Homework 1: Binomial Asset Pricing Model

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1. Suppose in the situation of Example 1 (see notes) that the option sells for 1.20 at time zero. Consider an agent who begins with wealth $X_0 = 0$ and at time zero buys $\Delta_0$ shares of stock and $\Gamma_0$ options. The numbers $\Delta_0$ and $\Gamma_0$ can be either positive or negative or zero. This leaves the agent with a cash position of $-4\Delta_0 - 1.20\Gamma_0$. If this number is positive, it is invested in the money market; if it is negative, it represents money borrowed from the money market. At time one, the value of the agent’s portfolio of stock, option, and money market assets is

$$X_1 = \Delta_0 S_1 + \Gamma_0 (S_1 - 5)^+ - \frac{5}{4} (4\Delta_0 + 1.20\Gamma_0).$$

Assume that both $H$ and $T$ have positive probability of occurring. Show that if there is a positive probability that $X_1$ is positive, then there is a positive probability that $X_1$ is negative. In other words, one cannot find an arbitrage when the time-zero price of the option is 1.20.

2. Please refer to Example 1 (see notes). At time zero, a bank owns that option with strike $K = 5$ and maturity $T = 1$. This ties up capital $V_0 = 1.20$ for the bank. The bank wants to earn the interest rate 25% on this capital until time one (i.e., without investing more money, and regardless of how the coin tossing turns out, the bank wants to have $\frac{5}{4} \cdot 1.20 = 1.50$ at time one, after collecting the payoff from the option (if any) at time one). Specify how the bank’s trader should invest in the stock and money markets to accomplish this. (Hint: obviously, the trader can sell the option and deposit 1.20 into the money market. But what if the trader wants to keep this long position?)