You must answer all six questions, and show all your work in order to receive full credit. Partial credit will be given. The test is closed book, but a formula sheet is provided on the last page. Start each question on a new page, and remember that your answers must indicate the units and directions of vectors where appropriate. You have 3 hours to complete the test. Good Luck and Happy Holidays!

1. A bullet is fired at an initial angle, $\theta_0 = 40^\circ$, into an open window. The bullet is fired from the ground at a distance $D = 40\,m$ from the base of the building. The window is located at a height $h = 20\,m$ from the ground. Ignore air resistance.

A. (10pts) If the bullet is seen to pass through the open window, what is the initial speed, $v_0$, of the bullet?

B. (10pts) Will this same bullet reach its maximum height before it passes through the window or after it passes through the window? Justify your answer.

C. (10pts) What is the range of angles, $\theta$, for which the bullet will never pass through the open window regardless of the initial speed $v_0$?

2. A student decides that it is possible to carry his luggage on the front of his car by simply placing it on the front of the car and maintaining a constant acceleration, $a$. This is shown in the sketch at right. Ignore air resistance, and assume the front of the car is a flat surface that makes an angle $\theta = 45^\circ$ with the horizontal. The mass of the luggage is $m$.

A. (10pts) If it is a rainy day and the coefficient of friction between the front of the car and the luggage is zero, draw the free-body diagram for the luggage, and briefly explain what each force is.

B. (10pts) What acceleration, $a$, is required to keep the luggage from sliding?

C. (10pts) Suppose that the rain eventually dries up, and the surface is no longer frictionless, but instead there is a coefficient of static friction $\mu_s = 0.2$ between the front of the car and the luggage. What is the maximum acceleration the car can have now without the luggage slipping?
3. A satellite is placed into a geosynchronous orbit around the earth’s equator [a geosynchronous orbit is one where the satellite has the same period as the earth’s rotation, thereby keeping the satellite always above a fixed location on the equator]. The mass of the earth is \( M_e = 5.98 \times 10^{24} \) (kg). The mass of the satellite is \( m = 100 \) (kg).

A. (10pts) What is the radius of the satellite’s circular orbit (measured from the center of the earth)?

B. (10pts) What is the tangential speed, \( v \), of the satellite?

C. (10pts) Suppose that by accident, a second identical satellite is placed into an orbit at the same radius – in fact one that will cause the two satellites to collide head-on. Assume these two satellites collide completely inelastically. What happens to this lump of satellite wreckage? For example: does it continue in the same orbit? Does the radius of the orbit change? Or does something else happen? Justify your answer. Also, for this part assume the wreckage forms one lump of metal. [Hint: what is the speed after the collision?]

D. (10pts) Now let’s us consider what happens more realistically to the wreckage. Suppose each satellite is a solid ball of titanium (melting temp = 1941 K, boiling temp = 3533 K, \( L_F = 4.4 \times 10^5 \) (J/kg), \( L_V = 9.83 \times 10^6 \) (J/kg), and \( C = 669 \) (J kg\(^{-1}\) K\(^{-1}\)). Assume that the initial temp is approximately 0 K. What is the final phase (solid, liquid, vapor or a combination of those) of the wreckage, and what is its temperature assuming all the energy “lost” in the collision goes into heating the titanium (i.e. ignore radiation heat loss)?

4. A mass, \( m = 1 \) (kg), is attached to a steel bar and the system is in equilibrium when a force \( F = 50,000 \) (N) is applied to it. The stretched length of the bar at that time is \( L = 1.5 \) (m). The situation is shown in the sketch, where the bar is attached to the mass and the wall. The mass rests on a frictionless surface and the mass of the bar is negligible. The wire is made of steel (Young’s modulus of 2 \times 10^{11} \) (Pa)), and constant circular cross section of radius \( r = 1 \) mm (assume the radius remains constant). Gravity acts down as usual.

A. (10pts) Draw the free-body diagram for the mass. Indicate the nature or a brief description of the forces on your diagram (for example the weight).

B. (10pts) How much is the bar stretched from its natural unstretched length? Specify your answer to three significant figures.

C. (10pts) When the force \( F \) is removed, the mass starts to oscillate. What is the period of that oscillation?
D. (10pts) What is the maximum magnitude of acceleration achieved by the mass? Where, in its motion, does it achieve that value?

5. A horizontal section of pipe (it is a long pipe) as shown in the sketch has a cross-sectional area of $A = 40 \text{ cm}^2$ in the wider section and $a = 10 \text{ cm}^2$ in the narrower constriction. Water is flowing in the pipe which it enters in the wide section at a steady rate of $0.005 (m^3/s)$. A U-tube with mercury is attached to two openings as shown. The density of water is $1 \text{ g/cc}$ and the density of mercury is $13.6 \text{ g/cc}$.

A. (10pts) What are the speeds in the wide and in the narrow portions of the pipe?

B. (10pts) What is the magnitude of the pressure difference between these to portions?

C. (10pts) What is the difference in height between the mercury columns in the U-shaped tube? Is the drawing, which shows the higher mercury column on the right, correct? Justify your answer.

6. Consider the heat engine shown. It consists of 6 moles of an ideal gas following a straight line in the $p-V$ plane and then returning via an isothermal. The volume, $V_1 = 0.1 (m^3)$, the temperature $T_1 = 27 ^\circ C$, and $V_2 = 3 V_1$. Note that the work done in going from state $a$ to state $b$ in an isothermal process is $nRT[\ln(V_b)-\ln(V_a)]$.

A. (10pts) What is the pressure $p_2$?

B. (10pts) What is the work done in 1 cycle?

C. (10pts) What is the efficiency of this engine?