Strategic Ratification

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Abstract

Previous models of ratifier effects have relied on restrictive assumptions regarding the dimensionality of the choice space and the preferences and rationality of actors. These assumptions can lead to an underestimation of the importance of ratifiers and a mischaracterization of their role. A model presented here responds to these concerns and finds that when ratifiers are strategic ratification requirements consistently aid negotiators in settings where previous models find no effects. Furthermore, new relations between the types of ratifier and the strength of their influence are identified. Negotiators benefit from in-group homogeneity on dimensions along which negotiators agree and from internal dissension on dimensions along which negotiators disagree.

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1 Introduction

A well known conjecture in the study of relations between domestic and international levels of policy making suggests that international negotiators can benefit from constraints imposed upon them by hawkish domestic constituencies. Such constraints allow a negotiator credibly to claim that some unfavorable proposals that she would otherwise find acceptable are not ratifiable at home. An early statement of the conjecture is given by Thomas Schelling (1960):

[If the executive branch negotiates under legislative authority, with its position constrained by law and it is evident that Congress will not be reconvened to change the law within the necessary time period, then the executive branch has a firm position that is visible to its negotiating partners.]

The “Schelling conjecture” (Milner 1997)—that such constraints benefit the negotiator—though simple in its statement, is rich in implications. Stated here in terms of the way in which an inanimate constraint—a law—affects a negotiator’s calculus, Schelling’s conjecture is also relevant to situations in which a negotiator is constrained by the need to win the support of some third party, who may or may not be strategic. The logic applies as well to subnational negotiations as it does to international politics. Indeed, Schelling motivates the claim initially with reference to negotiations between management and labor. Other applications range across such diverse issues as trade policy (Mansfield et al 2000) and the politics of referenda (Hug and König 2002), to neocorporatist bargaining and civil war termination.

The wide field of application of the conjecture has led to multiple attempts to assess the generality of its logic and to pin down the mechanism through which the logic works. In one early attempt to provide greater formalism to the conjecture, Putnam (1988) made use of the idea of “winsets”—the set of points that the domestic constituency prefers to the status quo—to argue that the smaller the winset, the greater the advantage to the negotiator. This formulation of the problem has structured most subsequent attempts to analyze the role of ratifiers. Implicit is the idea that smaller winsets rule out bad outcomes. However, since smaller winsets may also rule out good outcomes, it is not clear whether a small winset will in fact benefit or harm a negotiator. These possibly ambiguous effects led to a series of more formal attempts to identify when and how ratifiers matter (see for example Iida 1993; Mo 1994; Mo 1995; Iida 1996; Milner and Rosendorf 1997; Haller and Holden 1997; Tarar 2001; Butler 2004).
Many of these more recent models have made in-roads by examining a series of extensions to a basic model of ratification. The extensions examine the effects of one sided constraints, of two sided constraints, of domestic uncertainty, and of a variety of different types of domestic institutions, including institutions for information aggregation (Iida 1996), supermajority rules (Haller and Holden 1997), presidential versus parliamentary systems (Tarar 2005), and possibilities for domestic amendments (Tarar 2004). However, although there has been much work done to develop the institutional environment, this work has relied on very stylized models that put severe constraints on three key aspects of a model’s design: the nature of the set of outcomes over which players make choices, the preferences of players over feasible outcomes, and the strategic behavior of the players.

The institutional complexity coupled with the simplified nature of the modelling assumptions make it difficult to see whether or not the basic intuition behind the conjecture is correct and to what extent existing answers depend on institutional assumptions rather than arbitrary modelling assumptions about player preferences and behavior. In a number of cases, as I argue below, this lack of clarity can lead to a misinterpretation of model predictions.

Here, rather than increasing the institutional complexity of the decision making environment, I return to an institutionally simple structure but greatly expand the generality of the model by allowing for multidimensional choice environments, a very general class of policy preferences and the possibility for strategic action on the part of ratifiers. I focus especially on a class of cases where many previous models suggest that ratifiers are unlikely to have any effect at all. In this more general environment the model provides surprisingly clear answers despite what appears to be the highly contingent nature of previous results. In situations in which the status quo is costly, the Schelling conjecture holds for a very general class of games. Exceptions occur only for extreme cases in which negotiators are very impatient and the pie is relatively “indivisible.” Beyond the qualitative nature of the Schelling effect, these results also allow for the identification of the role of relations between player preferences in determining ratifier influence. Such effects are impossible to identify in simpler environments.

That a clear result can be found in an environment in which past models have produced a heterogeneity of outcomes is gratifying, but it also poses a challenge. The results here require us to revisit other models to better understand the differences in their logics. I do this in the concluding section and demonstrate how the logic highlighted by this more general model allows us fruitfully to reinterpret the dynamics of
two previous models, more clearly identifying which institutional features matter to negate a Schelling effect or produce one where it would otherwise not arise.

I proceed as follows. In the next section I discuss the ways that previous models have handled the three key modelling decisions noted above, highlighting areas in which these decisions have consequential effects. In Section 3 I provide a formal description of the model. I present the main results in Section 4. Section 5 provides a series of examples that demonstrate the core logic underlying the propositions. A concluding section emphasizes some of the insights gained from the model and identifies ways in which these results force us to rethink previous results.

2 Policy Options, Political Preferences and Strategic Behavior

Recent developments in the study of ratifier effects have focused on institutional details but have done so by relying on highly restrictive assumptions regarding three core features of the basic model: the nature of the outcome space, the preferences of political actors and the rationality of actors. These assumptions are not without loss of generality. I discuss in turn the ways that outcome spaces, preferences, and behavior have been modelled in past work, in each case identifying ways in which the limitations are likely to alter the conclusions of models.

One Dimensional Choice Spaces. In most models of ratifier effects, feasible elements for negotiation at the international level lie in a one dimensional space. In a small number of models the space appears to be somewhat richer, with negotiators having Euclidean preferences over spaces of two or more dimensions (Pahre 1997; Milner and Rosendorff 1997; Hug and König 2002; Mansfield et al 2000). However, even in these cases it is often assumed that negotiators confine their attention to points on a one dimensional space (for example Pahre 1997; Mansfield et al 2000, Mansfield et al 2002). This is in fact an unusual modelling assumption. As emphasized by Dai (2002) the common assumption that negotiators select only from a one dimensional set is unmotivated; it implies in effect that negotiators do not look down the game tree. By confining attention to the set of efficient policies that obtain in cases where there is no ratifier rather than from the set of efficient policies that obtain when there is a ratifier (in multidimensional bargaining situations the latter set is not necessarily a subset of the former) they often allow negotiators to select sub-optimal strategies. While Dai correctly demonstrates that a proper consideration of the multidimensional nature of the problem produces results that differ markedly from those that ob-
tain in one dimensional spaces, no general multidimensional model has yet been developed to capture these effects in ratification games (although for some initial attempts in this direction see Hammond and Prins, 1999). Lamenting the point, Hug (2004) argues that when third parties are involved in ratification, a one-dimensional representation “inevitably breaks down [...] quite obviously the theoretical models and derived implications we currently use to study the effect of ratification constraints on international negotiations are inadequate.”

**Private Valuations.** Many models of ratifier effects assume that the negotiators propose divisions of a pie and that an individual’s preferences are a function only of their own cut (Iida 1993; Mo 1994; Mo 1995; Iida 1996; Haller and Holden 1997, Tarar 2005). In some ways such situations approximate a multidimensional environment since the set of possible divisions of a pie can be represented as a multidimensional simplex. However by assuming that players care only about their own share, these models implicitly impose a strong assumption on player preferences, one that violates the strict quasiconcavity assumptions found in many multidimensional spatial models developed in other spheres of political analysis. The assumption of purely private benefits is obviously an unrealistic one. In many instances the objects of negotiation—trade policies, access to markets, decisions to go to war—are of a distinctly public nature; but even in those situations that most closely approximate divide the dollar decisions at the international level—for example divisions of rights to oil fields, the location of borders, or troop or cash contributions to joint projects—commonalities of interest and externalities can render the costs and benefits of decisions less than fully private. That there is a loss of realism entailed by assuming purely private preferences presents a problem only if the assumption does not substantively affect model outcomes. Below however I demonstrate that it does. In the settings I consider below, ratifier effects may exist with preferences that are arbitrarily “close” to private valuations but fail to exist if valuations are strictly private.

**Myopic Ratifiers.** Most existing models inherit a feature from Putnam’s formulation of the conjecture that is rarely made explicit but that is now worth emphasizing: Putnam’s formulation builds in the idea that ratifiers cannot, or need not, take account of the effects of their decisions on other actors. This could be because they are assumed not to be strategic—they are endowed with parametric but not necessarily strategic rationality—or it could also be because it is assumed that they only take a single action in the final period of the game. One consequence of this assumption is that ratifiers prove to be relevant in these models if and only if they prefer the status quo to outcomes that would otherwise
be agreed upon by negotiators. Ratifiers are constrained to take the present—the status quo—as the reference point, rather than expectations of possible future deals. They act as if they invariably receive take it or leave it offers. It is easy to demonstrate that if all that matters is the determination of the set of feasible offers, then anything can happen (Hammond and Prins 1999): a ratifier can benefit or hurt either or both negotiators; in some settings the presence of a ratifier might benefit a negotiator if the negotiator is a weak bargainer but hurt her if she is strong. In contrast the logic suggests that whenever the status quo is painful to ratifiers relative to what would be negotiated in the absence of ratification requirement, the requirement of ratification is irrelevant.

How plausible is the assumption that negotiators are strategic but that ratifiers are not (or, at least, that they have no opportunity to employ strategies)? The assumption may be plausible if the “ratifier” is in fact inanimate and incapable of strategy or if ratifiers are indeed only ever faced with take-it-or-leave-it offers. For a large class of situations, however, the assumption is hard to defend. In many applications—international trade negotiations, peace negotiations, bargaining in bicameral legislatures with a presidential veto—rejection by the ratifier can lead to a re-opening of negotiations rather than simply to an end of negotiations.

Despite the central role of strategy, the preponderance of models have not allowed strategic behavior. A small number of exceptions stand out.

In one very rich paper studying endogenous domestic coalitions Mo (1994) introduces the possibility that ratifiers reject divisions of a pie in anticipation of being in a position to themselves make proposals in the future or receive proposals from other offerers. Schelling-like results can emerge in this model since stronger domestic players that would require large payoffs from foreign actors reduce the expected benefits to foreign actors of delay, all to the benefit of domestic proposers. Although this model captures striking features of domestic-international linkages, the setup is somewhat far removed from standard descriptions of ratification processes. In effect, in this model the formal distinction between negotiators and ratifiers is removed. In a pure ratification model, by contrast ratifiers have no probability of becoming negotiators. We return to examine the implications of these modelling decisions in the concluding section.

In a related model, Hayes and Smith (1997) examine a game in which there is no ratifier but there is a domestic electorate. Although the electorate does not ratify offers they can replace a negotiator with one who can subsequently modify the deal. This captures a very important feature of international negotiation and introduces a limited degree of
strategy (the electorate chooses negotiators in anticipation of how they will play, but they cannot take actions that will alter the way different individuals will respond). A logic is produced that is somewhat akin to ratification, and one which can produce Schelling-like effects in the sense that negotiators can claim that if a bad deal is produced (and the negotiator is sufficiently unpopular for other reasons) then they can be turned out of office and replaced by a more hawkish negotiator. The model is highly suggestive although the introduction of multiple motives makes it difficult to ascertain what effects should be attributed to ratification constraints and which are due directly to the vulnerability of a negotiator’s position. Indeed, in this model, the introduction of a strong domestic competitor can be damaging to a negotiator even if it has no impact on his bargaining behavior.

A third study that examines the possibility of strategic ratifiers is found in Iida (1996). Like Mo (1994), Iida’s model provides an institutionally rich representation of domestic politics but does so by placing severe constraints on the outcome space and on player preferences. As in Mo (1994) the game is a divide the dollar game but whereas in Mo’s model the dollar is divided in four parts, in Iida it is divided simply in two with domestic ratifiers having individual requirements for minimal country allocations. Even though the model allows formally for multiple ratifiers, since the cutoffs of the individual ratifiers are distributed along a line, the relevant question comes down to where the median is located. For the most part the model assumes sincere voting but Iida develops one example of a situation with strategic voting. His treatment, while informative, is partial: it is assumed that only one voter acts strategically and that she does so in only one round of bargaining, acting sincerely thereafter; a complete model of strategic ratification is not presented.

A final recent study (Tarar 2005) allows for sophisticated ratifiers but the game form limits the scope for strategic action. Ratifiers can act strategically to determine how to divide a fixed sum among themselves and they can choose strategically between accepting a given deal or instead dividing some fixed sum among themselves. They cannot however strategically employ their powers of ratification to alter the behavior of negotiators in this model. Hence, though strategic, ratifiers do not in fact interact strategically with negotiators.

Again, the lack of realism of the assumption that ratifiers do not act strategically is innocuous if it does not qualitatively affect results. The analysis below suggests however that it does. Models that ignore the potential for strategic ratification conclude that in situations in which all outcomes are preferred by the ratifier to the status quo, ratifiers have no effects. Below I demonstrate that this claim is not robust to changes.
3 The Model

The model presented here responds to the three concerns discussed in the previous section. It does so by employing an alternating offers model of bargaining, as developed by Ståhl (1972) and Rubinstein (1982), over public goods. The major features of the model presented here that differ from previous work are the following:

- The outcome space is multidimensional. This allows the model to capture situations in which groups bargain over pure public goods. The space can also represent choices of public goods coupled with transfers, or it can represent distinct allocations of divisible goods.

- Only weak assumptions are imposed on the preferences of players. Player preferences are not constrained to be linear or Euclidean. To allow for sharper contrast with past work, I restrict attention to situations in which ratifiers find all outcomes preferable to the status quo. Winset effects may still obtain of course when in fact the status quo is not costly to the ratifier. Our purpose here however is to examine whether and how the logic of ratification can function even when such effects are removed and hence in cases where present approaches suggest ratifier irrelevance. In doing so we extend the domain of our knowledge of ratifier effects to settings that are central to much of the classic bargaining literature (see for example Banks and Duggan 2000) and that approximate well many actual environments.

- The game form allows the ratifier to be just as strategic as the other players, and, like other actors, she may elect to forego acceptable offers in expectation of more favorable future rewards.

The game involves three players: Players 1 and 2 are negotiators; Player 3 is a ratifier. The game form is as follows. In every odd (alt. even) period $t \in T = \{0, 1, 2, \ldots, \infty\}$ in which an agreement has not already been reached, Negotiator $i$ (alt. $j$) proposes an offer $x \in X$, where $X$ is a convex and compact subset of $\mathbb{R}^n$. If $x$ is accepted by $j$ (alt. $i$), it is put to the ratifier. If $x$ is ratified it is implemented immediately. The new policy remains in place thereafter and all bargaining ends. If it is rejected by $j$ (alt. $i$) or fails to receive ratification, then the status quo
remains in place, the game moves into period \( t + 1 \) and bargaining continues. The “disagreement outcome,” in which no agreement is reached at any stage, is given by \( D \).

Each player \( i \) possesses a rational preference relation \( \succeq_i \) defined over \( \{D\} \cup (X \times T) \) (that is, over the disagreement outcome and over pairs in which the first element records the element of \( X \) and the second the time period in which the agreement is implemented). These preference relations satisfy the following conditions:

1. Disagreement is the worst outcome: \( (x, t) \succ_i D \) for all \( x \in X \), \( t \in T \).

2. Time is valuable: \( (x, t) \succ_i (x, t + 1) \) for all \( x \in X \), \( t \in T \).

3. Stationarity: \( (a, t) \succeq_i (b, t + r) \) if and only if \( (a, s) \succeq_i (b, s + r) \) for arbitrary non-negative integers \( s, t \) and \( r \).

4. Continuity, Convexity and Correspondence of instantaneous utility: In any period, \( s \), each player \( i \) has preferences over elements in \( X \) that may be represented by a continuous strictly quasiconcave von Neumann-Morgenstern utility function, \( u_i : X \rightarrow \mathbb{R}^1 \). In addition, instantaneous utility corresponds with preferences over outcomes in \( X \times T \) in the sense that for any \( s \) and any points \( x, y \):

\[
u_i(x) \geq u_i(y) \iff (x, s) \succeq_i (y, s).
\]

From these assumptions we have that for any \( s \), \( (x, s) \sim_i (y, s) \) implies \( (z, s) \succ_i (x, s) \) for every point \( z \) in the interior of the convex hull of \( x \) and \( y \). Since \( X \) is compact and convex, each player has a most-preferred point, or “ideal point” in \( X \). I use \( \hat{x} \) to denote the ideal point for Player 1; and \( \hat{y} \) for Player 2. The negotiation set (contract curve) for Players 1 and 2, denoted by \( C \), is defined as \( C = \{x \in X | \exists y \in X \text{ such that } u_i(y) \geq u_i(x) \text{ for all } i \text{ and } u_i(y) > u_i(x) \text{ for some } i \in \{1, 2\} \} \). Furthermore, \( C \) is non-empty, one dimensional, connected, compact and strictly monotone in the sense that for \( x, y \in C \), \( u_i(y) \geq u_i(x) \) implies \( u_j(y) < u_j(x) \). Monotonicity in turn implies that there is no pair \( (x, y) \) on \( C \) such that \( u_i(x) = u_i(y) \) for any \( i \).

I add two further conditions.

**Condition (\( \ast \))** For points \( a, b, c, d \) on \( C \), with \( u_i(c) > u_i(d) \), \( (d, 0) \sim_i (b, 1) \) and \( (b, 0) \sim_j (d, 1) \):

- (I) \( (a, 0) \sim_j (c, 1) \) implies \( (c, 0) \succ_i (a, 1) \)
- (II) \( (c, 0) \sim_i (a, 1) \) implies \( (c, 1) \succ_j (a, 0) \)

Informally Condition (\( \ast \)), related to the notion in Osborne and Rubinstein (1994) of “increasing costs to delay,” requires that compensating
satisfied players for delay is more difficult than compensating dissatisfied players. Hence for example, players who are as dissatisfied with an outcome as they are with the status quo are indifferent whether or not they receive that outcome with delay whereas players that prefer an outcome to the status quo prefer consuming it immediately to waiting. This assumption holds for example whenever $C$ is linear and players have discounted utility with concave utility functions.\(^8\)

The second additional condition places a weak restriction on relations between the ratified’s preferences and those of the negotiators:

Condition (**) The ratified has single peaked preferences over $C$.

Like Condition (*), Condition (**) can be satisfied very generally, including in all cases in which $C$ is linear and preferences are convex. It may fail to obtain however in cases in which ratifiers find compromise outcomes painful in the sense of finding some more extreme outcomes that benefit either negotiator preferable to a compromise.

## 4 Results

I derive results that allow comparison between the outcomes of an alternating bargaining game with and without a ratifier. In Proposition 1 I extend the Ståhl-Rubinstein framework to allow for the possibility of a ratifier and identify constraints on players’ strategies that guarantee them to be sub-game perfect equilibrium strategies in the presence of a ratifier. In Proposition 2 I identify conditions under which ratifiers are irrelevant to bargained outcomes. In the final two propositions I characterize the effects of ratifiers in cases where they do alter the outcome.

**Proposition 1** [Existence] There exists a pair $(\bar{x}, \bar{y})$ such that:

- $\bar{x}$ maximizes $u_1(x)$ s.t. $(C_1) (x, 0) \succ_2 (\bar{y}, 1)$ and $(C_2) (x, 0) \succ_3 (\bar{y}, 1)$
- $\bar{y}$ maximizes $u_2(y)$ s.t. $(C_3) (y, 0) \succ_1 (\bar{x}, 1)$ and $(C_4) (y, 0) \succ_3 (\bar{x}, 1)$

Furthermore the following constitutes a sub-game perfect equilibrium set of strategies: Player 1 always proposes $\bar{x}$ and accepts any proposal $y$ if and only if $(y, 0) \succ_1 (\bar{x}, 1)$; Player 2 always proposes $\bar{y}$, and accepts any proposal $x$ if and only if $(x, 0) \succ_2 (\bar{y}, 1)$. The ratifier ratifies proposal $x$ from Player 1 iff $(x, 0) \succ_3 (\bar{y}, 1)$ and $y$ from Player 2 iff $(y, 0) \succ_3 (\bar{x}, 1)$.

**Proof.** Let $f_1(y) : X \to X$ denote the solution to the problem of maximizing $u_1(x)$ s.t. $(x, 0) \succ_2 (y, 1)$ and $(x, 0) \succ_3 (y, 1)$. With strictly quasiconcave preferences, the set satisfying $(x, 0) \succ_2 (y, 1)$ and $(x, 0) \succ_3 (y, 1)$ is strictly convex and hence $f_1(y)$ is unique. Furthermore, with $u_i$ continuous the sets satisfying $(x, 0) \succ_2 (y, 1)$ and $(x, 0) \succ_3 (y, 1)$ are continuous in $y$ and hence $f_1(y)$ is a continuous function in $y$. The analogous function $f_2(x) : X \to X$, with $f_2(x) = \arg \max_y (u_2(y) |
that \( (y,0) \succeq_1 (x,1) \), \( (y,0) \succeq_3 (x,1) \), is also continuous. Hence the function \( f(x,y) = (f_1(y), f_2(x)) : X \times X \rightarrow X \times X \) is a continuous mapping from a compact convex set into itself, and, from Brouwer’s fixed point theorem, has a fixed point. The existence of such a point \( (\bar{x}, \bar{y}) = f(\bar{x}, \bar{y}) \) establishes the first part of the claim.

For the remainder of the Proposition, we use the one stage deviation principle and check that a deviation from the prescribed strategies in any single stage does not improve the payoff of any player (see Fudenberg and Tirole 1995: 108-10).

For the ratifier, rejecting an offer \( (\bar{x},0) \) from Player 1 that she weakly prefers to \( (\bar{x},0) \) improves the ratifier’s payoff only if \( (\bar{y},1) \succ_3 (\bar{x},0) \), which, from the definition of \( \bar{x} \) is not the case. Accepting any offer \( (\bar{x},0) \) for which \( (\bar{x},0) \prec_3 (\bar{y},1) \) is sub-optimal since rejecting guarantees \( (\bar{y},1) \). An analogous argument applies for offers from Player 2.

For Player 1, offering any \( x \neq f_1(\bar{y}) \) must either be an offer for which \( u_1(\bar{x}) < u_1(x) \) or else it must be that \( x \) is not acceptable to at least one of Player 2 or Player 3. The former is clearly sub-optimal. In the latter case, Player 1 receives \( (\bar{y},1) \) instead of \( (\bar{x},0) \). Note however that if \( u_1(\bar{y}) > u_1(\bar{x}) \), then, since \( (\bar{y},0) \succeq_2 (\bar{y},1) \) and \( (\bar{y},0) \succeq_3 (\bar{y},1) \), necessarily, \( x \neq f(\bar{y}) \), a contradiction. It follows then that \( (\bar{y},1) \prec_1 (\bar{y},0) \) and hence that choosing \( (\bar{y},1) \) over \( (\bar{x},0) \) is suboptimal. Deviation in any stage in which Player 1 has to choose whether to accept Player 2’s offer occurs if Player 1 accepts an offer \( y \) with \( (y,0) \prec_1 (x,1) \), or rejects an offer \( y \) for which \( (y,0) \succeq_1 (x,1) \). In the latter case the ratifier receives \( (x,1) \) which is no better than \( (y,0) \). Similarly in the former case, Player 1 does not improve upon the return she would get from rejecting, since rejecting yields \( (x,1) \succ_1 (y,0) \). An analogous argument demonstrates that a one stage deviation is also sub-optimal for Player 2.

Note that if conditions \( (C_2) \) and \( (C_4) \) above are removed then the equilibrium pair of strategies, \( (\bar{x}, \bar{y}) \), are the standard solutions to the Ståhl-Rubinstein bargaining game. Henceforth I use \( (x^*, y^*) \) to label these equilibrium offers in the “unconstrained” game and \( (\bar{x}, \bar{y}) \) to label the equilibrium offers identified above for the constrained game. Note that \( (x^*, y^*) \) both lie on \( \mathcal{C} \). In the remaining sections, I ask: when and how do the \( (\bar{x}, \bar{y}) \) defined in Proposition 2 differ from benchmark outcomes \( (x^*, y^*) \).

I begin by describing cases where the ratifier has no effect.

**Proposition 2** [Ratifier Irrelevance] Equilibrium offers in the game without a ratifier, \( (x^*, y^*) \), are also equilibrium offers in the game with a ratifier if and only if \( (x^*, 0) \succeq_3 (y^*, 1) \succeq_3 (x^*, 1) \) or \( (y^*, 0) \succeq_3 (x^*, 1) \succeq_3 (y^*, 1) \).
\((y^*, 1)\). In this case the strategies described in the previous proposition with \(\bar{x} = x^*\) and \(\bar{y} = y^*\) are sub-game perfect equilibrium strategies.

**Proof.** Assume without loss of generality that \((y^*, 1) \succeq_3 (x^*, 1)\). To check the if part, note that \((x^*, 0) \succeq_3 (y^*, 1) \succeq_3 (x^*, 1)\) implies that rejecting an offer of \(x^*\) from Player 1 and then accepting \(y^*\) from Player 2 one period later does not improve the ratifier’s payoff. With \((y^*, 1) \succeq_3 (x^*, 1)\), condition \((y^*, 0) \succeq_3 (x^*, 1) \succeq_3 (y^*, 1)\) implies that \((x^*, 0) \sim_3 (y^*, 0) \succ_3 (x^*, 1) \sim_3 (y^*, 1)\) and hence \((x^*, 0) \succ_3 (y^*, 1)\) and so in this case also, forgoing \(x^*\) to accept \(y^*\) one period later does not improve the ratifier’s payoff. Similarly \((y^*, 1) \succeq_3 (x^*, 1)\) implies \((y^*, 0) \succ_3 (x^*, 1)\) and hence deviating for one stage after an offer from Player 2 is sub-optimal. With the ratifier accepting their offers from the unconstrained game, the incentives faced by negotiators are unaltered from the unconstrained game. Since no player has an incentive to deviate from their strategies in any single period, the one stage deviation principle implies that these strategies are indeed sub-game perfect.

For the only if part, note that with \((y^*, 1) \succeq_3 (x^*, 1)\), both conditions fail if and only if \((y^*, 1) \succ_3 (x^*, 0)\). In this case we need to ascertain that \(x^*\) and \(y^*\) are not equilibrium offers. Note first that in this case rejection of \(x^*\) and subsequent acceptance of \(y^*\) is preferable to accepting \(x^*\) immediately. But in this case, the offers \(x^*\) and \(y^*\) are not optimal offers. To check this, assume that they are. In this case, Player 2’s “acceptance set,” \(A_2^\ast\), is given by all points \(x\) such that \((x, 0) \succeq_2 (y^*, 1)\). In any sub-game perfect equilibrium in which Player 2 offers \(y^*\), Player 3’s acceptance set for offers from Player 1 is the set \(A_3^\ast = \{x \mid (x, 0) \succeq_3 (y^*, 1)\}\). Hence, Player 1 should deviate from her strategy if there exists any point \(x'\) such that \(x' \in A_3^\ast \cap A_2^\ast\) and \((x', 0) \succ_1 (y^*, 1)\). But \(x' = y^*\) is one such point and so \((x^*, y^*)\) are not optimal strategies. •

One important application of Proposition 2 is worth emphasizing. The conditions for ratifier irrelevance hold in cases in which the ratifier is approximately indifferent between the two offers that would have been made in the game without the ratifier. This situation may arise when negotiators with selfish preferences divide a private good. If in both offers zero allocations are made to the ratifier, then, although dissatisfied, there is no gain to the ratifier in blocking the proposal. Hence in games with selfish negotiators dividing a pie, we expect ratifiers to be irrelevant.9

In our discussion of Example 5 below we highlight the importance of preference dependencies for generating ratifier effects in a divide the dollar game.

While the last proposition identifies the conditions that must be satisfied for the ratifier to matter sufficiently to alter the outcome of negotiations, the final two propositions identify how she matters. In these I
compare players’ attitudes to the pair $(\bar{x}, \bar{y})$ relative to the pair $(x^*, y^*)$. I distinguish between two cases. In the first, the constraints that negotiators place on each other in the unconstrained games bind. In these cases we may expect players to make offers that compromise their ideals. In the second case, the first mover advantage is such that for at least one of the players, when making offers she can afford to offer her ideal.

The next Proposition considers the first case. It finds that when the ratifier prefers Player $i$’s unconstrained offer to Player $j$’s unconstrained offer, then the equilibrium offers in the constrained game are such that Player $i$ is made strictly better off both by her own offer and by Player $j$’s offer relative to offers from the unconstrained game. Player $j$ however is made strictly worse off by both offers, relative to the corresponding offers in the unconstrained game. The Proposition is written for the case where Player 3 prefers Player 2’s unconstrained offer. An analogous result when the ratifier prefers Player 1’s offer.

**Proposition 3** If:

1. $(x^*, 0) \sim_2 (y^*, 1)$ and $(y^*, 0) \sim_1 (x^*, 1)$ but $(x^*, 0) \prec_3 (y^*, 1)$
2. $y^* \neq \bar{y}$

Then: $u_2(\bar{y}) > u_2(y^*), u_1(\bar{y}) < u_1(y^*), u_2(\bar{x}) > u_2(x^*)$ and $u_1(\bar{x}) < u_1(x^*)$.

**Proof.** To prove the proposition it is sufficient to establish that $u_2(\bar{y}) > u_2(y^*)$. To see this, observe that if $u_2(\bar{y}) > u_2(y^*)$, then, $(\bar{y}, 1) \succ_2 (y^*, 1)$ and from $C_1$ of Proposition 1 we have $(\bar{x}, 0) \succeq_2 (\bar{y}, 1) \succ_2 (y^*, 1) \sim_2 (x^*, 0)$ and hence $u_2(\bar{x}) > u_2(x^*)$. But then, from the definition of $x^*$, $u_1(\bar{x}) < u_1(x^*)$ (as otherwise there would exist a better offer than $x^*$ for player 1 in the unconstrained game). Finally from the definition of $y^*$, $u_2(\bar{y}) > u_2(y^*)$ implies $u_1(\bar{y}) < u_1(y^*)$.

Exactly one of $C_2$ or $C_4$ from Proposition 1 is binding. This follows since if both are binding then $(\bar{x}, 0) \sim_3 (\bar{y}, 1) \sim_3 (\bar{x}, 2)$ contradicting the assumption that time is valuable. If neither is binding then neither player is constrained in equilibrium and hence $\bar{y} = y^*$ and $\bar{x} = x^*$; but then since $(x^*, 0) \prec_3 (y^*, 1)$, $C_2$ is not satisfied. With exactly one of $C_2$, $C_4$ binding we can divide all possible equilibriums into three classes: (i) where both $C_4$ and $C_3$ are slack (ii) where $C_4$ is slack but $C_3$ is binding (iii) where $C_4$ is binding (and $C_2$ is slack). Consider each in turn:

(i) With neither $C_3$ nor $C_4$ binding, Player 2 is unconstrained in equilibrium and in these instances $\bar{y} = \bar{x}$ and so $u_2(\bar{y}) > u_2(y^*)$.

(ii) If $C_4$ is slack but $C_3$ is binding then $y^*$ and $\bar{y}$ both lie on $\mathcal{C}$. Assume first that $u_2(y^*) = u_2(\bar{y})$. With $y^*$ and $\bar{y}$ on $\mathcal{C}$, $u_2(y^*) = u_2(\bar{y})$ implies $y^* = \bar{y}$. However $\bar{x} \neq x^*$ (as in this case $C_2$ would be violated). If $x^*$ maximizes $u_1$ subject to $C_1$ (uniquely) then since $\bar{x}$ solves the
The Proposition highlights two features. The first is the role of interests. The ratifier alters the outcome because she strictly prefers what one negotiator offers in the unconstrained game to the offer of the other negotiator. If the ratifier is more likely to accept the bargained outcome of “her own” team than that proposed by the other team, then the proposition provides strong support for the Schelling conjecture. If the interests of the ratifier were orthogonal to those of the negotiators, then the ratifier is irrelevant. The second is the role of time: more patient ratifiers have a greater impact.
The final proposition considers the second case in which time is highly valuable to negotiators but the divisibility of the pie is low.\textsuperscript{10}

**Proposition 4** Assume \((x^*, 0) \succeq_2 (y^*, 1)\) and \((y^*, 0) \succeq_1 (x^*, 1); y^* = \tilde{y}\) and \(x^* = \tilde{x}\). Then:

\(i\) If \((x^*, 0) \prec_3 (y^*, 1)\) then \(\tilde{y} = y^*\) and \(u_1(\tilde{x}) < u_1(x^*)\).

\(ii\) If \((y^*, 0) \prec_3 (x^*, 1)\) then \(\tilde{x} = x^*\) and \(u_2(\tilde{y}) < u_2(y^*)\).

**Proof.** Consider (i). Since exactly one of \(C_2\) or \(C_4\) is binding in any equilibrium either \(\tilde{x} = x^*\) or \(\tilde{y} = y^*\). Assume first that \(\tilde{x} = x^*\). Then since \((y^*, 1) \succ_3 (x^*, 0) \succeq_3 (\tilde{y}, 1)\) we have \(u_3(y^*) > u_3(\tilde{y})\). But then a deviation by Player 2 to offer \(y^*\) instead of \(\tilde{y}\) is optimal. Therefore \(\tilde{y} = y^*\) and \(\tilde{x} \neq x^*\) as, otherwise \(C_2\) would not be satisfied. Since \(x^*\) uniquely maximizes \(u_1\) we then have \(u_1(\tilde{x}) < u_1(x^*)\). The proof of (ii) is identical subject to a re-labelling of players. ■

Proposition 4 shows that in situations where the ratifier prefers Player 2’s unconstrained offer with delay to Player 1’s offer without delay, Player 2 does as well as before in games in which she offers first. Furthermore, in games in which Player 1 offers first Player 1 certainly loses out. But unlike the previous case we cannot be sure that Player 2 does better in these situations (the proposition establishes that \(u_1(\tilde{x}) < u_1(x^*)\) but not that \(u_2(\tilde{x}) > u_2(x^*)\)). Indeed Player 2 may also lose out when Player 1 offers first if Player 1’s attempts to placate the ratifier lead her to propose a point \(\tilde{x}\) that, though acceptable, is worse for Player 2 than \(x^*\).

In such games then the presence of a ratifier may make matters worse for both players. The fact that Player 2 can make Player 1 an offer that is worse for both than the offer in the unconstrained game is due to the fact that, in these games, both players have complementary interests insofar as each strictly prefers the other’s offer to waiting a period for her own offer. In other words, the constraints that they impose on each other are slack—that they do not fare worse in the unconstrained game is because there is sufficient commonality of interests for each not to desire to push the other to the boundaries of their acceptance set. This slack however may be consumed when Player 2 is required to make an offer that is acceptable to the ratifier.

5 **Examples**

I end this article by considering three examples that illustrate some of these results on bargaining in the presence of ratifiers. The first
example, of a Divide-the-Dollar game, illustrates the roles of preference
dependence and patience. The second provides a case where the pie is
partly indivisible and shows how both negotiators may lose out due to
the presence of a ratiﬁer. While Propositions 3 and 4 determine when a
ratiﬁer matters and who beneﬁts as a result, without further constraining
utility functions we are unable to say how much different players beneﬁt
from the presence of a ratiﬁer. The ﬁnal example provides an answer to
this question for a common class of games. In all three cases I assume
that each player’s preferences over $X \times T$ may be represented with a
utility function over $X$ and time-constant discount rate, $\delta_i$.

**Example 5 Benefits to a Negotiator in Divide-the-Dollar Games.**

Two risk neutral players negotiate over a three-way split of a dollar sub-
ject to ratiﬁcation. The (risk neutral) ratiﬁer receives some share, $\varepsilon$, of
Player 2’s share.

In a Divide-the-Dollar game, if gains are purely private ($\varepsilon = 0$), then
a ratiﬁer has no effect on the bargained outcome and achieves nothing.
In this instance the Rubinstein solution applies and, assuming linear
utility and discount rate, $\delta_i$ for each negotiator, Player 1 oﬀers
$\delta_2 - \delta_1 \delta_2 \over 1 - \delta_1 \delta_2$ to Player 2; Player 2 oﬀers $\delta_1 - \delta_1 \delta_2$; each oﬀers nothing to the ratiﬁer and
each accepts the other’s oﬀer (or better). If instead the gains are at least
partly public ($\varepsilon > 0$), then a patient ratiﬁer improves the eﬀectiveness
of a negotiator. In this case ratiﬁcation will be relevant if and only if
$\delta_R > \delta_2$. If $\delta_R > \delta_2$ then Player 2 oﬀers $\delta_1 - \delta_1 \delta_R \over 1 - \delta_1 \delta_R$ while Player 1 oﬀers
$\delta_2 - \delta_1 \delta_R \over 1 - \delta_1 \delta_R$. The outcome is exactly as if Player 1 were negotiating directly
with the ratiﬁer, ignoring Player 2. With $\varepsilon$ small, Player 2 does strictly
better relative to the negotiations without a ratiﬁer whenever the ratiﬁer
is more patient than she is. A negotiator then would do well to promise
a share of his takings to a patient ratiﬁer.

**Example 6 Indivisibility and Adverse Eﬀects of Ratiﬁcation.**

Consider the game played over $[0, 1]^2$ in which Player 1’s utility is given
by $u_1(x) = 10 - (1 - x_1)^2 - 5(x_2)^2$, Player 2’s utility is $u_2(x) = 10 - 5(x_1)^2 - (x_2)^2$ and the Ratiﬁer’s utility is $u_3(x) = 10 - (1 - x_1)^2 - (1 - x_2)^2$. Set discount rates $\delta_1 = \delta_2 = 0.4$ and $\delta_3 = 0.99$.

In this example preferences are such that the relative salience of
Dimension 2 to Dimension 1 is highest for Player 1 and lowest for Player
2. In the unconstrained game, Players 1 and 2 oﬀer their ideals, $(1, 0)$
and $(0, 0)$ respectively and accept any oﬀer $x$ such that $u_i(x) \geq 4$, for
$i \in \{1, 2\}$. When Player 2 oﬀers, the negotiators receive $u_1 = 9$ and $u_2 =$
10. In the constrained game, since \( \delta_3 u_3((1, 0)) = 8.91 > u_3((0, 0)) = 8 \), the ratifier would rather wait to receive an offer of \((1, 0)\) from Player 1 than to accept an offer of \((0, 0)\) from Player 2. In this case, while Player 1 may play the same strategies as before, Player 2 must alter her strategy in order to make an offer acceptable to the ratifier, at least cost to herself. This is done by offering \((0.13, 0.42)\). This offer just satisfies the ratifier. And it does so by yielding on dimension 2, which, while optimal for Player 2, is especially costly for Player 1. When Player 2 makes this offer, the negotiators receive \(u_1 = 8.3\) and \(u_2 = 9.7\), a worsening for both players relative to the unconstrained game.

Example 7 Euclidean Spatial Games. Consider the game where \(X = [0, 30] \times [0, 30] \subset \mathbb{R}^2\), players \(i\) and \(j\) have ideals \(\hat{x}_i = (20, 15)\) and \(\hat{x}_j = (10, 15)\) and all players have instantaneous utility \(u_k(x) = 500 - |x - \hat{x}_k|^2\) and common discount rate \(\delta = 0.95\). Let \(\hat{x}_3\) vary over the range of \(X\).

The choice of units and ideals in this example is arbitrary but in this instance allows us to distinguish easily between the dimension along which negotiators disagree (the first dimension) and the dimension along which negotiators agree (the second).

Figure 1 reports the payoffs from this game to Players \(i\) and \(j\) from \(i\)'s equilibrium offer given the presence of a ratifier with ideal \(\hat{x}_3 \in X\). The results show that “hawkish” ratifiers are better than dovish ratifiers—where by hawkish we mean that ratifiers have extreme preferences on the dimension along which negotiators disagree. Indeed as long as all points in \(X\) are preferred by the ratifier to the status quo, the more hawkish the better—in these instances the negotiator benefits from disagreement within her own camp. Conversely, a negotiator benefits from homogeneity within the opposing camp. The effects of discord along the dimension along which negotiators agree are just the opposite: Negotiators benefit more from ratifiers that agree with them along this dimension. If there is discord within a camp along the dimension of agreement, then the opposing camp can satisfy ratifiers by making concessions to the ratifier that do not benefit the negotiator. This counterintuitive result implies that a negotiator can benefit in particular from her own disagreement with the other camp’s ratifier.

A benefit of the model presented here is that in calculating equilibrium offers, we may, for any given game, observe which of Constraints \(C_1 - C_4\) are binding for different values of \(\hat{x}_3\). In other words we can tell to whose attention small adjustments in offers are made, and address the questions: When is an offer adjusted in order to satisfy the
Figure 1: Payoffs to negotiators as a function of the location of the ratifier’s ideal. This figure shows payoffs to $i$ and $j$ from $i$’s equilibrium offer as a function of the location of the ideal of the ratifier on $X$. The floor of the graph represents $X$ and has sample indifference curves marked for $i$ and $j$, with $i$’s ideal to the right of $j$’s. The graph that is increasing from left to right is $i$’s payoff for every position of the ratifier’s ideal. The graph decreasing from left to right is $j$’s payoff.

ratifier? and When are the preferences of the opposite negotiator most important? As seen in the proofs of the proposition, cases may arise in which each negotiator is constrained by the other and one of them is also constrained by the ratifier; in which one negotiator is constrained by the ratifier but not by the other negotiator; and in which one or other negotiator is entirely unconstrained in equilibrium.

Figure 2 shows the cases for which these constraints are binding on negotiator $i$ for the case studied in Example 7. We see that when the ratifier’s preferences are more extreme than $j$’s preferences on the dimension of disagreement, only the constraints imposed by the ratifier on negotiator $i$ are binding in equilibrium. In principle in this case $i$ could make a weaker offer to $j$ that $j$ would find preferable to waiting out a turn, but this offer would not be acceptable to the ratifier. When the ratifier is somewhat less extreme on this dimension but has divergent preferences on the dimension of agreement between the negotiators, $i$’s optimal strategy is to make an offer for which the acceptance conditions are binding for both the ratifier and the other negotiator. A trade-off point is identified that makes both players just indifferent between ac-
Figure 2: Constraints on negotiators as a function of the location of the ratifier’s ideal. This figure shows the constraints that are binding on $i$ as she makes her equilibrium offers for the same parameter values as used in Figure 1. The ideals of the two negotiators are marked, with $i$’s to the right of $j$’s accepting and rejecting the offer. For more moderate ratifiers, only the opposite negotiator’s constraint is binding in equilibrium—although in such cases the ratifier may still affect outcomes through the constraints he imposes on the opposite negotiator. Finally, when the ratifier is in fact an ally of $i$, and is indeed more extreme than $i$ on the dimension of disagreement, $i$ may be able to make an offer with all constraints slack—in such cases he chooses his ideal point, knowing that this offer will be acceptable to the opposite negotiator precisely because the opposite negotiator will be hard pressed to satisfy the ratifier. Hence, for empirical work, such variation gives clear predictions of which players “matter” during particular negotiation rounds.

6 Conclusions

In this article I consider games in which bilateral bargainers negotiate simultaneously over multiple issue areas but do so subject to the constraint that a strategic ratifier will sign off on the final deal. The model studied is appropriate for contexts in which failure to reach agreements is costly for all parties—including the ratifier—but in which the content of agreements is sufficiently important for ratifiers that they may try to
use their power of ratification to alter the behavior of negotiators.

The results provide insights that differ markedly from existing models. Surprisingly, when the mechanism that has traditionally been used to explain the effect of ratifiers—winsets—is removed we find strong support for the notion that ratification constraints help negotiators. Hence the models predict ratifier effects even where standard models suggest that ratifiers are irrelevant.

The models also provide insights regarding where the action is during multiparty negotiations. When the ratifier alters outcomes, she imposes constraints that are binding in equilibrium. In some instances the opposition negotiator has to take account of information regarding both the preferences of the home negotiating team and of the domestic ratifier. In other instances however, the ratifier constraints may be so severe that the constraint imposed by the domestic negotiating team on the opposition negotiators is slack. For offers in such cases it is as if the opposition were negotiating directly with the ratifier: the preferences of the home negotiator—perfectly satiated at the ensuing bargaining point—are irrelevant on the margin. This possibility, dismissed by Mansfield et al (2002), turns out to be a key determinant of ratifier power.

Finally, the model sheds new light on past models, clarifying which features of these models are salient in producing their results and opening new doors for theoretical and empirical research.

To consider the implications of our discussion for the interpretation of older models, return first to the model provided in Mo (1994). In the context of a divide the dollar game, Mo suggests that if one domestic actor has veto power, a rise in the constraints imposed by that player can have harmful effects. This result sits uncomfortably with our claim that in pure divide the dollar games with private valuations, ratifiers should be irrelevant. On closer examination however we find that the result in Mo is due entirely to the fact that the model does not distinguish functionally between ratifiers and negotiators. It is possible to examine the special case of Mo’s model in which ratifiers become proposers with 0 probability. When we do so we find that in this special case ratification constraints have no impact on outcomes one way or the other.\textsuperscript{11} This clarifies two issues. First, it highlights the role that the restrictive assumption on preferences make in this model: that ratifiers are strictly irrelevant in this representation of the ratification game follows not from the institutional structure but from the strong, albeit common, assumption that ratifiers treat the outcome of negotiations as a private good. Second, this now provides a handle on what in the model does induce the Schelling (and anti-Schelling) effects. Although Mo’s model introduces a multiplicity of domestic actors it is not this feature that produces the
result; rather, the results arise from the prospect that negotiators can be replaced by their domestic rivals—a possibility that some find implausible (Tarar 2005), but which may nonetheless reflect relevant aspects of domestic politics over the long run.

Consider next the model and discussion provided in Tarar (2005). Tarar describes his study as an examination of differences between presidential systems and parliamentary systems. He finds that constraints can benefit executives in the first but harm them in the second. On close reading, however, the differences between the two systems is summarized in the differences in the preferences of the leaders: under presidential systems the executive benefits from all returns to the country; under parliamentary systems the executive gains a private benefit from returns to his local constituency. Since in this game negotiations take place over divisions of a pie and the worst outcome obtains when a deal is not made, this anti-Schelling effect appears at first inconsistent with the results presented above. In the model presented here no such anti-Schelling effect arises in this context and Schelling effects when they arise can do so over a broad class of preferences.

One possible explanation for the differences in outcomes is that Tarar’s model, like Mo’s, has a multiplicity of domestic actors. In fact however it is easy to check that this is not the case; the anti-Schelling effects obtain in the parliamentary case even with only one member of parliament. On further examination we can see that it is another feature of Tarar’s model that drives the differences. In the model, constraints on executives are driven by the fact that ratifiers have an outside option, a payoff that they gain if they refuse to ratify. Introducing an outside option is akin to rendering ratifiers dissatisfied with feasible negotiated outcomes relative to the status quo. Hence this result turns out to be another application of the constrained winset approach discussed in Hammond and Prins (1999). It is easy to check that without this outside option ratifiers have no effect whatsoever in the parliamentary case, despite their numbers and sophistication. The empirical relevance of the results then are constrained to situations where negotiation failure is not costly to ratifiers (just as the empirical relevance of the results presented here obtains in cases where bargaining failure is costly).

In closing, I emphasize one of the newer directions that this model opens up for the study of ratification processes. The model presented here, by studying the Schelling conjecture in a general multidimensional environment, allows us to examine a feature that is central to the political processes under study but which cannot be analyzed in one dimensional frameworks, namely the effects of preference dependency and the role of different forms of within-group and between-group homo-
geneity. The model suggests that such features, although they do not qualitatively affect the applicability of the Schelling conjecture, do have quantitative effects. We have examined these most carefully here for one special case of the model. Negotiators, we found, may benefit from internal dissension within their own group when that dissension produces ratifiers that are extreme, relative to the negotiator, on the dimension along which negotiators disagree with each other; however they benefit from group homogeneity on the dimension along which negotiators agree—internal dissension on this dimension may allow the rival team to placate ratifiers without providing significant benefits to the negotiator. These predictions are directly testable. They are important too at the theoretical level: in starting to model such effects we move away from the representation of states as fundamentally discrete, if internally complex, units, and allow for more nuanced relations that allow for subnational domestic actors to have interests along multiple issues areas some of which may be held in common with foreign groups while others may be in conflict.
7 Endnotes

1. An important exception is Hayes and Smith (1997). In this model negotiators move last and this justifies a restriction of final outcomes to the (unrestricted) contract curve.

2. There exist more general studies of bargaining in a multidimensional context but these do not analyze the particular effects of ratification. See for example Banks and Duggan (2000).

3. In Iida (1993) the ratifier is partly strategic insofar as she can determine reservation prices. The negotiators’ problem however is solved cooperatively and the ratifier’s role is, as in previous work, limited to an up-down vote.

4. The assumptions used here are closest to those in Osborne and Rubinstein (1994). While much of the model is general the underlying alternating offers protocol is not.

5. To confirm that a unique \( \bar{x}_i \) exists note that with \( u_i \) continuous and \( X \) compact, \( u_i \) attains a maximum on \( X \). Uniqueness follows from the fact that with \( X \) convex and \( u_i \) quasiconcave any point between two maxima is preferred to either.

6. To see that \( C \) is strictly monotone, consider two distinct points \( x, y \in C \) such that \( u_i(y) \geq u_i(x) \) and \( u_j(y) = u_j(x) \). With strict quasiconcavity there exists a point \( z \in \text{Co}(x, y) \) that both players prefer to \( x \). But then \( x \notin C \), a contradiction.

7. Since \( u_i(y) = u_i(x) \) implies \( u_j(y) < u_j(x) \) which implies \( u_i(x) < u_i(y) \), a contradiction.


9. Although we examine strictly quasiconcave preferences, the proof of Proposition 2 does not depend on this as long as \( x^* \) and \( y^* \) are well defined as they are, for example, in a pure distributive game.

10. The divisibility of a pie refers here to the range of efficient divisions of surplus. If preferences were such that only divisions that awarded between \$.40 and \$.60 of a dollar to a player were efficient, the pie would be considered less divisible than in cases where all divisions are efficient.

11. This special case can be recovered from Mo’s model by examining the model in Appendix D for the special case in which \( p = 1 \). In this case the equilibrium offers of the Domestic negotiator (player 3) and Foreign reduce exactly to the Rubinstein offers of a two player game.

12. In Tarar’s model this is done by setting \( \Delta_i = 0 \) for all \( i \), which produces constraints \( C_i = 0 \) for all \( i \).
References


