1. A company has an outstanding bond with face value $F = 1\,000$, bond interest rate $r = 8\%$, with $m = 1$ payment per year, that matures in $N = 8$ years. Suppose the bond is now selling for $P = 837.50$:

(a) Find the bond's yield to maturity $i$.
Answer: The equation is

$$837.50 = 80(P=A;i;8) + 1000(P=F;i;8):$$

Using solver in Excel, we find: $i = 11.18\%$.

NOTE: For those of you who are using Excel built-in functions, it is also possible to use RATE in order to find $i$. Then the formula would be: $\text{RATE}(N\text{per}, P\text{mt}, P\text{v}, F\text{v}) = \text{RATE}(8, 80, -837.50, 1000) = 11.18\%$. (Do not forget 'minus' before 837.5 as you first need to buy the bond). YIELD will also work when used properly, but you should be aware that you need to specify a Day Count convention. This is beyond our scope. Therefore, I would recommend RATE rather than YIELD.

(b) Suppose the company has excess cash, describe an investment the company can make that earns interest rate $i$.
Answer: The company can buy back its own bonds and earn $i$ by doing so.

(c) Suppose the company is considering raising new capital by issuing new bonds that will mature in eight years. Argue why the company cannot expect to sell these bonds at par value unless the interest rate is at least equal to the yield $i$ found in part (a).
Answer: Suppose the interest rate is $j$ and $j < i$. Now investors have a choice of buying the new bonds at par, or the old bonds at $837.50$. The yield on the new bonds is $j$, while the yield on the old bonds is $i$. All investors would prefer to buy the old bonds, so the company will be unable to sell the new bonds at par unless $j = i$. The company does not need to offer a higher yield unless issuing new bonds will make the company more risky and investors insist on a higher yield. Under these conditions $j = i$ for the bonds to sell at par.

(d) Suppose the company sells new bonds that mature in eight years, with $F = 1\,000$; $m = 1$, and bond interest rate $r = i$. Suppose further that the bonds sell at par value, and that the unit cost of issuing these new bonds is $S = 75:50$. Find the yield of the new bonds taking into account the issuing costs.
Answer: The yield to investors is of course $r = i$. For the company, however, the cost of debt capital (before taxes) is now higher because of issuing costs. The problem asks you to compute the yield from the point of view of the company. The equation is

$$924.50 = 111.18(P=A;j;8) + 1000(P=F;j;8)$$

where $111.18 = 100 + 11.18$ (rate found in a) resulting in $j = 12.67\%$. (using Excel). Thus, the before cost of raising capital through the new bonds, assuming no taxes and an immediate write-off of the selling expenses, is 12.67%.

2. Recall that under the growth model

$$i_e = \frac{DPS_1}{P_o} + g,$$

where $DPS_1$ is the dividend per share at the end of year 1, and $P_o$ is the ex-post current price of the stock.
(a) Find an expression for \( P_0 \) in terms of \( DPS_1; i_e; \) and \( g \). We will be using this formula to study the sensitivity of the price \( P_0 \) to changes in \( DPS_1; i_e; \) and \( g \).

Answer:

\[
P_0 = \frac{DPS_1}{i_e + g}
\]

(b) Suppose that your current estimates of \( DPS_1 \) and \( g \) are $1, and \( g = 10\% \) respectively. What is the value of \( P_0 \) if \( i_e = 15\% \)?

Answer:

\[
P_0 = \frac{1}{0.15 - 0.10} = 20
\]

(c) Now assume that you revise your estimates of \( DPS_1 \) and \( g \) to $0.90, and \( g = 8\% \) respectively. At the same time, investors perceive the company to be more risky and demand \( i_e = 17\% \). Find the new value of \( P_0 \).

Answer:

\[
P_0 = \frac{0.9}{0.17 - 0.08} = \frac{0.9}{0.09} = 10
\]

(d) Find the ratio of the \( P_0's \) obtain in parts (b) and (c).

Answer: The ratio is 2. This explains that modest changes in the estimates of \( DPS_1; g; \) and \( i_e \) can lead to large changes in \( P_0 \).

3. Consider a variation of the growth model for valuating common stocks. Under this model, the analyst forecasts dividends per share \( DPS_n \) for \( n = 1; \ldots; T \), after which he or she assumes that \( D_k = D_T(1 + g)^{k-T} \) for \( k = T + 1; \ldots; \infty \). That is, dividends are expected to grow at a constant rate \( g \) after period \( T \). Let \( S^+(i_e) \) be the present value of the dividends up to time \( T \), and let \( S^+(i_e) \) be the present value of the dividends after time \( T \).

(a) Find a formula for \( S(i_e) = S^+(i_e) + S^+(i_e) \).

Answer: \( S^+(i_e) = \sum_{n=1}^{T} DPS_n (1 + i_e)^n \). For \( n > T \) we have

\[
S^+(i_e) = DPS_T (1 + i_e)^T \frac{1 + g}{i_e + g}
\]

(b) Let \( P_o \) denote the ex-post price of the stock at time zero. The value \( i_e \) that makes \( S(i_e) = P_o \) is then the cost of equity capital. Find \( i_e \) for \( P_o = $55, \) \( DPS_1 = $2, \) \( DPS_2 = $3, \) \( T = 2, \) and \( g = 10\% \).

Answer:

\[
S(i_e) = 2(1 + i_e)^0 + 3(1 + i_e)^1 + 3(1 + i_e)^2 + \frac{1.1}{i_e + 10} = 55
\]

Solving \( S(i_e) = 55 \) results in \( i_e = 14.90\% \):

4. Consider a company that uses half debt and half equity as its capital structure. Assume that lenders require a return of \( i_b = 10\% \) and equity investors require a return of \( i_e = 14\% \), and that the company pays no taxes (so \( k_b = i_b \)).

(a) Find the WACC.

Answer: The assumption is that \( S = B = 0.5V \) at all times. Then \( B = V = S = V = 0.5 \). Hence the WACC is 0.5(1.10) + 0.5(1.14) = 0.12 = 12\%:

(b) An investment costing $1,000 generates cash flow of $700 a year for two years, so the cash flow is \( F_0 = i = $1; \) \( F_1 = F_2 = $700 \). Find the present value of this cash at the WACC.

Answer: \( PV(12\%) = i 1000 + 700(1.12) + 700(1.12)^2 = 183.04 \):
(c) Find the equity cash flow assuming that debt and equity are kept in the same proportion every year. Thus at any time, debt should be equal to one half the present value of the remaining cash in flows discounted at the WACC.

We must keep $B = S$ at all times. The value $V$ at time zero is $V = 1; 183:04$, which is the present value of the cash flows discounted at the WACC. Consequently $B = 591:52 = 1183:04 = 2$. Notice that the value of the equity holders is also $S = 591:52$ although they only invest 408.48. The net equity flow at the EOY 0 is therefore $F_0 = i 408:48$: At the EOY 1, $V = 700 = 1:12 = 625$, so $B = 312:50$: To reduce the debt to that level, equity holders must pay 59.15 on interest, and 279.02 of principal. Therefore the equity flow at the EOY 1 is $F_1 = 700 i 279:02 i 59:15 = 361:83$: Finally, at the EOY 2, $V$ will be zero, so we need to pay off the debt. This requires 312:50 to pay the principal. Consequently $F_2 = 700 i 31:25 i 312:50 = 356:25$.

(d) Find the present value of the equity cash flow at rate $i_e$.

Answer: The equity cash flow is $F_0 = i 408:48$; $F_1 = 361:83$; and $F_2 = 356:25$. The present value of the equity flow at interest rate $i_e$ is equal to 183:04.

(e) Compare parts (b) and (d). Is this a coincidence? We get the same results. No, it is not a coincidence. It happens whenever $B = V$ remains constant. See, example 5.10. There the present value of the cash flows at time zero are equal to the cost of the asset. That is, $B + S = 150; 000$.

5. Suppose you can borrow money at $i = 25\%$ per year. Currently you have no cash, but you anticipate that you can earn $j = 5\%$ per year on cash holdings. You want to determine whether or not it is profitable for you to borrow money to invest it in a project with the following cash flows: $F_0 = i 1; 000; F_1 = 1; 300; F_2 = i 1; 000; F_3 = 1; 250$.

(a) Suppose you borrow $1,000 and invest this money in the project. What is your balance at the end of year 1? Is it positive?

Answer: $P_B = i 1000(1:25) + 1300 = 50$:

Yes, it is positive.

(b) How much do you need to borrow at the end of year 2?

Answer: $P_B = 50(1:05) i 1000 = i 947:50$:

I need to borrow 947.50.

(c) What is your balance at the end of year 3? Is it positive?

Answer: $P_B = i 947:5(1:25) + 1250 = 65:625$:

Yes, the balance is positive.

(d) Interpret the meaning of the balance at the end of year 3.

Answer: The ending balance is the amount I can keep at the end of the project after paying debt and interest.

(e) Should you borrow to invest in this project?

Answer: Yes. Here, I am assuming that either this is my only investment opportunity, or I can borrow unlimited amounts at 25%.

(e) What is the meaning of a positive balance at the end of year 3 if you are borrowing from yourself at rate $i = 25\%$?

Answer: The ending balance is what I can keep in addition to the interest I pay myself.