slides
chapter 9
nominal rigidity
Introduction

• develop a theoretical framework with nominal rigidities that result in inefficient adjustment to aggregate disturbances

• framework can be used in an intuitive manner to demonstrate how nominal rigidities amplify the business cycle in open economies

• but framework can also be used to derive quantitative prediction useful for policy evaluation
Some Motivation: Emerging Europe and the Global Crisis of 2008

Take a look at the next slide.

- The inception of the Euro in 1999, was followed by massive capital inflows into the region, possibly driven by expectations of quick convergence of peripheral and core Europe.

- Large current account deficits and large increases in nominal hourly wages, with declining rates of unemployment between 2000 and 2008.

- With the Global crisis, capital inflows dried up abruptly. The region suffered a severe **sudden stop** (sharp reductions in current account deficits).

- In spite of the collapse in aggregate demand and the lack of a devaluation, nominal hourly wages remained as high as at the peak of the boom.

- Massive unemployment affected all countries in the region.
**Figure 9.1 Boom-Bust Cycle in Peripheral Europe: 2000-2011**

Data Source: Eurostat. Labor Cost Index, Nominal, is the nominal hourly wage rate in manufacturing, construction and services (including the public sector, but for Spain.)

Data represents arithmetic mean of Bulgaria, Cyprus, Estonia, Greece, Ireland, Lithuania, Latvia, Portugal, Spain, Slovenia, and Slovakia.
This suggests the following narrative:

Countries in the periphery of the European Union, such as Ireland, Portugal, Greece, and a number of small eastern European countries adopted a fixed exchange rate regime by joining the Euro area. Most of these countries experienced an initial transition into the Euro characterized by low inflation, low interest rates, and economic expansion.

**However, history has shown time and again that fixed exchange rate arrangements are easy to adopt but difficult to maintain. (Example: Argentina’s 1991 convertibility plan.)**

The Achilles’ heel of currency pegs is that they hinder the efficient adjustment of the economy to negative external shocks, such as drops in the terms of trade or hikes in the interest-rate. Such shocks produce a contraction in aggregate demand that requires a decrease in the relative price of nontradables, that is, a real depreciation of the domestic currency, in order to bring about an expenditure switch away from tradables and toward nontradables. In turn, the required real depreciation may come about via a nominal devaluation of the domestic currency or via a fall in nominal prices or both.

The currency peg rules out a devaluation. Thus, the only way the necessary real depreciation can occur is through a decline in the nominal price of nontradables. However, when nominal wages are downwardly rigid, producers of nontradables are reluctant to lower prices, for doing so might render their enterprises no longer profitable. As a result, the necessary real depreciation takes place too slowly, causing recession and unemployment along the way.

This narrative goes back at least to Keynes (1925) who argued that Britain's 1925 decision to return to the gold standard at the 1913 parity despite the significant increase in the aggregate price level that took place during World War I would force deflation in nominal wages with deleterious consequences for unemployment and economic activity. Similarly, Friedman’s (1953) seminal essay points at downward nominal wage rigidity as the central argument against fixed exchange rates.
To formalize this narrative we build an open economy model with:

- downward nominal wage rigidity
- a traded and a nontraded sector
- involuntary unemployment

To produce quantitative predictions:

- Estimate the key parameters of the model (with particular attention on the parameter governing downward wage rigidity) and estimate the driving forces.
- Characterize response to large negative external shock under a peg and show that the model can explain the observed sudden stop.
- Characterize optimal exchange rate policy.
- Quantify the costs of currency pegs in terms of unemployment and welfare.

The material is based on Schmitt-Grohé and Uribe (JPE, 2016).
An Open Economy Model with Involuntary Unemployment due to Downward Nominal Wage Rigidity

(Section 9.1, Chapter 9)
\textbf{Downward Nominal Wage Rigidity (DNWR)}

\[ W_t \geq \gamma W_{t-1} \]

\( W_t = \) nominal wage rate in period \( t \)

\( \gamma = \) degree of downward wage rigidity.

\( \gamma = 0 \Rightarrow \) fully flexible wages.

Think of \( \gamma \) as being around 1. The empirical evidence presented later in this chapter suggests \( \gamma = 0.99 \) at quarterly frequency.
Traded and Nontraded Goods

Stochastic endowment of tradable goods: $y^T_t$.

Nontraded goods, $y^N_t$, produced with labor, $h_t$: $y^N_t = F(h_t)$

Law of one price holds for tradables: $P^T_t = E_t P^*_t$.

$P^T_t$, nominal price of tradable goods.

$E_t$, nominal exchange rate, domestic-currency price of one unit of foreign currency ($E_t \uparrow$ depreciation of domestic currency).

$P^*_t$, foreign currency price of tradable goods.

Assume that $P^*_t = 1$, so that $P^T_t = E_t$
The Nontraded Sector

Profits, $\Phi_t$:

$$\Phi_t = P_t^N F(h_t) - W_t h_t$$

$P_t^N$, nominal price of nontradables.

Firms maximize profits taking as given $P_t^N$ and $W_t$. Optimality Condition:

$$P_t^N F'(h_t) = W_t$$

Divide by $P_t^T = 1 + \epsilon_t$ and rearrange

$$p_t = \frac{W_t/\epsilon_t}{F'(h_t)}$$

$p_t \equiv \frac{P_t^N}{P_t^T}$, relative price of nontradables in terms of tradables. Interpret this optimality conditions as a supply schedule for nontradables, see next slide.
Figure 9.3 The Supply Of Nontradables

\[ p_t = \frac{W_t/\varepsilon_t}{F'(F^{-1}(y^N_t))} \]

- A decrease in nominal wage from \( W_1 \) to \( W_0 < W_1 \) shifts the supply schedule down.
- A devaluation \( \varepsilon_t \uparrow \) (not shown) shifts the supply schedule in the same manner as a nominal wage cut.
Households

$$\max_{\{c_t^T, c_t^N, d_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t)$$

subject to

$$P_t^T c_t^T + P_t^N c_t^N + \varepsilon_t d_t = P_t^T y_t + W_t h_t + \varepsilon_t \frac{d_{t+1}}{1 + r_t} + \Phi_t$$

$$h_t \leq \bar{h}$$

$$c_t = A(c_t^T, c_t^N)$$

- First constraint: $d_t = \text{one-period debt chosen in } t, \text{ due in } t + 1$. Debt is denominated in units of foreign currency → full liability dollarization. → *Original Sin*: In emerging countries almost 100% of external debt issued in foreign currency (Eichengreen, Hausmann, and Panizza, 2005).

- Second constraint: Workers supply $\bar{h}$ hours inelastically, but may not be able to sell them all. They take $h_t \leq \bar{h}$ as given.

- Third constraint: Consumption is a composite of traded and non-traded goods. $A(., .)$ increasing, concave, and HD1
Optimality Conditions associated with the Household Problem

\[ P_t^T c_t^T + P_t^N c_t^N + \varepsilon_t d_t = P_t^T y_t^T + W_t h_t + \varepsilon_t \frac{d_t+1}{1 + r_t} + \Phi_t \]

\[ h_t \leq \bar{h} \]

\[ \frac{A_2(c_t^T, c_t^N)}{A_1(c_t^T, c_t^N)} = p_t \]

\[ \lambda_t = U'(A(c_t^T, c_t^N))A_1(c_t^T, c_t^N) \]

\[ \lambda_t = \beta(1 + r_t)E_t \lambda_{t+1} \]
The Demand For Nontradables

Look again at the optimality condition

\[
\frac{A_2(c^T_t, c^N_t)}{A_1(c^T_t, c^N_t)} = p_t.
\]

If \( A(c^T, c^N) \) is concave and HD1, then given \( c^T_t \), the left-hand side is decreasing in \( c^N_t \). This means that, all other things equal, an increase in \( p_t \) reduces the desired demand for nontradables, giving rise to the downward sloping demand schedule shown in the next slide.

Note that \( c^T_t \) acts as a shifter of the demand schedule for nontradables: given \( p_t \), an increase in \( c^T_t \) is associated with an equiproportional desired increase in \( c^N_t \). Of course, this shifter is endogenously determined.
\[ p_t = \frac{A_2(c_t^T, c_t^N)}{A_1(c_t^T, c_t^N)} \]

- Here we treat $c_t^T$ as a shifter of the demand schedule.
- A increase in $c_t^T$ from $c_0^T$ to $c_1^T > c_0^T$, shifts the demand schedule up and to the right.
Closing of the Labor Market

Impose the following slackness condition:

\[(\bar{h} - h_t)(W_t - \gamma W_{t-1}) = 0\]

This slackness condition says that, if there is involuntary unemployment \((h_t < \bar{h})\), then the lower bound on nominal wages must be binding. It also says that if the lower bound on nominal wages is not binding \((W_t > \gamma W_{t-1})\), then the labor market must feature full employment.

Market clearing in the Nontraded Sector

\[c_t^N = y_t^N = F(h_t)\]
A competitive equilibrium is a set of stochastic processes \( \{c^T_t, h_t, w_t, d_{t+1}, p_t, \lambda_t\}_{t=0}^{\infty} \) satisfying

\[
\begin{align*}
  c^T_t + d_t &= y^T_t + \frac{d_{t+1}}{1 + r_t} \\
  \lambda_t &= U'(A(c^T_t, F(h_t)))A_1(c^T_t, F(h_t)) \\
  \frac{\lambda_t}{1 + r_t} &= \beta \mathbb{E}_t \lambda_{t+1} \\
  p_t &= \frac{A_2(c^T_t, F(h_t))}{A_1(c^T_t, F(h_t))} \\
  p_t &= \frac{w_t}{F'(h_t)} \\
  w_t &\geq \gamma \frac{w_{t-1}}{\epsilon_t} \\
  h_t &\leq \bar{h} \\
  (\bar{h} - h_t) \left( w_t - \gamma \frac{w_{t-1}}{\epsilon_t} \right) &= 0
\end{align*}
\]

given an exchange rate policy \( \{\epsilon_t\}_{t=0}^{\infty} \), initial conditions \( w_{-1} \) and \( d_0 \), and exogenous stochastic processes \( \{r_t, y^T_t\}_{t=0}^{\infty} \).
Currency Pegs

(Section 9.2 of Chapter 9)
\[ \varepsilon_t = \varepsilon_0; \quad \forall t \geq 0. \]

Use the graphical apparatus just developed to show that a \textbf{boom-bust cycle} leads to

- nominal wage growth and real appreciation during the boom phase
- involuntary unemployment and insufficient real depreciation during the bust phase

For the moment we treat a boom-bust cycle as a rise in \( c^T \) followed by a fall in \( c^T \).
Adjust to a Boom-Bust Cycle under a Currency Peg

\[
\frac{A_2(c^T_1, F(h))}{A_1(c^T_0, F(h))}
\]

\[
\frac{W_1/\varepsilon_0}{F'(h)} = \frac{W_0/\varepsilon_0}{F'(h)} = \frac{W_1/\varepsilon_1}{F'(h)}
\]

\[
c^T_1 < c^T_0
\]

negative external shock possibly caused by \( r_t \uparrow \)
The boom bust cycle, observations on figure 9.4. The initial situation is point $A$. There is full employment, $h = \bar{h}$.

Now a boom starts. We capture this by an increase in $c^T$ (perhaps because $r$ falls). Given nominal wages the economy moves to point $B$. But at point $B$, there is excess demand for labor. Thus nominal wages will rise. By how much? Until the excess demand for labor has disappeared. That will be at point $C$. Thus the boom leads to an increase in nominal wages ($W \uparrow$) and a real appreciation ($p \uparrow$). The economy continues to operate at full employment.

Next the boom is over and the bust comes. We capture this by assuming that $c^T$ falls back to its original level, $c^T_0$.

This shifts the demand for nontradables back to its original position. The new intersection between supply and demand is at point $D$. At $D$, labor supply exceeds labor demand. However, because nominal wages are downwardly rigid and the nominal exchange rate is fixed, the supply schedule does not shift, (here we assume $\gamma = 1$). Thus the economy is stuck at point $D$. At point $D$ there is involuntary unemployment $(\bar{h} - h^{\text{bust}})$ and there is insufficient real depreciation, ie, $p$ falls to decline enough to bring about full employment.
Section 9.2.2 Volatility and Average Unemployment

The model predicts that aggregate volatility increases the mean level of unemployment.

This prediction gives rise to large welfare benefits of stabilization policy.

— This prediction is not due to the assumption of downward nominal wage rigidity, but due to the assumption that employment is determined by the minimum of labor demand and labor supply. (Note: key difference with Calvo-style sticky wage models in which employment is always demand determined.)

— Downward nominal wage rigidity amplifies the connection between aggregate volatility and mean unemployment.
To see this consider the following example:

\[ U(A(c_t^T, c_t^N)) = \ln c_t^T + \ln c_t^N \]

\[ d_t = 0 \text{ (no access to international financial markets)} \]

\[ \Rightarrow c_t^T = y_t^T. \]

\[ y_t^T = \begin{cases} 
1 + \sigma \text{ prob } \frac{1}{2} \\
1 - \sigma \text{ prob } \frac{1}{2}
\end{cases} \]

\[ E(y_t^T) = 1 \text{ and } \text{var}(y_t^T) = \sigma^2. \]

\[ F(h_t) = h_t^\alpha \]

\[ \bar{h} = 1 \]

\[ \mathcal{E}_t = \bar{\mathcal{E}} \text{ (currency peg)} \]

\[ W_{-1}/\mathcal{E} = \alpha \]
The equilibrium conditions associated with this economy are

\[
\frac{c_t^T}{c_t^N} = p_t
\]

\[
\alpha p_t (h_t)^{\alpha-1} = W_t / \mathcal{E}
\]

\[
c_t^T = y_t^T
\]

\[
c_t^N = h_t^\alpha
\]

\[
W_t = \alpha \mathcal{E}
\]

Step 1: Find labor demand: \( h_t^d = \alpha y_t^T / (W_t / \mathcal{E}) \)

Step 2: Find equilibrium labor as \( h_t = \min \{ \bar{h}, h_t^d \} \)
Case 1: Assume bi-directional nominal wage rigidity.

\[ W_t = \alpha \xi \]

Then,

\[ h_t = \begin{cases} 
1 - \sigma & \text{if } y_t^T = 1 - \sigma \\
1 & \text{if } y_t^T = 1 + \sigma 
\end{cases} \]

Let \( u_t \equiv \bar{h} - h_t \) denote the unemployment rate. It follows that the equilibrium distribution of \( u_t \) is given by

\[ u_t = \begin{cases} 
\sigma & \text{with probability } \frac{1}{2} \\
0 & \text{with probability } \frac{1}{2} 
\end{cases} \]

The unconditional mean of the unemployment rate is then given by

\[ E(u_t) = \frac{\sigma}{2}. \]

Average level of unemployment increases linearly with the volatility of tradable endowment, in spite of the fact that wage rigidity is symmetric!
Case 2: assume ‘only’ downward nominal wage rigidity, $W_t \geq W_{t-1}$

Then, $W_t = \alpha(1 + \sigma) > \alpha$ and

$$h_t = \begin{cases} 
\frac{1-\sigma}{1+\sigma} & \text{if } y_t^T = 1 - \sigma \\
1 & \text{if } y_t^T = 1 + \sigma
\end{cases}$$

$E(u_t) = \sigma/(1 + \sigma) > \sigma/2$ (recall that $\sigma$ must be less than 1).

Thus uni-directional wage rigidity strengthens the link between mean unemployment and volatility.
Section 9.3 Optimal Exchange Rate Policy
An important reference point: The **full-employment real wage**, denoted $\omega(c_t^T)$, defined as the real wage that clears the labor market,

$$
\omega(c_t^T) \equiv \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))} F'(\bar{h}); \quad \omega'(c_t^T) > 0
$$

Set the (gross) devaluation rate, $\epsilon_t = \mathcal{E}_t/\mathcal{E}_{t-1}$, to eliminate unemployment:

$$
\epsilon_t \geq \frac{\gamma W_{t-1}/\mathcal{E}_{t-1}}{\omega(c_t^T)}
$$

Note: There is a whole family of optimal exchange-rate policies. Under any member of this policy, $h_t = \bar{h}$ and $w_t = \omega(c_t^T)$ for all $t$. External debt and tradable consumption are determined by the solution to

$$
v^{OPT}(y_t^T, r_t, d_t) = \max_{\{d_{t+1}, c_t^T\}} \left\{ U(A(c_t^T, F(\bar{h}))) + \beta \mathbb{E}_t v^{OPT}(y_{t+1}^T, r_{t+1}, d_{t+1}) \right\}
$$

subject to $d_{t+1} \leq \bar{d}$ and

$$
y_t^T + \frac{d_t+1}{1 + r_t} = d_t + c_t^T
$$
Optimal Exchange-Rate Policy

(again, assume $\gamma = 1$)

$c_1^T < c_0^T$ (negative shock, possibly $r_t \uparrow$)

$\varepsilon_1 > \varepsilon_0$ (optimal devaluation)
Optimal Exchange-Rate Policy (continued)

Comments on the Graph: The initial situation is point $A$. Suppose a negative shock (possibly an increase in the country interest rate), shifts the demand schedule down and to the left. Without government intervention, the supply schedule does not move, because $W$ is downwardly rigid. The equilibrium would then be point $B$, with involuntary unemployment equal to $\bar{h} - h_{PEG}$.

Suppose now that the government devalues the domestic currency from $E_0$ to $E_1 > E_0$. The devaluation lowers the real wage from $W_0/E_0$ to $W_0/E_1$, causing the supply schedule to shift down and to the right.

If the devaluation is just right, the new supply schedule will cross the new demand schedule at point $C$, preserving full employment (we say preserving and not restoring full employment because the economy jumps from $A$ to $C$, without visiting $B$).

Note: the fall in labor cost caused by the drop in the real wage allows firms to cut prices from $p_0$ to $p^{OPT}$ and induces households to switch expenditure away from tradables and toward nontradables.

Take another look at the graph for the analytical example in of a temporary fall in $r_0$. The broken lines display the equilibrium under optimal exchange-rate policy.
An Analytical Example

Adjustment To A Temporary Interest Rate Fall:

Here is an example that shows that under a currency peg and downward nominal wage rigidity, a good shock, in this case a fall in the country interest rate $r_t$, can be the prelude to very bad things to happen later.

Consider the following environment:

$$U(A(c_t^T, c_t^N)) = \ln c_t^T + \ln c_t^N$$

$$F(h_t) = h_t^\alpha$$

$$\bar{h} = 1; \quad y_t^T = y^T > 0; \quad \gamma = 1; \quad d_0 = 0; \quad w_{-1} = \alpha y^T$$

$$r_t = \begin{cases} r & t > 0 \\ \bar{r} < r & t = 0 \end{cases}$$
A Temporary Interest Rate Fall (Continued)

In a couple of slides, you’ll find the solution of the equilibrium in algebraic and graphical form. To help the interpretation of those slides, it is of use to discuss intuitively what is going on in this economy:

— The fall in the interest rate induces an expansion in the desired demand for consumption goods, of all types, tradables and nontradable.

— The increase demand for tradables causes a trade balance deficit, a deficit in the current account, and an increase in external debt in period 0.

— The increased demand for nontradables cause a rise in wages and a rise in the relative price of nontradables (i.e., an appreciation of the real exchange rate).
— In period 1, the interest rate goes back up to its permanent value $r$, causing a contraction in the demand for consumption goods (both tradables and nontradables), and a reversal in the trade balance and the current account.

— The contraction in the demand for nontradables causes a derived contraction in the demand for labor. However, because nominal wages are downwardly rigid and the exchange rate is fixed, the real wage fails to fall, causing involuntary unemployment.

— Involuntary unemployment is highly persistent. (In fact, because $\gamma = 1$, it never disappears.)
A Temporary Interest Rate Fall (Continued)

The equilibrium has the following closed-form solution:

\[ c_0^T = y^T \left[ \frac{1}{1 + r} + \frac{r}{1 + r} \right] > y^T \]

\[ c_t^T = y^T \left[ \frac{1}{1 + r} + \frac{r}{1 + r} \frac{1 + r}{1 + r} \right] < y^T, \]

\[ d_t = y^T \left[ 1 - \frac{1 + r}{1 + r} \right] > 0, \]

\[ h_0 = 1; \]

\[ h_1 = h_2 \cdots = \frac{1 + r}{1 + r} < 1 \]

The following slide displays the same information graphically.
A Temporary Decline in the Country Interest Rate

- Country Interest Rate, $r_t$
- Consumption of Tradables, $c_t^T$
- Debt, $d_t$
- Unemployment, $(\bar{h} - h)/\bar{h}$
- Real Wage, $w_t$
- Real Exchange Rate, $P_t^N/P_t^T$

currency peg flexible wage economy or optimal exchange rate economy
Empirical Evidence On

Downward Nominal Wage Rigidity

(Section 9.4 of Chapter 9)
• Downward nominal wage rigidity is the central friction in the present model ⇒ natural to ask if it is empirically relevant.

• Downward wage rigidity is a widespread phenomenon:
  — Evident in micro and macro data.
  — Rich, emerging, and poor countries.
  — Developed and underdeveloped regions of the world.

• By product: Will obtain an estimate of the parameter $\gamma$ governing wage stickiness in the model (useful for quantitative analysis).
Downward Nominal Wage Rigidity:

A.) Evidence From Micro Data from Developed Countries

1. United States, 1986-1993

2. United States, 1996-1999

3. United States, 2011

4. Other Developed Countries
1.) United States, 1986-1993

Probability of Decline, Increase, or No Change in Wages

<table>
<thead>
<tr>
<th></th>
<th>Interviews One Year apart</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Males</td>
</tr>
<tr>
<td>Decline</td>
<td>5.1%</td>
</tr>
<tr>
<td><strong>Constant</strong></td>
<td>53.7%</td>
</tr>
<tr>
<td>Increase</td>
<td>41.2%</td>
</tr>
</tbody>
</table>

Source: Gottschalk (2005). Note: Male and female hourly workers not in school, 18 to 55 at some point during the panel. All nominal-wage changes are within-job wage changes, defined as changes while working for the same employer. SIPP panel data.

- Large mass at ‘Constant’ suggests nominal wage rigidity.
- Small mass at 'Decline' suggests downward nominal wage rigidity.
2.) United States 1996-1999

Distribution of Non-Zero Nominal Wage Changes

Source: Barattieri, Basu, and Gottschalk (2012). SIPP panel data.
3.) United States 2011

Distribution of Nominal Wage Changes, U.S. 2011

Source: Daly, Hobijn, and Lucking (2012).
4.) Micro Evidence On Downward Nominal Wage Rigidity From Other Developed Countries

- Switzerland: Fehr and Goette (2005).
B.) Evidence From Informal Labor Markets

- Kaur (2012) examines the behavior of nominal wages, employment, and rainfall in casual daily agricultural labor markets in rural India (500 districts from 1956 to 2008).

- Finds asymmetric nominal wage adjustment:
  - $W_t$ increases in response to positive rainfall shocks
  - $W_t$ fails to fall, labor rationing, and unemployment are observed in response to negative rain shocks.

- Inflation (uncorrelated with local rain shocks) tends to moderate rationing and unemployment during negative rain shocks, suggesting downward rigidity in nominal rather than real wages.
C.) Evidence From the U.S. Great Depression, 1929-1933

- Enormous contraction in employment: 31% between 1929 and 1931.

- Nonetheless, during this period nominal hourly wages fell by 0.6% per year, while consumer prices fell by 6.6% per year. See the figure on the next slide.

- A similar pattern is observed during the second half of the Depression. By 1933, real wages were 26% higher than in 1929, in spite of a highly distressed labor market.
Nominal Wage Rate and Consumer Prices, United States
1923:1-1935:7

D.) Evidence From the Great Depression In Europe

- Countries that left the gold standard earlier recovered faster than countries that remained on gold.

  — Left Gold Early (sterling bloc): United Kingdom, Sweden, Finland, Norway, and Denmark.

  — Countries That Stuck To Gold (gold bloc): France, Belgium, the Netherlands, and Italy.

- Think of the gold standard as a currency peg (a peg not to a currency, but to gold).

- When the sterling-bloc left gold, they effectively devalued, as their currencies lost value against gold.

- Look at the figure on the next slide. Between 1929 and 1935, sterling-bloc countries experienced less real wage growth and larger increases in industrial production than gold-bloc countries.
Changes In Real Wages and Industrial Production, 1929-1935

Figure 2: Changes in Real Wages and Industrial Production, 1929-1935
E.) Evidence From Emerging Countries

- Argentina: pegged the peso at a 1-to-1 rate to the dollar between 1991 and 2001.
- Starting in 1998, the economy was buffeted by a number of large negative shocks (weak commodity prices, large devaluation in Brazil, large increase in country premium, etc.).
- Nonetheless, nominal wages remained remarkably flat.
- This evidence suggests that nominal wages are downwardly rigid, and that $\gamma$ is about 1.
- Why $\gamma \approx 1$? The slackness condition $(\bar{h} - h_t)(W_t - \gamma W_{t-1})$ (recall $\epsilon_t = 1$ during this period), implies that if unemployment is growing, wages must grow at the gross rate $\gamma$. 
Argentina 1996-2006

Nominal Exchange Rate ($E_t$)

Unemployment Rate + Underemployment Rate

Nominal Wage ($W_t$)

Real Wage ($W_t/E_t$)

Implied Value of $\gamma$: Around unity.
Evidence From Peripheral Europe (2008-2011)

• Look at the table on the next slide.

• Between 2008 and 2011, all countries in the periphery of Europe experienced increases in unemployment; Some very large increases.

• In spite of extreme duress in the labor market, nominal hourly wages experienced increases in most countries and modest declines in only a few.

• The slide following the table explains how to use the information in the table to infer a range for $\gamma$. 
### Unemployment, Nominal Wages, and $\gamma$
#### Evidence from the Eurozone

<table>
<thead>
<tr>
<th>Country</th>
<th>Unemployment Rate 2008Q1 (in percent)</th>
<th>Unemployment Rate 2011Q2 (in percent)</th>
<th>Wage Growth $\frac{W_{2011Q2}}{W_{2008Q1}}$ (in percent)</th>
<th>Implied Value of $\gamma$</th>
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</thead>
<tbody>
<tr>
<td>Bulgaria</td>
<td>6.1</td>
<td>11.3</td>
<td>43.3</td>
<td>1.028</td>
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<td>Cyprus</td>
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<td>6.9</td>
<td>10.7</td>
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<td>12.8</td>
<td>2.5</td>
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<td>16.7</td>
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<td>0.5</td>
<td>1.0004</td>
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<td>Italy</td>
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<td>8.2</td>
<td>10.0</td>
<td>1.007</td>
</tr>
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<td>Lithuania</td>
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<td>15.6</td>
<td>-5.1</td>
<td>0.996</td>
</tr>
<tr>
<td>Latvia</td>
<td>6.1</td>
<td>16.2</td>
<td>-0.6</td>
<td>0.9995</td>
</tr>
<tr>
<td>Portugal</td>
<td>8.3</td>
<td>12.5</td>
<td>1.91</td>
<td>1.001</td>
</tr>
<tr>
<td>Spain</td>
<td>9.2</td>
<td>20.8</td>
<td>8.0</td>
<td>1.006</td>
</tr>
<tr>
<td>Slovenia</td>
<td>4.7</td>
<td>7.9</td>
<td>12.5</td>
<td>1.009</td>
</tr>
<tr>
<td>Slovakia</td>
<td>10.2</td>
<td>13.3</td>
<td>13.4</td>
<td>1.010</td>
</tr>
</tbody>
</table>

How To Infer $\gamma$

The model implies that if unemployment increases from one period to the next, then nominal wages must be growing at the rate $\gamma$.

How to calculate $\gamma$:

$$\gamma = \left( \frac{W_{2011:Q2}}{W_{2008:Q1}} \right)^{\frac{1}{13}}$$

Subtract 0.6% per quarter to adjust for foreign inflation and long-run growth (because they are not explicitly incorporated in the model) to obtain the estimate:

$$\gamma \in [0.99, 1.022]$$
Quantitative Analysis

Replication files: usg_dnwr.zip available online with the materials for this chapter.
Functional Forms

Assume a CRRA form for preferences, a CES form for the aggregator of tradables and nontradables, and an isoelastic form for the production function of nontradables:

$$U(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

$$A(c^T, c^N) = \left[ a(c^T)^{1-\frac{1}{\xi}} + (1 - a)(c^N)^{1-\frac{1}{\xi}} \right]^{1-\frac{1}{\xi}}$$

$$F(h) = h^\alpha,$$

with $\sigma, \xi, a, \alpha > 0$. 
The case of Equal Intra- and Intertemporal Elasticities of Substitution

Consider the case

\[ \xi = \frac{1}{\sigma} \]

Why this case is of interest:

- It makes the determination of the equilibrium levels of debt, \( d_t \), and consumption of tradables, \( c^T_t \), independent of the level of activity in the nontraded sector (see the next slide). As a result, the welfare consequences of exchange-rate policy or nominal wage rigidity are fully attributable to their effect on unemployment, and not on their effect on the accumulation of external debt.

- It facilitates the computation of equilibrium, as the equilibrium dynamics of \( d_t \) and \( c^T_t \) can be computed separately.

- As we will argue shortly, \( \sigma = 1/\xi = 2 \) is empirically plausible.
Determination of Debt and Tradable Consumption when $\xi = \frac{1}{\sigma}$

In this case,

$$U(A(c_t^T, c_t^N)) = \frac{ac_t^{1-\sigma} + (1-a)c_t^{N1-\sigma} - 1}{1-\sigma},$$

which is separable in $c_t^T$ and $c_t^N$. Then $d_t$ and $c_t^T$ solve

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t},$$

$$(c_t^T)^{-\sigma} = \beta(1 + r_t)E_t(c_{t+1}^T)^{-\sigma}$$

This subsystem is independent of $h_t$, $w_t$, and $c_t^N$. 
Approximating Equilibrium Dynamics

Optimal Exchange-Rate Policy: Equilibrium processes \( \{c_t^T, d_{t+1}\} \) solve the Bellman equation problem

\[
v^{OPT}(y_t^T, r_t, d_t) = \max_{\{d_{t+1}, c_t^T\}} \left\{ U(A(c_t^T, F(\bar{h}))) + \beta E_t v^{OPT}(y_{t+1}^T, r_{t+1}, d_{t+1}) \right\}
\]

subject to

\[c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t}; \quad \text{and} \quad d_{t+1} \leq \bar{d}.
\]

Approximate by value function iteration over a discretized state space \((y_t^T, r_t, d_t)\). Use 21 values for \(y_t^T\) and 11 for \(r_t\) (the estimated joint process \((y_t^T, r_t)\) is given below). Use 501 equally spaced points for \(d_t\) between 1 and 8.34.

Given approximated solutions to \(d_{t+1}\) and \(c_t^T\), all other variables of the model can be easily backed out:

\[h_t = \bar{h},\]

\[p_t = \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))}, \quad \text{and}\]

\[w_t = p_t F'(\bar{h}),\]
Approximating Equilibrium Dynamics (continued)

A Currency Peg, $\epsilon_t = 1$: Given $\xi = 1/\sigma$, the solution to $d_{t+1}$ and $c_t^T$ for the optimal exchange-rate policy applies to currency pegs.

The determination of $w_t$ requires knowledge of past real wages. So, $w_{t-1}$ is a state variable (along with $y_t^T$, $r_t$, and $d_t$).

But given $d_{t+1}$ and $c_t^T$, the solution for $w_t$ is static (simple): First, conjecture that $h_t = \bar{h}$, and obtain $w_t = \frac{A_2(c_t^T,F(\bar{h}))}{A_1(c_t^T,F(\bar{h}))} F'(\bar{h})$. If $w_t \geq \gamma w_{t-1}$, this is the solution. Otherwise, $w_t = \gamma w_{t-1}$, and $h_t$ solves $w_t = \frac{A_2(c_t^T,F(h_t))}{A_1(c_t^T,F(h_t))} F'(h_t)$.*

Discretization of $w_{t-1}$ grid: 500 points between 0.25 and 6, equally spaced in logs.

*When $\xi \neq 1/\sigma$ the solution is more complicated. See Schmitt-Grohé and Uribe (JPE, 2016).
The Driving Process:

Empirical Measure of $y_T^T$: sum of GDP in agriculture, manufacturing, fishing, forestry, and mining. Quadratically detrended.

Empirical Measure of $r_t$: Sum of Argentine EMBI+ plus 90-day Treasury-Bill rate minus a measure of U.S. expected inflation.
Estimate the following AR(1) system using Argentine data over the period 1983:Q1—2001:Q3:

\[
\begin{bmatrix}
\ln y^T_t \\
\ln \frac{1 + r_t}{1 + r}
\end{bmatrix} = A \begin{bmatrix}
\ln y^T_{t-1} \\
\ln \frac{1 + r_{t-1}}{1 + r}
\end{bmatrix} + \epsilon_t,
\]

Note: exclude the period 2001:Q4 to present, because of the default episode in 2002 (the present model assumes that debts are always honored).

**OLS Estimate of the Driving Process**

\[
A = \begin{bmatrix}
0.79 & -1.36 \\
-0.01 & 0.86
\end{bmatrix}; \quad \Sigma \epsilon = \begin{bmatrix}
0.00123 & -0.00008 \\
-0.00008 & 0.00004
\end{bmatrix};
\]

\[
r = 0.0316 \text{ (3.16% per quarter).}
\]
## Some Unconditional Summary Statistics

<table>
<thead>
<tr>
<th>Statistic</th>
<th>$y^T$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. Dev.</td>
<td>12%</td>
<td>6%yr</td>
</tr>
<tr>
<td>Serial Corr.</td>
<td>0.95</td>
<td>0.93</td>
</tr>
<tr>
<td>$\text{Corr}(y^T_t, r_t)$</td>
<td>-0.86</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1</td>
<td>12%yr</td>
</tr>
</tbody>
</table>

Comments: (1) High volatility of both $y^T_t$ and $r_t$; (2) negative correlation between $y^T_t$ and $r_t$ (when it rains it pours); (3) High mean country interest rate.
Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>0.99</td>
<td>Degree of downward nominal wage rigidity</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2</td>
<td>Inverse Intertemp. elast. of subst.</td>
</tr>
<tr>
<td>$y^T$</td>
<td>1</td>
<td>Steady-state tradable output</td>
</tr>
<tr>
<td>$\bar{h}$</td>
<td>1</td>
<td>Labor endowment</td>
</tr>
<tr>
<td>$a$</td>
<td>0.26</td>
<td>Share of tradables</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.5</td>
<td>Intratemp. elast. of subst.</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.75</td>
<td>Labor share in nontraded sector</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9635</td>
<td>Quarterly subjective discount factor</td>
</tr>
</tbody>
</table>

Note: $\sigma = 2$ is widely used in business cycle analysis, and $\xi = 0.5$ is within the range of values estimated for emerging countries (see the survey by Akinci, 2011). Consequently, the restriction $\xi = 1/\sigma$ is quite compelling on empirical and computational grounds.
Crisis Dynamics Under A Currency Peg and the Optimal Exchange-Rate Policy

We are interested in characterizing quantitatively the response of the model economy to a large contraction like the ones observed in Argentina in 2001 and in the periphery of Europe in 2008. In Argentina, for instance, traded output fell by 2 standard deviations in a period of two and a half years.

Definition of a Crisis: It’s a situation in which in period $t$ $y_{tT}$ is at or above average, and in period $t + 10$ it is at least two standard deviations below trend.

The Typical Response to a Crisis: Simulate the model for 20 million periods. Extract all windows of time in which $y_{tT}$ conforms to the definition of a crisis. For each variable of interest, average all windows and subtract its unconditional mean (i.e., the mean taken over the 20 million observations).
The Source of a Crisis

Tradable Output

Interest Rate Minus Mean

Note. Replication file plot_ir.m in usg_dnwr.zip.

Comments: (1) Because \( y_t^T \) and \( r_t \) are negatively correlated, the collapse in \( y_t^T \) coincides with a sharp increase in the country interest rate. (2) Recall that the response of \( y_t^T \) and \( r_t \) are exogenous to the model. The next slide displays the response of the endogenous variables.
An Optimal Exchange-Rate Policy

From the family of optimal exchange-rate policies, we pick

$$\epsilon_t = \frac{w_{t-1}}{\omega(c^T_t)}$$

Properties of this policy:

(1) It implies that the nominal wage rate, $W_t$, and the nominal price of nontradables, $P^N_t$, are constant at all times. Note: It fully stabilizes the (factor) price that suffers from nominal rigidity.

(2) It induces zero inflation and zero devaluation on average.
Pegs Amplify Negative External Shocks

\[ \text{Unemployment Rate} \quad \text{Real Wage (in terms of tradable)} \quad \text{Annualized Devaluation Rate} \quad \text{Relative Price of Nontradables (} \frac{P_t^N}{E_t} \text{)} \]

\[ \text{Annual CPI Inflation Rate} \quad \text{Consumption of Tradables} \quad \text{Trade-Balance-To-Output Ratio} \quad \text{Debt-To-Output Ratio (Annual)} \]

---

Currency Peg

Optimal Exchange-Rate Policy

Note. Replication file plot_ir.m in usg_dnwr.zip.
Observations

- Large contraction in $c^T$, driven primarily by the hike in the country interest rate. The fall in $c^T$ is so pronounced that the trade balance actually improves, in spite of the fact that $y^T$ falls sharply. The response of $c^T$ is independent of exchange-rate policy due to the restriction $\xi = 1/\sigma$.

- **Currency Pegs:** large increase in unemployment (25%), because the real wage does not fall sufficiently (stays 40% above the full-employment real wage). Firms don’t cut prices because labor cost remains high. As a result, consumers don’t switch spending away from tradables and toward nontradables.

- **Optimal Exchange-Rate Policy:** Full employment throughout the crisis (this result was established theoretically). Large devaluations of around 30% per year for 2.5 years. Consistent with devaluations post Convertibility in Argentina. Devaluations bring the real wage down, fostering employment and allowing the real exchange to depreciates. Real depreciation facilitates expenditure switch toward nontradables.
Devaluations and Revaluations in Reality

Under the assumed optimal exchange-rate policy, the large devaluations during the crisis must be followed by revaluations once the crisis is over, to guarantee zero inflation and zero devaluation on average (recall that these are two properties of the assumed optimal exchange-rate policy $\epsilon_t = w_{t-1}/\omega(c^T_t)$).

Do countries devalue during crises and revalue when the crisis is over? Look at the next graph. It displays the devaluation rate and the inflation rate for two sets of Latin American countries during the global crisis of 2008. One set is Argentina and the other includes Chile, Colombia, Mexico, Peru, and Uruguay.

During the crisis, all countries devalued significantly. However, during the recovery, all countries but Argentina revalued their currencies. The countries that revalued experienced lower inflation than Argentina.
Devaluation and Inflation In Latin America: 2006-2011

Devaluation Rate

Inflation Rate

Argentina

Others

NBER reference dates

% per year

% per year

2006 2007 2008 2009 2010 2011
The Welfare Costs of Currency Pegs

Find the compensation, measured as percent increase in the stream of consumption in the peg economy, denoted $\Lambda(s_t)$, that makes agents indifferent between living under a peg or under the optimal exchange-rate policy, given the current state $s_t = (y_t^T, r_t, d_t, w_{t-1})$. This compensation is implicitly given by

$$
E \left\{ \sum_{j=0}^{\infty} \beta^j U \left( c_{t+j}^{PEG} \frac{1 + \Lambda(s_t)}{100} \right) \mid s_t \right\} = E \left\{ \sum_{j=0}^{\infty} \beta^j U \left( c_{t+j}^{OPT} \right) \mid s_t \right\},
$$

where $s_t = \{y_t^T, r_t, d_t, w_{t-1}\}$.

Solve for $\Lambda(s_t)$ to obtain

$$
\Lambda(s_t) = 100 \left\{ \frac{v^{OPT}(y_t^T, r_t, d_t)(1 - \sigma) + (1 - \beta)^{-1}}{v^{PEG}(y_t^T, r_t, d_t, w_{t-1})(1 - \sigma) + (1 - \beta)^{-1}} \right\}^{1/(1-\sigma)} - 1.
$$

Note that $\Lambda(s_t)$ is a random variable as it is a function of the state in period $t$, $s_t$. When $\sigma = 1/\xi$ only state variable that is policy dependent is $w_{t-1}$ and $c_t^N$ is policy independent. Thus only source of welfare loss of suboptimal exchange rate policy stems from the dynamics of $c_t^N$. 

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The Welfare Costs of Currency Pegs

<table>
<thead>
<tr>
<th>Model</th>
<th>Welfare Cost</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Median</td>
</tr>
<tr>
<td>Baseline ($\gamma = 0.99$)</td>
<td>7.8</td>
<td>7.2</td>
</tr>
</tbody>
</table>

Note. The welfare cost of a currency peg is expressed in percent of consumption. Welfare costs are computed over the distribution of the state ($y_t^T, r_t, d_t, w_{t-1}$) induced by the peg economy. Replication files: simu_welf.m (welfare cost) and simu.m (unemployment) in usg_dnwr.zip.

Observation: Large welfare costs of currency pegs. All of the cost is explained by lost consumption of nontradables due to unemployment in that sector.
Observation: The distribution of welfare costs of pegs is highly skewed to the right, suggesting the existence of initial states, \((y_t^T, r_t, d_t, w_{t-1})\) in which pegs are highly costly in terms of unemployment. The next slide identifies such states.
Welfare Cost of Currency Pegs and the Initial State

Replication file plot_welf.m in usg_dnwr.zip.

**Observation:** Currency pegs are more costly the higher the initial past wage, the higher the initial stock of external debt, the lower the initial endowment of tradables, and the higher the initial country interest rate.
### Sensitivity Analysis I: Varying the Degree of Downward Wage Rigidity

<table>
<thead>
<tr>
<th>Model</th>
<th>Welfare Cost</th>
<th></th>
<th>Unempl. Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Median</td>
<td></td>
</tr>
<tr>
<td>Baseline ((\gamma = 0.99))</td>
<td>7.8</td>
<td>7.2</td>
<td>11.7</td>
</tr>
<tr>
<td>Lower Downward Wage Rigidity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\gamma = 0.98)</td>
<td>5.7</td>
<td>5.3</td>
<td>8.9</td>
</tr>
<tr>
<td>(\gamma = 0.97)</td>
<td>3.5</td>
<td>3.3</td>
<td>5.6</td>
</tr>
<tr>
<td>(\gamma = 0.96)</td>
<td>2.8</td>
<td>2.7</td>
<td>4.6</td>
</tr>
<tr>
<td>Higher Downward Wage Rigidity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\gamma = 0.995)</td>
<td>14.3</td>
<td>13.0</td>
<td>19.5</td>
</tr>
</tbody>
</table>

**Observation:** Sizable welfare costs and unemployment even for highly flexible wages, e.g., \(\gamma = 0.96\). Recall, \(\gamma = 0.96\) means that wages can fall frictionlessly by 16% per year.
Sensitivity Analysis II: Symmetric Wage Rigidity

Q: Is more wage flexibility always welfare increasing?

We have just seen that the welfare costs of a currency peg increase as $\gamma$, the parameter governing the degree of downward nominal wage rigidity, increases. Thus, that particular sensitivity analysis shows that more wage flexibility (in the form of a lower value for $\gamma$) is welfare increasing under a peg.

We now consider a case of less wage flexibility by assuming bi-directional, that is, upward and downward, wage rigidity:

$$\frac{1}{\gamma} \geq \frac{W_t}{W_{t-1}} \geq \gamma$$
The Welfare Costs of Pegs: Symmetric Wage Rigidity

\((\gamma = 0.99)\)

<table>
<thead>
<tr>
<th></th>
<th>Welfare Cost</th>
<th>Unempl.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Rate</td>
</tr>
<tr>
<td>Downward only: (\frac{W_t}{W_{t-1}} \geq \gamma)</td>
<td>7.8</td>
<td>11.7</td>
</tr>
<tr>
<td>Upward and downward: (\frac{1}{\gamma} \geq \frac{W_t}{W_{t-1}} \geq \gamma)</td>
<td>3.3</td>
<td>5.2</td>
</tr>
</tbody>
</table>

- Welfare costs under symmetric rigidity, while still large, are half that under downward wage rigidity. Thus greater wage flexibility is welfare decreasing. Why? Symmetric wage rigidity alleviates the peg-induced externality.

- To the extend that downward wage rigidity is case of greatest empirical relevance, this suggests that models with upward and downward wage rigidities underestimate the welfare costs of external shocks in peg economies.
Sensitivity Analysis III: Endogenous Labor Supply

How should this assumption affect the results?

- Negative income shocks, by increasing labor supply, may increase the level of involuntary unemployment.

- If involuntary and voluntary leisure perfect substitutes, then utility in period of unemployment higher than in model with inelastic labor supply. However, this effect may be small if voluntary and involuntary leisure are poor substitutes (which as we will show is the case of greatest empirical relevance).

- Thus, on the one hand, welfare costs of pegs may increase because involuntary leisure will rise, but on the other hand, welfare costs may decrease if the additional forced leisure due to unemployment provides a lot of utility.

Let’s introduce this change in the model ...
Assume that period utility is increasing in consumption, $c_t$, and leisure, $\ell_t$,

$$U(c_t, \ell_t) = \frac{c_t^{1-\sigma} - 1}{1 - \sigma} + \frac{\ell_t^{1-\theta} - 1}{1 - \theta}$$

New first-order condition: The desired demand for leisure

$$\varphi(\ell^v_t)^{-\theta} = w_t \lambda_t,$$

where $\ell^v_t$ denotes voluntary leisure. Let $\bar{h}$ denote the total endowment of hours per period. The desired (or voluntary) labor supply is given by

$$h^v_t = \bar{h} - \ell^v_t.$$

Let $h_t$ denote actual hours worked.
Impose

\[ h_t^v \geq h_t, \]

\[ (h_t^v - h_t) \left( w_t - \gamma \frac{w_{t-1}}{\epsilon_t} \right) = 0. \]

The above four equations replace the conditions \( h_t \leq \bar{h} \) and \( (\bar{h} - h_t)(w_t - \gamma w_{t-1}/\epsilon_t) = 0 \) of the baseline economy with inelastic labor supply. The rest of the equilibrium conditions are unchanged.

Involuntary unemployment, or, synonymously, involuntary leisure, denoted \( u_t \), is given by

\[ u_t = h_t^v - h_t \]
Policy evaluation requires addressing an important question:

**How should voluntary and involuntary leisure enter in the period utility function?**

One possibility is to assume that $\ell_t^v$ and $u_t$ are perfect substitutes. In this case, the second argument of the utility function becomes $\ell_t = \ell_t^v + u_t$. 
The existing empirical literature, however, strongly rejects this assumption:

- Krueger and Mueller (2012): the unemployed enjoy leisure activities to a lesser degree than the employed and on a typical day report higher levels of sadness than the employed.

- Winkelmann and Winkelmann (1998): Unemployment has a large non-pecuniary detrimental effect on life satisfaction.

- Krueger and Mueller (2012): Unemployed spend 101 minutes more per day on job search than employed (not surprising). However, job search generates the highest feeling of sadness after personal care out of 13 time-use categories.

⇒ Better specification: $l_t = l_t^v + \delta u_t$

with $\delta < 1$. Will consider three values, 1, 0.75, and 0.5.
## Endogenous Labor Supply And The Welfare Costs of Currency Pegs

<table>
<thead>
<tr>
<th>Parameterization</th>
<th>Welfare Cost Mean</th>
<th>Welfare Cost Median</th>
<th>Unemployment Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline (inelastic labor supply)</td>
<td>7.8</td>
<td>7.2</td>
<td>11.7</td>
</tr>
<tr>
<td>Endogenous Labor Supply $\ell_t = \ell_t^v + \delta u_t$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta = 0.5$</td>
<td>16.5</td>
<td>15.2</td>
<td>30.9</td>
</tr>
<tr>
<td>$\delta = 0.75$</td>
<td>8.2</td>
<td>7.5</td>
<td>30.9</td>
</tr>
<tr>
<td>$\delta = 1$</td>
<td>1.7</td>
<td>1.5</td>
<td>30.9</td>
</tr>
</tbody>
</table>

**Observations:** (1) Unemployment larger under endogenous labor supply specification. (2) Welfare cost of peg larger or smaller depending on preferences about involuntary leisure, $\delta$.  

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Sensitivity Analysis IV: Product Price Rigidity

Assume now that nominal wages are fully flexible and that instead prices are sticky. Consider downward nominal price rigidity and symmetric price rigidity.

\[
\text{downward price rigidity: } \frac{P_t^N}{P_{t-1}^N} \geq \gamma_p
\]

\[
\text{symmetric price rigidity: } \frac{1}{\gamma_p} \geq \frac{P_t^N}{P_{t-1}^N} \geq \gamma_p
\]

Calibrate models as before, but set \( \gamma = 0 \) and \( \gamma_p = 0.99 \).
Adjustment to a Negative External Shock with Downward Price Rigidity Under A Currency Peg

\[
\begin{align*}
\frac{P^N}{e} &= \frac{A_2(c_0^T, F(h))}{A_1(c_0^T, F(h))} \\
\frac{P^N}{e} &= \frac{A_2(c_0^T, F(h))}{A_1(c_0^T, F(h))}
\end{align*}
\]

\(c_1^T < c_0^T; \quad \gamma_p = 1\)
Observations: When $c^T$ falls from $c^T_0$ to $c^T_1$, full employment would occur if prices could decline to $\rho(c^T_1)$, point C in the figure. But because of downward nominal price rigidity, $P^N_t/\varepsilon_t$ remains at $\rho(c^T_0)$. At this price households only demand $F(h^{bust})$ units of nontradables and the economy suffers of unemployment due to weak demand.

Optimal policy calls for a devaluation that lowers $P^N_t/\varepsilon_t$ down to $\rho(c^T_1)$ and restores full employment. Hence contractions continue to be devaluatory!

The economy continues to suffer from a peg induced externality. Increases in $P^N_t$ during booms should be limited to avoid unemployment during the recession phase of the cycle.
## Price Rigidity And The Welfare Costs of Currency Pegs

<table>
<thead>
<tr>
<th>Parameterization</th>
<th>Welf Cost</th>
<th>Unempl Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline (wage rigidity, $\gamma = 0.99$ and $\gamma_p = 0.$)</td>
<td>7.8</td>
<td>11.7</td>
</tr>
<tr>
<td>Nominal Price Rigidity ($\gamma = 0$, $\gamma_p = 0.99$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Downward Price Rigidity, $P_t^N / P_{t-1}^N \geq \gamma_p$</td>
<td>9.9</td>
<td>14.1</td>
</tr>
<tr>
<td>Symmetric Price Rigidity, $1/\gamma_p \geq P_t^N / P_{t-1}^N \geq \gamma_p$</td>
<td>4.4</td>
<td>6.6</td>
</tr>
<tr>
<td>Calvo Price Rigidity, $\theta = 0.7$</td>
<td>3.6</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Note. The welfare cost of a currency peg is expressed in percent of consumption per quarter. Unemployment rates are expressed in percent.
Observations. The welfare costs of pegs under downward price rigidity are as large as under downward wage rigidity. The welfare costs under symmetric price rigidity are only about half as large as under downward price rigidity. It follows that adding upward price rigidity ameliorates the peg induced externality, just like adding upward wage rigidity ameliorates it in the model model with wage rigidity.

The last line, labeled Calvo pricing, pertains to an economy with Calvo-type price rigidity in the nontraded sector. The probability of not being able to change the nominal price is $\theta = 0.7$ per period. The calibration of the shocks is the same as in the baseline model. The Calvo model is one of symmetric price rigidity and so it is not surprising that the welfare costs of pegs are most similar to those associated with the economy with the bi-directional price rigidity studied earlier.
Summary

● **Theoretical results:**

— The combination of a currency peg and downward nominal wage rigidity creates an externality that amplifies the severity of contractions.
— The model predicts that the average rate of unemployment is increasing in the degree of aggregate volatility.
— The model can account for boom bust dynamics.

● **Quantitative Results:** The costs of currency pegs due to downward nominal wage rigidity are large,

— in terms of welfare, 4 to 10% of consumption per period
— and in terms of unemployment, 10 to 30%.

— Results appear to be robust to a variety of changes parameter values and model specifications.
Section 9.12 Mussa 1986
Mussa (1986) is an empirical paper that provides evidence against the “nominal exchange rate regime neutrality” hypothesis.


Observables:
\[ E_t = \text{nominal exchange rate vis-à-vis the U.S. dollar} \]
\[ P^*_t = \text{U.S. Consumer Price Index} \]
\[ P_t = \text{Domestic Consumer Price Index} \]
\[ RER_t = \frac{E_t P^*_t}{P_t} \]

Take a look at the next figure. It shows the natural logarithms of the dollar French franc nominal and real exchange rates as well as of the relative CPIs.
Source: This is figure 1 of Mussa 1986. Solid line: nominal exchange rate, $\varepsilon_t$; plus-line: ratio of CPIs, $P_t^*/P_t$; diamond line: real exchange rate, $RER_t = \varepsilon_t P_t^*/P_t$; variables are shown in natural logarithms.
Comments on the figure:


Selection of sample period: Why a gap in the sample period? Mussa argues that 1970Q4 to 1973Q1 is a transition period. The float starts in March 1973. Why does Mussa start his sample in 1957, this was afterall not the beginning of the Bretton Woods agreement? Mussa justifies the start date by saying that this is when the IFS tapes start having the raw data. Why does Mussa end the sample 1983Q3? This must be the most recent observation when he wrote the paper.
• The graph shows that during the Bretton Woods period (1957-1970) the nominal exchange rate was basically constant. In the post-Bretton Woods period (1973-1984) it fluctuates quite a bit. This confirms treating these two periods as having a different nominal exchange rate regime.

• The striking feature of the graph is that the real exchange rate mimics the behavior of the nominal exchange rate throughout.

• The flipside of this observation is that the times series properties of relative prices, $P_t^*/P_t$, has remained remarkable constant across the two periods.

• Mussa argues that under the hypothesis nominal exchange rate regime neutrality one should not observe such a high correlation between nominal and real exchange rates.
Mussa documents three facts:
1.) The standard deviation of the real depreciation rate is smaller during the peg era.
2.) The standard deviations and serial correlations of the nominal and real depreciation rates are similar to each other in each exchange rate regime.
3.) The volatility of CPI inflation is about the same during the fixed and the floating regime sample period.

Let

$$\epsilon^{RER}_t \equiv \ln RER_t - \ln RER_{t-1}$$
$$\epsilon_t \equiv \ln \mathcal{E}_t - \ln \mathcal{E}_{t-1}$$
$$\epsilon^p_t \equiv \ln P^*_t / P_t - \ln P^*_{t-1} / P_{t-1}$$

Illustrate these 3 facts for France

<table>
<thead>
<tr>
<th></th>
<th>1957Q2-1970Q4</th>
<th>1973Q1-1984Q3</th>
</tr>
</thead>
<tbody>
<tr>
<td>var($\epsilon^{RER}_t$)</td>
<td>5.258</td>
<td>23.590</td>
</tr>
<tr>
<td>var($\epsilon_t$)</td>
<td>8.197</td>
<td>24.275</td>
</tr>
<tr>
<td>corr($\epsilon^{RER}<em>t$, $\epsilon^{RER}</em>{t-1}$)</td>
<td>0.1150</td>
<td>0.3376</td>
</tr>
<tr>
<td>corr($\epsilon_t$, $\epsilon_{t-1}$)</td>
<td>0.1776</td>
<td>0.4317</td>
</tr>
<tr>
<td>var($\epsilon^p_t$)</td>
<td>1.543</td>
<td>0.540</td>
</tr>
</tbody>
</table>

Taken from Table 1.4 of Mussa (1986)
Empirical Evidence on Real Effects of Monetary Instability

1. Industrialized countries, Mussa 1986, method, compare fixed and float.


3. small open economies, to be added, method, VAR identification.
Real Effects of Monetary Instability: Empirical Evidence from VARs

The dynamic effects of a shock to monetary policy •

\[ R_t = f(\Omega_t) + \epsilon_t \]

\( R_t \) the federal funds rate

\( f(.) \) is a linear function

\( \epsilon_t \) is a monetary policy shock

• Identification assumption: \( \epsilon_t \perp \Omega_t \)

\[ Y_t = \begin{bmatrix} Y_{1t} \\ R_t \\ Y_{2t} \end{bmatrix} \]
$Y_{1t}$ is part of $\Omega_t$, which does not respond contemporaneously to monetary policy shocks

\[
Y_{1t6\times1} = \begin{bmatrix}
\log \text{ of real } GDP_t \\
\log \text{ of real } C_t \\
\log \text{ of GDP deflator} \\
\log \text{ of real investment} \\
\log \text{ of real wage} \\
\log \text{ of labor productivity}
\end{bmatrix}
\]

\[
Y_{2t2\times1} = \begin{bmatrix}
\log \text{ of real profits} \\
\text{growth rate of M2}
\end{bmatrix}
\]

\[
\Omega_t = Y_{1t}, \ Y_{1t-1}, \ Y_{1t-2}, \ Y_{1t-3}, \ Y_{1t-4} \\
R_{t-1}, \ R_{t-2}, \ R_{t-3}, \ R_{t-4} \\
Y_{2t-1}, \ Y_{1t-2}, \ Y_{2t-3}, \ Y_{2t-4}
\]
• Sample: 1965:3-1995:3

• Ignoring a constant the VAR is

\[ Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + A_3 Y_{t-3} + A_4 Y_{t-4} + C \eta_t \]

Let \( C \eta_t = u_t \)

• \( C \) is 9 by 9 and lower triangular (zeros above the diagonal)
• \( C \) has ones on the diagonal
• \( \eta_{t9 \times 1} \sim \text{mean 0, serially uncorrelated, diagonal covariance matrix} \)
• Element 7 of \( \eta_t \) is \( \epsilon_t \), the monetary policy shock.
• \( \epsilon_t > 0 \) means a contractionary policy shock
• Estimate \( A_i \) for \( i = 1, 2, 3, 4 \)
• Estimate \( C \), how? \( \text{var}(u_t) = C \Sigma \eta C' \)
• Shaded area is the 95 percent confidence interval computed using the methodology of Sims and Zha
These plots are taken from Figure 1 of Christiano, Eichenbaum, Evans, JPE 2005.
Real Effects of an expansionary monetary policy shock from CEE

1.) Y, C, I

- Y, C, I respond in a hump-shaped fashion
- Y peaks 4-6 quarters after the shock
- C peaks 4 quarters after the monetary policy shock
- I peaks 5 quarters after the monetary policy shock
- The impulse response is significant until $t = 8$.

2.) R

- increases by 60 basis points
- peak response on impact
- is over after 4-5 quarters [long before peak output]

While there is quite some debate about what are the correct identification assumptions, there is considerable agreement about the qualitative effects of a monetary policy shock in the sense that inference is robust across a large subset of the identification schemes that have been considered in the literature.
Section 9.16: Staggered Price Setting: The Calvo Model
The Calvo model:

— is perhaps *the* canonical models used in monetary economics

— assumes staggered price setting

— proposed by Calvo (1983) and later refined by Woodford (1996) and Yun (1996)

— price rigidity is bidirectional, that is, the upward and downward adjustment of nominal prices is sluggish
Differences to the models of nominal rigidities studied earlier in the chapter:

(1) firms are forced to satisfy demand even if the price is below marginal cost. An implication of this difference is that in the Calvo model the labor supply must be wage elastic for price stickiness to have first-order effects.

(2) Calvo model assumes imperfect competition in product markets. This assumption allows firms be price setters and to have nonzero finite demand even when their prices differ from those of their closest competitors.

(3) wages are flexible and there is no involuntary unemployment.
Households

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t [U(c_t) - V(h_t)] \tag{9.41}
\]

subject to

\[
c_t = A(c_t^T, c_t^N) \tag{9.42}
\]

\[
P_t^T c_t^T + P_t^N c_t^N + \varepsilon_t d_t = P_t^T y_t^T + W_t h_t + \Phi_t + T_t + \frac{\varepsilon_t d_{t+1}}{1 + r_t} \tag{9.43}
\]

\[
d_{t+1} \leq \bar{d} \tag{9.44}
\]
Assume law of one price holds for tradables

\[ P_t^T = \varepsilon_t \]

Optimality conditions of the household’s problem are: (9.42)-(9.44)

\[ \frac{A_2(c_t^T, c_t^N)}{A_1(c_t^T, c_t^N)} = p_t \]

\[ \lambda_t = U'(c_t)A_1(c_t^T, c_t^N) \]

\[ \lambda_t \frac{1}{1 + r_t} = \beta E_t \lambda_{t+1} + \mu_t \]

\[ \mu_t \geq 0 \]

\[ \mu_t(d_{t+1} - \bar{d}) = 0 \]

\[ V'(h_t) = \lambda_t \frac{W_t}{P_t^T} \]
9.16.2 Firms Producing Final Nontraded Goods

Production technology:

$$y^N_t = \left[ \int_0^1 (a^N_{it})^{1-\frac{1}{\mu}} di \right]^{\frac{1}{1-\frac{1}{\mu}}}$$  \hspace{1cm} (9.45)

$y^N_t$ = output of the final nontraded good
$a^N_{it}$ = quantity of intermediate goods of type $i \in [0, 1]$ used in the production of the final nontraded good
$\mu > 1$ elasticity of substitution across varieties

Environment: perfect competition
Firm profits:

\[ P_t^N y_t^N - \int_0^1 P_{it}^N a_{it}^N di, \]

Profit maximization implies intermediate input demand of the form

\[ a_{it}^N = y_t^N \left( \frac{P_{it}^N}{P_t^N} \right)^{-\mu} \]  

(9.46)

- demand for the intermediate good of variety \( i \) is increasing in the level of final output and decreasing in the relative price of the variety in terms of the final good, with a price elasticity of \(-\mu\).

Using this expression to eliminate \( a_{it}^N \) from the Dixit-Stiglitz aggregator (9.45) yields

\[ P_t^N = \left[ \int_0^1 (P_{it}^N)^{1-\mu} di \right]^{\frac{1}{1-\mu}} \]  

(9.47)

- price of the final nontraded good is increasing and homogeneous of degree one in the price of the intermediate nontraded goods
9.16.2 Firms Producing Nontraded Intermediate Goods

Production technology for variety $i$

\[ y_{it}^N = h_{it}^\alpha; \quad \alpha \in (0, 1] \]  \hspace{1cm} (9.48)

Environment: Monopolistic competition, firms are price setters.

Production is demand determined. This means that, given the posted price $P_{it}^N$, firms must set production to ensure that all customers are served, that is,

\[ y_{it}^N = a_{it}^N \]  \hspace{1cm} (9.49)
Profits

\[ \Phi_{it} = P^N_{it} a^N_{it} - (1 - \tau)W_th_{it} \]

\( \tau \) = a labor subsidy to offset distortions introduced by imperfect competition. Its presence facilitates the characterization of optimal monetary policy, as it results in a model with a single distortion, namely the one stemming from price rigidity. That is, monetary policy here will not be called up to correct distortions stemming from imperfect competition.
Price setting problem of producer of variety $i$

Use (9.46), (9.48), and (9.49) to eliminate $a^N_{it}$, $h_{it}$, and $y^N_{it}$, respectively, from the expression for profits in period $t$:

$$P^N_{it} y^N_{it} \left( \frac{P^N_{it}}{P^N_t} \right)^{-\mu} - (1 - \tau) W_t y^N_{it} \frac{1}{\alpha} \left( \frac{P^N_{it}}{P^N_t} \right)^{-\frac{\mu}{\alpha}}.$$

Under price flexibility, the optimal pricing decision consists in choosing $P^N_{it}$ to maximize the above expression.

Under price stickiness, the pricing problem is different because firms, by assumption, cannot reoptimize prices every period.

With probability $\theta \in (0, 1)$ a firm cannot reset its price in period $t$ and must charge the same price as in the previous period, and with probability $1 - \theta$ it can adjust the price freely. Consider the pricing decision of a firm that can reoptimize its price in period $t$. 
Let $\tilde{P}_{it}^N$ denote the price chosen in $t$. Then, with probability $\theta$, the price will continue to be $\tilde{P}_{it}^N$ in period $t + 1$. With probability $\theta^2$, the price will continue to be $\tilde{P}_{it}^N$ in period $t + 2$, and so on. In general, with probability $\theta^s$ the price will continue to be $\tilde{P}_{it}^N$ in period $t + s$.

The original Calvo (1983) formulation assumes that $\tilde{P}_{it}^N$ is set following an ad hoc rule of thumb. The innovation introduced by Yun (1996) is to assume that the firm picks $\tilde{P}_{it}^N$ in a profit-maximizing fashion.

As in much of the modern new-Keynesian literature (e.g., Woodford, 2003), we adopt Yun’s approach. Specifically, the present discounted value of profits associated with $\tilde{P}_{it}^N$ is given by

$$E_t \sum_{s=0}^{\infty} Q_{t,t+s} \theta^s \left[ \tilde{P}_{it}^N y_{t+s} \left( \frac{\tilde{P}_{it}^N}{P_{t+s}^N} \right)^{-\mu} - (1 - \tau)W_{t+s} y_{t+s} \frac{1}{\alpha} \left( \frac{\tilde{P}_{it}^N}{P_{t+s}^N} \right)^{-\frac{\mu}{\alpha}} \right],$$

$Q_{t,t+s} = \text{state-contingent nominal discount factor that converts nominal payments in period } t + s \text{ into a nominal payment in period } t.$
Chapter 9: Nominal Rigidity, Exchange Rates, And Unemployment

The firm picks $\tilde{P}_{it}^N$ to maximize the pdv of profits. The FOC is:

$$E_t \sum_{s=0}^{\infty} Q_{t,s} \theta^s y_{t+s}^N \left( \frac{\tilde{P}_{it}^N}{P_t^N} \right)^{-\mu} \left\{ \frac{\mu - 1}{\mu} \tilde{P}_{it}^N - \frac{1}{\alpha}(1 - \tau)W_{t+s} \left[ y_{t+s}^N \left( \frac{\tilde{P}_{it}^N}{P_t^N} \right)^{-\mu} \right]^{\frac{1-\alpha}{\alpha}} \right\} = 0.$$  \hspace{1cm} (9.50)

$\frac{\mu - 1}{\mu} \tilde{P}_{it}^N$ = marginal revenue in period $t + s$

$\frac{1}{\alpha}(1 - \tau)W_{t+s} \left[ y_{t+s}^N \left( \frac{\tilde{P}_{it}^N}{P_t^N} \right)^{-\mu} \right]^{\frac{1-\alpha}{\alpha}} = \frac{(1 - \tau)W_{t+s}}{\alpha(h_{it+s}^N)^{\alpha-1}}$ = marginal cost in period $t + s$.

$y_{t+s}^N \left( \frac{\tilde{P}_{it}^N}{P_t^N} \right)^{-\mu}$ = quantity sold in period $t + s$

• price set in period $t$ equates the present discounted values of marginal revenues and marginal costs weighted by the level of production.
Crisis Dynamics in the Calvo Model: The Role Of Exchange-Rate Policy

Note. Replication file typical_crisis.m in calvo.zip available online with the materials for this chapter.
To be continued ...