Hints for the Final Exam

December 13, 2010

We will use two benchmark models: the binomial models and the Black-Scholes model. No arbitrage will be a standard assumption unless otherwise stated. Here are some points you need to pay attention.

• Know how to price an European call and a lookback option in a two-period binomial model. Both the backward recursion and the risk-neutral expectation are required. In Black-Scholes model, you can derive the Black-Scholes PDE for European call price, and you know Black-Scholes formula.

• Know how to price a forward contract with risk-neutral expectation, and be able to use Put-Call parity. Notice: put-call parity is only true for European options.

• Know how to price an American put/straddle in a two-period binomial model using backward recursion.

• For the above American derivatives, be able to find the optimal exercise time for each contract.

• The physical probabilities \((p, q)\) in binomial models, and the physical mean rate of return \((\alpha)\) in Black-Scholes model, are not relevant in derivative pricing, because risk-neutral measure is the correct measure for derivative pricing.

• In BS model, stock A has an annualized mean rate of return \(\alpha_1 = 30\%\) and volatility \(\sigma_1 = 10\%\), stock B has an annualized mean rate of return \(\alpha_2 = 15\%\) and volatility \(\sigma_2 = 20\%\). Assume both stocks are worth 100 now, compare the prices of co-terminal at-the-money calls on these two stocks.

• Is it ever optimal to early exercise an American call option?

• A put and a call on a stock struck at the forward price have the same value by put-call parity. Yet the value of a put is bounded and the value of a call is unbounded. Explain how they can have the same value?

• Which is riskier: a call or the underlying?

• Do you gain anything over the risk-free rate by continuously delta-hedge a call option in a ideal B-S model?

• You can solve for Vasicek model and work out the mean and variance.

• Assume we have a risk-neutral measure (so no arbitrage is allowed), and the underlying price process is continuous (not necessarily Black-Scholes). The interest rate is always zero, and the present stock price is 100, what is the price of a up-and-out put struck with strike at 120 and knockout barrier at 120?

• Interpretation of Black-Scholes formula: the time-0 price of a call struck at \(K\) is given by \(S_0N(d_+) - Ke^{-rT}N(d_-)\). Show that \(N(d_-)\) is the risk-neutral probability that the underlying will finish in the money. Construct another probability \(\tilde{P}^S\) and show that

\[N(d_+) = \tilde{P}^S(\text{"the underlying will finish in the money")}\]

• Sketch the graph of delta, gamma and theta of an European call.

• Assume that there exists a risk-neutral measure. Also assume the underlying process is continuous (but not necessarily B-S model). Consider a digital option that pays $1 if the price increases to 200. What is the present price of the option if the current stock price is 100.

• What is the distribution of \(\int_0^1 W_t dt\), where \(\{W_t\}\) is a standard Brownian motion.