Appendix To “Real Business Cycles in Emerging Countries?” *

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1 Optimality Conditions of the Household’s Problem

Letting $\lambda_t X_{t-1}^{-\gamma}$ denote the Lagrange multiplier associated with the sequential budget constraint, the optimality conditions associated with this problem are (??), (??), the no-Ponzi-game constraint holding with equality, and

$$[C_t/X_{t-1} - \theta \omega^{-1} h_t^\omega]^{-\gamma} = \lambda_t$$

$$[C_t/X_{t-1} - \theta \omega^{-1} h_t^\omega]^{-\gamma} \theta h_t^{\omega-1} = (1 - \alpha) a_t \left( K_t / X_{t-1} h_t \right)^{\alpha} \left( X_t / X_{t-1} \right)^{1-\alpha} \lambda_t$$

$$\lambda_t = \beta \frac{1 + r_t}{g_t^\gamma} E_t \lambda_{t+1}$$

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\[ 1 + \phi \left( \frac{K_{t+1}}{K_t} - g \right) \lambda_t = \frac{\beta}{g_t} E_t \lambda_{t+1} \left[ 1 - \delta + \alpha a_{t+1} \left( \frac{X_{t+1} h_{t+1}}{K_{t+1}} \right)^{1-\alpha} 
+ \phi \left( \frac{K_{t+2}}{K_{t+1}} \right) \left( \frac{K_{t+2}}{K_{t+1}} - g \right) - \frac{\phi}{2} \left( \frac{K_{t+2}}{K_{t+1}} - g \right)^2 \right] \]

2 Equilibrium Conditions in Stationary Form

Define \( y_t = Y_t / X_{t-1} \), \( c_t = C_t / X_{t-1} \), \( d_t = D_t / X_{t-1} \), and \( k_t = K_t / X_{t-1} \). Then, a stationary competitive equilibrium is give by a set of processes stationary solution to the following equations:

\[ [c_t - \theta \omega^{-1} h_t^\omega]^{-\gamma} = \lambda_t \]
\[ \theta h_t^\omega = (1 - \alpha) a_t g_t^{1-\alpha} \left( \frac{k_t}{h_t} \right)^\alpha \]
\[ \lambda_t = \frac{\beta}{g_t} \left[ 1 + r^* + \psi \left( e^{d_t - \bar{d}} - 1 \right) \right] E_t \lambda_{t+1} \]

\[ [1 + \phi \left( \frac{k_{t+1}}{k_t} g_t - g \right)] \lambda_t = \frac{\beta}{g_t} E_t \lambda_{t+1} \left[ 1 - \delta + \alpha a_{t+1} \left( \frac{g_{t+1} h_{t+1}}{k_{t+1}} \right)^{1-\alpha} 
+ \phi \frac{k_{t+2}}{k_{t+1}} g_{t+1} \left( \frac{k_{t+2}}{k_{t+1}} g_{t+1} - g \right) - \frac{\phi}{2} \left( \frac{k_{t+2}}{k_{t+1}} g_{t+1} - g \right)^2 \right] \]

\[ \frac{d_{t+1}}{1 + r_t} g_t = d_t - y_t + c_t + i_t + \phi \left( \frac{k_{t+1}}{k_t} g_t - g \right)^2 k_t, \]
\[ r_t = r^* + \psi \left( e^{d_t - \bar{d}} - 1 \right), \]
\[ k_{t+1} g_t = (1 - \delta) k_t + i_t \]
\[ y_t = a_t k_t^\alpha (g_t h_t)^{1-\alpha} \]

3 GMM Estimation Procedure

Let \( b \equiv [g \sigma_g \sigma_p \sigma_a \rho_a] \)' be the 6x1 vector of structural parameters to be estimated. We write the moment conditions as:\footnote{The estimation results are little changed if in writing the moment conditions we replace the empirical moments \( \bar{g}' Y, \bar{g}' C, \) and \( \bar{g}' I \) by their theoretical counterpart \( E_{gy}(b) \), and the empirical moment \( \bar{tb}_y \) by its theoretical counterpart \( E_{tby}(b) \). Specifically, the parameter estimates using annual Mexican data from 1900 to 2005 are}

\[ \frac{d_{t+1}}{1 + r_t} g_t = d_t - Y_t + C_t + I_t + \phi \left( \frac{k_{t+1}}{k_t} g_t - g \right)^2 k_t, \]
\[ r_t = r^* + \psi \left( e^{d_t - \bar{d}} - 1 \right), \]
\[ k_{t+1} g_t = (1 - \delta) k_t + I_t \]
\[ y_t = a_t k_t^\alpha (g_t h_t)^{1-\alpha} \]
\[
\begin{pmatrix}
E_{yy}(b) - g_t \bar{Y} \\
\sigma_{yy}(b) - (g_t \bar{Y} - \bar{g} \bar{Y})^2 \\
\sigma_{ge}(b) - (g_t \bar{C} - \bar{g} \bar{C})^2 \\
\sigma_{gI}(b) - (g_t I - \bar{g} I)^2 \\
\sigma_{by}(b) - (tby_t - \bar{tby})^2 \\
\rho_{gy,gc} - \frac{(g_t \bar{Y} - \bar{g} \bar{Y})(g_t \bar{C} - \bar{g} \bar{C})}{\sigma_{gy}(b)\sigma_{gc}(b)} \\
\rho_{gy,gi} - \frac{(g_t \bar{Y} - \bar{g} \bar{Y})(g_t I - \bar{g} I)}{\sigma_{gy}(b)\sigma_{gi}(b)} \\
\rho_{gy,by} - \frac{(g_t \bar{Y} - \bar{g} \bar{Y})(tby_t - \bar{tby})}{\sigma_{gy}(b)\sigma_{by}(b)} \\
\rho_{gy1}(b) - \frac{(g_t \bar{Y} - \bar{g} \bar{Y})(g_t Y_{t-1} - \bar{g} Y_{t-1})}{\sigma_{gy}(b)\sigma_{y1}(b)} \\
\rho_{gy2}(b) - \frac{(g_t \bar{Y} - \bar{g} \bar{Y})(g_t Y_{t-2} - \bar{g} Y_{t-2})}{\sigma_{gy}(b)\sigma_{y2}(b)} \\
\rho_{gy3}(b) - \frac{(g_t \bar{Y} - \bar{g} \bar{Y})(g_t Y_{t-3} - \bar{g} Y_{t-3})}{\sigma_{gy}(b)\sigma_{y3}(b)} \\
\rho_{gi1}(b) - \frac{(g_t I - \bar{g} I)(g_t Y_{t-1} - \bar{g} Y_{t-1})}{\sigma_{gi}(b)\sigma_{y1}(b)} \\
\rho_{gi2}(b) - \frac{(g_t I - \bar{g} I)(g_t Y_{t-2} - \bar{g} Y_{t-2})}{\sigma_{gi}(b)\sigma_{y2}(b)} \\
\rho_{gi3}(b) - \frac{(g_t I - \bar{g} I)(g_t Y_{t-3} - \bar{g} Y_{t-3})}{\sigma_{gi}(b)\sigma_{y3}(b)} \\
\rho_{by1}(b) - \frac{(tby_t - \bar{tby})(tby_{t-1} - \bar{tby})}{\sigma_{by}(b)\sigma_{y1}(b)} \\
\rho_{by2}(b) - \frac{(tby_t - \bar{tby})(tby_{t-2} - \bar{tby})}{\sigma_{by}(b)\sigma_{y2}(b)} \\
\rho_{by3}(b) - \frac{(tby_t - \bar{tby})(tby_{t-3} - \bar{tby})}{\sigma_{by}(b)\sigma_{y3}(b)} \\
\end{pmatrix},
\]

where \(Ex(b)\) denotes the expected value of the variable \(x_t\) implied by the theoretical model, \(\sigma_x(b)\) denotes the standard deviation of \(x_t\) implied by the theoretical model, \(\rho_{xy}(b)\) denotes the correlation between \(x_t\) and \(y_t\) implied by the theoretical model, and \(\rho_{xj}\) denotes the autocorrelation of order \(j\) of \(x_t\) implied by the theoretical model. All of these statistics are functions of the vector \(b\) of structural parameters. We denote by \(\bar{x} \equiv T^{-1} \sum_{t=1}^{T} x_t\) the sample mean of \(x_t\), where \(T\) is the sample size. We compute moments implied by the theoretical model by solving a linearized version of the system of equilibrium conditions with respect to the logarithm of all variables except the trade-balance share in GDP, which we keep in levels.

Define \(J(b, W) = \bar{u}'W\bar{u}\), where \(\bar{u}(b)\) denotes the sample mean of \(u_t(b)\) and \(W\) is a sym-
Table 1: Mexico 1980:Q1-2003:Q3: Estimated Structural Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Point Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g$</td>
<td>1.000</td>
<td>0.001</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>0.008</td>
<td>0.001</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>0.715</td>
<td>0.038</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>0.006</td>
<td>0.001</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.508</td>
<td>0.147</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1.150</td>
<td>0.128</td>
</tr>
</tbody>
</table>

Overidentifying Restrictions Test

<table>
<thead>
<tr>
<th>Test</th>
<th>p value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$ value</td>
<td>0.189</td>
</tr>
</tbody>
</table>

metric positive definite matrix compatible with $\bar{u}(b)$. The GMM estimate of $b$, denoted $\hat{b}$, is given by

$$\hat{b} = \arg\min_b J(b,W).$$

The matrix $W$ is estimated in two steps. For more details see Burnside (1999).2

4 GMM Estimation: Mexico 1980:Q1 2003:Q2

The estimation of the RBC model using quarterly Mexican data from 1980:1 to 2003:2 is shown in table Table 1. The fit of the model, as measured by the $p$ value of the test of overidentifying restrictions is much better than the one obtained using the long sample 1900-2005. This is reflected in a better matching of the second moments of interest, as shown in table 2 and figure 1.

Table 2: Mexico 1980:Q1-2003:Q2

<table>
<thead>
<tr>
<th>Statistic</th>
<th>( g^Y )</th>
<th>( g^C )</th>
<th>( g^I )</th>
<th>tby</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standard Deviation</strong></td>
<td>1.6</td>
<td>1.6</td>
<td>7.4</td>
<td>4.0</td>
</tr>
<tr>
<td>—Model</td>
<td>1.5</td>
<td>1.9</td>
<td>5.7</td>
<td>3.7</td>
</tr>
<tr>
<td>—Data</td>
<td>(0.3)</td>
<td>(0.2)</td>
<td>(1.0)</td>
<td>(0.4)</td>
</tr>
<tr>
<td><strong>Correlation with ( g^Y )</strong></td>
<td>0.91</td>
<td>0.65</td>
<td>-0.41</td>
<td></td>
</tr>
<tr>
<td>—Model</td>
<td>0.76</td>
<td>0.75</td>
<td>-0.19</td>
<td></td>
</tr>
<tr>
<td>—Data</td>
<td>(0.07)</td>
<td>(0.07)</td>
<td>(0.09)</td>
<td></td>
</tr>
<tr>
<td><strong>Correlation with tby</strong></td>
<td>-0.45</td>
<td>-0.32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>—Model</td>
<td>-0.23</td>
<td>-0.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>—Data</td>
<td>0.07</td>
<td>0.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Serial Correlation</strong></td>
<td>0.07</td>
<td>0.08</td>
<td>-0.06</td>
<td>0.89</td>
</tr>
<tr>
<td>—Model</td>
<td>0.25</td>
<td>0.19</td>
<td>0.44</td>
<td>0.95</td>
</tr>
<tr>
<td>—Data</td>
<td>(0.10)</td>
<td>(0.14)</td>
<td>(0.10)</td>
<td>(0.03)</td>
</tr>
</tbody>
</table>

Note: Standard deviations are reported in percentage points. Standard errors of sample-moment estimates are shown in parenthesis.
Figure 1: Mexico 1980:Q1-2003:Q2: The Autocorrelation Function of the Trade Balance-to-Output Ratio