Lecture 5: Product Differentiation

(This week’s lecture is based on Tirole, Chap. 7.1 & 7.5)

1. Overview

Recall that Bertrand competition leads to intense price competition and prices get driven down to cost. Three critical assumptions were made: a) goods homogeneous, b) MC constant and, in terms of information, c) price information readily available to consumers (this is one hallmark of the internet). Will keep b) & c) but now we will modify a).

Two modifications: 1. **Horizontal Differentiation** = consumers have differences of opinion; even if prices are the same, some consumers prefer buying from firm 1 and some from firm 2. 2. **Vertical Differentiation** = goods have different quality; if prices are the same then every consumer prefers buying from the higher quality firm and latter holds onto some consumers even if its price is higher than its competitors.

Note both prices equal to MC is not a Nash eq. anymore - under either kind of differentiation. Consider 1. If p = MC (& less than or equal to AC), firm 1 can raise price a little. It loses some consumers especially those who were pretty much indifferent between the two firms to begin with. But it does not lose the customers who vastly preferred its product at the earlier price. Furthermore it makes a positive (variable) profit (on the consumers who still buy) versus the zero it was making before.

Consider 2. If the higher quality firm raises its price it loses some but not all of the customers it had. Same reason as previous paragraph leads it to prefer to do so.

General conclusions that will emerge from both classes of models: a. distinctiveness (= differentiation) helps make profits, i.e., it ameliorates the intensity of price competition. b. This leads firms to plan on being distinct (since that is the only way to make money). But, c. easy entry whittles away at profits as more and more firms show up with similar products.

2. Horizontal Differentiation I – Model

Consumers differ in their taste along a single-dimensional parameter x. One extreme of taste is x = 0 and the other is x = 1. Taste can also be interpreted as individual’s location (in a long narrow city!). Consumer located at x has a “travel cost” of going to location 0 that is equal to tx^2, where t is a parameter that indexes how severe this cost is.

Suppose that consumers are uniformly distributed in the city, i.e., between locations 0 and 0.5 there are exactly 50% of the consumers, between locations 0 and 0.8 there are exactly 80% of the consumers, etc. Suppose also that each consumer wants to buy one unit of the good.

Two firms located respectively at points a & b within that city. To begin with, a =0 and b = 1. The firms charge prices p_1 and p_2 respectively.

Cost to consumer at location x in buying from firm 1 (located at 0) = p_1 + tx^2
Cost to consumer at location x in buying from firm 2 (located at 1) = p_2 + t(1-x)^2

Note t can also be interpreted as a measure of how different the two products really are in the consumers’ minds; t = 0 means no difference (homogeneous goods) while t = infinity means the two goods are completely different.

3. Horizontal Differentiation I – Price Competition

Consumer at location x would rather buy from firm 1 if and only if p_1 + tx^2 > p_2 + t(1-x)^2
Consumer at location x is indifferent between the two firms if and only if p_1 + tx^2 = p_2 + t(1-x)^2
Denote that consumer X. Hence firm 1’s consumers are those in locations between 0 and X. Note 2tX = t - p_1 + p_2.
By the assumptions above, firm 1 sells $X$ units (and 2 sells $1 - X$ units). Hence, firm 1’s profits are $2t$ times

$$(p_1 - c)(t - p_1 + p_2)$$

These profits are maximized when marginal profits are zero, i.e., at

$$2p_1 = c + t + p_2$$

This maximum profit price is called firm 1’s best response (to $p_2$) price. A similar exercise yields firm 1’s best response price to be given by

$$2p_2 = c + t + p_1$$

Recall that a Nash equilibrium is a pair of prices such that each firm is playing a best response. Hence, it is a solution to the above best response price equations. It follows that the equilibrium prices are

$$p_1 = p_2 = c + t$$

Note that prices are above cost. That follows from the differentiation in products. (What happens when the goods are homogeneous in this model? What about completely differentiated?)

Show that equilibrium profits equal $0.5t$.

4. **Horizontal Differentiation II – Different Locations**

Suppose instead that the two firms are located at points $a$ and $b$ ($a < b$) instead, i.e., they are not maximally differentiated. All consumers at locations $x < a$ prefer firm 1 & those at $x > b$ prefer firm 2 (why?). The competition is for the souls of those in between $a$ and $b$. By the same reasoning as above it can be shown that in fact all consumers to the left of $X$ prefer firm 1 – and only those consumers prefer firm 1. $X$ is given by

$$2t(b-a)X = t(b^2-a^2) - p_1 + p_2.$$ 

Hence, firm 1’s profits are $2(b-a)t$ times

$$(p_1 - c)[t(b^2-a^2) - p_1 + p_2]$$

These profits are maximized when marginal profits are zero, i.e., at

$$2p_1 = c + t(b^2-a^2) + p_2$$

This maximum profit price is called firm 1’s best response (to $p_2$) price. A similar exercise yields firm 2’s best response price to be given by

$$2p_2 = c + t(b-a)(2-(a+b)) + p_1$$

Since a Nash equilibrium is a pair of prices such that each firm is playing a best response, it is a solution to the above best response price equations. It follows that the equilibrium prices are

$$p_1 = c + \frac{1}{3}t(b-a)(2+b+a)$$

$$p_2 = c + t(b-a)(\frac{4}{3}-\frac{1}{3}(b+a))$$

Note that prices are not necessarily the same. (Why?) Also make sure we get the same equilibrium as above if $a = 0$ and $b = 1$. Prices are again above cost but lower than in the maximal differentiation case. (Show!)

4. **Horizontal Differentiation III – Location Competition**

Where should firm 1 locate? Closer to firm 2 increases its captive market but intensifies price comp. Note that firm 1’s profits are $(p_1 - c)X$. Also $p_1 - c = \frac{1}{3}t(b-a)(2+b+a)$ – from the previous subsection. Substituting for the formula for the market share we get $6X = 2 + b + a$.

With firm 2 located at $b$, 1’s profits are then proportional to

$$(2+b+a)^2t(b-a)$$

In turn simple differentiation shows that those profits decrease as $a$ increases. Put differently, no matter what firm 2’s location $b$ is anticipated to be, firm 1 is best off by picking a location – designing a product – furthest away from firm2. Put yet another way, firm 1’s best response location is always $a = 0$. 


A similar argument can be used to show that firm 2’s best response location (no matter what a is) is \( b = 1 \). Hence, the Nash equilibrium in locations in this models is maximal differentiation, \( a = 0 \) and \( b = 1 \).

Remark: That inching closer to a rival steals some of his market away from him but can engender bruising price competition, is true for many models of horizontal differentiation. What is special about this model is that the second effect is always stronger – lower prices more than offset any increase in market share. In some other models the first effect predominates at least initially and firms prefer to locate closer to each other. But never right on top of each other (why?).

5. Entry

Two firms make profits by keeping price competition amongst themselves low. But what happens if there is a third firm (located at say the point 0.5) charging the same price as the first two? Well this third firm will take away some consumers from firm 1 – those who are between 0.25 and 0.5 in tastes/locations. Consequently, firm 1 will fight to hold onto some of those customers by lowering its price. That will induce firm 3 to lower its price as well (by reasoning identical to the two firm case). So firm 1 will be unambiguously worse off than when there was only one other firm. After all, its market share has fallen and it is selling at a lower price. Similar considerations show that firm 1 will also be unambiguously worse off.

A third firm will enter – and possibly a fourth and fifth as well – as long as they make positive profits that cover any start-up costs. Entry stops when profits just about cover start-up costs. So in a market with easy entry this is what an incumbent expects to make – profits equal to start-up costs.