Assignment 2 is due September 17, 2002

1. Suppose that $B$ dollars, borrowed at time zero, are to be paid in payments of equal size over $N$ periods and the interest rate per period is $i_b$. Then $A = B(A/P, i_b, N)$ is the amount to be paid at the end of periods 1, 2, ..., $N$. Find $A$ if $B = 100,000$, $i_b = .5\%$, and $N = 360$. $A$ is the monthly payment on a 30 year $100,000 mortgage at a 6\% interest rate compounded monthly.

2. Each payment $A$ can be decomposed into two parts: interest and principal. If $B_n$ is the balance owed at the beginning of period $n = 0, 1, \ldots, N + 1$, then $i_bB_n$ is the interest paid in period $n$, and $D_n = A - i_bB_n$ is the principal paid in period $n$. Consequently,

$$B_{n+1} = B_n - D_n = B_n(1 + i_b) - A,$$

so the balance $B_{n+1}$ owed at the beginning of period $n + 1$ is the balance $B_n$ owed at the beginning of period $n$, plus the interest $i_bB_n$ on that balance, minus the payment $A$. Find $B_{17}$ and $D_{32}$ using the data of Problem 1. Notice that $B_0 = B$.

3. The interest payments $I_n = i_bB_n, n = 1, \ldots, N$ are tax deductible, so it is important to compute the present value of interest payments. Develop a formula to compute

$$PV(i) = \sum_{n=1}^{N} I_n(1 + i)^{-n}$$

where $i$ is the interest per period and $i$ need not be equal to $i_b$. Use this formula to compute $PV(i)$ for the data of Problem 1 when $i = 1\%$, and for $i = i_b = .5\%$.


5. Problem 2.19 in Textbook.