Answer all 6 questions. You must show all your work to receive full credit. An formula sheet is provided on the last page. Start each question on a new page.

1. A particle of mass, \( m \), is launched at an angle of 45° from the edge of a 5m cliff as shown. The initial velocity is \( v = 20 \text{ m/s} \).

   (10 pts) A. How high above ground level does the particle go?

   (5 pts) B. How long is the particle in the air?

   (5 pts) C. What is the range (i.e. total horizontal distance from launch to impact) of the particle?

   (10 pts) D. What is the final velocity just before impact?

   (10 pts) E. For the same initial velocity and position how should you change the angle, \( \theta \), to increase the range (increase, decrease, or keep \( \theta \) the same)? Justify your answer.

2. A solid cylinder, initially at rest, rolls down an inclined plane that makes an angle \( \theta = 45^\circ \) with the horizontal. The radius of the cylinder is 5 cm and its mass is 2 kg.

   (10 pts) A. What is the minimum coefficient of friction required to insure that the cylinder rolls down the slope?

   (10 pts) B. If it rolls down a slope of length \( L = 8 \text{ m} \), how long does it take to reach the bottom of the slope?

   (10 pts) C. What is the translational kinetic energy and the rotational kinetic energy of the cylinder at the bottom of the slope? How much energy is lost to friction?

   (10 pts) D. If the coefficient of friction were zero would the time taken to reach the bottom of the slope change from that obtained in part (B)? If so, by how much? Which way is faster?
3. Consider a ballistic pendulum of the type shown at right, consisting of a 395g lead block attached to a spring and sitting on a frictionless table. A 5g lead bullet with a velocity of 80 m/s embeds itself in the block which is initially at rest. The spring constant is .4 N/m.

(10 pts) A. What is the velocity of the combined bullet and block system immediately after impact?

(10 pts) B. How much does the spring compress?

(10 pts) C. How long does it take the spring to reach maximum compression?

(10 pts) D. If the bullet and the block are at 20°C to start with, what is the final temperature of the system.

(10 pts) E. What is the equation of motion for the bullet+block system after impact (i.e. what is x(t))? Assume the block starts at x=0 at time t=0.

4. If a rock is thrown overboard from a ship which is floating in a fixed tank of water, and the rock sinks to the bottom, does the water level in the tank rise, fall or remain the same? Justify your answer quantitatively.

(10 pts)

5. A large 10m deep tank of mercury has a small hole located 5m below the top surface.

(10 pts) A. What is the velocity of the mercury as it leaves the hole?

(5 pts) B. What is the maximum range of the mercury that squirts out?
6. A heat engine consisting of 1 mole of N₂ gas undergoes the cycle shown in the diagram, where process 1-2 occurs at constant volume, 2-3 is adiabatic, and 3-1 is at constant pressure. Assume N₂ behaves like an ideal gas, and that T₁ = 27°C, T₂ = 327°C, p₁ = 2 atm.

(10 pts) A. Find p, V, T, and U (in Joules) at point 1, 2, and 3?

(10 pts) B. What is the total work done by the gas? Give your answer in Joules.

(10 pts) C. What is the efficiency of this engine if all the heat from 3-1 is considered lost?

(5 pts) D. What is the maximum efficiency possible for a heat engine operating between T₁ and T₂? Compare this with the efficiency of part (C).

(10 pts) E. What is root mean square velocity (v_{rms}) for the N₂ molecules at point 3.
FORMULA SHEET

\[ v = \Delta x / \Delta t = (x_2 - x_1) / (t_2 - t_1) \quad v_k = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \quad a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \quad a_\perp = \frac{v^2}{r} \quad w = mg \]

\[ I_cylinder = \frac{1}{2} mr^2 \quad I_sphere = \frac{2}{5} mr^2 \quad I_cylindrical = \frac{1}{2} ml^2 \quad I_{plate} = \frac{1}{2} m(a^2 + b^2) \]

\[ P = mv \quad F = ma \quad \Gamma = I \alpha = Fr \sin \theta \]

\[ \sum F = 0 \quad \sum \Gamma = 0 \quad (equilibrium) \]

\[ K = \frac{1}{2} mv^2 \quad K_{rot} = \frac{1}{2} I \omega^2 \quad W = F \cdot \delta = F \cos \theta \]

\[ P = F_k (F\text{ const}) \quad P = \Gamma \omega (\Gamma\text{ const}) \]

\[ F_{\text{static friction}} \leq \mu_s N \quad F_{\text{kinetic friction}} = \mu_k N \]

\[ F_{\text{spring}} = -kx \quad U_{\text{spring}} = \frac{1}{2} kx^2 \]

\[ F_{\text{static friction}} \leq \mu_s N \quad \mu = \frac{F_{\text{friction}}}{N} \]

\[ \text{SHM:} \quad x = A \cos \omega t \quad c = fL \quad w = \frac{2\pi}{\lambda} \]

\[ v = -\omega A \sin \omega t \quad a = -\omega^2 A \cos \omega t = -\omega^2 x \quad \psi = \frac{1}{2} \omega^2 = 2\pi f \]

\[ y(x, t) = A \sin (\omega t - kx) \]

\[ y(x, t) = 2A \cos \omega t \sin kx \]

\[ \Delta y = \frac{w + w_2}{2} \quad \Delta x = \frac{w^2 - w_1^2}{2} \]

\[ B = 10 \log (I/I_0) \quad f = \frac{nc}{v_L} \quad f_{\text{beat}} = f_1 - f_2 \quad f_{\text{cylinder}} = \frac{f_s (v_c + v_L)}{c + v_L} \quad \text{rad} = 57.3^\circ = 180^\circ / \pi \]

\[ g = 9.8 \text{ (m/s}^2 \text{)} \quad 1 \text{ rev} = 360^\circ = 2\pi \text{ (rad)} \quad 1 \text{ watt} = 1 J/s \quad 1 \text{ Newton} = 1 \text{ kg} \cdot \text{m/s}^2 \]

\[ a = -b + \sqrt{b^2 - 4ac} \]

\[ \sin^2 \theta + \cos^2 \theta = 1 \quad \sin 2\theta = 2 \sin \theta \cos \theta \]

\[ \sin(a \pm b) = \sin a \cos b \pm \cos a \sin b \quad \cos(a \pm b) = \cos a \cos b \mp \sin a \sin b \]
FORMULA SHEET (Cont'd)

\[ \rho = \frac{m}{V} \quad p = p_o + \rho g h \quad A v_1 = A v_2 \]
\[ p + \rho g y + \frac{1}{2} \rho v^2 = \text{constant} \quad v = \frac{p_1 - p_2}{4 \eta L} \]

\[ \Delta L = \alpha L_0 \Delta T \quad \Delta V = \beta V_0 \Delta T \]
\[ \Delta Q = mc \Delta T \quad \Delta Q = nC_p \Delta T \]
\[ pV = nRT = NK \] \[ \Delta W = p \Delta V \quad pV^2 = \text{const} \]
\[ Q = \Delta U + W \]
\[ Q = nC_v \Delta T \quad Y = C_p / C_v \]
\[ C_p = C_v + R \]

\[ e = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H} = 1 - \frac{Q_c}{Q_H} \]
\[ K = -\frac{Q_C}{W} = -\frac{Q_c}{Q_H + Q_c} \]
\[ K = \frac{T_c}{T_H - T_c} \]
\[ T = \frac{T_c}{T_H - T_c} \]

\[ \Delta S = \frac{Q}{T} \]
\[ M = N_A m \quad pV = \frac{3}{2} U \quad \text{(monatomic)} \]
\[ pV = \frac{3}{2} U \quad \text{(diatomic)} \]

\[ U = \frac{3}{2} nRT = \frac{3}{2} NK \quad \text{(monatomic)} \]
\[ U = \frac{5}{2} nRT = \frac{5}{2} NK \quad \text{(diatomic)} \]

\[ \frac{1}{2} m v_{rms}^2 = \frac{3}{2} kT \]

\[ R = 8.314 \frac{\text{cal mol}^{-1} \text{K}^{-1}}{\text{mol}} = 1.98 cal \text{mol}^{-1} \text{K}^{-1} = 0.08207 \frac{\text{l atm}}{\text{mol} \text{K}} \]

1 Btu = 252 cal
1 cal = 4.186 joules
1 liter = 10^{-3} m^3
1 cc = 1 cm^3

C water = 4.19 \frac{\text{J g}^{-1} \text{C}^{-1}}{\text{g} \text{C}} = 1 \text{cal g}^{-1} \text{C}^{-1}

C lead = 0.031 \text{cal g}^{-1} \text{C}^{-1}
C Hg = 0.033 \text{cal g}^{-1} \text{C}^{-1}

\[ L_f = 79.7 \text{ cal g}^{-1} \]
\[ L_v = 539 \text{ cal g}^{-1} \]
\[ \rho_{Hg} = 13.6 \text{ g cc} \]
\[ \rho_{water} = 1 \text{ g cc} \]
\[ \rho_{lead} = 11.3 \text{ g cc} \]