INSTRUCTIONS

1. The exam is closed book. You are allowed one 8.5 by 11 sheet with notes on one side.

2. Write all answers on the question sheets. Answers written on scrap paper will not be evaluated. Show and explain your work.

3. Sign the honor pledge at the bottom of this page.

- Last name:______________________
- First name:______________________
- Social SN:______________________

HONOR PLEDGE: I pledge that I have neither given nor received unauthorized aid during this examination.

__________________________
Signature
1. (15 points, 10 minutes) Answer the following questions assuming the interest rate is 14%. You don’t have to write down a numerical answer. It is enough to write down the solution in terms of the time value of money factors studied in class.

(a) What is the present value of a cash flow of $1,000 to be received in 5 years?

\[
P V(1000) = 1000(P|F, 14\%, 5) = \frac{1000}{(1 + 0.14)^5}
\]

(b) How much is a bond worth that has 10 years to maturity, a $1,000 face value, and a bond interest rate of 10% if interest payments are made annually?

\[
1000(P|F, 14\%, 10) + 100(P|A, 14\%, 10)
\]

(c) A couple wants to save $100,000 over the next 18 years for their child’s college education. What uniform annual amount must they deposit at the end of each year to achieve their objective?

\[
100000(A|P, 14\%, 18)
\]
2. (20 points, 15 minutes) You want to buy or lease a 2003 Honda Civic which you plan to use for four years. The dealer offers you three options:

- Pay $17,000.00 cash.
- Buy the car by giving a down payment of $2,000.00 and making 48 monthly payments of $350 each.
- Lease the car by paying a non-refundable fee of $1,500.00 at inception and 48 monthly payments of $200.00 each.

Assume that if you buy the car (first two options), you will sell it for $8,000.00 after 4 years. Write down the cash flow for each alternative and rank the alternatives by their monthly equivalent cost using a discount rate of 5% compounded monthly. Note: \((A/P, \frac{5\%}{12}, 48) = 0.023\).

**Option 1:**
Month 0: $-17000
Month 1-48: $0
Month 48: $8000

\[
A_1 = P(A/P, 5\%/12, 48) = -17000 \cdot 0.023 = -391 \\
A_2 = 8000(A/F, 5\%/12, 48) = 151 \\
A^1 = A_1 + A_2 = -391 + 151 = $ -240
\]

**Option 2:**
Month 0: $-2000
Month 1-48: $-350
Month 48: $8000

\[
A_1 = P(A/P, 5\%/12, 48) = -2000 \cdot 0.023 = $ -46 \\
A_2 = $ -350 \\
A_3 = 8000(A/F, 5\%/12, 48) = 151 \\
A^2 = A_1 + A_2 + A_3 = -46 -350 + 151 = $ -245
\]

**Option 3:**
Month 0: $-1500
Month 1-48: $-200

\[
A_1 = P(A/P, 5\%/12, 48) = -1500 \cdot 0.023 = $ -34.5 \\
A_2 = $ -200 \\
A^3 = A_1 + A_2 = -200 -34.5 = $ -234.5
\]

Option 3 has the smallest annual payments.

\[A^3 < A^1 < A^2\]
3. (20 points, 15 minutes) True or False.

(a) ( ) The effective interest rate per year is always larger than the nominal interest rate when the number of compounding periods per year \( m > 1 \).
True.

(b) ( ) It is always best to depreciate an asset as quickly as possible.
False.

(c) ( ) If the inflation rate over a three year period is respectively 5\%, 3\%, and 7\%, then the average inflation rate is at least 5\%.
False.

(d) ( ) The present value criterion gives the correct answer if you can reinvest over-recovered balances at the MARR or if the MARR is at least as large as \( i_{\text{min}} \).
True.
4. (20 points, 15 minutes) Consider a project with cash flow $F_0 = -$10,000, $F_1 = $14,000, $F_2 = -$3,000. Compute the project balances $PB(i, j)_n$, $n = 0, 1, 2$ at $i = 20\%$ and $j = 10\%$, where $i$ denotes the MARR, and $j$ denotes the reinvestment rate.

(a) Would you recommend this project?

\[
P_B(20\%, 10\%)_0 = -10000
\]
\[
P_B(20\%, 10\%)_1 = -10000(1 + 20\%) + 14000 = 2000
\]
\[
P_B(20\%, 10\%)_2 = 2000(1 + 10\%) - 3000 = -800
\]
since $PB_2 < 0$, we would not recommend this project.

(b) Is the investment mixed?

\[
P_B(i_{min})_0 = -10000
\]
\[
P_B(i_{min})_1 = -10000i + 14000
\]
to keep $PB(i_{min})_0$ and $PB(i_{min})_1$ non positive,

\[
i_{min} \geq 1.4
\]
and

\[
P_B(1.4)_2 = -3000 < 0
\]
This investment is mixed.

(c) What is the maximum rate $i$ at which you can borrow money and break even when $j = 15\%$?

\[
P_B(i, 15\%)_0 = -10000
\]
\[
P_B(i, 15\%)_1 = -10000(1 + i) + 14000 = 4000 - 10000i
\]
\[
P_B(i, 15\%)_2 = (4000 - 10000i)(1 + 15\%) - 3000 = 1600 - 11500i
\]
We should keep $PB_2 = 0$, i.e., $i = \frac{16}{115} \approx 13.9\%$

(d) What is the minimum rate $j$ at which you need to invest over-recovered balances when $i = 25\%$ in order to break even?

\[
P_B(25\%, j)_0 = -10000
\]
\[
P_B(25\%, j)_1 = -10000(1 + 25\%) + 14000 = 1500
\]
\[
P_B(25\%, j)_2 = 1500(1 + j) - 3000
\]
To keep $PB_2 = 0$, we should keep $j = 1$ as the minimum rate.
5. (15 pts. 15 min.)

The dual variables for a post-horizon cash flow model \((N = 3)\) are given by \(\rho_0 = 2.088, \rho_1 = 1.74, \rho_2 = 1.3, \rho_3 = 1\). The lending rate is 20% and the borrowing rate is 30%. Moreover, there is a maximum amount that can be borrowed in any given period. Determine whether it is optimal to borrow or lend in each period, and whether the borrowing limit is in effect. In addition, determine the marginal value of increasing the credit line by one dollar in each period.

\[
\rho_{i+1}(1 + \gamma_l) \leq \rho_i \leq \rho_{i+1}(1 + \gamma_b) + \beta_i
\]

where \(\beta_i\) is the dual variable corresponding to the constraint \(w_i \leq W_i\).

\(\frac{\rho_0}{\rho_1} = 1.2 = (1 + \gamma_l)\) It is optimal to lend in period 0, and the bound is not in effect. Since it is a lending period, increasing the credit line would not effect its action, so the marginal value is 0.

\(\rho_1 = (1 + \gamma_b)\rho_2 + 0.05\) It is optimal to borrow in period 1 and bound is in effect. The marginal value would be 0.05.

\(\rho_2 = (1 + \gamma_b)\rho_3\) It is optimal to borrow in period 2 and the bound is not in effect. Since the bound is not in effect, increasing the credit line would not effect the whole action. And the marginal value is 0.