1. (15 points, 10 minutes) Answer the following questions assuming the interest rate is 14%. You don't have to write down a numerical answer. It is enough to write down the solution in terms of the time value of money factors studied in class.

(a) What is the present value of $1,000 in 5 years?
Answer: \( PV = 1,000 \times (P/F, 14\%, 5) \)

(b) How much is a bond worth that has 10 years to maturity, a $1,000 face value, and a bond interest rate of 10% if interest payments are made annually?
Answer: \( P = 100 \times (P/A, 14\%, 10) + 1,000 \times (P/F, 14\%, 10) \)

(c) A couple wants to save $100,000 over the next 18 years for their child's college education. What uniform annual amount must they deposit at the end of each year to achieve their objective?
Answer: \( A = 100,000 \times (A/F, 14\%, 18) \)
2. (20 points, 15 minutes) You want to buy or lease a 1999 Honda Accord EX which you plan to use for four years. The dealer offers you three options:

- Pay $21,000.00 cash.
- Give a down payment of $4,200.00 and make 48 monthly payments of $400 each.
- Lease the car by paying a non-refundable fee of $1,500.00 at inception and 48 monthly payments of $250.00 each.

Assume that the car will sell for $12,500.00 after 4 years. Write down the cash flow for each alternative and rank the alternatives by their monthly equivalent cost using a discount rate of 5% compounded monthly. Note: $(A=P; 5\%/12; 48) = 0.023:

Answer:
Cash flows:

<table>
<thead>
<tr>
<th>Option</th>
<th>Month 0</th>
<th>Month 1,...,47</th>
<th>Month 48</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opt:1</td>
<td>-$21,000</td>
<td>$0</td>
<td>$12,500</td>
</tr>
<tr>
<td>Opt:2</td>
<td>-$4,200</td>
<td>-$400</td>
<td>-$400 + $12,500</td>
</tr>
<tr>
<td>Opt:3</td>
<td>-$1,500</td>
<td>-$250</td>
<td>-$250</td>
</tr>
</tbody>
</table>

Let $A_1$ be the monthly equivalent for option 1

$A_1 = 21,000 \times (A=P; 5\%/12; 48) \downarrow 12; 500 \times (A=F; 5\%/12; 48)$

$A_1 = 21,000 \times 0.023 \downarrow 12; 500 \times 0.023 = (1 + 0.05 = 12)^{48}$

$A_1 = $247.52

Option 2:

$A_2 = 4,200 \times (A/P, 5\%/12, 48) + 400 - 12,500 \times (A/F, 5\%/12, 48)$

$A_2 = 4200 \times 0.023 + 400 \downarrow 12; 500 \times 0.023 = (1 + 0.05 = 12)^{48}$

$A_2 = $261.11

Option 3:

$A_3 = 1,500 \times (A/P, 5\%/12, 48) + 250$

$A_3 = $284.5

Therefore Option 1 is better than option 2 and Option 2 is better than Option 3.
3. (20 points, 15 minutes) True or False.

(a) ( ) The effective interest rate is never larger than the nominal interest rate when the number of compounding periods per year, say \( m \), is at least one.

Answer: False. It is larger. The effective interest rate is \((1 + (\text{nominal rate})^m - 1)\) which is larger than (nominal rate). This actually means that when \( m \) is at least one you get interest on the interest, therefore your effective rate is larger than the nominal rate.

(b) ( ) It is always best to depreciate an asset as quickly as possible.

Answer: False. It might not be the case if tax rates are not constant over the life of the asset.

(c) ( ) If the inflation rate over a three year period is respectively 5%, 3%, and 7%, then the average inflation rate is at least 5%.

Answer: False. The average inflation is \((1.05 \times 1.03 \times 1.07)^{1/3} - 1 = 4.99\%

(d) ( ) The present value criterion gives the correct answer if either \( i \) is equal to \( j \) or if \( i > i_{\text{min}} \).

Answer: True
4. (20 points, 15 minutes) Consider a project with cash flow \( F_0 = \$1,000, F_1 = \$1,400; F_2 = \$300 \): Compute the project balances \( PB(i;j)_n \) at \( i = 20\% \) and \( j = 10\% \); where \( i \) denotes the M A R R, and \( j \) denotes the reinvestment rate.

(a) Would you recommend this project?
Answer: We need to compute the project balances: \( PB(0) = -$1,000. \)
\[
PB(1) = -1,000 \times 1.2 + 1,400 = \$200. \]
\[
PB(2) = 200 \times 1.1 - 300 = -$80. \]
\( PB(2) \) is negative, therefore we would not recommend the project.

(b) Is the investment mixed?
Answer: Computed with \( i \) and \( j \), some project balances are positive, some negative, the investment is mixed.

(c) What is the maximum rate \( i \) at which you can borrow money and break even when \( j = 15\% \)?
Answer: \( PB(0) = -$1,000. \) \( PB(0) \) is negative therefore we will use \( i \) to compute \( PB(1). \)
\[
PB(1) = -1,000 \times (1+i) + 1,400 = 400 - 1,000 \times i. \]
We need to consider 2 cases depending on whether \( i <= 40\% \) or \( i > 40\% \)
If \( i > 40\% \) then \( PB(1) < 0 \), therefore we would use \( i \) to compute \( PB(2) \) but we would end up with a negative value for \( PB(2). \)
Now if \( i <= 40\% \) then \( PB(1) >= 0 \) and \( PB(2) = (400 - 1,000 \times i) \times (1.15) - 300 = 160 - 1,150 \times i. \) \( PB(2) \) is nonnegative if and only if \( i \cdot 13.91\% \)
This means \( i_{max} = 13.91\% \)

(d) What is the minimum rate \( j \) at which you need to invest over-recovered balances when \( i = 25\% \) in order to break even?
Answer: \( PB(0) = -$1,000. \) \( PB(1) = -1,000 \times 1.25 + 1,400 = $150. \) \( PB(2) = 150 \times (1+j) - 300. \)

We want \( PB(2) \) to be nonnegative, this implies \( 150 + 150j \geq 0 \) and \( j \geq 1 \)

Therefore \( j_{\text{min}} = 100\% \)

5. (15 pts. 15 min.) The dual variables for a post-horizon cash flow model \((N = 3)\) are given by \( \frac{1}{2}^0 = 2:088; \frac{1}{2}^1 = 1:74; \frac{1}{2}^2 = 1:3; \frac{1}{2}^3 = 1 \) The lending rate is 20% and the borrowing rate is 30%. Moreover, there is a maximum amount that can be borrowed in any given period. Determine whether it is optimal to borrow or lend in each period, and whether the borrowing limit is in effect. In addition, determine the marginal value of increasing the credit line by one dollar in each period.

\( \frac{1}{2}^0 = 1:2 = 1 + r_l \) Therefore we are lending money in period 0 in the optimal solution.

\( \frac{1}{2}^1 = 1:338 > 1 + r_b \) There is borrowing in period 1 and the limit is under effect.

\( \frac{1}{2}^2 = 1:3 = 1 + r_b \) There is borrowing in period 2.

If we increase the borrowing limit by 1 unit in period 0 or 2, the objective function will not increase. Therefore the marginal value of increasing the credit line by one dollar is 0 in period 0 and 2. In period 1 however, as the limit is reached, if we call \( - \) the dual variable corresponding to the borrowing limit constraint, we have: \( \frac{1}{2}^1 + (1 + r_b) \times \frac{1}{2}^2 + - = 0 \)

\( - = 1:74; \) \( 1:3 \times 1:3 = 0:05 \), this is the marginal value of increasing the credit line by one dollar in period 1.