Variance of Innovations

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This note describes the computations to obtain the variance-covariance matrix of the innovations in

\[
\begin{align*}
\rho_t & \equiv E_t \left\{ \hat{G}_{t+3} - \hat{G}_{t+2} \right\} \\
\bar{Y}_t & \equiv E_t \hat{Y}_{t+2}^S
\end{align*}
\]

in the Rotemberg-Woodford (1997, 1999) model. The matlab program that does the computations is \texttt{stat14.m}. It requires running \texttt{par13.m} and \texttt{var13.m} in advance.

In RW (1997), the evolution of \( \hat{G}_{t+1}, \hat{Y}_{t+1}^S \) is given by

\[
\begin{bmatrix}
\hat{G}_{t+1} \\
\hat{Y}_{t+1}^S
\end{bmatrix}
= C \bar{Z}_{t-1} + D \bar{e}_t
\]

\[
\bar{Z}_t = B \bar{Z}_{t-1} + U \bar{e}_t
\]

where \( E_t \bar{e}_{t+j} = 0 \) for all \( j > 0 \). Let \( \Omega \) be the VCV matrix of \( \bar{e}_t \) (no real shock \( e_1 \), no monetary shock; in program: VCVe2). Let \( c_1 \) and \( c_2 \) be respectively the first and second rows of \( C \), and let \( d_1 \) and \( d_2 \) be respectively the first and second rows of \( D \). We have

\[
\begin{align*}
\hat{G}_{t+1} & = c_1 \bar{Z}_{t-1} + d_1 \bar{e}_t \\
\hat{Y}_{t+1}^S & = c_2 \bar{Z}_{t-1} + d_2 \bar{e}_t.
\end{align*}
\]

Using this notation,

\[
\rho_t \equiv E_t \left\{ \hat{G}_{t+3} - \hat{G}_{t+2} \right\} = c_1 E_t \left\{ \bar{Z}_{t+1} - \bar{Z}_t \right\} = c_1 (B-I) \bar{Z}_t.
\]

So the innovation in \( \rho_t \) can be expressed as

\[
\rho_t - E_{t-1} \rho_t = c_1 (B-I) (\bar{Z}_t - B \bar{Z}_{t-1}) = c_1 (B-I) U \bar{e}_t.
\]

Similarly,

\[
\bar{Y}_t \equiv E_t \hat{Y}_{t+2} = c_2 \bar{Z}_t.
\]
and the innovation of $\tilde{Y}_t$ can be expressed as

$$\tilde{Y}_t - E_{t-1}\tilde{Y}_t = c_2(\tilde{Z}_t - B\tilde{Z}_{t-1}) = c_2 U\bar{e}_t.$$ 

The innovations can be written compactly as follows

$$\begin{bmatrix} \rho_t - E_{t-1}\rho_t \\ \tilde{Y}_t - E_{t-1}\tilde{Y}_t \end{bmatrix} = H\bar{e}_t$$

where

$$H \equiv \begin{bmatrix} c_1 (B - I) & c_2 \\ c_1 & c_2 \end{bmatrix} U.$$ 

Finally, the variance covariance matrix of the innovations is $H\Omega H'$. 

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