1. **Capital Market Line** Assume the expected rate of return on the market portfolio is 12% and the rate of return on T-bills is 5%. The standard deviation of the market is 14%. Assume the market portfolio is efficient.

   (a) What is the equation of the capital market line.
   
   (b) If an expected return of 15% is desired, what is the standard deviation of this position? If you had $1,000 to invest, how should you allocate it to achieve the above position if you can borrow and lend at 5%?
   
   (c) If you invest $300 in the risk-free asset and $700 in the market portfolio, how much money would you expect to have at the end of the year.

2. **Two Asset Economy** Consider an economy with only two risky stocks, A and B. There are 100 shares outstanding of Stock A, and 150 shares outstanding of Stock B. The price per share of stock A is $1.50, while that of stock B is $2.00.

   (a) Find the market capitalization of each stock, and form the market portfolio using market capitalization weights.
   
   (b) The expected return of stock A is 15%, and that of stock B is 12%. Find the expected return of the market portfolio.
   
   (c) The standard deviation of returns are 15% and 9% respectively for stock A and stock B. Assume that the correlation coefficient between the returns of stocks A and B is $\rho = 1/3$.
   
   (d) Find the standard deviation of the market portfolio.
   
   (e) Find the Beta of stock A.
   
   (f) Find the Beta of stock B. Compute the expected return of stock B using the CAPM.

3. **Residual Risk under the CAPM**

   Under the CAPM excess returns are given by
   
   $$R_t - R_f = \beta_t(R_m - R_f) + \epsilon_t,$$
   
   where $\beta_t = \sigma_{tm}/\sigma_m^2$, and the random variable $\epsilon_t$ is uncorrelated with the market portfolio.

   (a) Show that you can write the variance-covariance matrix as
   
   $$V = \sigma^2 \beta \beta' + T$$
   
   where $T$ is the variance-covariance matrix of residual risk, i.e., $T_{ij} = \text{Cov}(\epsilon_i, \epsilon_j)$.

   (b) Show that $Tx_m = 0$.

   (c) Show that for any portfolio $x_p$ the residual variance can be written as $\omega_p^2 \equiv x_p'Tx_p = \sigma_p^2 - \sigma_{m}^2 \beta_p^2$, where $\sigma_p^2 = x_p'Vx_p$ and $\beta_p = \beta'x_p$.

   (d) Argue that the market portfolio is the minimum variance portfolio with beta equal to one.

4. **Multi-Period Investments** Consider a security with annual total return $TR = 0.5$ with probability 0.5 and $TR = 1.8$ with probability 0.5.

   (a) Compute the mean $\mu$ and the standard deviation $\sigma$ of the return $R = TR - 1$.

   (b) Let $TR_n$ be the total return in year $n = 1, 2, \ldots, N$. Assume the returns over different years are independent and have the same distribution. The return over $N$ periods is given by $TR_1TR_2 \ldots TR_N - 1$. Find a formula for the mean and standard deviation of the return over $N$ years in terms of $\mu$ and $\sigma$. 
(c) Use the formula obtained in part (b) to compute the ratio of the mean return to the standard deviation of return for $N = 1, 2, \ldots, 30$ years. Is the ratio increasing or decreasing over time?

(d) Let $V = \ln(TR)$. Compute the mean $\nu$ and the standard deviation $\tau$ of $V$.

(e) Write $TR_1 TR_2 \ldots TR_N = \exp(V_1 + V_2 + \ldots V_N)$. By the central limit theorem, for large $N$,

\[ V_1 + V_2 + \ldots V_N \]

is approximately normal with mean $N\nu$ and variance $N\tau^2$. Use this approximation to compute the probability that the return over $N$ periods is negative, i.e.,

\[ \text{Prob}(TR_1 TR_2 \ldots TR_N \leq 1). \]

In particular, compute this probability for $N = 30$.

(f) What happens to

\[ \text{Prob}(TR_1 TR_2 \ldots TR_N \leq 1) \]

as $N \to \infty$.

(g) Would you invest in this security? Why or why not?

5. **Portfolio Rebalancing** Assume there are two independent securities with the same return characteristics as in problem 2. Consider the two investment strategies:

- **Buy-and-hold** Invest half your money on each of the two securities and hold this portfolio indefinitely.
- **Yearly Rebalance** Invest half your money on each of the two securities. At the end of each year rebalance your portfolio so that half the money is invested in each of the securities.

(a) Find the mean and variance of the return under the buy-and-hold strategy at the end of 30 years.

(b) Find the mean and variance of the return under the yearly rebalance strategy at the end of 30 years.

(c) Which of these strategies, if any, would you recommend?