Shifting Bottleneck Heuristic

Algorithm
1. Initialization
   - $M_0 = \emptyset$ (scheduled machines)
   - $G =$ only conjuctive arcs
   - $C_{\text{max}} =$ critical path in $G$.
2. (Choice of machine.) For each $M_i \in M - M_0$,
   - generate the $1|r_j|L_{\text{max}}$ schedule
   - compute $L_{|\text{max}}(i)$. 
3. Scheduling the bottleneck machine
   - Let $k$ be the machine that maximizes $L_{\text{max}}(i)$
   - Schedule $k$ by the $!|r_j|L_{\text{max}}$ solution
   - Update $G$
   - $M_0 = M_0 \cup \{k\}$. 
4. (Resequence already scheduled machines.) For each $M_i \in M_0 - \{k\}$
   - Delete disjunctive arcs for $M_i$ from $G$
   - Form the $1|r_j|L_{\text{max}}$
   - Reschedule $M_i$ according to this schedule
5. If $M = M_0$ stop, else go to 2
Example

<table>
<thead>
<tr>
<th>jobs</th>
<th>machine sequence</th>
<th>processing times</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,2,3</td>
<td>$p_{11} = 10, p_{21} = 8, p_{31} = 4$</td>
</tr>
<tr>
<td>2</td>
<td>2,1,4,3</td>
<td>$p_{22} = 8, p_{12} = 3, p_{42} = 5, p_{32} = 6$</td>
</tr>
<tr>
<td>3</td>
<td>1,2,4</td>
<td>$p_{13} = 4, p_{23} = 7, p_{43} = 3$</td>
</tr>
</tbody>
</table>
Iteration 1

Form the $1|r_j|\ell_{\text{max}}$ problems (Recall $d_j = LB - (CP - p_j)$)

Machine 1

<table>
<thead>
<tr>
<th>job</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_j$</td>
<td>0</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>$p_j$</td>
<td>10</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>$d_j$</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
</tbody>
</table>

Optimal schedule $1,2,3$, $L_{\text{max}}(1) = 5$
Form the $1|r_j|l_{\text{max}}$ problems

Machine 2

<table>
<thead>
<tr>
<th>job</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_j$</td>
<td>10</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>$p_j$</td>
<td>8</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>$d_j$</td>
<td>18</td>
<td>8</td>
<td>19</td>
</tr>
</tbody>
</table>

Optimal schedule $2,3,1$ $L_{\text{max}}(2) = 5$

Similarly $L_{\text{max}}(3) = 4$. $L_{\text{max}}(4) = 0$.

Schedule $M_1$ in the order 1,2,3.
Form the $1|r_j|l_{\text{max}}$ problems

**Machine 2**

<table>
<thead>
<tr>
<th>job</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_j$</td>
<td>10</td>
<td>0</td>
<td>17</td>
</tr>
<tr>
<td>$p_j$</td>
<td>8</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>$d_j$</td>
<td>23</td>
<td>10</td>
<td>14</td>
</tr>
</tbody>
</table>

**Optimal schedule** $2,1,3$, $L_{\text{max}}(2) = 1$
Iteration 2 (cont)

Form the $1|r_j|L_{\text{max}}$ problems

Machine 3

<table>
<thead>
<tr>
<th>job</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_j$</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>$p_j$</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>$d_j$</td>
<td>27</td>
<td>27</td>
</tr>
</tbody>
</table>

Optimal schedule (either), $L_{\text{max}}(3) = 1$

Similarly $L_{\text{max}}(4) = 0$.

Schedule $M_2$ in the order 2,1,3.

Resequence Trying to resequence machine 1 does not help.
Form the $1|r_j|\text{L}_{\text{max}}$ problems
Machine 3 and Machine 4 both have $\text{L}_{\text{max}}() = 0$.
Resequencing does not help.
Final schedule

Critical path length is 28. (Colors between pictures do not correspond)