Economics of the Internet (W4490)
Problem set 3-due Monday Feb 25th

Question 1

a) What kind of goods are sold through the Internet? What are their characteristics and differences?

b) What is the typical cost structure of the goods sold through the Internet? Does it differ with the type of good sold? Does it differ from the cost structure of the same type of goods not sold through the Internet?

Question 2

a) Give a definition of total cost function. What are the sources of costs we can identify in a total cost function? Give a definition of returns to scale and relate them to the shape of the marginal cost.

b) The production function for a firm is \( Y = 10K^{1/2}L^{1/2} \). It pays workers a wage of $1 and capital a rental rate of $1. Does the production function exhibit increasing, constant, or decreasing returns to scale? Justify your answer. In the short run, the firm’s capital input is fixed at \( K_s = 10 \) units and its labor input is variable. Find the firm’s short run cost function (cost as a function of output).

c) A competitive firm has the short-run cost function: \( C(Q) = Q^3 - 2Q^2 + 5Q + 6 \)
   Give the firm’s average cost function, average variable cost function (average cost without including fixed costs), and marginal cost function. At what level of output is average variable cost minimized? Explain how you arrived at your answer. Graph the firm’s average variable cost function, marginal cost function, and its short-run supply function.
Question 3

*(Bertrand Competition)*. Consider the price competition model of Problem Set 2 Question 5.

a) Show that the best response to your rival posting a price of 6 dollars is to post the monopoly price of 3 dollars. What is the best response against a price of 4 and 5 dollars?

b) Can you show that the best response to the monopoly price of 3 dollars is to post a price of 2 dollars instead?

c) Show that the Nash equilibrium of this price competition model is for each firm to post a price of 1 dollar. Why is it lower than the monopoly price?

d) Show that if the two firms could cooperate and agree on the prices they post, they could both increase their profits with respect to the Nash equilibrium prices. Why this cooperative solution is not sustainable? (Hint: show that both firms have a profitable deviation from the cooperative solution.)

Question 4

*(Cournot Competition)*. Now consider the same model of the previous exercise, but suppose that firms compete by choosing one of the quantity levels 0, 1, 2, 3, 4, 5, 6. Hence now the demand function is \( p=6-Q \), where \( Q=q_1+q_2 \), and from the demand function you get a unique price for both firms depending on the quantity they produce (suppose that \( p=0 \) whenever \( Q>6 \), so that negative prices are ruled out). Production costs are still zero.

a) Write down the strategic form of this game, i.e. the matrix with the profits for the two firms depending on the quantity levels they produce.

b) What is the best response for firm one to firm two producing a quantity of 4? And to firm two producing a quantity of 2?

c) Find the Nash equilibrium quantity levels and corresponding market price