1. The Current Account As Insurance Against Catastrophic Events

Consider a two-period endowment economy populated by identical households with preferences defined over consumption in period 1, $C_1$ and consumption in period 2, $C_2$, and described by the utility function

$$\ln C_1 + E \ln C_2,$$

where $E$ denotes the expected value operator. Each period, the economy receives an endowment of 10 units of food. Households start period 1 carrying no assets or debts from the past. The interest rate on financial assets held between periods 1 and 2 is zero.

(a) Compute consumption, the trade balance, the current account, and national savings in period 1.

(b) Assume now that the endowment in period 1 continues to be 10, but that the economy is prone to severe natural disasters in period 2. Suppose that these negative events are very rare, but have catastrophic effects on the country’s output. Specifically, assume that with probability 0.01 the economy suffers an earthquake in period 2 that causes the endowment to drop by 90 percent with respect to period 1. With probability 0.99, the endowment in period 2 increases to $111/11$.

i. What is the expected endowment in period 2? How does it compare to that of period 1?

ii. What percent of period-1 endowment will the country export? Compare this answer to what happens under certainty and provide intuition.

2. An Economy With Investment

Consider a two-period model of a small open economy with a single good each period. Let preferences of the representative household be described by the utility function

$$\ln(C_1) + \ln(C_2),$$

where $C_1$ and $C_2$ denote consumption in periods 1 and 2, respectively, and $\ln$ denotes the natural logarithm. In period 1, the household receives an endowment of $Q_1 = 10$. In period 2, the household receives profits, denoted by $\Pi_2$, from the firms it owns. Households and firms have access to financial markets where they can borrow or lend at the interest rate $r_1$. ($r_1$ is the interest rate on assets held between periods 1 and 2.)

Firms invest in period 1 to be able to produce goods in period 2. The production technology in period 2 is given by

$$Q_2 = \sqrt{I_1},$$

where $Q_2$ and $I_1$ denote, respectively, output in period 2 and investment in period 1.

Assume that there exists free international capital mobility and that the world interest rate, $r^*$, is 10% per period (i.e., $r^* = 0.1$). Finally, assume that the economy’s initial net foreign asset position is zero ($B_0^* = 0$).

(a) Compute the firm’s optimal levels of period-1 investment and period-2 profits.

(b) State the maximization problem of the representative household and solve for the optimal levels of consumption in periods 1 and 2.

(c) Find the country’s net foreign asset position at the end of period 1, the trade balance in periods 1 and 2, and the current account in periods 1 and 2.
(d) Now consider an investment surge. Specifically, assume that as a result of a technological improvement, the production technology becomes $Q_2 = 2\sqrt{T_1}$. Find the equilibrium levels of savings, investment, the trade balance, the current account, and the country’s net foreign asset position in period 1. Compare your results with those obtained in items (a)-(c) providing interpretation and intuition.