An algorithm for $1 \mid \sum T_j$.

Use dynamic programming, from the following formulas:

- Let $V(J, t)$ be the optimal schedule for job set $J$ starting at time $t$.

- Let $J(j, l; k)$ be the set of job with indices between $j$ and $l$ and which have processing time less than $p_k$. Note that $k$ is excluded from this set.

Recursion:

\[
V(\emptyset, t) = 0.
\]

\[
V(\{j\}, t) = \max(0, t + p_j - d_j)
\]

\[
V(J(j, l; k), t) = \min_{\delta} \left( V(J(j, k' + \delta; k'), t) + \max(0, C_{k'}(\delta) - d_{k'}) + V(J(k' + \delta + 1, l, k'), C_{k'}(\delta)) \right)
\]

where $k'$ is the job with minimum processing time in $J(j, l; k)$. 
Example

<table>
<thead>
<tr>
<th>$j$</th>
<th>$p_j$</th>
<th>$d_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>121</td>
<td>260</td>
</tr>
<tr>
<td>2</td>
<td>79</td>
<td>266</td>
</tr>
<tr>
<td>3</td>
<td>147</td>
<td>266</td>
</tr>
<tr>
<td>4</td>
<td>83</td>
<td>336</td>
</tr>
<tr>
<td>5</td>
<td>130</td>
<td>337</td>
</tr>
</tbody>
</table>

$$V(1, \ldots, 5, 0) = \min \left\{ \begin{array}{l}
V(J(1, 3, 3), 0) + 81 + V(J(4, 5, 3), 347), \\
V(J(1, 4, 3), 0) + 164 + V(J(5, 5, 3), 430), \\
V(J(1, 5, 3), 0) + 294 + V(\emptyset, 560)
\end{array} \right\}$$

- $J(1, 3, 3) = \{1, 2\}$. These can be scheduled in order 1,2, with objective value 0.
- $J(1, 4, 3) = \{1, 2, 4\}$. We compute this recursively and get that we can schedule (1, 2, 4) with objective value 0.
- $J(1, 5, 3) = \{1, 2, 4, 5\}$. We compute this recursively and get the schedule (1, 2, 4, 5) with value 347.
• $J(4, 5, 3) = \{4, 5\}$. $V(J(4, 5, 3), 347)$ is the optimal schedule for 4 and 5 starting at time 347. This order is (4, 5) with value $94 + 223 = 317$.

• $J(5, 5, 3) = \{5\}$. $V(J(5, 5, 3), 430)$ is the optimal way to schedule job 5 starting at time 430.

so we get

$$\min \begin{cases} 
0 + 81 + 317 \\
0 + 164 + 223, \\
76 + 294 + 0 
\end{cases} = 370$$

So optimal schedule is 1, 2, 4, 5, 3
Recursion for $J(1, 4, 3)$

$k' = 4$

\[ J(1, 4, 3) = \min \{ V(J(1, 4, 4), 0) + 0 + V(\emptyset, 283) \} \]

Recursion for $J(1, 5, 3)$

$k' = 5$

\[ J(1, 5, 3) = \min \{ V(J(1, 5, 5), 0) + 76 + V(\emptyset, 413) \} \]