IEOR 4003 INDUSTRIAL ECONOMICS

Solution for Homework Set 2.

1. The equal payment amount $A$ for a loan of size $B$ is calculated by setting $A = B(A/P, i_b, N)$. Substituting the values given in the problem,

\[
A = 100,000(A/P, 0.5\%, 360) = \$599.55.
\]

2. We write the loan balance for each year starting from year 1,

\[
\begin{align*}
B_1 &= B \\
B_2 &= B(1 + i_b) - A \\
B_3 &= (B(1 + i_b) - A)(1 + i_b) - A \\
&= B(1 + i_b)^2 - A((1 + i_b) + 1) \\
&\vdots \\
B_n &= B(1 + i_b)^{n-1} - A((1 + i_b)^{n-2} + (1 + i_b)^{n-1} + \ldots + 1) \\
&= B(1 + i_b)^{n-1} - A \left( \frac{(1 + i_b)^{n-1} - 1}{i_b} \right) \\
&= B(F/P, i_b, n - 1) - A(F/A, i_b, n - 1).
\end{align*}
\]

Then we have

\[
B_{17} = 100,000(F/P, 0.5\%, 16) - 599.55(F/A, 0.5\%, 16) = \$98,346.05,
\]

and

\[
D_{32} = A - i_bB_{32} \\
= 599.55 - (0.005)\{100,000(F/P, 0.5\%, 31) - 599.55(F/A, 0.5\%, 31)\} \\
= \$116.19
\]

3. We can write $I_n = i_bB_n$ as

\[
I_n = i_b \left( B(1 + i_b)^{n-1} - A \left( \frac{(1 + i_b)^{n-1} - 1}{i_b} \right) \right) \\
= (i_bB - A)(1 + i_b)^{n-1} + A.
\]
The present value of \( \{I_n\} \) at interest rate \( i \) becomes

\[
PV(i) = \sum_{n=1}^{N} I_n (1+i)^{-n}
\]

\[
= \sum_{n=1}^{N} \left( i_b B - A \right) \frac{(1+i)^{n-1}}{(1+i)^n} + A \sum_{n=1}^{N} \frac{1}{(1+i)^n}
\]

\[
= \frac{(i_b B - A)}{(1+i_b)} \sum_{n=1}^{N} \frac{(1+i)^n}{(1+i)^n} + A \sum_{n=1}^{N} \frac{1}{(1+i)^n}
\]

Now let

\[
(1+g) = \frac{(1+i)}{(1+i_b)}.
\]

Then,

\[
PV(i) = \frac{(i_b B - A)}{(1+i_b)} \sum_{n=1}^{N} \frac{1}{(1+g)^n} + A \sum_{n=1}^{N} \frac{1}{(1+i)^n}
\]

\[
= \frac{(i_b B - A)}{(1+i_b)} (P/A, g, N) + A(P/A, i, N).
\]

(a) When \( i = 1 \% \) and \( i_b = 0.5 \%, g = 0.495 \%. \) Then,

\[
PV(1\%) = \frac{(0.005)(100,000) - 599.55}{(1+0.005)} (P/A, 0.495\%, 360) + 599.55( P/A, 1\%, 360)
\]

\[
= $41,712.74.
\]

(b) When \( i = i_b = 0.5 \%, g = 0. \) Then,

\[
PV(1\%) = 360 \frac{(0.005)(100,000) - 599.55}{(1+0.005)} + 599.55(P/A, 0.5\%, 360)
\]

\[
= $64,340.12.
\]

4. (a) 10,000 = 460 (P/A, i, 30). Solving for \( i \) gives \( i = 2.22 \% \).

(b) \( i_a = (1 + 0.0222)^{12} - 1 = 30.12 \% \).

(c) From a strict discounting point of view the answer is 460 + 460 (P/A, 2.22\%, 14) = $5944.34. However, this may not be acceptable to the lender and he may insist on other forms of discounting, e.g., 460 times 15.

5. The heating oil price in a year from now is $1.10. The total amount required to buy 1000 gallons is $1100 during the first year. Hence,

\[
P = 1100(P/A, 10, 6, 10)
\]

\[
= 1100 \frac{1 - (1 + 0.1)^{10}(1 + 0.06)^{-10}}{0.06 - 0.1}
\]

\[
= 12,329.13
\]