Hints for the Second Test

November 9, 2010

The second middle term is on Nov 15th. Below are some points you need to pay attention.

• Reflection principle of Brownian motion. You do not have to remember the exact formula for the joint density of Brownian motion and its running maximum. But you need to be able to use this principle. (What is the relationship between $P($a BM starting at 0 hits $m > 0$ by time $t$) and $P(B_t \geq m)$, etc.)

• You know properties of Itô integral and Itô isometry. You know how to show an Itô process is a martingale. (no drift in its differential)

• You can apply Itô’s formula to solve Vasicek model (see Exercise 4.8).

• As long as option pricing is concerned, you can tell if the actual drift part of the stock price dynamics matters or not. (see Lecture notes in change of measure, or Lecture notes 15)

• Assuming interest rate is 0. If you short a call option at time 0, and then purchase $\Delta(t, S_t)$ shares of stocks to continuously hedge that short position from time 0 to time $T$. At time $T$, are you going to lose money? make money? sometimes make sometime lose, but on average get nothing? or just get nothing no matter what? (see Lecture notes 16)

• You can derive Black-Scholes PDE, and compute delta of an European call. (see Lecture notes 15.)

• Take a look at Exercise 4.11.

• Which is risky? Stock or option? (see Lecture notes 16.)

• You can sketch the graph of delta, gamma of an European call at a given time $t$. (see Lecture notes 16.)

• (Butterfly spread) Which one should be more expensive? Two calls struck at 100, or one call struck at 90 plus one call struck at 110, assuming they are co-terminal. (Hint: you can treat call price as a function of strike $K$, and show it is a convex function of $K$ by computing $\frac{\partial^2 C}{\partial K^2}$ with B-S formula.) If the reality is that they are equal, do you think this is an arbitrage or not. If yes, please construct your arbitrage strategy.