1. **A Two-Country Economy**

Consider a two-period, two-country, endowment economy. Let one of the countries be the United States and the other Europe. Households in the United States have preferences described by the utility function

\[ \ln C_U^1 + \ln C_U^2, \]

where \( C_U^1 \) and \( C_U^2 \) denote consumption of U.S. households in periods 1 and 2, respectively. Europeans have identical preferences, given by

\[ \ln C_E^1 + \ln C_E^2, \]

where \( C_E^1 \) and \( C_E^2 \) denote consumption of European households in periods 1 and 2, respectively. Let \( Q_U^1 \) and \( Q_U^2 \) denote the U.S. endowments of goods in periods 1 and 2, respectively. Similarly, let \( Q_E^1 \) and \( Q_E^2 \) denote the European endowments of goods in periods 1 and 2, respectively. Assume further that the endowments are nonstorable, that the U.S. and Europe are of equal size, and that there is free capital mobility between the two economies. The United States starts period 1 with a zero net foreign asset position carried over from period 0.

(a) **Symmetric Equilibrium** Suppose that \( Q_U^1 = Q_U^2 = Q_E^1 = Q_E^2 = 10 \). Calculate the equilibrium world interest rate, and the current accounts in the United States and Europe in period 1.

(b) **US-Originated Contraction # 1** Suppose that a contraction originates in the United States. Specifically, assume that \( Q_U^1 \) drops from 10 to 8. All other endowments (\( Q_U^2, Q_E^1, \) and \( Q_E^2 \)) remain unchanged at 10. This contraction in output has two characteristics: First, it originates in the United States (the European endowments are unchanged.) Second, it is temporary (the U.S. endowment is expected to return to its normal value of 10 after one period). Calculate the equilibrium interest rate and the two current accounts in period 1. Provide intuition.

(c) **US-Originated Contraction # 2** Suppose now a second type of contraction in which the U.S. endowment falls from 10 to 8 in the first period but is expected to continue falling to 6 in the second period (\( Q_U^1 = 8 \) and \( Q_U^2 = 6 \)). The endowments in Europe remain unchanged at 10 each period (\( Q_E^1 = Q_E^2 = 10 \)). Like the one described in the previous item, this contraction originates in the United States. However, it differs from the one described in the previous paragraph in the fact that it displays a more protracted string of negative output growth rates. Calculate again the equilibrium interest rate and the two current accounts in period 1. Point out differences in the effects of the two types of contraction and provide intuition.

(d) At the beginning of the great contraction of 2008, interest rates fell sharply around the world. What does the model above say about people’s expectations around 2008 about the future path of real activity.

2. **An Small Open Economy With Distortionary Taxes**

Consider a two-period economy populated by a large number of households with preferences described by the utility function

\[ \ln C_1 + \beta \ln C_2, \]

where \( C_1 \) and \( C_2 \) denote consumption in periods 1 and 2, respectively, and \( \beta = 1/1.1 \) is a subjective discount factor. Households receive endowments \( Q_1 \) in period 1 and \( Q_2 \) in period 2, with \( Q_1 = Q_2 = 10 \) and can borrow or lend in international financial markets at the interest rate \( r^* = 0.1 \). The government
imposes taxes $T_1 = T^L + \tau_1 C_1$ in period 1 and $T_2 = \tau_2 C_2$ in period 2 and consumes $G_1$ units of goods in period 1 and $G_2$ units in period 2. Finally, households and the government start period 1 with no assets or debts carried over from the past.

(a) Derive the intertemporal budget constraint of the household, the intertemporal budget constraint of the government, and the intertemporal resource constraint of the economy as a whole.

(b) Derive the optimality condition that results from choosing $C_1$ and $C_2$ to maximize the household’s utility function subject to its intertemporal budget constraint.

(c) Suppose $G_1 = G_2 = 2$ and $\tau_1 = \tau_2 = 0.2$. Find the equilibrium levels of consumption and the trade balance in periods 1 and 2, and the equilibrium level of lump-sum taxes $T^L$. Report the primary and secondary fiscal deficits in period 1.

(d) Continue to assume that $G_1 = G_2 = 2$. Suppose that the government implements a tax cut in period 1 consisting in lowering the consumption tax rate from 20 to 10 percent. Suppose further that lump-sum taxes, $T^L$, are kept at the level found in the previous item. Find consumption, the trade balance, the primary fiscal deficit in period 1, and the consumption tax rate in period 2.

(e) Now answer the previous question assuming that the cut in consumption taxes in period 1 from 20 to 10 percent is financed with an appropriate change in lump-sum taxes in the same period, while the consumption tax rate in period 2 is kept constant at its initial level of 20 percent. Compare your answer with the one for the previous item and provide intuition.

(f) Suppose that $G_1 = 2$, $G_2 = 1$, and $T^L = 0$. Clearly, there are many possible equilibrium tax schemes $(\tau_1, \tau_2)$. Find the pair $(\tau^1, \tau^2)$ that maximizes the household’s lifetime utility. Show your derivation. Refer to your solution as the Ramsey optimal tax policy.