IEOR 4003 INDUSTRIAL ECONOMICS

Solution for Homework Set 2.

1. The equal payment amount $A$ for a loan of size $B$ is calculated by setting $A = B(A/P, i_b, N)$. Substituting the values given in the problem,

$$A = 100,000(A/P, 0.5\%, 360)$$
$$= 599.55.$$ 

2. We write the loan balance for each year starting from year 1,

$$B_1 = B$$
$$B_2 = B(1 + i_b) - A$$
$$B_3 = (B(1 + i_b) - A)(1 + i_b) - A$$
$$= B(1 + i_b)^2 - A((1 + i_b) + 1)$$
$$\vdots$$
$$B_n = B(1 + i_b)^{n-1} - A\left\{(1 + i_b)^{n-2} + \frac{(1 + i_b)^{n-1} - 1}{i_b}\right\}$$
$$= B(F/P, i_b, n - 1) - A(F/A, i_b, n - 1).$$

Then we have

$$B_{17} = 100,000(F/P, 0.5\%, 16) - 599.55(F/A, 0.5\%, 16)$$
$$= 98,346.05,$$

and

$$D_{32} = A - i_b B_{32}$$
$$= 599.55 - (0.005)\{100,000(F/P, 0.5\%, 31) - 599.55(F/A, 0.5\%, 31)\}$$
$$= 116.19.$$

3. We can write $I_n = i_b B_n$ as

$$I_n = i_b \left\{B(1 + i_b)^{n-1} - A\left\{(1 + i_b)^{n-1} - 1\right\}\right\}$$
$$= (i_b B - A)(1 + i_b)^{n-1} + A.$$
The present value of \( \{I_n\} \) at interest rate \( i \) becomes

\[
PV(i) = \sum_{n=1}^{n=N} I_n(1+i)^{-n}
\]

\[
= \sum_{n=1}^{n=N} \left( i_b B - A \right) \frac{(1+i_b)^{n-1}}{(1+i)^n} + A \sum_{n=1}^{n=N} \frac{1}{(1+i)^n}
\]

Now let

\[
(1+g) = \frac{(1+i)}{(1+i_b)}.
\]

Then,

\[
PV(i) = \left( \frac{i_b B - A}{1+i_b} \right) \sum_{n=1}^{n=N} \frac{1}{(1+g)^n} + A \sum_{n=1}^{n=N} \frac{1}{(1+i)^n}
\]

\[
= \left( \frac{i_b B - A}{1+i_b} \right) (P/A, g, N) + A(P/A, i, N).
\]

(a) When \( i = 1\% \) and \( i_b = 0.5\% \), \( g = 0.495\% \). Then,

\[
PV(1\%) = \frac{(0.005)(100,000) - 599.55}{1 + 0.005} \frac{1}{(P/A, i = 1\%, N)} = 41,712.74.
\]

(b) When \( i = i_b = 0.5\% \), \( g = 0 \). Then,

\[
PV(1\%) = 360 \frac{(0.005)(100,000) - 599.55}{1 + 0.005} = 64,340.12.
\]

4. (a) \( 10,000 = 460 \) (P/A, i, 30). Solving for \( i \) gives \( i = 2.22\% \).

(b) \( i_a = (1 + 0.0222)^{12} - 1 = 30.12\% \).

(c) \( 460 + 460 \) (P/A, 2.22\%, 14) = 5944.34.

5. The heating oil price in a year from now is $1.10. The total amount required to buy 1000 gallons is $1100 during the first year. Hence,

\[
P = 1100(P/A, 10, 6, 10)
\]

\[
= 1100 \frac{1 - (1+0.1)^{10}(1+0.06)^{-10}}{0.06 - 0.1}
\]

\[
= 12,329.13
\]