Experiment 9: AC circuits

PHYS C1493/C1494/C2699
Introduction

- **Last week (RC circuit):**
  - *Constant* Voltage power source (constant over time)
- **This week:**
  - *Alternating current (AC) circuits*
    - Time dependent voltage source $V(t)$
  - Leads to:
    - *Time dependent currents* (alternating currents)
    - *Phase shifts in voltage and currents* in components with respect to one another.
    - *Impedance Z* (generalization of resistance)
    - *Resonance*
Why AC circuits?

- Sensitive to input frequency (i.e. function generator frequency)
- Serve as signal frequency filters:
  - High-frequency filters
  - Low-frequency filters
  - Band-pass filters
- Transformers
  - Induction effects - Ability to raise or lower the voltage amplitude.
- Generators and Motors

e.g. Radios

e.g. Speakers
AC circuits: resistors

- Of course, not going to be able to cover all aspects of AC circuits
- For this lab:
  - Consider sources only sources that vary **sinusoidally**:
    \[ I(t) = I_{\text{max}} \sin(\omega t) \]
  - Simple example:
    - Function generator + resistor
  - **Ohms Law**: voltage across the resistor is just \( V_R = I_R \)}
AC circuits: capacitors

- More interesting case: connect capacitor the AC voltage source

- Voltage rule for capacitor:
  
  \[ Q = CV \]
  \[ \int I(t)\,dt = CV \]

- When driven by sinusoidal current source:

  \[ V_c = \frac{1}{C} \int I(t)\,dt \]

  \[ V_c = \frac{1}{C} \int I_{\text{max}} \sin(\omega t)\,dt \]

  \[ V_c = -\frac{I_{\text{max}}}{\omega C} \cos(\omega t) \]
AC circuits: capacitors

- The voltage is sinusoidal:
  
  \[ V_C = -\frac{I_{\text{max}}}{\omega C} \cos(\omega t) \]
  
  \[ V_C = -V_{\text{max}} \cos(\omega t) \]
  
  \[ V_C = V_{\text{max}} \sin(\omega t - \frac{\pi}{2}) \]

- Extra \( \pi/2 \) in the expression is the **phase of the voltage**.

- Voltage in capacitor **lags behind** the current by:
  
  \[ \phi = -\pi/2 \]
Inductors

- **Store energy** in form of magnetic fields (similar to capacitors)
- Voltage drop across inductor:

\[ V_L = -L \frac{dI}{dt} \]

- Negative sign indicates it opposes any change in current (**Lenz's Law**).
AC circuits: Inductors

- The voltage is still sinusoidal:
  \[ I(t) = I_{\text{max}} \sin(\omega t) \]
  \[ V_L = \omega L I_{\text{max}} \cos(\omega t) \]
  \[ V_L = V_{\text{max}} \cos(\omega t) \]
  \[ V_L = V_{\text{max}} \sin(\omega t + \frac{\pi}{2}) \]

- Inductor voltage is also phase shifted w.r.t. current.

- Voltage across inductor leads the current through it by:
  \[ \phi = +\frac{\pi}{2} \]
Voltage maxima: A closer look

- Given our expression for $V_R$, the **maximum value** of the voltage across the resistor is just given by **Ohm's Law**:

$$V_R = I_{\text{max}} R$$

- But, the **maximum** voltage across the inductor is a function of the **driving frequency**:

$$V_L = \omega L I_{\text{max}} = I_{\text{max}} X_L$$

  **Inductive reactance**

$$X_L = \omega L$$

- The **maximum** voltage across the capacitor is also a function of $\omega$. Unlike $V_L$, $V_C$ actually **decreases** as $\omega$ gets larger:

$$V_C = \frac{1}{\omega C} I_{\text{max}} = I_{\text{max}} X_C$$

  **Capacitive reactance**

$$X_C = \frac{1}{\omega C}$$
Physical explanation: capacitors

- Why does the capacitor *resist low-frequency signals* more than high-frequency ones?

- Last time: when charging/discharging the capacitor, the current – *the rate at which you can charge it* – decreases exponentially. *It becomes harder and harder to push in more charge as the capacitor fills up.*

![Graph showing current over time with labels for low and high reactance (X_c)]
Physical explanation: capacitors

- **Rapidly varying signals** (high frequency) quickly charge/discharge capacitor before it fills with charge → low impedance.

- **Slowly varying signals** (low frequency) charge the capacitor to its limit, slowing down the rate: that is, decreasing the current!

- Also, looking at \[ Q = \int I(t) \, dt = CV \quad I = C \frac{dV}{dt} \]
  - \( I \) is maximum when \( dV/dt \) is maximum (as \( V \) crosses zero)
  - \( I \) is minimum when \( dV/dt \) is zero (i.e. when \( V \) is maximized)
Physical explanation: Inductors

- Why does the inductor resist high-frequency signals more than low-frequency ones?

- Think about what an inductor is: a coil of wire. If the current in the wire changes, then the magnetic flux through the coil changes → induction

- **Lenz’s Law:** a coil will oppose changes in magnetic flux. Self-induced EMF is

\[ \varepsilon = -N \frac{d\Phi}{dt} = -L \frac{dI}{dt} \]

- **Rapidly varying** signals strongly change the flux, so the inductor “pushes back” harder against the flow of current!
  - Voltage is maximum (and opposing) when \( I \) changing most rapidly (i.e. when it crosses zero)
  - **Voltage = 0** when \( I \) is constant (i.e. at either maximum extreme)
**RLC circuits**

- Combining all three components in series:

\[ V(t) = V_R + V_L + V_C \]

- Applying Kirchoff's loop:

\[ V(t) = IR + L \frac{dI}{dt} + \frac{q}{C} \]
**RLC circuits**

- Recasting using inductive and capacitive reactances:

\[
V(t) = I_{\text{max}} \left[ R \sin(\omega t) + X_L \cos(\omega t) - X_C \cos(\omega t) \right]
\]

\[
V(t) = I_{\text{max}} \left[ R \sin(\omega t) + (X_L - X_C) \cos(\omega t) \right]
\]

- Let solution to voltage be sinusoidal, but with phase shift (\(\phi\))

\[
V(t) = I_{\text{max}} Z \sin(\omega t + \phi)
\]

\[
V(t) = I_{\text{max}} \left[ Z \cos(\phi) \sin(\omega t) + Z \sin(\phi) \cos(\omega t) \right]
\]
RLC circuits

- Recasting using inductive and capacitive reactances:

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\[ V(t) = I_{max} \left[ Z \cos(\phi) \sin(\omega t) + Z \sin(\phi) \cos(\omega t) \right] \]
Series RLC circuit: band-pass filter

- What happens when you set up an inductor, capacitor, and resistor in series?

- The entire circuit has a very interesting frequency-dependent “effective resistance” to signals, called the impedance of the circuit:

  \[ Z_{RLC} = \sqrt{R^2 + (X_L - X_C)^2} \]

- The inductor attenuates high-frequency signals while capacitor attenuates low-frequency, but lets through signals around a particular frequency \( \omega_0 \) called the resonance frequency.
Resonant frequency

- The resonance occurs when the current in the RLC circuit is a **maximum**:

\[ I_{\text{max}} = \frac{V_{\text{max}}}{Z_{\text{max}}} = \frac{V_{\text{max}}}{\sqrt{R^2 + (X_L - X_C)^2}} \]

- For a given input voltage, the current in the circuit is a maximum when \( Z \) is a **minimum** i.e. when \( X_L = X_C \).

- The **resonance frequency** is given by:

\[ \omega_o L = \frac{1}{\omega_o C} \quad \Rightarrow \quad \omega_o = \frac{1}{\sqrt{LC}} \]
Recall that the resistor voltage $V_R$ is directly proportional to the magnitude of current.

**Terminology**

FWHM: **Full Width at Half Maximum**

*Is the full width of the resonance peak at the point where its height is halfway between zero and the maximum.*
Measurements

- Resonance of RLC circuit:
  - Set resistance in circuit to **three** different values.
  - Observe **resonant frequency** $\omega_0$, **FWHM** of circuit for each resistor values.
  - Replace known inductor with large copper coil. Find its **inductance** $L$ by observing $\omega_0$ for the new circuit.

- Observe **phase difference** $\phi$ between driving voltage and $V_R$ for **five values** of driving frequency $\omega$ (close to $\omega_0$)

- Observe phase difference between driving voltage and $V_R$ at low and high frequency limits; compare to predictions of **phase shift** for capacitor, inductor.
Experimental setup
Experimental setup

• Recommendations:
  
  ● Set the function generator peak-to-peak voltage to **20 V**, the maximum allowed.
    
    – *There is a 0-2V / 0-20V selector button in addition to the voltage knob.*

  ● Make sure the oscilloscope is set to trigger on channel 1, the function generator signal. You can do this by pressing the **TRIGGER** button and checking in the window menu that CH 1 is selected.

  ● Use the **MEASURE** tools to observe peak-peak amplitudes, signal periods, and signal frequencies. Let the scope do the work for you!

  ● Make sure that both peak's are in the viewable range of the scope!
Resonance measurements

- For about 20 frequencies above and below $\omega_0$, record peak-peak voltage across resistor.
- Normalize table with respect to maximum of each measurement set.
- Plot resonance curves obtained for three values of $R$ set with decade resistor box.
Resonance measurements

- Use the plots to determine approximately the value of the resonance frequency, with errors.

- Also use the plots to measure the FWHM of the three curves.

- Compare the results from the three measurements.

- When you’re finished with this part, swap in the large copper coil to determine its inductance by finding the new resonant frequency.
Phase measurement

- Use the scope cursors to measure the phase of the signals.

- Set up the cursors to measure time differences on the horizontal axis.

- Time difference between signal peaks can be read directly from cursor menu.

\[ \phi = 2\pi \frac{t_R - t_d}{T_d} \]

Compare to the predicted \( \phi \) from L, R, and C