \sigma^d_{pp} \omega^d_{p,t+1}, \\
    p^s_{t+1} &= \tilde{p}^s + \rho^s_p (p^s_t - \tilde{p}^s) + \sigma^s_{pp} \omega^s_{p,t+1}, \\
    n^d_{t+1} &= \tilde{n}^d + \rho^d_n (n^d_t - \tilde{n}^d) + \sigma^d_{nn} \omega^d_{n,t+1}, \\
    n^s_{t+1} &= \tilde{n}^s + \rho^s_n (n^s_t - \tilde{n}^s) + \sigma^s_{nn} \omega^s_{n,t+1}.
\end{align*}
\]
Table 4: Consumption, Dividends, and Inflation Dynamics. Data are the US quarterly observations from 1970Q1 to 2012Q4. Parameters are OLS parameter estimates. Standard errors in parentheses are bootstrap standard errors computed from 10,000 bootstrap samples of historical length.

<table>
<thead>
<tr>
<th>Consumption and dividends growth</th>
<th>Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g$ 0.42%</td>
<td>$\bar{\pi}$ 1.06%</td>
</tr>
<tr>
<td>(0.04%)</td>
<td>(0.07%)</td>
</tr>
<tr>
<td>$\gamma_d$ 1.35</td>
<td>$\rho_{x\pi}$ 0.93</td>
</tr>
<tr>
<td>(1.73)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>$\gamma_{d\pi}$ 4.24</td>
<td>$\gamma_\pi$ 0.22</td>
</tr>
<tr>
<td>(5.83)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>$\sigma_d$ 0.06</td>
<td>$\gamma_{\pi d}$ 0.09</td>
</tr>
<tr>
<td>(0.03)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>$\sigma_{x\pi}$ 0.0011</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0005)</td>
</tr>
</tbody>
</table>

The dynamics are the following:

\[
\begin{align*}
g_{t+1} &= \bar{g} + \epsilon_{g,t+1}, \\
\pi_{t+1} &= \bar{\pi} + x_{\pi} + \epsilon_{\pi,t+1}, \\
\Delta d_{t+1} &= \bar{g} + \gamma_d (\sigma_d^d u_{t+1}^d + \sigma_d^s u_{t+1}^s) + \gamma_{d\pi} \sigma_d^d u_{t+1} + \sigma_d \epsilon_{d,t+1}, \\
x_{t+1} &= \rho_{x\pi} x_{t} + \gamma_{\pi} (\sigma_{\pi\pi}^d u_{t+1}^d + \sigma_{\pi\pi}^s u_{t+1}^s) + \gamma_{\pi d} \sigma_{\pi}^d u_{t+1} + \sigma_{x\pi} \epsilon_{\pi,t+1}. 
\end{align*}
\]
Table 5: Implied Consumption Growth and Inflation Moments. Model implied moments are obtained by simulating a long time series of 100,000 observations using the estimated parameters. Bootstrap standard errors in parentheses are computed from 10,000 bootstrap samples of the historical length.

<table>
<thead>
<tr>
<th></th>
<th>Consumption growth</th>
<th>Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>Mean</td>
<td>0.42%</td>
<td>0.42%</td>
</tr>
<tr>
<td></td>
<td>(0.04%)</td>
<td>(0.07%)</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.41%</td>
<td>0.44%</td>
</tr>
<tr>
<td></td>
<td>(0.03%)</td>
<td>(0.08%)</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.41</td>
<td>-0.37</td>
</tr>
<tr>
<td></td>
<td>(0.26)</td>
<td>(0.78)</td>
</tr>
<tr>
<td>Excess kurtosis</td>
<td>1.24</td>
<td>1.75</td>
</tr>
<tr>
<td></td>
<td>(0.56)</td>
<td>(2.53)</td>
</tr>
<tr>
<td>Pr(≤mean-2-Standard deviations)</td>
<td>2.91%</td>
<td>3.11%</td>
</tr>
<tr>
<td></td>
<td>(0.97%)</td>
<td>(0.60%)</td>
</tr>
<tr>
<td>Pr(≤mean-4-Standard deviations)</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td></td>
<td>(0.12%)</td>
<td>(0.60%)</td>
</tr>
<tr>
<td>Pr(≥mean+2-Standard deviations)</td>
<td>2.91%</td>
<td>2.05%</td>
</tr>
<tr>
<td></td>
<td>(1.04%)</td>
<td>(1.64%)</td>
</tr>
<tr>
<td>Pr(≥mean+4-Standard deviations)</td>
<td>0.00%</td>
<td>0.03%</td>
</tr>
<tr>
<td></td>
<td>(0.00%)</td>
<td>(0.14%)</td>
</tr>
<tr>
<td>Corr($g_t, \pi_t$)</td>
<td>-0.14</td>
<td>-0.22</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td></td>
</tr>
</tbody>
</table>
Table 6: Preference Parameter Estimates. Data are the US quarterly observations from 1970Q1 to 2012Q4. Parameters are estimated with GMM using a diagonal weighting matrix. The moments are quarterly nominal risk-free rate and its variance, the 5 year bond expected excess holding return and the variance of this return, price-dividend ratio and its variance, the average equity premium, the variance of excess equity returns, and the unconditional covariance between 5 year bond and stock returns. The standard errors are in parentheses.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta)</td>
<td>0.9920</td>
<td>(fixed)</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>4.1222</td>
<td>(0.5125)</td>
</tr>
<tr>
<td>(\bar{q})</td>
<td>1.0000</td>
<td>(fixed)</td>
</tr>
<tr>
<td>(\rho_q)</td>
<td>0.9890</td>
<td>(0.0249)</td>
</tr>
<tr>
<td>(\gamma_q)</td>
<td>-9.5067</td>
<td>(0.8402)</td>
</tr>
</tbody>
</table>

The preference specification is:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t - H_t}{1 - \gamma} \right)^{1 - \gamma} - 1, \\
Q_t = \frac{C_t}{C_t - H_t}, \\
M_{t+1} = \beta e^{-\gamma g_{t+1} + \gamma (q_{t+1} - q_t)}, \\
q_{t+1} = \bar{q} + \rho_q (q_t - \bar{q}) + \gamma_q \epsilon_{t+1}.
\]
Table 7: Unconditional Local Risk-aversion Distribution. Unconditional local risk-aversion is defined as $\gamma e^a$. The unconditional distribution is computed from simulated time series of 100,000 observations.

<table>
<thead>
<tr>
<th>Percentile</th>
<th>1%</th>
<th>5%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>95%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>6.33</td>
<td>7.30</td>
<td>8.99</td>
<td>10.58</td>
<td>13.02</td>
<td>19.85</td>
<td>29.23</td>
</tr>
</tbody>
</table>
Table 8: Asset Pricing Moments Fit: Moments Used in the GMM Estimation. Data are US quarterly observations from 1970Q1 to 2012Q4. Parameters are estimated using GMM with a diagonal weighting matrix. The standard errors in parentheses are computed from 10,000 bootstrap samples of historical length. $y_{1q}$ is the nominal 1 quarter interest rate, $r_{5y}^{bx}$ is the excess holding period return on the 5 year nominal bond, $pd$ is the log price-dividend ratio, and the $r^{ex}$ is the excess holding period return on equity.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(y_{1q})$</td>
<td>1.33%</td>
<td>1.53%</td>
</tr>
<tr>
<td></td>
<td>(0.18%)</td>
<td></td>
</tr>
<tr>
<td>$Var(y_{1q})$</td>
<td>6.48E-05</td>
<td>7.74E-05</td>
</tr>
<tr>
<td></td>
<td>(2.00E-05)</td>
<td></td>
</tr>
<tr>
<td>$E(r_{5y}^{bx})$</td>
<td>0.49%</td>
<td>0.62%</td>
</tr>
<tr>
<td></td>
<td>(0.24%)</td>
<td></td>
</tr>
<tr>
<td>$Var(r_{5y}^{bx})$</td>
<td>0.0011</td>
<td>0.0008</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td></td>
</tr>
<tr>
<td>$E(pd)$</td>
<td>5.01</td>
<td>5.09</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td></td>
</tr>
<tr>
<td>$Var(pd)$</td>
<td>0.18</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td></td>
</tr>
<tr>
<td>$E(r^{ex})$</td>
<td>1.08%</td>
<td>0.90%</td>
</tr>
<tr>
<td></td>
<td>(0.58%)</td>
<td></td>
</tr>
<tr>
<td>$Var(r^{ex})$</td>
<td>0.0085</td>
<td>0.0074</td>
</tr>
<tr>
<td></td>
<td>(0.0013)</td>
<td></td>
</tr>
<tr>
<td>$Cov(r^{ex}, r_{5y}^{bx})$</td>
<td>0.0002</td>
<td>0.0007</td>
</tr>
<tr>
<td></td>
<td>(0.0005)</td>
<td></td>
</tr>
<tr>
<td>Overidentification test p-value</td>
<td>0.2406</td>
<td></td>
</tr>
</tbody>
</table>
Table 9: Implied Pricing Moments. Data are the US quarterly observations from 1970Q1 to 2012Q4. In-sample standard errors are in parentheses. $y$ are the real yields. Data real yields are from Chernov and Mueller (2012) for 1971-2002 and from Gürkaynak, Sack, and Wright (2010b) for 2003-2012. $y^s$ are the nominal yields. $pd$ is the log price-dividend ratio. In Fama-Bliss (1987) the log excess holding return is regressed on a constant and the excess log forward rate. $AC_1$ is the lag 1 autocorrelation. $r^{ex}$ is the equity premium.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y^s_{5y} - y^s_{1y}$</td>
<td>0.18%</td>
<td>0.12%</td>
</tr>
<tr>
<td></td>
<td>(0.04%)</td>
<td></td>
</tr>
<tr>
<td>$y_{5y} - y_{1y}$</td>
<td>0.11%</td>
<td>0.09%</td>
</tr>
<tr>
<td></td>
<td>(0.02%)</td>
<td></td>
</tr>
<tr>
<td>Fama-Bliss (1987) slope: 5 year bond vs 1 year bond</td>
<td>0.77</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>(0.36)</td>
<td></td>
</tr>
<tr>
<td>$AC_1(pd)$</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td></td>
</tr>
<tr>
<td>Slope $r^{ex}_{t+1}$ wrt $pd_t$</td>
<td>-0.0204</td>
<td>-0.0056</td>
</tr>
<tr>
<td></td>
<td>(0.0171)</td>
<td></td>
</tr>
</tbody>
</table>
Table 10: Stock and Bond Return Correlations. Data are the US quarterly aggregate stock and 5 year Treasury bond return correlations from 1970Q1 to 2012Q4. The min and the max are the minimum and the maximum from the historical time series for both the data and the model. The percentiles for the data are from the historical time series and for the model from the unconditional distribution. Data expectations are the long-run (quarterly) conditional correlations from the MIDAS-dynamic conditional correlation model of Colacito, Engle, and Ghysels (2011). Bootstrap standard errors obtained by sampling 10,000 time series of historical length are in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>Data (expectations)</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Min</strong></td>
<td>-0.71</td>
<td>-0.48</td>
</tr>
<tr>
<td><strong>Max</strong></td>
<td>0.60</td>
<td>0.55</td>
</tr>
<tr>
<td>1\textsuperscript{st} percentile</td>
<td>-0.68</td>
<td>-0.19</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td></td>
</tr>
<tr>
<td>2.5\textsuperscript{th} percentile</td>
<td>-0.60</td>
<td>-0.10</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td></td>
</tr>
<tr>
<td>97.5\textsuperscript{th} percentile</td>
<td>0.55</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td></td>
</tr>
<tr>
<td>99\textsuperscript{th} percentile</td>
<td>0.57</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unconditional correlation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td></td>
</tr>
</tbody>
</table>
Table 11: Changes in Historical Stock and Bond Return Correlations. Data are the US quarterly aggregate stock and 5 year Treasury bond return correlations from 1970Q1 to 2012Q4. Data expectations are the long-run (quarterly) conditional correlations from the MIDAS-dynamic conditional correlation model of Colacito, Engle, and Ghysels (2011). The standard errors are in sample standard errors. The difference test (1970-2000 versus 2001-2012) is the Student’s $t$ difference in means test assuming unequal variances for the subsamples. *** indicates the significance at the 1% confidence level.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Data: expectations</td>
<td>0.27</td>
<td>-0.32</td>
<td>-0.59***</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.22)</td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>0.30</td>
<td>0.06</td>
<td>-0.23***</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.15)</td>
<td></td>
</tr>
</tbody>
</table>