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Attainment of Statistical Equilibrium in Excited Nuclei*

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The usual interpretation of high-energy nuclear reactions entails a two-step mechanism: (1) a first step, which includes the emission of knock-on particles in an intranuclear cascade generated by the incident particle, followed by (2) the "evaporation" of particles from the residual excited nucleus, which is assumed to be at statistical equilibrium. If the independent-particle model with residual two-body interaction is taken as a description of the residual excited nucleus, the assumption in the second step requires that the nonequilibrium distribution of nucleons and holes that are produced in the fast step approach the equilibrium distribution before more particles leave the nucleus. This requirement has been investigated and shown to be valid by numerically solving a Boltzmann-like master equation for a Fermi-gas system.

I. INTRODUCTION

HIGH-ENERGY nuclear reactions have usually been interpreted in terms of a two-step mechanism.[1] In the first, or fast, step the bombarding particle develops a cascade in the nucleus through a series of binary nucleon-nucleon collisions in which some nucleons escape. Then in the second, or slow, step the unstable residual excited nucleus de-excites through the emission of nucleons, clusters of nucleons, and γ rays. Viewing the nucelus as a Fermi gas, the state of the residual excited nucleus that remains after the fast cascade can be described by the occupation numbers of single-particle nucleon states. The levels in the Fermi gas are thus occupied by either particles or holes.

Past calculations of the de-excitation of the residual excited nucleus have been made on the assumption that the residual nucleus comes to statistical equilibrium so rapidly that essentially no nucleons have a chance to escape during equilibration, and that thereafter the nucleons are emitted from the relaxed nucleus by evaporation. This model is analogous to the well-known phenomena of thermionic emission of electrons from metals. The equilibrium state of the excited residual nucleus is then characterized by the excitation energy and the particle numbers of the residual excited nucleus.

This view of the de-excitation process, particularly for highly excited nuclei, has met with considerable criticism. It has been asserted that the relaxation time of such residual nuclei is long compared with the lifetimes for nucleon emission and that therefore considerable particle emission and consequent loss of excitation energy will take place long before the residual nucleus comes to statistical equilibrium. Further, it is suggested that excitation energies that are of the order of the binding energy of the nucleus are somehow without meaning.

It is the purpose of this paper to offer evidence in support of the evaporation theory applied to even highly excited cascade products. Our procedure is to solve numerically a Boltzmann-like master equation for the time evolution of the occupation numbers of a Fermi system. In this computation, when allowance is made for the escape of nucleons, it is found that very few nucleons escape before the residual nucleus has relaxed to statistical equilibrium even when the excitation energy is of the order of the total binding energy of the nucleus. The analysis is based on a model in which only binary nucleon-nucleon interactions are important and in which the only particle correlations considered are those required by Fermi statistics. Thus the nucleus is viewed as a dilute gas of fermions. It should be noted that departures of the nucleus from this model will be such as only to decrease the relaxation time and thereby make our conclusions more convincing.

II. MODEL AND METHOD OF CALCULATION

For convenience, we shall consider the relaxation of a single-component Fermi gas. Assuming the gas is made up of independent fermions, the occupation numbers for the single-particle states of the gas completely specify its configuration. The problem is to determine these occupation numbers as the gas relaxes.

The gas is initially confined to translational states within a volume V of the order of nuclear dimensions, but with access to translational states in a volume Ω of the order of laboratory dimensions. The volume V is adjusted to give a Fermi energy for the gas of 40 MeV. The maximum energy of a "bound" state is taken as 48 MeV. These energies correspond approximately to the Fermi energy of neutrons and the Fermi energy plus binding energy of neutrons in nuclear matter. The gas is assumed to equilibrate within the states "inside the nucleus" lying between 0 and 96 MeV and to have access to all unbound states "outside the nucleus" with laboratory energies between 0 and 48 MeV. In other words, only particles whose energies "inside the nucleus" are above 48 MeV have access to the laboratory states, or equivalently, can escape from the "nucleus." It is also assumed that:

(a) The states within the "nucleus" interact pairwise.

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[1] For a review and general discussion see J. M. Miller and J. Hudis, Ann. Rev. Nucl. Sci. 9, 159 (1959).

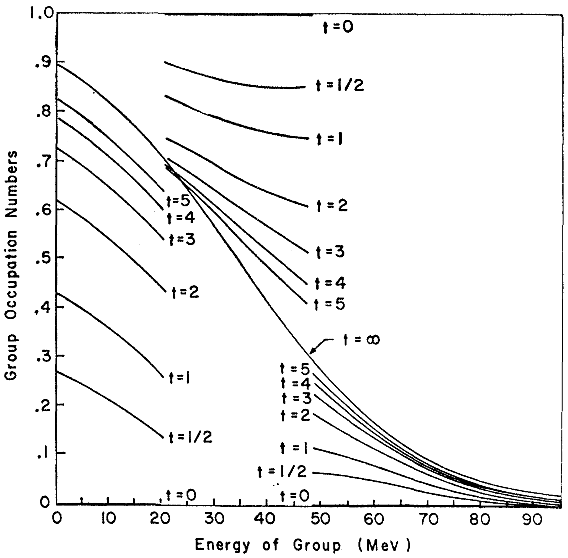

FIG. 1. Total configuration of a 100-particle Fermi system at various times. The initial excitation of the system was 1054 MeV. The time t is in units of $1/\langle\omega_i(0)\rangle$.

(b) The transition probabilities connecting either two different pairs of states inside or a state inside to a state outside depend only on energy.

(c) These transition probabilities vary slowly with energy over some energy interval $\Delta\epsilon$.

These assumptions allow the states to be grouped according to their energy. That is, all states within the nucleus whose energies were between $\epsilon_i-\frac{1}{2}\Delta\epsilon$ and $\epsilon_i+\frac{1}{2}\Delta\epsilon$ form the ith group of "nuclear" states where ϵ_i is the mean energy of the ith group. The total number of states in the ith group, g_i, is given by

$$g_i=\int_{\epsilon_i-\Delta\epsilon/2}^{\epsilon_i+\Delta\epsilon/2}\rho(\epsilon)d\epsilon,\quad 0\leq\epsilon_i\leq 96\text{ MeV}\qquad(1)$$

$$\rho(\epsilon)=4\pi V(2M)^{3/2}\epsilon^{1/2}/h^3.\qquad(2)$$

The quantity $\rho(\epsilon)$ is the density of translational states for the gas "inside the nucleus" and M is the neutron mass. Similarly, the laboratory states were grouped such that g_i' (primed quantities represent states of particles escaped from the nucleus), the total number of states in the group i', is given by

$$g_i'=\int_{\epsilon_i'-\Delta\epsilon/2}^{\epsilon_i'+\Delta\epsilon/2}\rho'(\epsilon)d\epsilon,\quad 0\leq\epsilon'\leq 48\text{ MeV}\qquad(3)$$

$$\rho'(\epsilon)=[4\pi\Omega(2M)^{3/2}/h^3]\epsilon^{1/2}.\qquad(4)$$

The quantity $\rho'(\epsilon)$ is the density of laboratory translational states; ρ and ρ' both include a spin degeneracy of two for fermions.

The occupation number of the ith group of "nuclear" states, n_i, is defined as follows:

N_i= the total number of occupied states in the ith group $=n_ig_i$. (5)

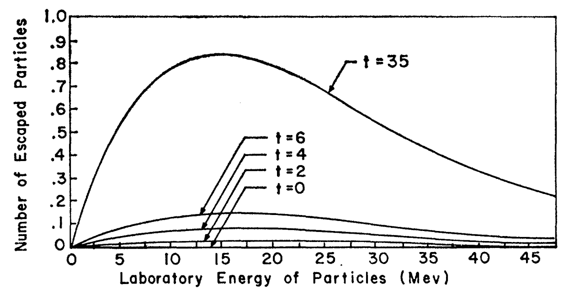

FIG. 2. Total number of escaped particles up to time t with energy E. t is in units of $1/\langle\omega_i(0)\rangle$.

The master equations describing the relaxation of this Fermi gas are, then[2]:

$$\frac{d(n_ig_i)}{dt}=\sum_{jkl}\omega_{kl\to ij}g_kn_kg_ln_l(1-n_i)(1-n_j)(g_ig_j/\Delta\epsilon)\times\delta(\epsilon_i+\epsilon_j-\epsilon_k-\epsilon_l)-\sum_{jkl}\omega_{ij\to kl}g_in_ig_jn_j(1-n_k)\times(1-n_l)(g_kg_l/\Delta\epsilon)\delta(\epsilon_i+\epsilon_j-\epsilon_k-\delta_l)-n_ig_i\omega_{i\to i'}g_i'\delta(\epsilon_i-48-\epsilon_i'),\qquad(6)$$

for all i such that $0\leq\epsilon_i\leq 96$ MeV.

$$dN_i'/dt=n_ig_i\omega_{i\to i'}g_i'\delta(\epsilon_i-48-\epsilon_i'),\qquad(7)$$

for all i' such that $0\leq\epsilon_i'\leq 48$ MeV.

The quantity $\omega_{i\to i'}$ is the probability per unit time that a particle in a particular state of the ith group escapes; $\omega_{i\to i'}=0$ if $\epsilon_i\leq 48$ MeV. The quantity $\omega_{kl\to ij}$ is the probability per unit time that a particle in a particular one of the states in the kth group scatters from a particle in a particular one of the states in the lth group such that one particle goes to the ith group, the other to the jth group. The δ functions ensure that there is conservation of energy in these transitions.

For a given set of transition probabilities and a given initial configuration, the above set of equations can be solved for the group occupation numbers n_i and the number of escaped particles with energy ϵ_i', N_i'.

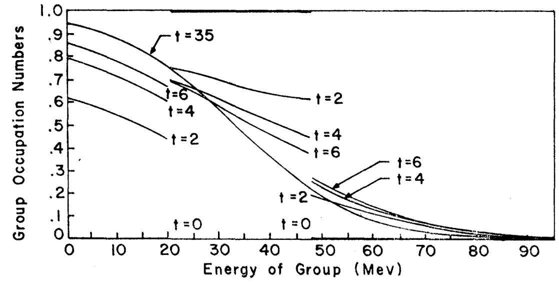

FIG. 3. Total configuration of the Fermi system inside the "nucleus" at various times. The time t is in units of $1/\langle\omega_i(0)\rangle$.

[2] R. C. Tolman, *The Principles of Statistical Mechanics* (Oxford University Press, London, 1938), Chap. V.

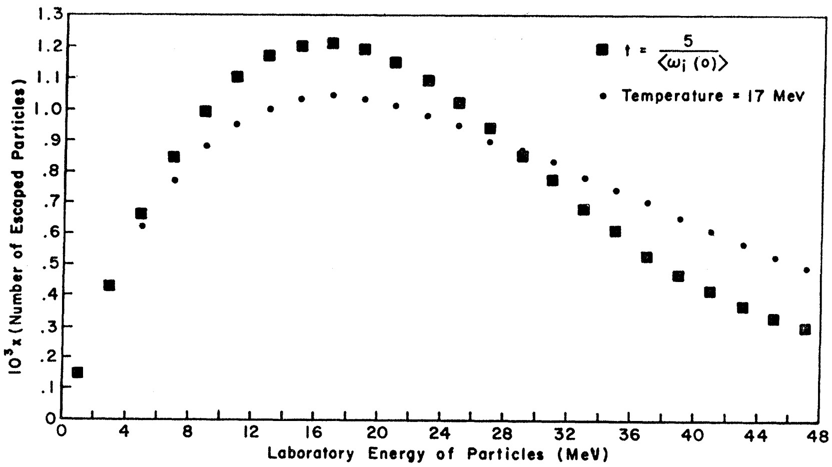

FIG. 4. Fit of spectra of escaped particles to a Maxwellian distribution at time $t=5/\langle\omega_i(0)\rangle$.

The transition probabilities $\omega_{ij\to kl}$ used are purely classical and are taken to be

$$\omega_{ij\to kl}=\frac{\sigma_{nn}(\epsilon_i+\epsilon_j)[(2/M)(\epsilon_i+\epsilon_j)]^{1/2}}{V\sum_{mn}[(g_mg_n/\Delta\epsilon)\delta(\epsilon_i+\epsilon_j-\epsilon_m-\epsilon_n)]}, \quad (8)$$

where $\sigma_{nn}(\epsilon)$ is the elementary neutron-neutron scattering cross section evaluated at an energy ϵ. The quantity σ_{nn} was determined from elementary proton-proton scattering cross sections after Coulomb effects had been removed at the lower energies.[3,4] The transition probability $\omega_{ij\to kl}$ corresponds to scattering between two particles whose initial velocity vectors were at 90° to one another, i.e., the most probable angle in a random distribution of orientation angles. This transition probability assigns equal weight to all pairs of final states that can be reached by scattering.

The $\omega_{i\to i'}$ are given by

$$\omega_{i\to i'}=\frac{\pi r_0^2A^{2/3}[(2/M)\epsilon_i']^{1/2}}{g_i\Omega}, \quad (9)$$

where $\pi r_0^2A^{2/3}$ is the geometrical cross section for the "nucleus," A is the original number of nucleons present, and r_0 is chosen such that a Fermi gas of A particles confined to a volume $V=\frac{4}{3}\pi r_0^3A$ has a Fermi energy of 40 MeV. This transition probability assigns equal wieght to all states in the ith group that can be reached by a particle striking the nucleus with a laboratory energy ϵ_i'.

III. RESULTS

The system of differential equations (6) and (7) with transition probabilities (8) and (9) were solved numerically using the method of Runga-Kutta-Gill.[5] The group energy width $\Delta\epsilon$ was taken as 2 MeV. The results presented here are for a 100-particle Fermi-gas system whose initial distribution of particles and holes corresponds to the maximum excitation possible—1054 MeV. If this were a real nuclear system, then it would have an excitation energy greater than its total binding energy.

The results of solving (6) and (7) with this distribution are presented in Figs. 1–6. The time scale used in

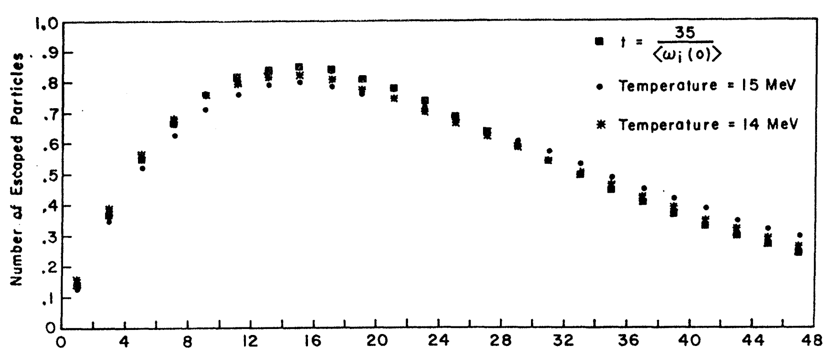

FIG. 5. Fit to spectra of escaped particles to a Maxwellian distribution at time $t=35/\langle\omega_i(0)\rangle$.

[3] N. Metropolis, R. Bivins, M. Storm, A. Turkevich, J. Miller, and G. Friedlander, Phys. Rev. **110**, 185 (1958).
[4] H. Bertini, Oak Ridge National Laboratory Report No. ORNL-3383 (unpublished).
[5] A. Ralston and H. Wilf, *Mathematical Methods for Digital Computers* (John Wiley & Sons, Inc., New York, 1966).

these figures is in units of $1/\langle\omega_i(0)\rangle$ where

$$\langle\omega_i(0)\rangle=\frac{P}{48}\sum_{i=1}^{48}\sum_{j=1}^{48}\sum_{k=1}^{48}\sum_{l=1}^{48}n_j(0)\omega_{ij\to kl}\frac{g_jg_kg_l}{\Delta\epsilon}\times\delta(\epsilon_i+\epsilon_j-\epsilon_k-\epsilon_l)\,.\quad(10)$$

That is, $1/\langle\omega_i(0)\rangle$ is approximately the initial average classical collision period for a particle in the gas: $1/\langle\omega_i(0)\rangle=2.31\times10^{-23}$ sec for this particular initial configuration.

Figure 1 shows the configuration of the gas "inside the nucleus" at various times during the relaxation process under the assumption that $\omega_{i\to i'}=0$ for all i and i'; i.e., that particles could not escape from the "nucleus" as it relaxed. This system relaxed approximately exponentially to a continuous or "equilibrium" distribution in about 11 collision periods. The final distribution ($t=\infty$) has the expected solutions:

$$n_i=\frac{1}{e^{\beta(\epsilon_i-\mu)}+1},\quad i=1,\cdots,48,$$

where $1/\beta=16$ MeV is the temperature of the gas and $\mu=34$ MeV is its chemical potential. This final distribution is, of course, the same for all transition probabilities used in (7) that satisfy

(a) $\omega_{ij\to kl}=\omega_{kl\to ij'}$

(b) $\omega_{i\to i'}=0$ for all i and i'.

The quantities β and μ have the same value when calculated independently through the conservation of energy and number of particles for the system.

Figures 2 and 3 contain plots of the total spectra of escaped particles and the configuration of the "nuclear" system at various times during the relaxation process when $\omega_{i\to i'}\neq0$, but is given by Eq. (9). The configuration of the "nuclear" system relaxed to a continuous or "quasiequilibrium" distribution again in approximately 11 collision periods. In Figs. 4 and 5 an attempt is made to fit the usual Maxwellian-type distribution to the spectra of emitted particles. These fitted spectra are normalized to the same number of particles as the calculated spectra. For the short time, $t=5$, the fit is not satisfactory because of the predominance of low-energy particles, but for the long time, $t=35$, the fit is very good and corresponds to the neutron spectra evaporated from a system whose temperature is approximately 14 MeV.

Finally, Fig. 6 contains a plot of the total number of particles that have been emitted as a function of time. The number of particles that escape before "equilibrium" has been reached is seen to be approximately 5% of the original number present.

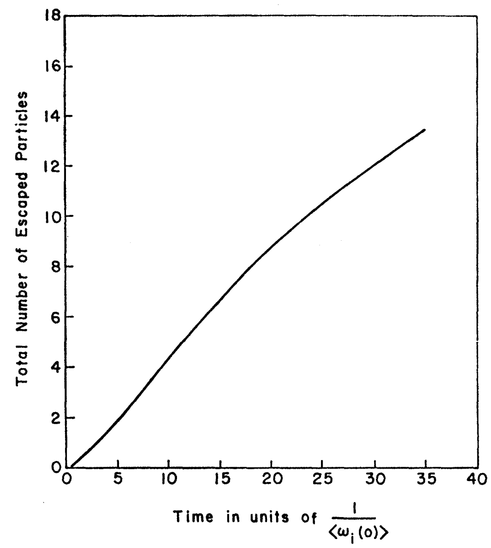

FIG. 6. Total number of escaped particles up to time t versus time.

IV. CONCLUSIONS

The system relaxes to an "equilibrium" distribution in about 11 collision periods, and it is seen from Fig. 4 that only about 5% of the particles are emitted during this time. From the energy spectra of the emitted particles given in Fig. 2, this corresponds to a loss of about 10% of the excitation energy during equilibration. Therefore, it is concluded in terms of this model of the relaxation process, that

(i) The high nuclear temperatures (7–12 MeV) and corresponding high excitation energies[6–8] that are required to explain the spectra of emitted clusters of nucleons such as Li^8 present no intrinsic difficulties.

(ii) The bulk of the de-excitation of nuclei that are excited even up to the vicinity of their total binding energy may be treated as emission from an equilibrium system.

ACKNOWLEDGMENT

We are indebted to R. Bersohn for many illuminating conversations.

[6] O. Skjeggestad and S. O. Sorenson, Phys. Rev. **113**, 1115 (1959).
[7] G. Baumann, H. Braun, and P. Cuer, Phys. Letters **8**, 146 (1964).
[8] W. Gajewski, J. Pniewski, J. Sieminsak, J. Suchorzewska, and Zielinski, Nucl. Phys. **58**, 17 (1964).